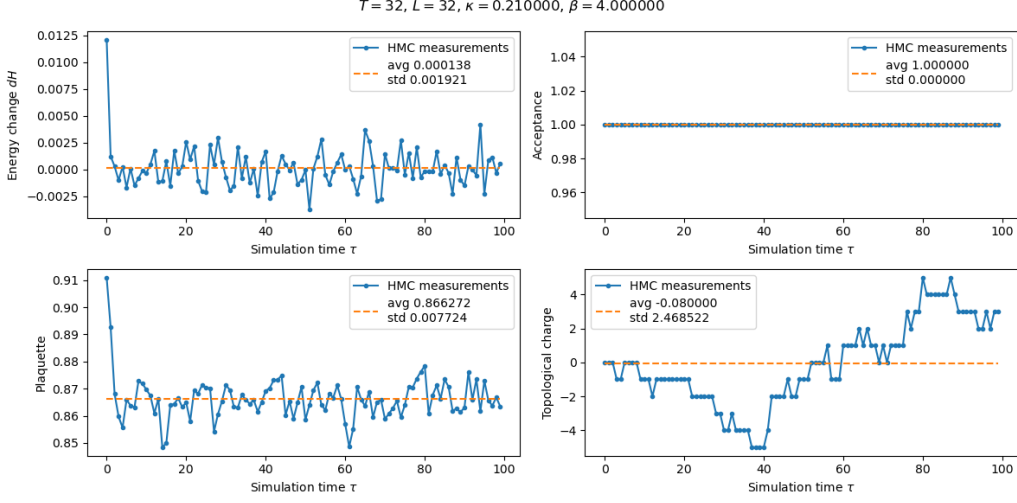


1 Simulation result



2 Computation of two-point correlators

We are interested to compute functions of the form $\langle j_X(r_1) j_Y(r_2) \rangle$, where r_1 and r_2 are positions and X, Y subscripts denote the current of interest.

- Vector current $j_V^{\mu,ab}(t, x) = \bar{\psi}_a(t, x) \gamma^\mu \psi_b(t, x)$
- Axial current $j_A^{\mu,ab}(t, x) = \bar{\psi}_a(t, x) \gamma^\mu \gamma^5 \psi_b(t, x)$
- Pseudoscalar current $j_P^{ab}(t, x) = \bar{\psi}_a(t, x) \gamma^5 \psi_b(t, x)$

Now for the computation:

- We choose a reference point $r_2 = 0$ on the lattice (e.g. middle)
- We can use Wick's theorem (we discard any global sign for now) to express these two-point functions as traces in spinor space.
- We use the CG algorithm to solve for the propagator $S(r_1, 0)$ (inverse Dirac operator).

We assume that the propagator is diagonal in flavour space and express the spinor components as

$$S(t, x; 0, 0) = \begin{pmatrix} S_{00} & S_{01} \\ S_{10} & S_{11} \end{pmatrix} \quad (1)$$

We also use γ^5 hermiticity (ref Gattigier & Lang), i.e. for arbitrary r_1, r_2

$$S(r_2, r_1) = \gamma^5 S(r_1, r_2)^\dagger \gamma^5 \quad (2)$$

Our representation of Euclidean gamma matrices:

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \gamma^1 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \gamma^5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (3)$$

2.1 P-P correlator

$$\begin{aligned}
\langle j_P^{12}(r) j_P^{21}(0) \rangle &= \langle \bar{\psi}_1(r) \gamma^5 \psi_2(r) \bar{\psi}_2(0) \gamma^5 \psi_1(0) \rangle \simeq \text{Tr} \{ S(r, 0) \gamma^5 S(0, r) \gamma^5 \} \\
&= \text{Tr} \{ S(r, 0) \gamma^5 \gamma^5 S(r, 0)^\dagger \gamma^5 \gamma^5 \} \\
&= \text{Tr} \{ S(r, 0) S(r, 0)^\dagger \} \\
&= \text{Tr} \left\{ \begin{pmatrix} S_{00} & S_{01} \\ S_{10} & S_{11} \end{pmatrix} \begin{pmatrix} S_{00}^* & S_{10}^* \\ S_{01}^* & S_{11}^* \end{pmatrix} \right\} \\
&= \text{Tr} \left\{ \begin{pmatrix} S_{00} S_{00}^* + S_{01} S_{01}^* & S_{10} S_{10}^* + S_{11} S_{11}^* \\ S_{10} S_{00}^* + S_{11} S_{01}^* & S_{00} S_{10}^* + S_{01} S_{11}^* \end{pmatrix} \right\} \\
&= |S_{00}|^2 + |S_{01}|^2 + |S_{10}|^2 + |S_{11}|^2
\end{aligned}$$

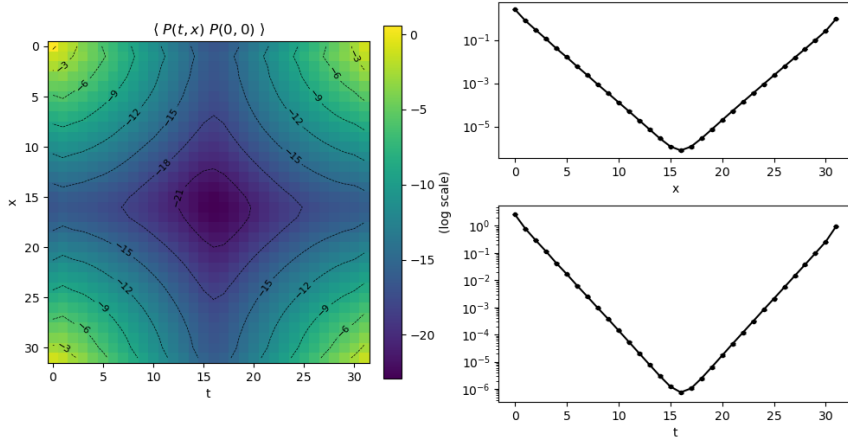


Figure 1

2.2 A-P correlators

$$\begin{aligned}
\langle j_A^{\mu, 12}(r) j_P^{21}(0) \rangle &= \langle \bar{\psi}_1(r) \gamma^\mu \gamma^5 \psi_2(r) \bar{\psi}_2(0) \gamma^5 \psi_1(0) \rangle \simeq \text{Tr} \{ S(r, 0) \gamma^\mu \gamma^5 S(0, r) \gamma^5 \} \\
&= \text{Tr} \{ S(r, 0) \gamma^\mu \gamma^5 \gamma^5 S(r, 0)^\dagger \gamma^5 \gamma^5 \} \\
&= \text{Tr} \{ S(r, 0) \gamma^\mu S(r, 0)^\dagger \}
\end{aligned}$$

For $\mu = 0$ we have

$$\begin{aligned}
\langle j_A^{0, 12}(r) j_P^{21}(0) \rangle &= \text{Tr} \left\{ \begin{pmatrix} S_{00} & S_{01} \\ S_{10} & S_{11} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} S_{00}^* & S_{10}^* \\ S_{01}^* & S_{11}^* \end{pmatrix} \right\} \\
&= \text{Tr} \left\{ \begin{pmatrix} S_{00} & S_{01} \\ S_{10} & S_{11} \end{pmatrix} \begin{pmatrix} S_{01}^* & S_{11}^* \\ S_{00}^* & S_{10}^* \end{pmatrix} \right\} \\
&= \text{Tr} \left\{ \begin{pmatrix} S_{00} S_{01}^* + S_{01} S_{00}^* & S_{10} S_{11}^* + S_{11} S_{10}^* \\ S_{10} S_{01}^* + S_{11} S_{00}^* & S_{00} S_{10}^* + S_{01} S_{11}^* \end{pmatrix} \right\} \\
&= 2 \Re [S_{00} S_{01}^* + S_{10} S_{11}^*]
\end{aligned}$$

For $\mu = 1$ we have

$$\begin{aligned}
\langle j_A^{1,12}(r) j_P^{21}(0) \rangle &= i \operatorname{Tr} \left\{ \begin{pmatrix} S_{00} & S_{01} \\ S_{10} & S_{11} \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} S_{00}^* & S_{10}^* \\ S_{01}^* & S_{11}^* \end{pmatrix} \right\} \\
&= i \operatorname{Tr} \left\{ \begin{pmatrix} S_{00} & S_{01} \\ S_{10} & S_{11} \end{pmatrix} \begin{pmatrix} -S_{01}^* & -S_{11}^* \\ S_{00}^* & S_{10}^* \end{pmatrix} \right\} \\
&= i \operatorname{Tr} \left\{ \begin{pmatrix} -S_{00}S_{01}^* + S_{01}S_{00}^* & \\ & -S_{10}S_{11}^* + S_{11}S_{10}^* \end{pmatrix} \right\} \\
&= 2 \operatorname{Im} [S_{00}S_{01}^* + S_{10}S_{11}^*]
\end{aligned}$$

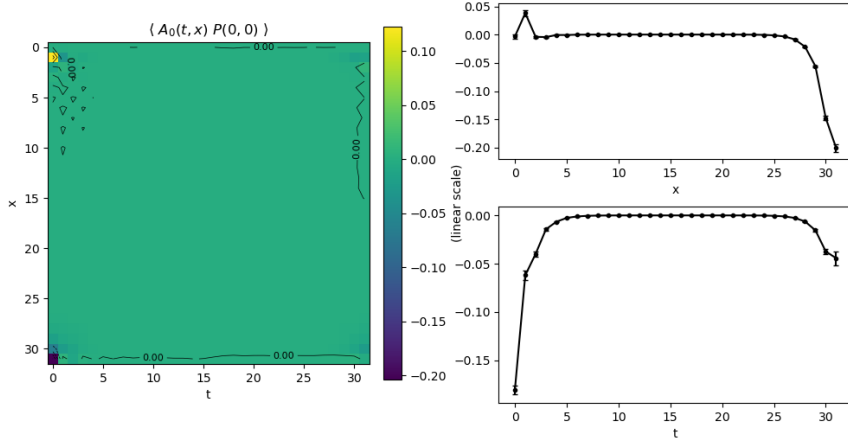


Figure 2

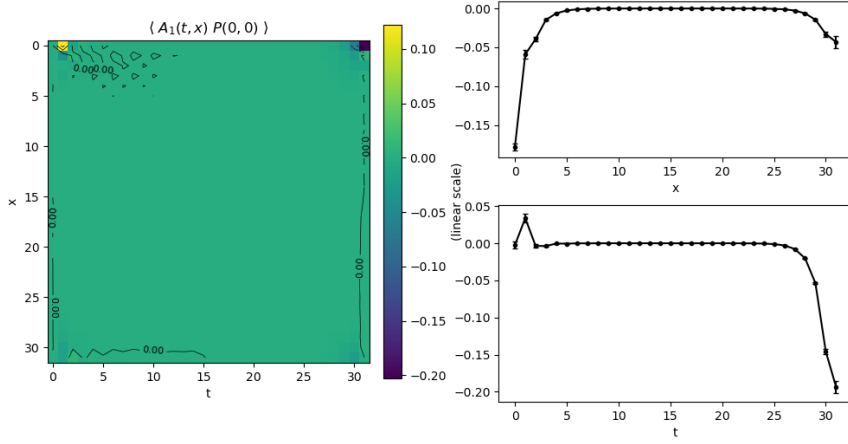


Figure 3

2.3 P-A correlators

$$\begin{aligned}
\langle j_P^{12}(r) j_A^{\mu,21}(0) \rangle &= \langle \bar{\psi}_1(r) \gamma^5 \psi_2(r) \bar{\psi}_2(0) \gamma^\mu \gamma^5 \psi_1(0) \rangle \simeq \text{Tr} \{ S(r,0) \gamma^5 S(0,r) \gamma^\mu \gamma^5 \} \\
&= \text{Tr} \{ S(r,0) \gamma^5 \gamma^5 S(r,0)^\dagger \gamma^5 \gamma^\mu \gamma^5 \} \\
&= -\text{Tr} \{ S(r,0) S(r,0)^\dagger \gamma^\mu \}
\end{aligned}$$

For $\mu = 0$ we have

$$\begin{aligned}
\langle j_P^{12}(r) j_A^{0,21}(0) \rangle &= -\text{Tr} \left\{ \begin{pmatrix} S_{00} & S_{01} \\ S_{10} & S_{11} \end{pmatrix} \begin{pmatrix} S_{00}^* & S_{10}^* \\ S_{01}^* & S_{11}^* \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \right\} \\
&= -\text{Tr} \left\{ \begin{pmatrix} S_{00} & S_{01} \\ S_{10} & S_{11} \end{pmatrix} \begin{pmatrix} S_{10}^* & S_{00}^* \\ S_{11}^* & S_{01}^* \end{pmatrix} \right\} \\
&= -\text{Tr} \left\{ \begin{pmatrix} S_{00} S_{10}^* + S_{01} S_{11}^* & \\ & S_{10} S_{00}^* + S_{11} S_{01}^* \end{pmatrix} \right\} \\
&= -2 \Re [S_{00} S_{10}^* + S_{01} S_{11}^*]
\end{aligned}$$

For $\mu = 1$ we have

$$\begin{aligned}
\langle j_P^{12}(r) j_A^{1,21}(0) \rangle &= -i \text{Tr} \left\{ \begin{pmatrix} S_{00} & S_{01} \\ S_{10} & S_{11} \end{pmatrix} \begin{pmatrix} S_{00}^* & S_{10}^* \\ S_{01}^* & S_{11}^* \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \right\} \\
&= -i \text{Tr} \left\{ \begin{pmatrix} S_{00} & S_{01} \\ S_{10} & S_{11} \end{pmatrix} \begin{pmatrix} S_{10}^* & -S_{00}^* \\ S_{11}^* & -S_{01}^* \end{pmatrix} \right\} \\
&= -i \text{Tr} \left\{ \begin{pmatrix} S_{00} S_{10}^* + S_{01} S_{11}^* & \\ & -S_{10} S_{00}^* - S_{11} S_{01}^* \end{pmatrix} \right\} \\
&= 2 \Im [S_{00} S_{10}^* + S_{01} S_{11}^*]
\end{aligned}$$

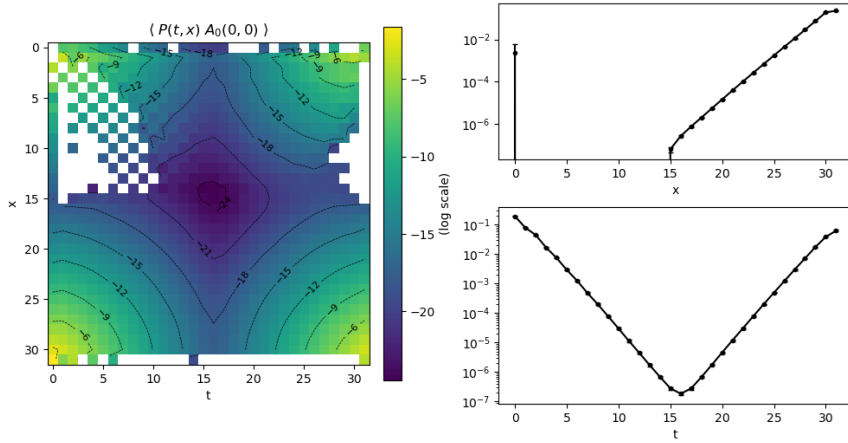


Figure 4

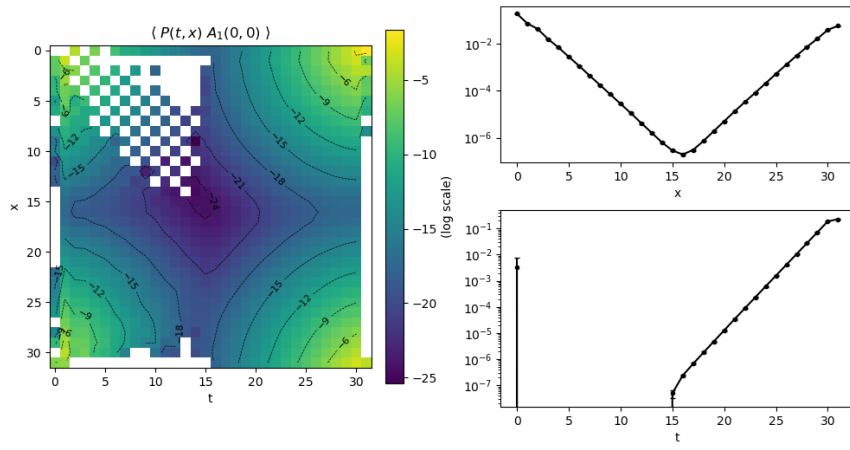


Figure 5

2.4 V-V correlators

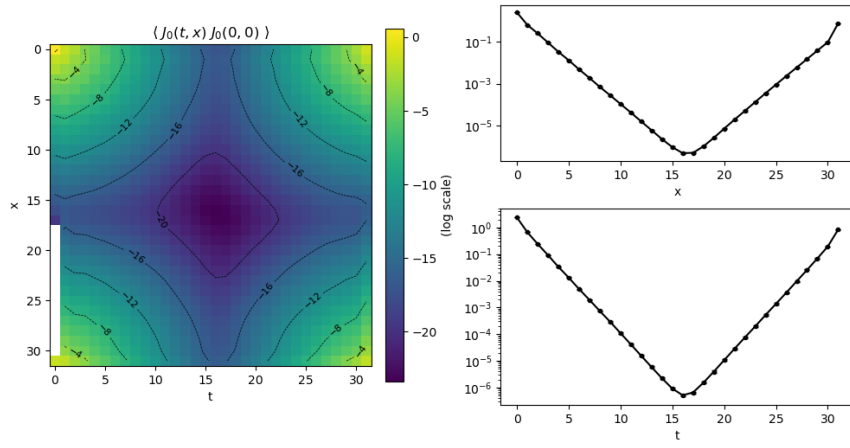


Figure 6

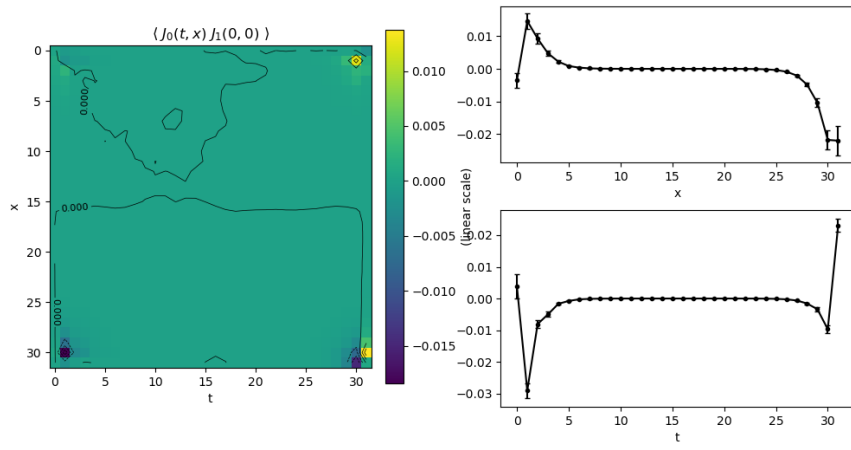


Figure 7

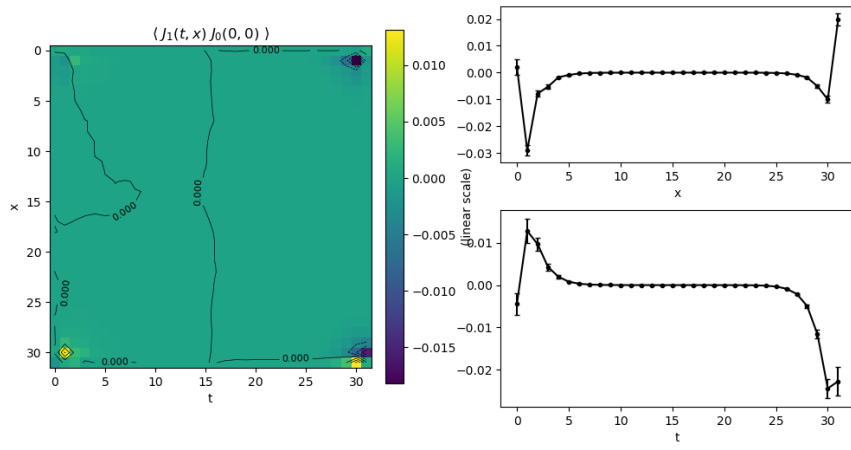


Figure 8

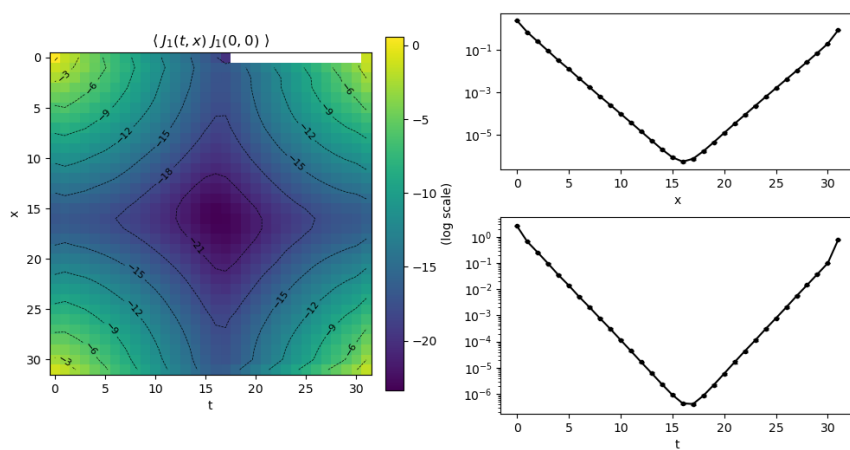


Figure 9