Unification with binding

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Syntax for a binding signature

you have two open terms t and u with set of metavariables $\{M_i\}$ and each metavariable M_i has an arity $a_i \in \mathbb{N}$.

The algorithm outputs the most general substitution of metavariables that unifies t and u.

Restriction: any metavariable is applied to variables only, and moreover they must be distinct.

1 Syntax

$t ::= x|o(\vec{t})|M_i(\vec{x})$

Assume M_0 has arity n_0 and u has free variables $y_1, \ldots, y_{n_0}$ (and possibly some metavariables)

$t[M_0 \mapsto u]$ is defined recursively:

- $M_i(\vec{x})[M_0 \mapsto u] = M_i(\vec{x})$ if $i \neq 0$
- $M_0(\vec{x})[M_0 \mapsto u] = u[y_i \mapsto x_i]$
- $\ldots$

2 Metavariable substitutions

A substitution of a metavariable M of arity n is a term t with n free variables $x_1, \ldots, x_n$ so that $M(z_1, \ldots, z_n) \mapsto t[x_i \mapsto z_i]$

3 Correctness statement

Let t and u the terms that we want to unify. Either there is no unifier for t and u and the algorithm fails, either there is at least one unifier, and the algorithm finds the most general unifying substitution σ in the following sense:

$$\forall \delta, s.t, \quad t[\delta] = u[\delta], \text{then } \exists! s \text{ such that } \delta = s \circ \sigma$$

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4 Case $M(x_1, \ldots, x_n) = M(y_1, \ldots, y_n)$

First compute the vector of "common" positions $(i_1, \ldots, i_m)$ i.e., $x_{i_j} = y_{i_j}$ and then create a new metavariable N of arity m, and outputs the substitution $M(z_1, \ldots, z_n) \mapsto N(z_{i_1}, \ldots, z_{i_m})$

4.1 Special case: $M(x_1, \ldots, x_n) = M(x_1, \ldots, x_n)$

In this case, the vector of common position is $(1, \ldots, n)$ so the output substitution is $M(z_1, \ldots, z_n) \mapsto N(z_1, \ldots, z_n)$

5 Case $M(x_1, \ldots, x_n) = M'(y_1, \ldots, y_{n'})$

First compute the vector of "common" values $(v_1, \ldots, v_m)$ i.e., $v_i \in \{x_i\} \cap \{y_i\}$ and then create a new metavariable N of arity m, and outputs the substitution $M(x_1, \ldots, x_n) \mapsto N(v_1, \ldots, v_m)$ and $M'(y_1, \ldots, y_{n'}) \mapsto N(v_1, \ldots, v_m)$

6 Case $M(\vec{x}) = t$

when t is not a metavariable application.

If M appears in t, then there is no unifier. If $fv(t) \subsetneq \vec{x}$, then there is no unifier.

We output the substitution $M(\vec{x}) \mapsto t$

7 Case $o(t_1, \ldots, t_n) = o(u_1, \ldots, u_n)$

- unify t_1 with $u_1 \to$ substitution σ_1
- unify $t_2[\sigma_1]$ with $u_2[\sigma_1] \to \sigma_2$
- unify $t_3[\sigma_1][\sigma_2]$ with $u_3[\sigma_1][\sigma_2] \to \sigma_3$
- $\ldots$

8 Case