Introduction to category theory

June 22, 2021

FAQ

- **What is category theory?**
  A common language that is suitable for various (seemingly unrelated) mathematical topics.
- **Is it hard?**
  It requires to learn an extended vocabulary that is abstract (because of its generality)
- **Is it necessary?**
  Maybe not: you could certainly always unfold categorical definitions and proofs
- **Is it useful?**
  Using a single language to talk about different topics can make you feel familiar quickly with new subjects, or give you ideas of proofs that you would not think of at first sight.
- **Doesn't set theory already serve that purpose?**
  Category theory can be seen as a (more structured?) layer on top of set theory, which seems particularly relevant for computer science

Some references

- **Categories for the working mathematician**, Mac Lane
  A useful reference, but I don't find it very nice to read.

- **Category theory**, Awodey
  I find it easier to read.

  "Category theory is the mathematical study of (abstract) algebras of functions."

- **The Joy of Cats**, Adamek et. al.
  Unconventional presentation of category theory, fun to read.

  "Category theory provides a language to describe precisely many similar phenomena that occur in different mathematical fields."

- **nlab wiki**: https://ncatlab.org
  A useful reference, more or less understandable depending on the topic. It can make you run into circles.

- Semantics of programming languages
  CBPV, lambda-calculus, dependent type theory
- Logic
  Forcing as a topos construction
- **Initial semantics** (my personal entrypoint to category theory)
  Formal account of inductive definitions
  - Example given later: $\mathbb{N}$.