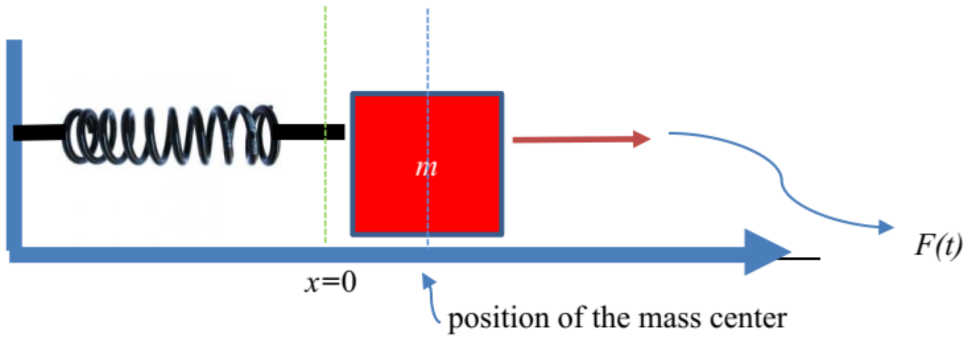
University of Genoa - DIBRIS

Master of Robotics Engineering 2023/2024

Control of Linear Multi-Variable System **Lab session 1**

Exercise 1: mass-spring-friction system

Let's study the following simple mechanical system



Where, m = mass of the block

K = elastic constant of the spring

f = viscous friction of the block

We apply the 2nd low of Newton projecting the equation on the x axis

$$m\ddot{x} = F(t) - kx - f\dot{x}$$

We can choose the state vector as following

$$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$

The state equation of the system is in this case

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{f}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} u$$

The equation matches with the general structure of linear time invariant system (LTI)

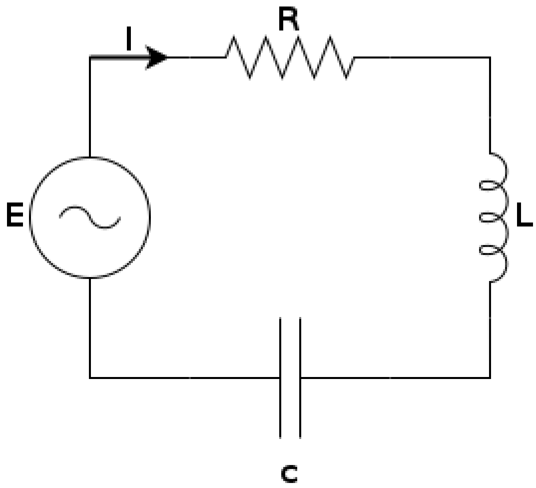
$$x\dot{(t)} = Ax(t) + Bu(t)$$

Q1. Define the system in matlab file: state vector, parameters (k,f,m,etc..) and matrices (A, B, etc).

Q2. Plot the position and the velocity of the block, suposing that $x(0) = 20cm, u(0) = 2N$.

Exercise 2: RLC circuit

We have the following RLC circuit:



Where, the basic elements of a linear circuit system are

1. Resistor $V_R = Ri$
2. Inductor $V_L = \frac{L(di)}{dt}$
3. Capacitor $i = C\frac{dV_C}{dt}$
4. Voltage generator E(t)

By applying the basis of Kirchhoff mesh law, we have

$$E(t) = V_R + V_L + V_C$$

After simplification, we obtain the final equation

$$LC\frac{d^2}{dt^2}V_C + RC\frac{d}{dt}V_C + V_C = E(t)$$

If we choose the state vector in the following way $x_1(t) = V_C$, $x_2(t) = i(t)$,

We can write the state equation as follows.

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} u$$

If we assume that we are able to measure the resistor voltage, the output signal is the voltage drop across the resistance $y = V_R = Ri = Rx_2$ which can matches $y = Cx + Du$

Q1. Define the system in matlab file: state vector, parameters and matrices.

Q2. Plot the state and the output, suposing that $x(0) = [0, 0], u(0) = 0$.

Exercise 3: Remotely Operated Vehicle (ROV)

Let suppose we have an underwater vehicle ROV and we study the motion along [x, y, w] which are called surge, sway and yaw. We have the following equations of motion written in the body frame of the ROV

$$\begin{cases} m\ddot{x} = -\alpha\dot{x} + F_X \\ m\ddot{y} = -\beta\dot{y} + F_Y \\ J\dot{w} = -\gamma w + N_z \end{cases}$$

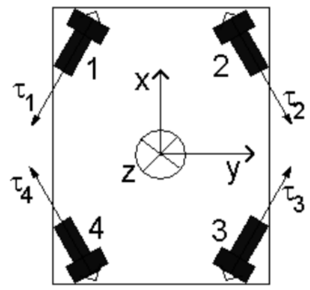


Fig. 1. Horizontal thruster configuration with respect to the body - fixed frame.

In this system, the input is not the force itself. The mapping matrix (allocation matrix) is given in the following equation

$$\begin{pmatrix} F_x \\ F_y \\ N_z \end{pmatrix} = \begin{bmatrix} c\theta_1 & c\theta_2 & c\theta_3 & c\theta_4 \\ s\theta_1 & s\theta_2 & s\theta_3 & s\theta_4 \\ J_{31} & J_{32} & J_{33} & J_{34} \end{bmatrix} \begin{pmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \\ \tau_4 \end{pmatrix}$$

$$\begin{aligned} c\psi &:= \cos\psi, \quad s\psi := \sin\psi \\ J_{31} &:= r_x\, s\theta_1 + r_y\, c\theta_1 \\ J_{32} &:= r_x\, s\theta_2 - r_y\, c\theta_2 \\ J_{33} &:= -r_x\, s\theta_3 - r_y\, c\theta_3 \\ J_{34} &:= -r_x\, s\theta_4 + r_y\, c\theta_4 \end{aligned}$$

Where θ_i is the orientation of thruster i with respect to the body frame axis x

$[r_x, r_y]$ is the position of the thruster.

Q1. Choose a convenient state vector and write the equation of motion in the general structure of LTI representation.

Q2. Define the system in matlab file: state vector, parameters and matrices.

Q3. Plot the state after initializing the state and the input vector.

Q4. Find an appropriate output for the system.