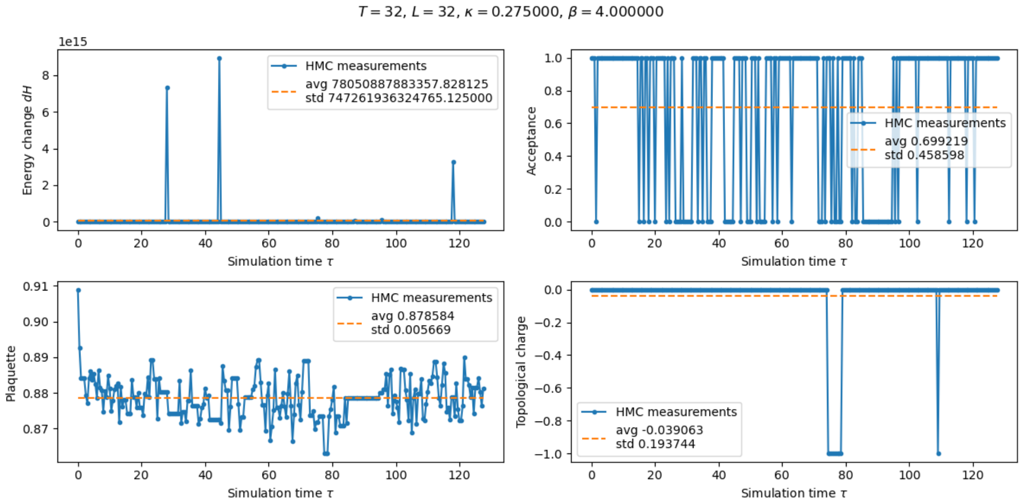
1 Simulation result



2 Computation of two-point correlators

We are interested to compute functions of the form $\langle j_X(r_1) j_Y(r_2)\rangle$, where r_1 and r_2 are positions and X, Y subscripts denote the current of interest.

- Vector current $j_V^{\mu,ab}(t,x) = \bar{\psi}_a(t,x)\gamma^\mu \psi_b(t,x)$
- Axial current $j_A^{\mu,ab}(t,x) = \bar{\psi}_a(t,x)\gamma^\mu\gamma^5 \psi_b(t,x)$
- Pseudoscalar current $j_P^{ab}(t,x) = \bar{\psi}_a(t,x)\gamma^5 \psi_b(t,x)$

Now for the computation:

- We choose a reference point $r_2 = 0$ on the lattice (e.g. middle)
- We can use Wick's theorem (we discard any global sign for now) to express these two-point functions as traces in spinor space.
- We use the CG algorithm to solve for the propagator $S(r_1, 0)$ (inverse Dirac operator).

We assume that the propagator is diagonal in flavour space and express the spinor components as

$$S(t,x;0,0) = \begin{pmatrix} S_{00} & S_{01} \\ S_{10} & S_{11} \end{pmatrix} \tag{1}$$

We also use γ^5 hermiticity (ref Gattriger & Lang), i.e. for arbitrary r_1, r_2

$$S(r_2, r_1) = \gamma^5 S(r_1, r_2)^\dagger \gamma^5 \tag{2}$$

Our representation of Euclidean gamma matrices:

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \gamma^1 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \gamma^5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{3}$$

2.1 P-P correlator

$$
\begin{aligned}
\langle j_P^{12}(r) j_P^{21}(0)\rangle &= \langle \bar\psi_1(r)\gamma^5\psi_2(r)\bar\psi_2(0)\gamma^5\psi_1(0)\rangle \simeq \mathrm{Tr}\left\{S(r,0)\gamma^5 S(0,r)\gamma^5\right\} \\
&= \mathrm{Tr}\left\{S(r,0)\gamma^5\gamma^5 S(r,0)^\dagger\gamma^5\gamma^5\right\} \\
&= \mathrm{Tr}\left\{S(r,0)S(r,0)^\dagger\right\} \\
&= \mathrm{Tr}\left\{\begin{pmatrix} S_{00} & S_{01} \\ S_{10} & S_{11}\end{pmatrix}\begin{pmatrix} S_{00}^* & S_{10}^* \\ S_{01}^* & S_{11}^*\end{pmatrix}\right\} \\
&= \mathrm{Tr}\left\{\begin{pmatrix} S_{00}S_{00}^* + S_{01}S_{01}^* & \\ & S_{10}S_{10}^* + S_{11}S_{11}^*\end{pmatrix}\right\} \\
&= |S_{00}|^2 + |S_{01}|^2 + |S_{10}|^2 + |S_{11}|^2
\end{aligned}
$$

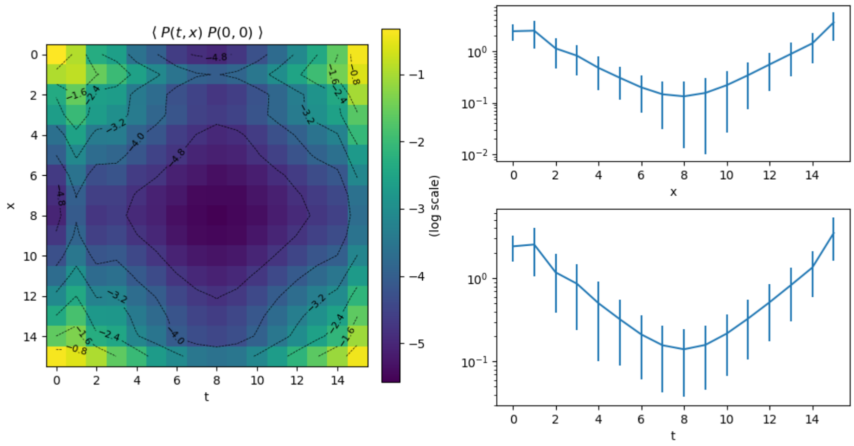


Figure 1

2.2 A-P correlators

$$
\begin{aligned}
\langle j_A^{\mu,12}(r) j_P^{21}(0)\rangle &= \langle \bar\psi_1(r)\gamma^\mu\gamma^5\psi_2(r)\bar\psi_2(0)\gamma^5\psi_1(0)\rangle \simeq \mathrm{Tr}\left\{S(r,0)\gamma^\mu\gamma^5 S(0,r)\gamma^5\right\} \\
&= \mathrm{Tr}\left\{S(r,0)\gamma^\mu\gamma^5\gamma^5 S(r,0)^\dagger\gamma^5\gamma^5\right\} \\
&= \mathrm{Tr}\left\{S(r,0)\gamma^\mu S(r,0)^\dagger\right\}
\end{aligned}
$$

For $\mu = 0$ we have

$$
\begin{aligned}
\langle j_A^{0,12}(r) j_P^{21}(0)\rangle &= \mathrm{Tr}\left\{\begin{pmatrix} S_{00} & S_{01} \\ S_{10} & S_{11}\end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0\end{pmatrix}\begin{pmatrix} S_{00}^* & S_{10}^* \\ S_{01}^* & S_{11}^*\end{pmatrix}\right\} \\
&= \mathrm{Tr}\left\{\begin{pmatrix} S_{00} & S_{01} \\ S_{10} & S_{11}\end{pmatrix}\begin{pmatrix} S_{01}^* & S_{11}^* \\ S_{00}^* & S_{10}^*\end{pmatrix}\right\} \\
&= \mathrm{Tr}\left\{\begin{pmatrix} S_{00}S_{01}^* + S_{01}S_{00}^* & \\ & S_{10}S_{11}^* + S_{11}S_{10}^*\end{pmatrix}\right\} \\
&= 2\,\mathfrak{Re}\left[S_{00}S_{01}^* + S_{10}S_{11}^*\right]
\end{aligned}
$$

For $\mu = 1$ we have

$$
\begin{aligned}
\langle j_A^{1,12}(r) j_P^{21}(0) \rangle &= i \ \mathrm{Tr} \left\{ \begin{pmatrix} S_{00} & S_{01} \\ S_{10} & S_{11} \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} S_{00}^* & S_{10}^* \\ S_{01}^* & S_{11}^* \end{pmatrix} \right\} \\
&= i \ \mathrm{Tr} \left\{ \begin{pmatrix} S_{00} & S_{01} \\ S_{10} & S_{11} \end{pmatrix} \begin{pmatrix} -S_{01}^* & -S_{11}^* \\ S_{00}^* & S_{10}^* \end{pmatrix} \right\} \\
&= i \ \mathrm{Tr} \left\{ \begin{pmatrix} -S_{00}S_{01}^* + S_{01}S_{00}^* & \\ & -S_{10}S_{11}^* + S_{11}S_{10}^* \end{pmatrix} \right\} \\
&= 2 \ \mathfrak{Im} \left[S_{00}S_{01}^* + S_{10}S_{11}^* \right]
\end{aligned}
$$

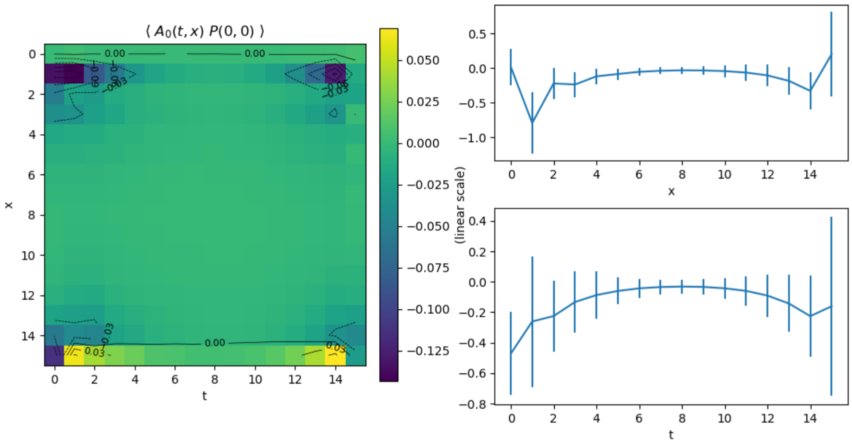


Figure 2

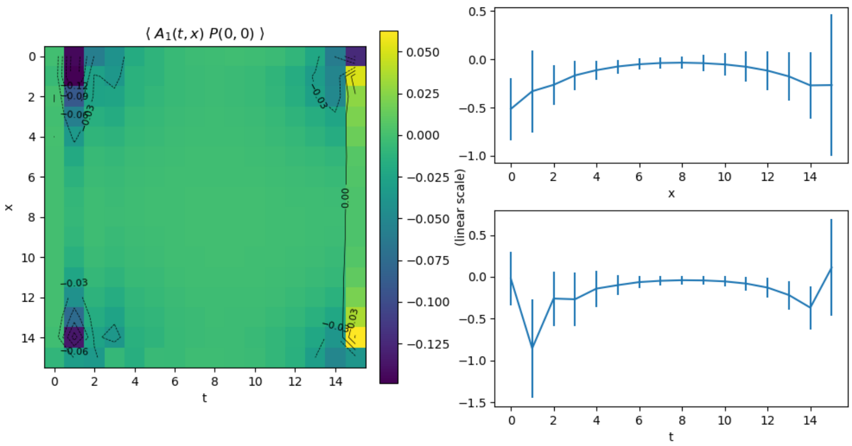


Figure 3

2.3 P-A correlators

$$
\begin{aligned}
\langle j_P^{12}(r) j_A^{\mu,21}(0)\rangle &= \langle \bar\psi_1(r)\gamma^5\psi_2(r)\bar\psi_2(0)\gamma^\mu\gamma^5\psi_1(0)\rangle \simeq \mathrm{Tr}\left\{S(r,0)\gamma^5 S(0,r)\gamma^\mu\gamma^5\right\} \\
&= \mathrm{Tr}\left\{S(r,0)\gamma^5\gamma^5 S(r,0)^\dagger\gamma^5\gamma^\mu\gamma^5\right\} \\
&= -\mathrm{Tr}\left\{S(r,0)S(r,0)^\dagger\gamma^\mu\right\}
\end{aligned}
$$

For $\mu = 0$ we have

$$
\begin{aligned}
\langle j_P^{12}(r) j_A^{0,21}(0)\rangle &= -\mathrm{Tr}\left\{\begin{pmatrix} S_{00} & S_{01} \\ S_{10} & S_{11}\end{pmatrix}\begin{pmatrix} S_{00}^* & S_{10}^* \\ S_{01}^* & S_{11}^*\end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0\end{pmatrix}\right\} \\
&= -\mathrm{Tr}\left\{\begin{pmatrix} S_{00} & S_{01} \\ S_{10} & S_{11}\end{pmatrix}\begin{pmatrix} S_{10}^* & S_{00}^* \\ S_{11}^* & S_{01}^*\end{pmatrix}\right\} \\
&= -\mathrm{Tr}\left\{\begin{pmatrix} S_{00}S_{10}^* + S_{01}S_{11}^* & \\ & S_{10}S_{00}^* + S_{11}S_{01}^*\end{pmatrix}\right\} \\
&= -2\ \mathfrak{Re}\left[S_{00}S_{10}^* + S_{01}S_{11}^*\right]
\end{aligned}
$$

For $\mu = 1$ we have

$$
\begin{aligned}
\langle j_P^{12}(r) j_A^{1,21}(0)\rangle &= -i\ \mathrm{Tr}\left\{\begin{pmatrix} S_{00} & S_{01} \\ S_{10} & S_{11}\end{pmatrix}\begin{pmatrix} S_{00}^* & S_{10}^* \\ S_{01}^* & S_{11}^*\end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0\end{pmatrix}\right\} \\
&= -i\ \mathrm{Tr}\left\{\begin{pmatrix} S_{00} & S_{01} \\ S_{10} & S_{11}\end{pmatrix}\begin{pmatrix} S_{10}^* & -S_{00}^* \\ S_{11}^* & -S_{01}^*\end{pmatrix}\right\} \\
&= -i\ \mathrm{Tr}\left\{\begin{pmatrix} S_{00}S_{10}^* + S_{01}S_{11}^* & \\ & -S_{10}S_{00}^* - S_{11}S_{01}^*\end{pmatrix}\right\} \\
&= 2\ \mathfrak{Im}\left[S_{00}S_{10}^* + S_{01}S_{11}^*\right]
\end{aligned}
$$

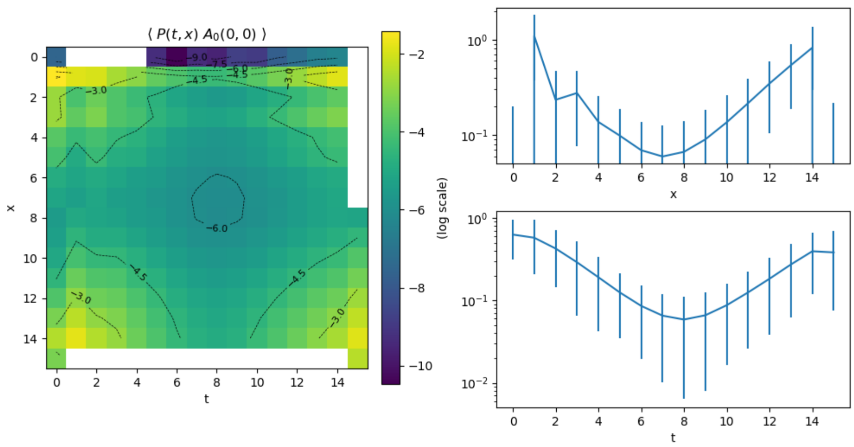


Figure 4

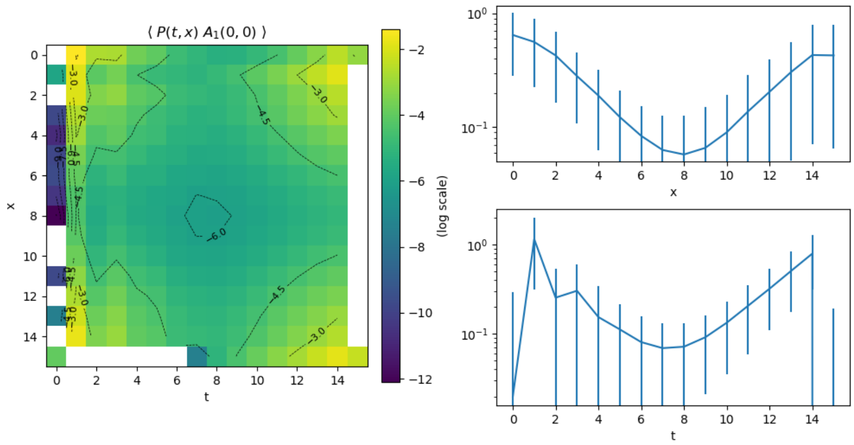


Figure 5

2.4 V-V correlators

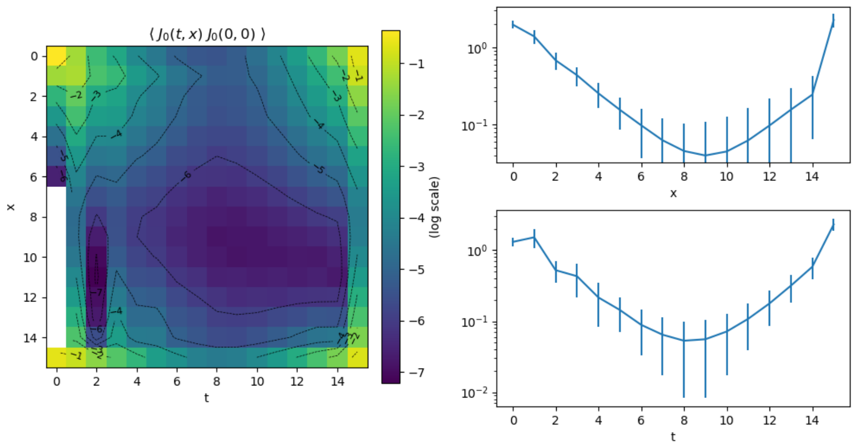


Figure 6

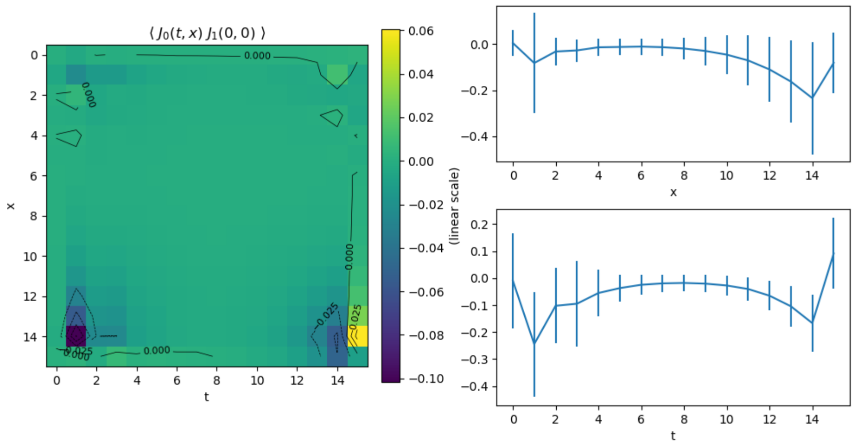


Figure 7

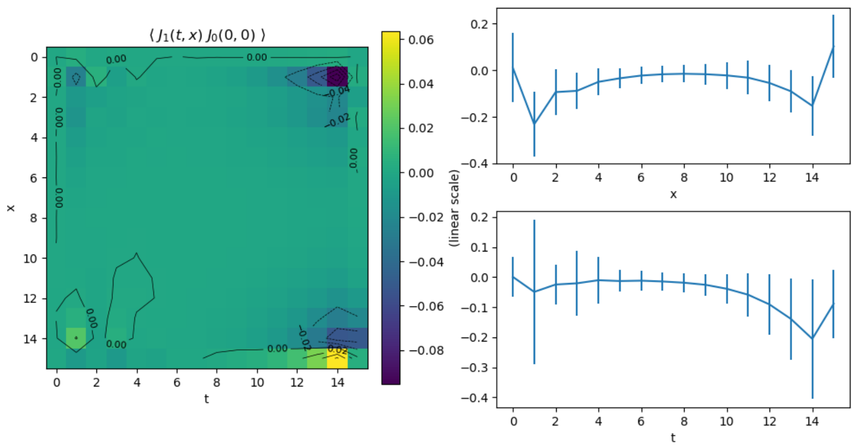


Figure 8

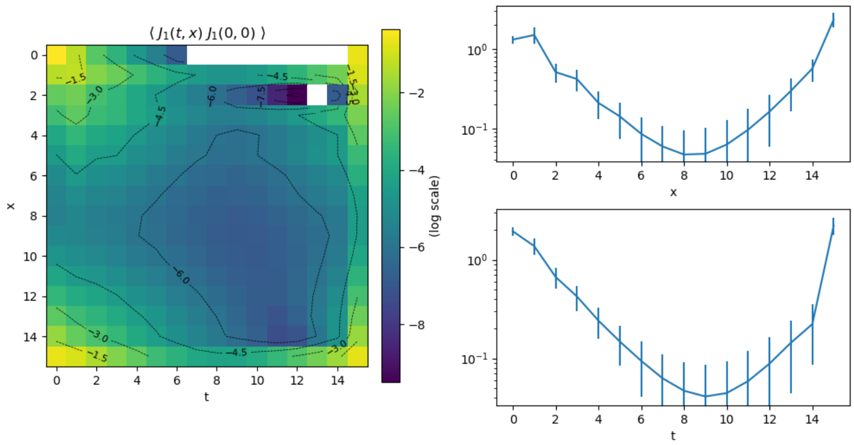


Figure 9