

SAMformer: Unlocking the Potential of Transformers in Time Series Forecasting

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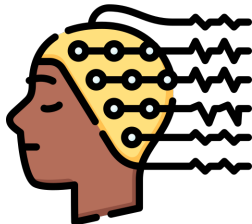


- ① Introduction
- ② Failure of Transformers
- ③ SAMformer
- ④ Experiments
- ⑤ Take Home Message



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In many applications, data are gathered sequentially.

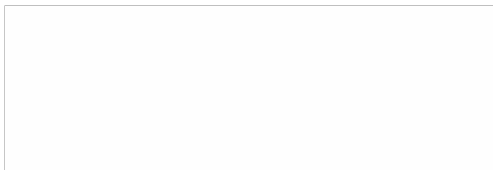
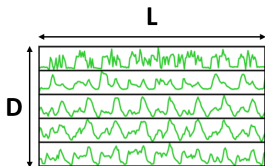




D -dimensional time series of length $L \rightarrow$ predict next H values.

- Training set of N observations $(\{\mathbf{X}^{(i)}\}_{i=0}^N, \{\mathbf{Y}^{(i)}\}_{i=0}^N)$,
- Find predictor $f_{\omega}: \mathbb{R}^{D \times L} \rightarrow \mathbb{R}^{D \times H}$ that minimizes the MSE

$$\mathcal{L}_{\text{train}}(\omega) = \frac{1}{ND} \sum_{i=0}^N \|\mathbf{Y}^{(i)} - f_{\omega}(\mathbf{X}^{(i)})\|_{\text{F}}^2.$$

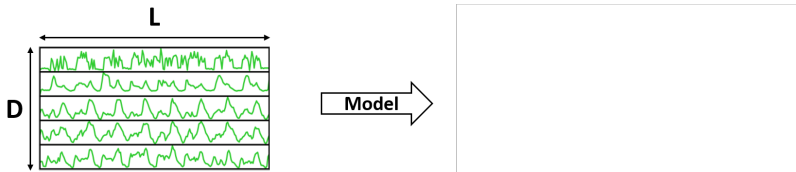




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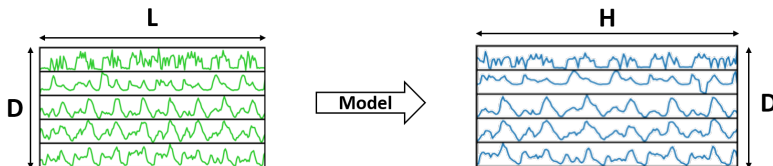




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Motivation

- Transformers tailored to deal with sequential data,
- Impressive results in NLP and Computer Vision.

Main challenges

- ① Quadratic computation of self-attention,
- ② Complex long-term dependencies.



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Main challenges

- ① Quadratic complexity of self-attention
 - Sparse attention: LogTrans [5], Informer [13]
 - Modified attention: Pyraformer [6]
- ② Complex long-term dependencies
 - Decomposition scheme: Autoformer [10], Pyraformer [6]
 - Fourier domain: FEDformer [14]

It leads to a wide range of Anything-formers with heavy and complex implementation and many parameters.

[11] showed that linear models outperform SOTA Anything-former.

Transformers in Computer Vision and NLP



**Commander of the Armies of GPT,
General of the Gemini Legions, loyal
servant to Claude, Llama3, Mixtral**

Transformers in Time Series Forecasting



**Please help, I just got
beaten by a linear model**



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- Generate toy data according to $\mathbf{Y} = \mathbf{X}\mathbf{W}_{\text{toy}} + \epsilon$,
- Design the simplest Transformer possible

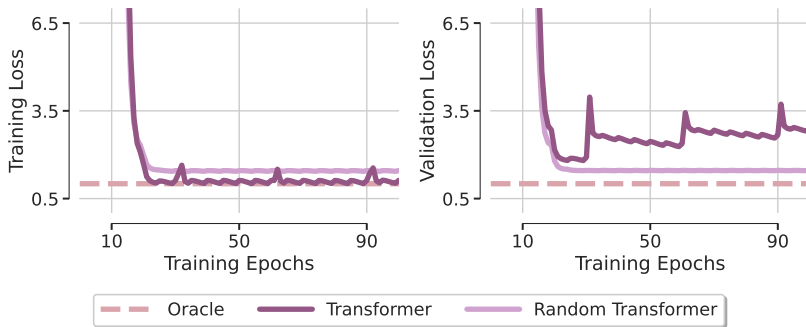
$$f(\mathbf{X}) = [\mathbf{X} + \mathbf{A}(\mathbf{X})\mathbf{X}\mathbf{W}_V\mathbf{W}_O]\mathbf{W}$$
$$\mathbf{A}(\mathbf{X}) = \text{softmax}\left(\frac{\mathbf{X}\mathbf{W}_Q\mathbf{W}_K^\top\mathbf{X}^\top}{\sqrt{d_m}}\right) \in \mathbb{R}^{D \times D}.$$

Theorem (Ilbert, O., Feofanov et al.)

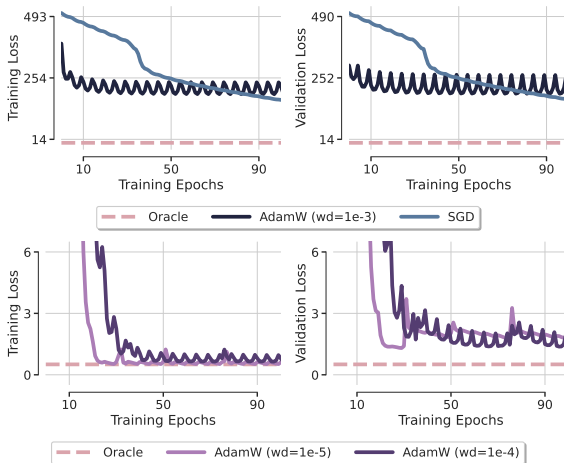
For $\mathbf{W}_Q, \mathbf{W}_K, \mathbf{W}_V, \mathbf{W}_O$ fixed, there exists an infinity \mathbf{W} such that $f(\mathbf{X}) = \mathbf{X}\mathbf{W}_{\text{toy}}$, i.e., the optimal solution (oracle) is reached.



- Oracle: optimal solution,
- Transformer with $\mathbf{W}_Q, \mathbf{W}_K, \mathbf{W}_V, \mathbf{W}_O, \mathbf{W}$ trainable,
- Random Transformer: only \mathbf{W} is trainable.



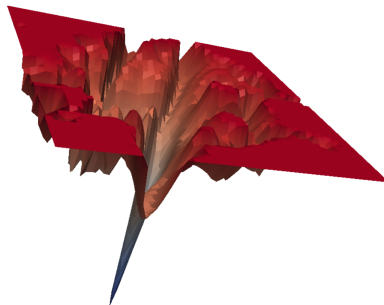
Despite its simplicity, Transformer overfits a lot. Fixing the attention weight improves generalization.



Poor generalization of Transformer with SGD, Adam, and AdamW.

Hypothesis from NLP and Computer Vision

- Transformers have **sharp loss landscape** [2]
 - Convergence to sharp minima,
 - Poor generalization.



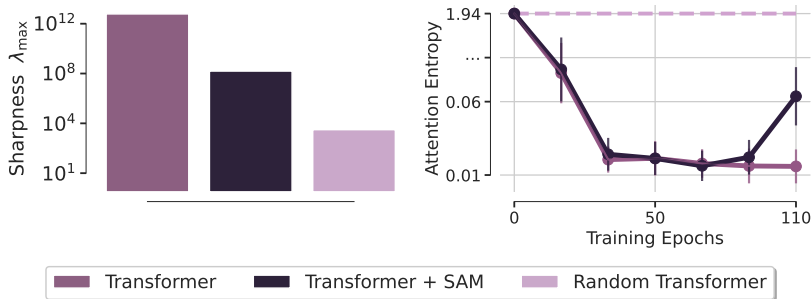


Hypothesis from NLP and Computer Vision

- Transformers have **sharp loss landscape** [2],
- Attention suffers from **entropy collapse** [12].
 - Entropy = average entropy of the rows,
 - It causes training instability.

Hypothesis from NLP and Computer Vision

- Transformers have **sharp loss landscape** [2],
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Training the attention induces an entropy collapse and a sharp loss landscape.



① σ Reparam [12]

Replace each weight matrix \mathbf{W} by

$$\widehat{\mathbf{W}} = \frac{\gamma}{\|\mathbf{W}\|_2} \mathbf{W}, \text{ with } \gamma \in \mathbb{R} \text{ learnable,}$$

② Sharpness-Aware Minimization (SAM) [3]

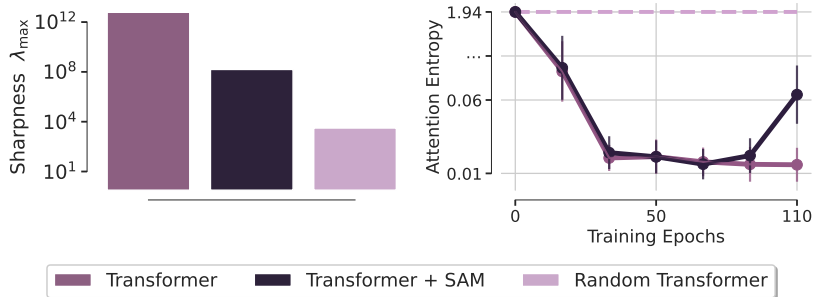
Replace the training loss $\mathcal{L}_{\text{train}}$ by

$$\mathcal{L}_{\text{train}}^{\text{SAM}}(\omega) = \max_{\|\epsilon\| < \rho} \mathcal{L}_{\text{train}}(\omega + \epsilon).$$

σ Reparam doesn't solve the problem, but SAM does.



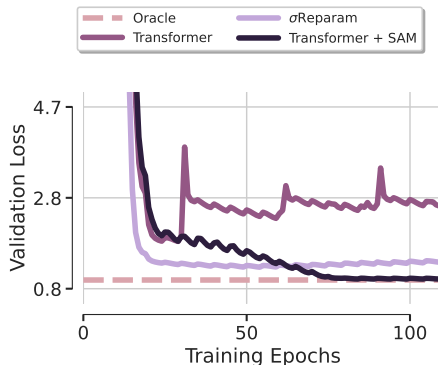
Contrary to NLP and Computer Vision, entropy collapse seems benign in time series forecasting while sharpness is harmful.



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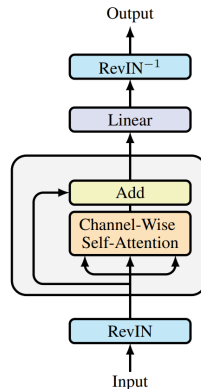


σ Reparam helps but is not sufficient while using SAM leads to the optimal solution (oracle).



- Input $\mathbf{X} \in \mathbb{R}^{D \times L}$, output $f(\mathbf{X}) \in \mathbb{R}^{D \times H}$,
- Reduce distribution shift with **RevIN** [4],
- **Channel-wise attention** $\mathbf{A}(\mathbf{X}) \in \mathbb{R}^{D \times D}$,
- **Smooth** loss landscape with SAM [3].

$$\mathbf{A}(\mathbf{X}) = \text{softmax}\left(\frac{\mathbf{X}\mathbf{W}_Q\mathbf{W}_K^\top\mathbf{X}^\top}{\sqrt{d_m}}\right)$$
$$f(\mathbf{X}) = [\mathbf{X} + \mathbf{A}(\mathbf{X})\mathbf{X}\mathbf{W}_V\mathbf{W}_O]\mathbf{W}$$



SAMformer is a shallow transformer trained with SAM.

→ One head, one encoder, 15 lines of code!



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All-MLP model (2023): TSMixer [1].

Transformers (2021-2022): FEDformer [14], Autoformer [10].

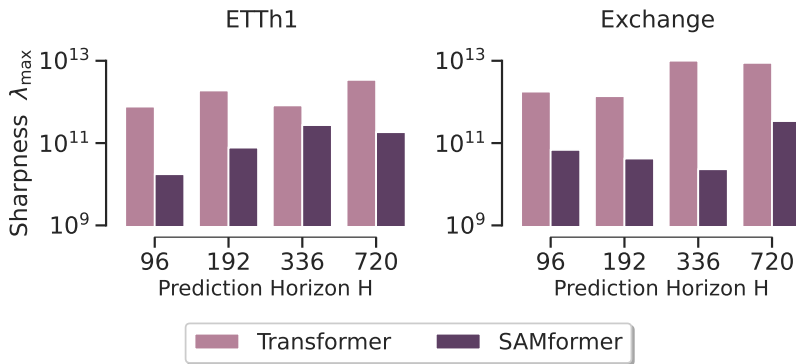
Recent Transformers (2023-2024): iTransformer [7], PatchTST [8].

Dataset	ETTh1/ETTh2	ETTM1/ETTM2	Electricity	Exchange	Traffic	Weather
# features	7	7	321	8	862	21
# time steps	17420	69680	26304	7588	17544	52696
Granularity	1 hour	15 minutes	1 hour	1 day	1 hour	10 minutes

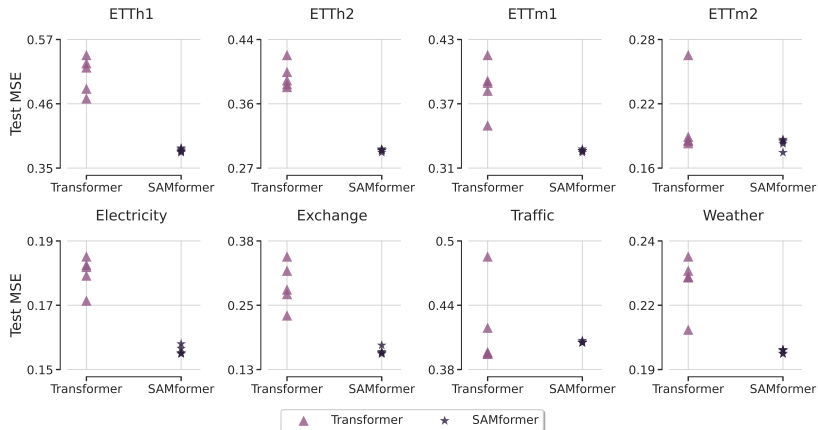


Dataset	SAMformer	iTransformer	PatchTST	TSMixer	FEDformer	Autoformer
	-	2024	2023	2023	2022	2021
ETTh1	0.410	0.454	0.469	0.437	0.440	0.496
ETTh2	0.344	0.383	0.387	0.357	0.437	0.450
ETTm1	0.373	0.407	0.387	0.385	0.448	0.588
ETTm2	0.269	0.288	0.281	0.289	0.305	0.327
Traffic	0.425	0.428	0.481	0.620	0.610	0.628
Weather	0.260	0.258	0.259	0.267	0.309	0.338
Overall improvement		6.58%	8.79%	13.2%	22.5%	35.9%

SAMformer outperforms all baselines while having significantly fewer parameters.



SAM provides a smoother loss landscape ...



... leading to better generalization and robustness.

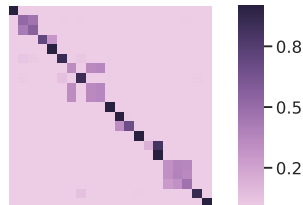
Transformer



σ Reparam



SAMformer

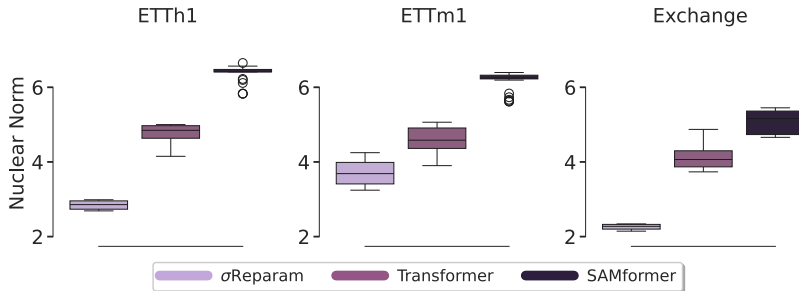


Channel-wise attention improves the propagation of the signal with self-feature correlations as in ViTs.

Theorem (Ilbert, O., Feofanov et al.)

Applying σ Reparam [12] leads to **attention rank collapse**.

$$\|\mathbf{X}\mathbf{W}_Q\mathbf{W}_K^\top\mathbf{X}^\top\|_* \leq \underbrace{\|\mathbf{W}_Q\mathbf{W}_K^\top\|_2}_{\text{goes to 0 with } \sigma\text{Reparam}} \|\mathbf{X}\|_F^2.$$





- MOIRAI [9]: foundation model trained on **27B samples**,
- Nb. params: small (**14M**), base (**91M**) and large (**314M**).

Dataset	Full-shot	Zero-shot		
	SAMformer	MOIRAI _{Small}	MOIRAI _{Base}	MOIRAI _{Large}
ETTh1	<u>0.410</u>	0.400	0.434	0.510
ETTh2	<u>0.344</u>	0.341	0.345	0.354
ETTm1	0.373	0.448	<u>0.381</u>	0.390
ETTm2	0.269	0.300	<u>0.272</u>	0.276
Electricity	0.181	0.233	<u>0.188</u>	<u>0.188</u>
Weather	0.260	<u>0.242</u>	0.238	0.259
Overall MSE improvement		6.9%	1.1%	7.6%

SAMformer outperforms MOIRAI while having significantly fewer parameters!



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Findings

- Transformer failure → trainability issues of the attention,
- In time series forecasting, entropy collapse is benign,
- But sharpness prevents good generalization.

Proposal

- **SAMformer**: RevIN + channel-wise attention + SAM,
- SOTA and lightest model,
- Strong competitor to MOIRAI [9].



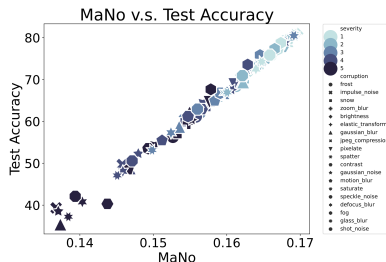
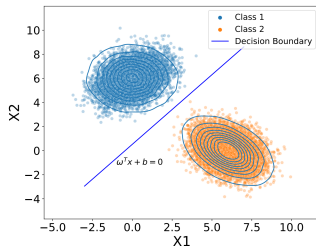
This work has been accepted as an **Oral at ICML 2024, Vienna**. You may find the links to the paper and the code below. To know more about my research, check my website: ambroiseodt.github.io and feel free to contact me.

- ★ Paper: <https://arxiv.org/pdf/2402.10198>
- ★ Code: <https://github.com/romilbert/samformer>



This project was led by [Romain Ilbert](#) and myself with our co-authors [Vasilii Feofanov](#), [Aladin Virmaux](#), [Giuseppe Paolo](#), [Themis Palpanas](#), and [Ievgen Redko](#).

MANO: Exploiting Matrix Norm for Unsupervised Accuracy Estimation Under Distribution Shifts



<https://arxiv.org/pdf/2405.18979>

Thanks for your attention !



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