

Factors Controlling the Statistics of Magnetic Reconnection in MHD Turbulence

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We study the statistics of dynamical quantities associated with magnetic reconnection events embedded in a sea of strong background magnetohydrodynamic (MHD) turbulence using direct numerical simulations. We focus on the relationship of the reconnection properties to the statistics of global turbulent fields. For the first time, we show that the distribution in turbulence of reconnection rates (determined by upstream fields) is strongly correlated with the magnitude of the global turbulent magnetic field at the *correlation* scale. The average reconnection rates, and associated dissipation rates, during turbulence are thus much larger than predicted by using turbulent magnetic field fluctuation amplitudes at the dissipation or kinetic scales. Magnetic reconnection may therefore be playing a major role in energy dissipation in astrophysical and heliospheric turbulence.

Introduction. Magnetic reconnection is a process by which stored magnetic energy is dynamically released as kinetic energy, which may be either flow energy or thermal energy [1, 2]. Reconnection facilitates charged particle energization and changes in magnetic topology, and is thus considered an essential process in many heliospheric and astrophysical settings. At the same time, the complex dynamical processes known as turbulence [3, 4] occur commonly in the same settings. It is an appealing question to understand the relation between these fundamental features of cosmic electrodynamics.

The problem is often posed in complementary ways: Does turbulence influence the reconnection process? And, what are the properties of reconnection occurring as an element of turbulence? There are several approaches that have addressed these questions [5–14]. These include adding low turbulence levels to standard reconnection geometry, imposing external driving in the reconnection region [9, 15] and evaluating mode-mode couplings among tearing modes. A different approach is to examine turbulence in a magnetofluid or plasma, and then identify and analyze reconnection activity embedded in the medium (e.g., [8]).

Here we adopt the latter approach wherein reconnection occurs within a sea of interacting magnetic flux tubes [12, 13, 16–19], a standard scenario in a large anisotropic turbulent plasma. We emphasize that in this case (and this study), the turbulence is dynamically determining the large MHD scale geometry which ultimately drives reconnection. This kind of study has been coined, “reconnection as an element of turbulence.” A fundamental question is: what controls the rate of reconnection and its associated energy release in such a system? [16]. We emphasize that the reconnection in such turbulence is local in nature, and may not be leading to a major change of magnetic connectivity at the global scales

(e.g. dayside magnetosphere reconnection(e.g., [20, 21]), and heliospheric current sheet reconnection (e.g., [22])). In this study, a two-dimensional (2D) magnetohydrodynamic (MHD) model of decaying turbulence is simulated. Reconnection X-points are diagnosed in the simulation, and their properties including reconnection rate are compared to the statistical properties of turbulence. The central finding, not anticipated in many standard treatments (e.g., [23]), is that the reconnection rates are controlled by the dynamics of the large magnetic flux tubes at the correlation scale of the turbulence. This result implies that the dissipation of turbulence due to magnetic reconnection may be orders of magnitudes larger than previously thought.

Simulations. We study the time evolution of a decaying turbulent system using 2D incompressible MHD simulations. For simplicity, we write the time evolution equations in terms of magnetic potential $a(x, y)$ and vorticity $\omega(x, y)$ with uniform density ($\rho = 1$) as follows [24]:

$$\frac{\partial \omega}{\partial t} = -(\mathbf{v} \cdot \nabla) \omega + (\mathbf{b} \cdot \nabla) j + R_\nu^{-1} \nabla^2 \omega, \quad (1)$$

$$\frac{\partial a}{\partial t} = -(\mathbf{v} \cdot \nabla) a + R_\mu^{-1} \nabla^2 a. \quad (2)$$

Here the magnetic field $\mathbf{b} = \nabla a \times \hat{z}$, the velocity $\mathbf{v} = \nabla \phi \times \hat{z}$, the current density $j = -\nabla^2 a$, and the stream function $\phi(x, y)$ is related to vorticity as $\omega = -\nabla^2 \phi$. The system of equations is written in Alfvén units. The length scales are normalized to a characteristic length scale of the system, L_0 . The velocity and magnetic fields are normalized to the root mean squared Alfvén speed C_A and the time is normalized to L_0/C_A . R_ν and R_μ are fluid and magnetic Reynolds numbers, respectively, and are also (non-dimensionalized) reciprocals of kinematic viscosity and resistivity. Note that any out-of-plane (\hat{z}) magnetic field drops out of the dynamical equations for 2D incompressible MHD.

We solve these equations in a 2D periodic box of size $2\pi \times 2\pi$, with equal values of magnetic and fluid Reynolds numbers $R_\mu = R_\nu = 5000$, using a strongly energy-conserving pseudo-spectral code [25], de-aliased using the 2/3 rule, on a real-space grid with resolution 8192×8192 . In code units, the wavenumbers range from 1 to $k_{\max} = (2/3) \times 4096$. We ensure that the inequality $k_{\max}/k_{\text{diss}}(t) \geq 3$, is maintained, where $k_{\text{diss}} = R_\nu^{1/2} \langle \omega^2 + j^2 \rangle^{1/4}$ is the Kolmogorov dissipation wavenumber, and $\langle \dots \rangle$ denotes averaging over the simulation domain; this condition provides accurate representation of statistics up to at least fourth order [26].

Initially, the energy is concentrated in a thin shell of k -radius $5 \leq k \leq 30$ and is equipartitioned between the fluid flow and magnetic fields. The system is time-advanced using a second-order Runge–Kutta scheme, and double precision is employed. We analyze the dynamics of our turbulent system at the time ($t = 0.3$) when the mean squared current density is near its maximum value. This is the point at which turbulence is well-developed, and energy is distributed across decades of scales as a near-power law. After this time, the global dynamics may be described approximately by an adaptation of the von Kármán similarity decay principle [27].

A large number of magnetic reconnection events emerge from interactions between large-scale magnetic structures (magnetic islands). Using techniques described in the Appendix, we find 625 X-points in the simulation, determine local the inflow and outflow direction of each X-point, record the physical quantities associated with each local reconnection geometry, and analyze their distribution within the context of the turbulence pervading the entire system. In particular, we are interested in the statistics of upstream reconnecting fields and their relationship with the statistics of global turbulent magnetic fields. (Three X-points have zero reconnection rate and are excluded from the analysis.)

Results. Fig. 1a illustrates an interaction between two large-scale magnetic structures, i.e., magnetic islands, during turbulence through the process of magnetic reconnection. A magnetic island is a coherent multi-scale structure within turbulence, encompassing a hierarchy of smaller-scale features. The typical size of a magnetic island in a turbulent system scales roughly with the correlation or energy-containing scale associated with the macroscopic system. The process of magnetic reconnection between two coherent magnetic structures, which might themselves be initially well decorrelated from one another, typically begins when their distance of closest approach nears the dissipation scale, $\lambda_{\text{diss}} \equiv k_{\text{diss}}^{-1}$. The current sheet thickness, i.e., the scale transverse to the reconnecting field, is usually found to be close to but somewhat larger than the dissipation scale of the system [16]. The length of the reconnection regions is larger and broadly distributed around the correlation scale λ_c [16].

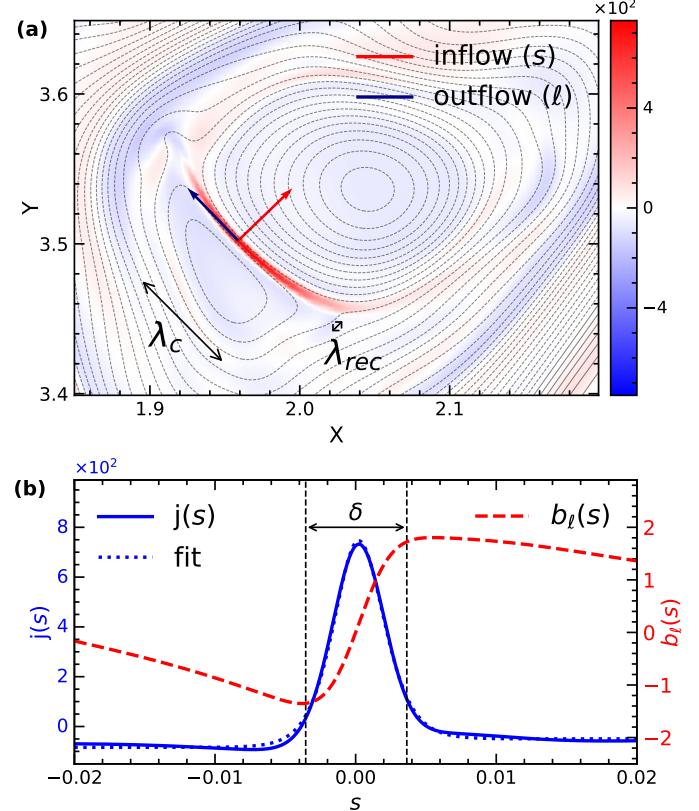


FIG. 1. (a) Current density j with contours of magnetic vector potential. λ_c is the correlation length and λ_{rec} is the average reconnection diffusion region thickness. Red and black arrows denote inflow and outflow directions associated with the diffusion region. (b) One-dimensional cuts of current density j (solid) and reconnecting magnetic field b_ℓ (dashed) along the inflow direction, for the current sheet shown in panel (a), along with best fit line for $j(s)$ (dotted). Locations where b_{up} is determined are shown by vertical dashed lines.

The relevance of both correlation scale and the inner scales of turbulence in the local reconnection process has been described previously [16], but the quantitative implications for reconnection rates have not been addressed until the present study as far as we are aware.

To reveal these connections, we identify the properties of all reconnection sites, as described in detail in the Appendix. For orientation, we refer to Fig. 1a. The positive directions of magnetic flux inflow (s) and outflow (ℓ) at the reconnection site are marked by red and blue vector arrows, respectively. The correlation length λ_c and average current sheet thickness λ_{rec} , i.e., the average transverse scale associated with reconnection, are also shown for scale comparison. Fig. 1b indicates the diffusion region boundaries (vertical lines), superimposed on magnetic field and current profiles along the inflow direction for the reconnection site in Fig. 1a. The upstream field strengths, b_{up_1} and b_{up_2} , are evaluated at these boundaries, which are determined using the techniques outlined

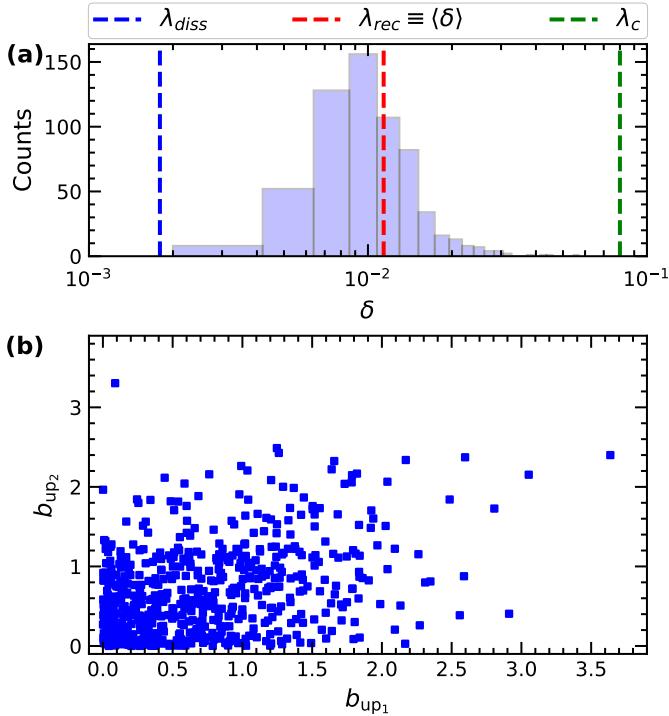


FIG. 2. (a) Distribution of thicknesses of diffusion regions, with vertical lines showing the average thickness λ_{rec} , the dissipation scale λ_{diss} and the correlation scale λ_c . (b) Scatter plot of b_{up1} vs b_{up2} sampled at each reconnection site.

in the Appendix. The profiles are typical of what is seen in a laminar magnetic reconnection setting, even though the system is turbulent (i.e., far from being a laminar flow).

Fig. 2a shows the histogram of the diffusion region thickness, i.e., the transverse scale δ associated with the reconnection sites, computed using the curve-fitting technique. The histogram displays a relatively narrow distribution, centered around a mean value of $\lambda_{rec} \approx 0.01 \pm 0.001$. Error estimation of λ_{rec} is described in the Appendix. Consequently, $\lambda_{rec} \equiv \langle \delta \rangle$ provides a reliable estimate of the diffusion region thickness. The average transverse scale is a little more than 5 times the dissipation scale, indicating that reconnection typically initiates well above the dissipation scale (see also [16]).

A scatter plot of upstream reconnecting fields b_{up1} and b_{up2} sampled at each reconnection site is shown in Fig. 2b. There is no significant correlation between the two upstream reconnecting fields, even though the upstream fields are separated by a distance much less than the correlation scale. An explanation is that these fields originate in two large, distinct, coherent island structures that are not strongly related to one another. At this point the hypothesis emerges that the global turbulent field statistics at the scale of reconnecting current sheets thicknesses may not be a good estimator of the recon-

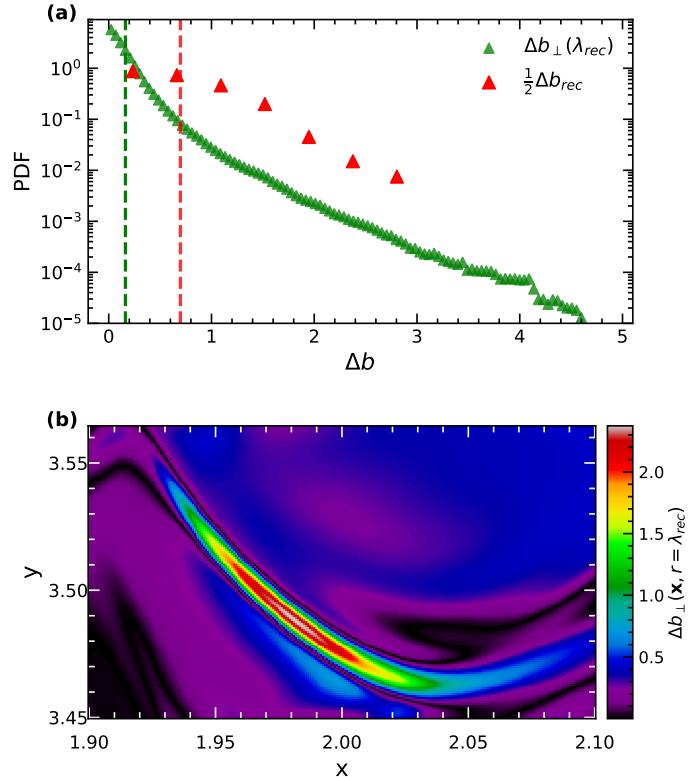


FIG. 3. (a) Comparison of turbulence increments and reconnection magnetic fields. PDF of transverse turbulent magnetic field increments taken with lag λ_{rec} sampled over entire simulation domain, $\Delta b_{\perp}(\mathbf{x}, \lambda_{rec})$. The reconnection magnetic field is defined as $\Delta b_{rec}/2$ for each X-point. Average values of each quantity are shown with vertical dashed lines with $\Delta b_{\perp}(\mathbf{x}, \lambda_{rec})$ averaged over \mathbf{x} and $\Delta b_{rec}/2$ averaged over X-points. (b) Turbulent field increments at transverse reconnection scale, $\Delta b_{\perp}(\mathbf{x}, \lambda_{rec})$, near the strongly reconnecting current sheet shown in Fig. 1a.

necting fields, and therefore will not provide an accurate basis for estimating reconnection rates.

To pursue this line of reasoning requires estimation of the correlation scale, by first computing the autocorrelation function $C(r) = \langle \mathbf{b}(\mathbf{x} + \mathbf{r}) \cdot \mathbf{b}(\mathbf{x}) \rangle / \langle \mathbf{b}(\mathbf{x}) \cdot \mathbf{b}(\mathbf{x}) \rangle$, where \mathbf{r} is the lag vector. The correlation scale is then obtained by integrating the autocorrelation function $\lambda_c = \int_0^\pi C(r) dr$, where π is half the size of the periodic box; for the simulation time studied, $\lambda_c \approx 0.08$. Since the turbulence is not biased to any position or direction, it may be viewed as homogeneous and isotropic. Therefore the direction of lag vector \mathbf{r} is not important in the computation of the correlation function, or of the global turbulence increments (introduced in what follows).

The next step is to explore the statistical relationship between the upstream reconnecting fields and the global turbulent fields. To achieve this, we compare increments of magnetic field components computed in two distinct ways: One, to make contact with standard theories of

reconnection, we compute the *reconnection increments*, defined as transverse magnetic field increments across the reconnecting current sheets, employing the average upstream field magnitude defined briefly in the next paragraph and in detail in the Appendix. Two, to characterize the turbulence we compute the turbulence increments i.e., the increments of *global* turbulent magnetic fields over the entire simulation, as defined below.

One, The reconnection increments Δb_{rec} , computed for each X-point, are calculated by taking the difference of upstream magnetic field evaluated at the boundaries of diffusion region as $\Delta b_{\text{rec}} = |\mathbf{b}_{\text{up}_2} - \mathbf{b}_{\text{up}_1}|$ or $\Delta b_{\text{rec}} = |\mathbf{b}_{\text{up}_2}| + |\mathbf{b}_{\text{up}_1}|$. The average upstream magnetic field at each X-point is then given by $\frac{1}{2}\Delta b_{\text{rec}}$. Note that at each reconnection site, the lag vector is aligned with the local inflow direction, i.e., perpendicular to the reconnecting magnetic field. This choice makes the reconnection increments inherently transverse.

Two, the *global turbulence increments* are defined as $\Delta\mathbf{b}(\mathbf{x}, \mathbf{r}) = \mathbf{b}(\mathbf{x} + \mathbf{r}) - \mathbf{b}(\mathbf{x})$. We define the *transverse increment* $\Delta b_{\perp}(\mathbf{x}, \mathbf{r})$ and the longitudinal increment $\Delta b_{\parallel}(\mathbf{x}, \mathbf{r})$ as the projection of $\Delta\mathbf{b}(\mathbf{x}, \mathbf{r})$ perpendicular and parallel to the lag vector \mathbf{r} , respectively. Note that different from reconnection increments, in this case the lag vector \mathbf{r} is a constant across the whole simulation. For simplicity and consistency with the reconnection increments, we will examine the absolute value of the global increments and discard the “ $|\dots|$.” For the present purposes the transverse turbulence increments are examined at two different lags: the average transverse reconnection scale λ_{rec} and the correlation scale λ_c .

We now proceed to compare statistics of the turbulence increments (at two scales) with statistics of the reconnection increments. Note that the transverse turbulence increments are effectively sampling only a single component of the turbulent magnetic field; therefore we compare this global increment with the average upstream reconnection field $\Delta b_{\text{rec}}/2$.

The red triangles in Fig. 3a are the probability density function (PDF) of $\Delta b_{\text{rec}}/2$ for all reconnection sites. The broad distribution reflects the wide variability of upstream reconnecting fields within the system. The distribution also exhibits a strong tail, suggesting the presence of significant intermittent reconnection events. These tail values indicate that while most field increments are moderate, there are occasional instances of particularly intense reconnection, contributing to the overall variability observed.

In Fig. 3a, this PDF of $\Delta b_{\text{rec}}/2$ is compared with the PDF of the transverse increments of turbulent fields at the average current sheet thickness λ_{rec} , computed globally. The mean values of the PDFs are indicated by vertical lines. Clearly, the turbulent magnetic fields have a very different statistical distribution than the reconnection fields. Both the shape and average values of the distributions do not match.

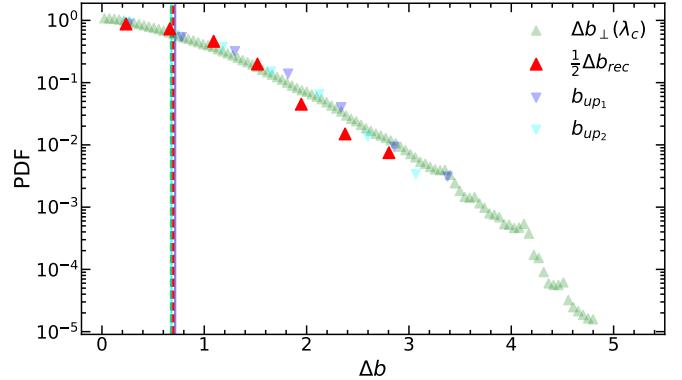


FIG. 4. PDFs of transverse turbulence increments at the correlation scale compared with PDFs of reconnection fields $\Delta b_{\text{rec}}/2$, b_1 , and b_2 . Average values of each quantity are shown as vertical lines with matching color. The agreement of these distributions is striking, and in significant contrast with Fig. 3(a).

We emphasize that it might seem reasonable to have postulated that the reconnecting fields correspond to the turbulence increments evaluated at the transverse reconnection scale, λ_{rec} . However, the result in Fig. 3a clearly demonstrates that this assumption fails. This mismatch between the turbulence and reconnection statistics occurs because at small lags the neighborhood near a current sheet exhibits a wide range of turbulent increments depending on where it is sampled. In Fig. 3b, the turbulent increments $\Delta b_{\perp}(\mathbf{x}, \lambda_{\text{rec}})$ are plotted in a close vicinity of the strongly reconnecting field line in Fig. 1a. The near-current sheet dynamics generate increments ranging in value from about 0.5 to 2.5. In contrast, this current sheet generates only a single reconnection increment near the peak value of that range. The PDF of turbulent magnetic field increments will thus be dominated by relatively small values, leading in Fig. 3a to a distribution more strongly peaked near $\Delta b_{\perp}(\lambda_{\text{rec}}) = 0$ and also a much smaller average value.

The above analysis hints that the reconnection may be fundamentally linked to larger scale turbulent dynamics. This idea is examined in Fig. 4, which shows PDFs of $\Delta b_{\text{rec}}/2$ and $\Delta b_{\perp}(\mathbf{x}, \lambda_c)$. Additionally, we plot the PDFs of the individual contributions to the upstream field increments at each reconnection site, namely b_{up_1} and b_{up_2} . The result is striking. The reconnection magnetic fields on each side of the X-point as well as their average value exhibit PDFs that match both the shape and average value of the PDFs of the turbulent magnetic fields.

Clearly, the statistics of reconnection are not coupled to the turbulence properties evaluated at the transverse reconnection scale, but instead are directly controlled by the statistics of the turbulent magnetic field at the correlation scale. This is the central result of this paper.

Discussion. The results of this study demonstrate that

the driver of reconnection, here shown to be the large scale, or *energy-containing* fluctuations, coincides with the driver of the turbulence cascade. To establish this connection, we analyze the statistics of small-scale reconnection events in a decaying turbulence simulation. Comparing the PDFs of (global) increments at both the correlation scale and at the average transverse reconnection scale with the PDF of the reconnection upstream field gives the key result of the study: the PDF of fields associated with reconnection aligns closely with the PDF of turbulence increments at the correlation scale, but does not align well with turbulence increments at the transverse reconnection scale. Additionally, the average value of the reconnection upstream fields matches well with the average value of turbulent fields at correlation scales, and is much greater than that at the transverse reconnection scale. This supports the conclusion that, for the present numerical experiment, the energy-containing scale dynamics are regulating the statistics of the reconnection. We suspect that this result may be applicable far beyond a 2D turbulent MHD system.

These results have significant implications for turbulent astrophysical plasmas. During reconnection, the reconnection rate and resultant heating are strongly controlled by the upstream reconnection field. A Sweet-Parker analysis shows that the reconnection rate is proportional to b_{up}^2 (e.g., [28]). The temperature change from the inflow region to the exhaust is also proportional to b_{up}^2 [29–31], while the heating rate of plasma due to reconnection is proportional to b_{up}^3 [23, 32]. A strong potential connection to classical turbulence theory becomes evident. The latter expression may be written, based on the findings above, in terms of r.m.s. fluctuation Alfvén speed C_A evaluated at the controlling length, here identified as the correlation scale λ_c . Then we can identify the heating rate per unit mass as $\propto C_A^3/\lambda_c$. This is the magnetofluid form of the classical von Kármán turbulence heating rate [33, 34].

The upstream reconnection magnetic field is often estimated as the amplitude of the turbulent field at the transverse reconnection scale (λ_{rec} in our notation) (e.g., [23]). Such a small scale leads to a very small reconnection field and thus small heating rates. However, the amplitude of the turbulent field at the correlation scale is significantly larger, with associated higher dissipation rates. To put this in perspective, we estimate the ratio of the turbulence amplitude in the solar wind for the correlation scale versus the transverse reconnection scale. Using a Kolmogorov type analysis, nominal solar wind conditions at 1 AU, and estimating the reconnection scale as the ion inertial length d_i ,

$$\frac{b_{\lambda_c}}{b_{\lambda_{\text{rec}}}} \sim \left(\frac{\lambda_c}{\lambda_{d_i}} \right)^{\frac{1}{3}} \sim \left(\frac{10^6 \text{ km}}{10^2 \text{ km}} \right)^{\frac{1}{3}} \sim 20. \quad (3)$$

If such a scaling is applicable, Eq. (3) implies that the

dissipation of turbulent fluctuations due to reconnection may be much larger than previous thought: reconnection rates and reconnection temperature changes may be about 500 times larger than previous estimates, while heating rates may be 10,000 times larger. Energy dissipation through reconnection therefore is likely to play a major role in dissipating the turbulence energy cascade.

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Data Availability Statement. The data that support the findings of this study are openly available in the Zenodo repository and can be accessed via Ref. [36].

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Appendix A: Determining Reconnection Statistics

Here we provide details of the technique used to determine the magnitude of the reconnection magnetic fields at the upstream edges of the diffusion region associated with each X-point. Because the reconnection is often asymmetric, each X-point has two distinct upstream fields, b_{up_1} and b_{up_2} . The values then give the reconnection increments $\Delta b_{\text{rec}} = |b_{\text{up}_2}| + |b_{\text{up}_1}|$.

First, the X-points are found by examining the extrema of the out-of-plane magnetic vector potential ($\nabla a = 0$), with X-points located at saddle points [8, 16, 35]. Because X-points are usually located between grid points, in order to determine their location with a higher accuracy, an interpolation is used to double the number of grid points in each direction ($2N \times 2N$) by padding the Fourier transform of the vector potential with zeros. Note that interpolation to an even larger grid than ($2N \times 2N$) does not yield any benefit; a test of $4N \times 4N$ interpolation revealed little or no change to the number of X-points, λ_{rec} , nor the primary conclusions. The “sea” of X-points and O-points which are found with this method are shown in Fig. 5a overlaid on a color map of j with contours of a . To highlight the large range of scales in the system, we show successive magnification of a small region of the total system (Fig. 5b and 5c). Ultimately, a single X-point is revealed in the presence of the global turbulence.

Second, using the Hessian matrix at each X-point, the local inflow (\hat{s}) and outflow ($\hat{\ell}$) directions are determined, as described in detail in [16]. An example of (\hat{s}) and ($\hat{\ell}$) for a single X-point is shown in Fig. 1a. We emphasize that these vectors are different for every X-point.

While some X-points have symmetric and simple variation in the magnetic field, like the case in Fig. 1b, other X-points can be much more complicated, an example of which is shown in Fig. 6. In this strongly asymmetric case, the peak of current j is slightly offset from the X-point and j has a complex structure as one moves away from the X-point. For $s < 0$ the current continuously increases, ultimately becoming positive for $s < -0.016$. For $s > 0$ it quickly changes sign leading to a plateau of positive j starting near $s = 0.01$.

There are two primary ways to determine the upstream edge of the diffusion region: (1) performing a fit to determine the approximate current width, or (2) taking the location where $j = 0$. However, for a complex case such as in Fig. 6a, we have found that a hybrid of the two methods is necessary to accurately classify complex X-points with relatively low reconnection rates.

In this hybrid method, the current $j(s)$ is fitted to a continuous but piecewise function broken into a left and right half at the location where j is maximum, denoted

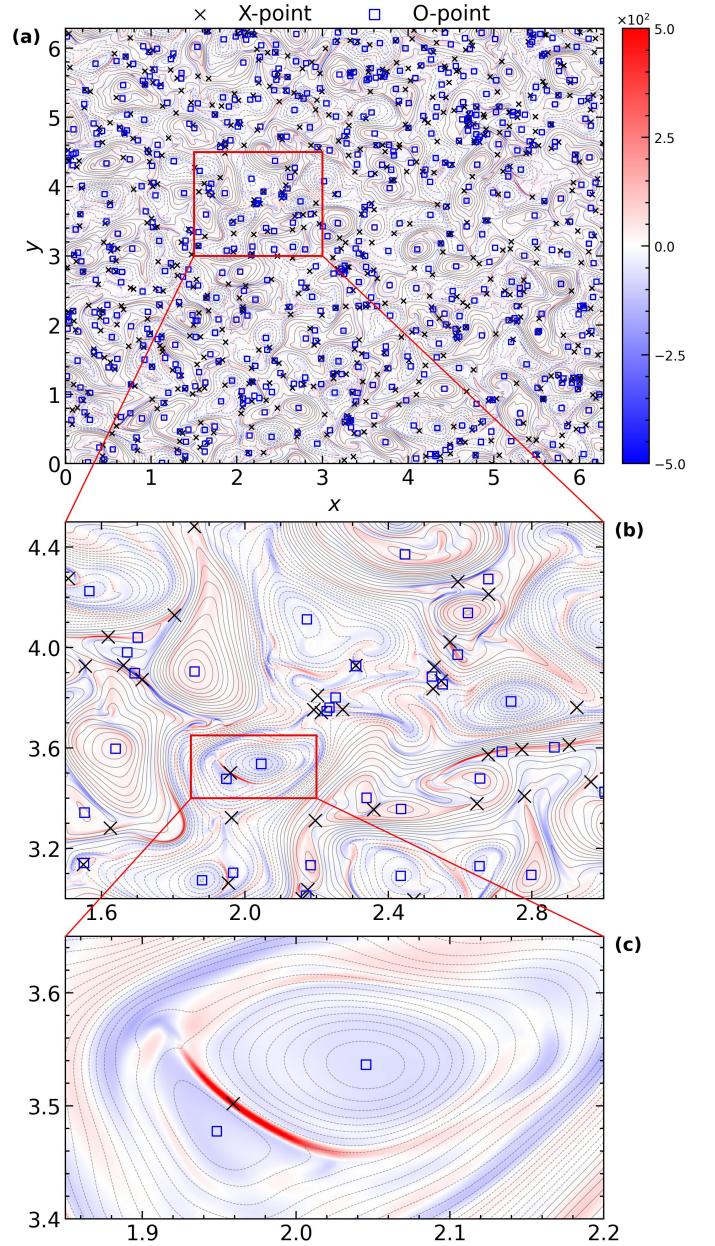


FIG. 5. Current density j at the time of analysis ($t = 0.3$) with contours of the magnetic potential a superimposed. The X-points and O-points are marked by ‘ \times ’ and ‘ \square ’ respectively. Insets highlight subsections of the system.

as $s = s_p$.

$$f(s) = \begin{cases} A_1 \operatorname{sech}^2 \left(\frac{s-s_p}{\delta_1} \right) - A_1 + C, & s < s_p \\ A_2 \operatorname{sech}^2 \left(\frac{s-s_p}{\delta_2} \right) - A_2 + C, & s > s_p \end{cases} \quad (4)$$

The peak current amplitude C , the function amplitudes A_1 and A_2 , and the current sheet thicknesses δ_1 and δ_2 are all fitting parameters. The current profile is iteratively fitted to the function for $|s| < 0.01$.

The fitted function is shown as the dashed blue line in

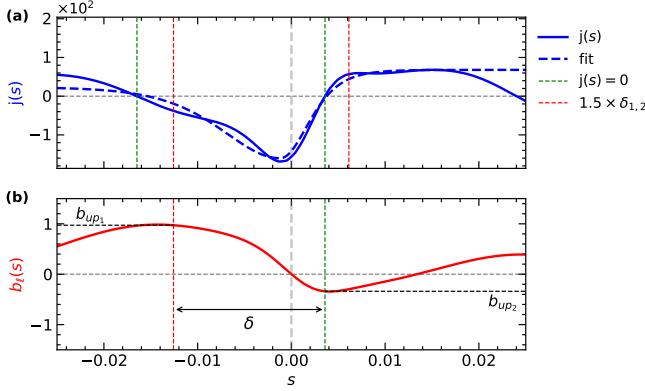


FIG. 6. Determination of upstream magnetic fields for a representative X-point. One dimensional cuts through the X-point along the inflow direction (\hat{s}). X-point location denoted by vertical gray lines at center. (a) current density $j(s)$ together with the best fit curve. Vertical green and red lines denote possible choices for upstream edges of diffusion region. (b) Reconnecting magnetic field $b_\ell(s)$. The chosen diffusion region boundaries are denoted by the green and red vertical lines ($1.5 \delta_1$ for $s < 0$ and $j = 0$ for $s > 0$). Horizontal black lines indicate the resultant b_{up1} and b_{up2} .

Fig. 6a. The two criterion for the diffusion region edge are also shown as vertical dashed colored lines: criterion A (green) is the location where $j = 0$ and Criterion B (red) is a distance $1.5 \delta_1$ or $1.5 \delta_2$ as appropriate. Independently on each side of the X-point, the diffusion region edges are chosen using the criterion which is closest to the X-point. The resulting diffusion region width δ and the values of b_{up1} and b_{up2} can be determined from $b_\ell(s)$, as shown in Fig. 6b.

The error in δ is estimated as follows. For criterion A the error is simply associated with the grid scale $\Delta = \frac{2\pi}{8192}$, so an error of $\Delta/2$ is used. For criterion B, the error is calculated from the covariance matrix of the curve fit. The uncertainty of the total diffusion width δ is then calculated through error propagation such that: $\sigma_\delta = \sqrt{(c_1 \sigma_{\delta_1})^2 + (c_2 \sigma_{\delta_2})^2 + 2c_1 c_2 \text{cov}(\delta_1, \delta_2)}$ where the values of $c_{1,2}$ depend upon the criterion used to determine the boundary at either side: $c_{1,2} = 1$ for criterion A and $c_{1,2} = 1.5$ for criterion B. The result is a diffusion region thickness and its error for each X-point, which when averaged appropriately, leads to an average diffusion region thickness of $\lambda_{rec} \equiv \langle \delta \rangle = 0.01 \pm 0.001$.

The errors associated with the probability distribution functions (PDFs) of the reconnection increments in Figs. 3 and 4 can also be estimated. The error in Δb is 0.21, which is half of the bin size used for the PDF. The vertical error is due to counting statistics. Note that the vertical error is too small to be visible for $\Delta b_{rec} \lesssim 1$. The PDFs with error bars are shown in Fig. 7.

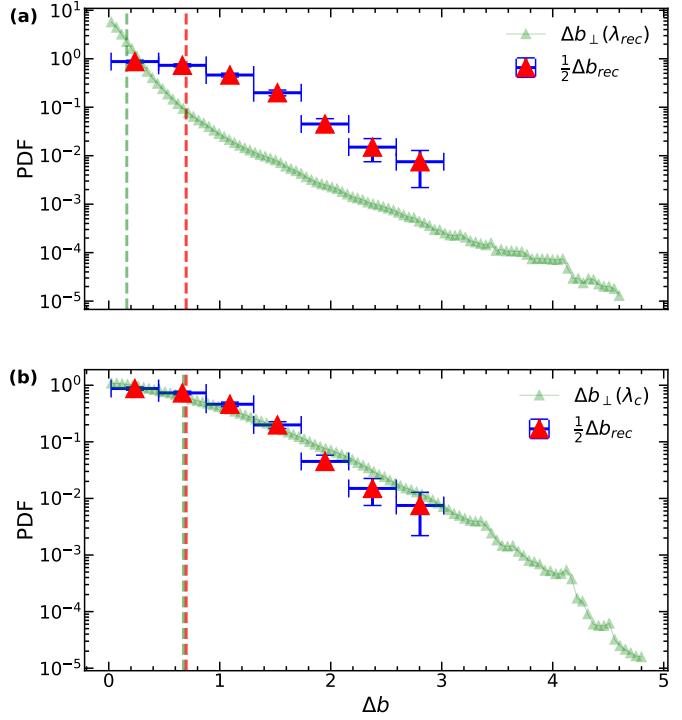


FIG. 7. PDFs of reconnection fields $\Delta b_{rec}/2$ with error bars compared with transverse turbulence increments with lags of (a) λ_{rec} and (b) the correlation scale λ_c . Average values of each quantity are shown as vertical lines with matching color.

Appendix B: Investigating the effects of selection bias

The central result of this paper is that the distribution of magnetic fields responsible for magnetic reconnection in a 2D MHD turbulent system correlates strongly with the global turbulent field at the *energy containing* scales of the system, as opposed to the turbulent fields at the transverse reconnection scale. The statistical analysis of magnetic reconnection in this letter is based on a total of 622 events. However, the statistical analysis of the global field consists of a significantly larger number of samples compared to the ensemble of reconnection events. This raises the question of whether the observed correlation is a consequence of selection bias. To address this concern, we studied the effect on the PDF of turbulence increments due to random downsampling of the dataset to 622 increments, equal to the total number of reconnection events in our system. In all cases we examined a lag equal to the average transverse reconnection scale. We downsampled the data using two different methods. First, we randomly sampled 622 increments multiple times from the ensemble of global increments at the transverse reconnection scale. Second, we sampled the transverse increments at 622 randomly chosen locations and directions within the system, using a procedure analogous to that

used for computing reconnection increments but without restricting the locations to X-points. In Fig 8, we then compared all of these randomly selected increments to the global probability density function (PDF) and found that they closely matched the global PDF. This analysis robustly eliminates the possibility of selection bias.

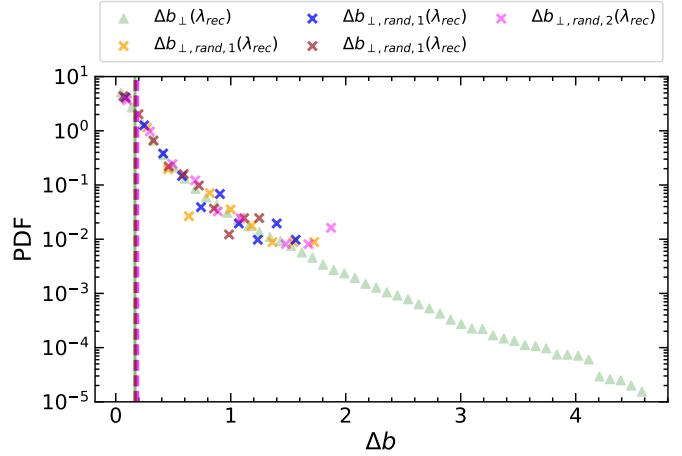


FIG. 8. Probability density function (PDF) of transverse turbulence increments Δb_{\perp} at the average transverse reconnection scale λ_{rec} . The global PDF (green) is compared with three independent random subsets of 622 samples (orange, blue, brown; first approach) and with 622 transverse increments sampled at random locations within the global system (magenta; second approach). Average values are shown as vertical lines with corresponding colors. Note that the average values are so similar that the vertical lines are nearly indistinguishable from each other.