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Going Ballistic: Bullet Trajectories

Amanda Wade
University of South Florida

Advisors:
Gerald Hefley, Mathematics and Statistics
Scott Campbell, Chemical & Biomedical Engineering

Problem Suggested By: Scott Campbell

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Going Ballistic: Bullet Trajectories

Abstract

This project seeks to answer at what angle does a gun marksman have to aim in order to hit the center of a target one meter off the ground and 1000 meters away? We begin by modeling the bullet's trajectory using Euler's method with the help of a Microsoft Excel spreadsheet solver, and then systematically search for the angle corresponding to the center of the target. It was found that a marksman shooting a target 1000 meters away and 1 meter off the ground has to aim the rifle 0.436° above horizontal to hit the center.

Keywords

Euler’s Method, Velocity, Projectile Motion

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PROBLEM STATEMENT

At what angle above the horizontal does a gun marksman shooting one meter off the ground in 27.2°C weather; have to aim a Remington 30-06 rifle to hit the center of a target that is 1000 meters away and one meter off the ground? Take into account air resistance and ignore the wind variable. You may assume a bullet mass of 150 grains, muzzle velocity of 887 m/s, bullet diameter of $0.008m$ and the shooter is at sea level.

MOTIVATION

The calculation of bullet trajectories is necessary help scientist and engineers develop equipment for the United States Department of Defense, particularly Marine Corp snipers. Snipers specialize in long distance shooting, both at targets of known and unknown distances. There are many important pieces that factor into the understanding of bullet trajectories: air resistance, angle, air pressure and temperature, muzzle velocity, bullet shape and drag coefficient. All of these factors contribute to the accuracy of the bullet hitting the desired target. Snipers usually have one chance to hit their target and knowing all the above pieces can make the job easier. Scientist and engineers work together to construct simulations, to develop data that can benefit snipers in real life situations. Ballistics data also goes into the design and construction of scopes, rangefinders and different types of ammunition.

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MATHEMATICAL DESCRIPTION AND SOLUTION APPROACH

Let $x(t)$ and $y(t)$ denote the horizontal and vertical displacement of the bullet at the time t. Similarly define $v_x(t)$ and $v_y(t)$ to be the horizontal and vertical velocity of the bullet at the time t. We use $f'(t)$ to denote the derivative of the function $f(t)$ with respect to t. We first use the density of air $\rho = 353.03\frac{P}{T}$ to calculate the drag,

$$D = \frac{1}{2}\,\rho\, C\, A, \tag{1}$$

where $C = 0.295$ is the drag coefficient of the bullet and A is the area of the bullet head in m^2. We now use the well-known motion equations for an object moving through a gravitation field to see:

$$v_x'(t) = -\frac{D}{M} v \cdot v_x(t) \tag{2}$$

$$v_y'(t) = -g - \frac{D}{M} v \cdot v_y(t) \tag{3}$$

where M is the mass of the bullet in kilograms, g is the acceleration due to gravity and v is the magnitude of the velocity vector. Using the velocity of the bullet given in the problem we can determine our initial conditions:

$$x'(0) = v_x(0) = V\ \cos\theta \tag{4}$$

$$y'(0) = v_y(0) = V \sin\theta \tag{5}$$

$$x(0) = 0 \tag{6}$$

$$y(0) = 1. \tag{7}$$

Now Euler's method gives:

$$F(t+\Delta t) \approx F(t) + \Delta t\, F'(t) \tag{8}$$

where $F(t)$ is any of the functions $x(t), y(t), v_x(t)$ or $v_y(t)$. We now use an Excel Spreadsheet solver and a guess and check method to determine the value of θ, the angle above the horizontal, which causes the bullet to strike the center of the target. The Excel Spreadsheets are given in the Appendix.

DISCUSSION

In order to hit the center of the target as stated in the problem, the gun marksman will have to aim the barrel of the Remington 30-06 rifle at $0.436°$ above the horizontal. The result was considerably smaller than the expected $30°$ above the horizontal. The data from the problem shows the time it took for the bullet to hit the target, the maximum height the bullet reaches while in flight and the velocity of the bullet as it hits the target. Although Euler's method only provides an approximation we believe the method is sufficient for the problem at hand, as our approximation takes air resistance into consideration.

To the problem at hand, there is a dramatic difference between methods that take into account the air resistance compared to methods that neglects the effect of air resistance. We provide Figure 1 and Figure 2 to highlight these differences in our situation.

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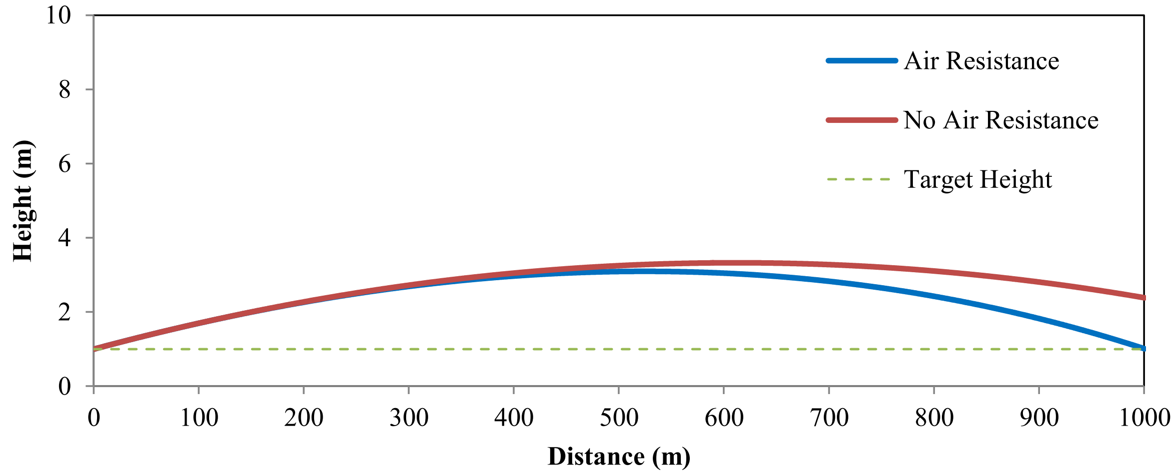

Figure 1: Trajectory of a bullet fired from a Remington 30-06 rifle at an angle of $\mathbf{0.436°}$ above the horizontal to hit a target sitting 1m off the ground 1000m away.

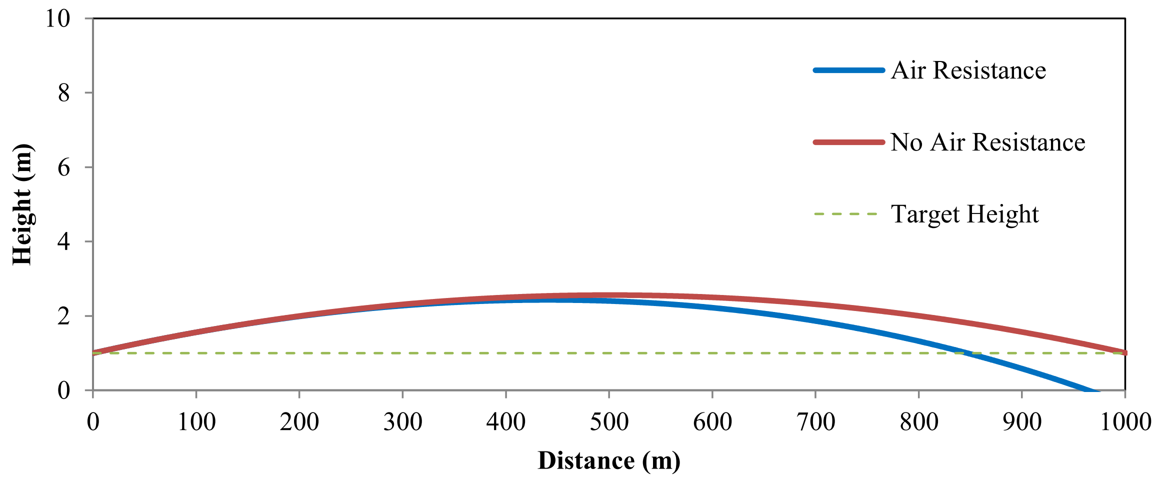

Figure 2: Trajectory of a bullet fired from a Remington 30-06 rifle at an angle of $\mathbf{0.357°}$ above the horizontal to hit a target sitting 1m off the ground 1000m away.

With air resistance the bullet launched at an angle of 0.436° above the horizontal travels a little over two meters vertically and remains in flight for 1.305 seconds before striking the target. Without air resistance the rifle must be aimed 0.357° above the horizontal to hit the center of the target and the bullet remains airborne for 1.128 seconds, travelling 1.5 meters vertically.

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Clearly air resistance should be considered for measuring a bullet's flight, otherwise the data will be distorted. This information is essential to engineers and scientists who develop guns, scopes, rangefinders and ammunition.

CONCLUSION AND RECOMMENDATIONS

In summary, we determined that a gun marksman has to aim the gun at $0.436°$ to hit the center of a target $1m$ off the ground and $1000m$ away with an air temperature of $27.2°C$. With the results stemming from our initial assumptions known, it would be interesting to run a similar problem with different assumptions and compare results. What is the effect of a different ambient temperature, elevation, or even bullet type? For our calculations we assumed a pointed soft point bullet. Pointed soft point bullets are solid compared to hollow point bullets (see Fig. 3) that have a small hollow point. As a result of the small indentation on the tip of the bullet, it is possible that hollow point bullet shot at the target in the same conditions would have more drag and require a different launch angle to hit the center of the target.

Figure 3: Comparison of soft point (left) and hollow point (right) bullets.

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NOMENCLATURE

Symbol	Definition	Units
P	Pressure	Atmospheres
V	Volume	Liters
T	Temperature	℃
θ	Angle off Horizontal	Degrees
v	Speed	$\frac{meters}{second}$
C	Drag Coefficient	No Units
M	Mass	Kilograms
A	Area	$meters^2$
D	Drag	No Units

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APPENDIX – EULER'S METHOD COMPUTATIONS

◇	A	B	C	D	E	F	G	H	I	J	K
1	Thetadeg	0.436			D	2.77E-06					
2	theta	0.00761			M	0.00972					
3	vx0	886.9743									
4	vy0	6.749676									
5	x0	0									
6	y0	1									
7											
8	t	x	y	vx	vy	v	x'	y'	vx'	vy'	
9	0	0	1	886.9743	6.749676	887	886.974319	6.749675879	-224.2102	-11.51	
10	0.001	0.886974	1.00675	886.7501	6.73817	886.7757	886.750108	6.73816969	-224.0968	-11.50	
11	0.002	1.773724	1.013488	886.526	6.726667	886.5515	886.526012	6.72666684	-223.9835	-11.50	
12	0.003	2.66025	1.020215	886.302	6.715167	886.3275	886.302028	6.715167327	-223.8704	-11.50	
13	0.004	3.546552	1.02693	886.0782	6.703671	886.1035	886.078158	6.703671148	-223.7573	-11.49	
14	0.005	4.432631	1.033633	885.8544	6.692178	885.8797	885.8544	6.692178301	-223.6443	-11.49	
15	0.006	5.318485	1.040326	885.6308	6.680689	885.656	885.630756	6.680688782	-223.5313	-11.49	
16	0.007	6.204116	1.047006	885.4072	6.669203	885.4323	885.407225	6.669202591	-223.4185	-11.48	
17	0.008	7.089523	1.053675	885.1838	6.65772	885.2088	885.183806	6.657719723	-223.3057	-11.48	

Figure 4: Screenshot of calculations for Euler's method using Microsoft Excel.

◇	A	B	C	D	E	F	G	H	I	J
1	Thetadeg	0.436			D	0.00000277				
2	theta	=B1/180*3.14159			M	0.00971984				
3	vx0	=887*COS(B2)								
4	vy0	=887*SIN(B2)								
5	x0	0								
6	y0	1								
7										
8	t	x	y	vx	vy	v	x'	y'	vx'	vy'
9	0	=B5	=B6	=B3	=B4	=SQRT(D9^2+E9^2)	=D9	=E9	=-$F$1/$F$2*F9*D9	=-9.8-$F$1/$F$2*F9*E9
10	0.001	=B9+(A10-A9)*G9	=C9+(A10-A9)*H9	=D9+(A10-A9)*I9	=E9+(A10-A9)*J9	=SQRT(D10^2+E10^2)	=D10	=E10	=-$F$1/$F$2*F10*D10	=-9.8-$F$1/$F$2*F10*E10
11	0.002	=B10+(A11-A10)*G10	=C10+(A11-A10)*H10	=D10+(A11-A10)*I10	=E10+(A11-A10)*J10	=SQRT(D11^2+E11^2)	=D11	=E11	=-$F$1/$F$2*F11*D11	=-9.8-$F$1/$F$2*F11*E11
12	0.003	=B11+(A12-A11)*G11	=C11+(A12-A11)*H11	=D11+(A12-A11)*I11	=E11+(A12-A11)*J11	=SQRT(D12^2+E12^2)	=D12	=E12	=-$F$1/$F$2*F12*D12	=-9.8-$F$1/$F$2*F12*E12
13	0.004	=B12+(A13-A12)*G12	=C12+(A13-A12)*H12	=D12+(A13-A12)*I12	=E12+(A13-A12)*J12	=SQRT(D13^2+E13^2)	=D13	=E13	=-$F$1/$F$2*F13*D13	=-9.8-$F$1/$F$2*F13*E13
14	0.005	=B13+(A14-A13)*G13	=C13+(A14-A13)*H13	=D13+(A14-A13)*I13	=E13+(A14-A13)*J13	=SQRT(D14^2+E14^2)	=D14	=E14	=-$F$1/$F$2*F14*D14	=-9.8-$F$1/$F$2*F14*E14
15	0.006	=B14+(A15-A14)*G14	=C14+(A15-A14)*H14	=D14+(A15-A14)*I14	=E14+(A15-A14)*J14	=SQRT(D15^2+E15^2)	=D15	=E15	=-$F$1/$F$2*F15*D15	=-9.8-$F$1/$F$2*F15*E15
16	0.007	=B15+(A16-A15)*G15	=C15+(A16-A15)*H15	=D15+(A16-A15)*I15	=E15+(A16-A15)*J15	=SQRT(D16^2+E16^2)	=D16	=E16	=-$F$1/$F$2*F16*D16	=-9.8-$F$1/$F$2*F16*E16
17	0.008	=B16+(A17-A16)*G16	=C16+(A17-A16)*H16	=D16+(A17-A16)*I16	=E16+(A17-A16)*J16	=SQRT(D17^2+E17^2)	=D17	=E17	=-$F$1/$F$2*F17*D17	=-9.8-$F$1/$F$2*F17*E17
18	0.009	=B17+(A18-A17)*G17	=C17+(A18-A17)*H17	=D17+(A18-A17)*I17	=E17+(A18-A17)*J17	=SQRT(D18^2+E18^2)	=D18	=E18	=-$F$1/$F$2*F18*D18	=-9.8-$F$1/$F$2*F18*E18
19	0.01	=B18+(A19-A18)*G18	=C18+(A19-A18)*H18	=D18+(A19-A18)*I18	=E18+(A19-A18)*J18	=SQRT(D19^2+E19^2)	=D19	=E19	=-$F$1/$F$2*F19*D19	=-9.8-$F$1/$F$2*F19*E19

Figure 5: Screenshot of formulas used in the calculation of Euler's method.

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