

# Parallelization of the FICO Xpress-Optimizer

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- Objective
  - Linear objective
  - Convex quadratic objective
- Constraint types
  - Linear constraints
  - Convex quadratic constraints
  - Second order cones
  - Indicator constraints
  - Special ordered sets (SOS1 and SOS2)
- Column types
  - Continuous
  - Integer
  - Semi-continuous
  - Semi-integer
  - Partial-integer

Barrier Optimizer

Simplex Optimizer

MIP Optimizer

# Feature of Xpress solvers

## Predictability and reproducibility

- All solver components deterministic by default.
- **Platform independence:**
  - win64, linux64 and mac64 will produce identical solves
- Minimal dependence on threads and architecture.

## Dual Simplex:

- Dependence only on single versus multi-threaded.
- Force multi-threading [`FORCEPARALLELDUAL=1`] to remove all threading dependencies.

## Barrier:

- Independent of cache size
- Mostly independent of vectorization support [`CPUPLATFORM`]
  - None, SSE2, AVX identical. Only AVX2 differs.
- Dependent on `BARTHREADS` and `BARCORES`.

# Parallel technology: Shared memory

## My laptop

- One 4-core socket
- 16 GB of RAM

## My server

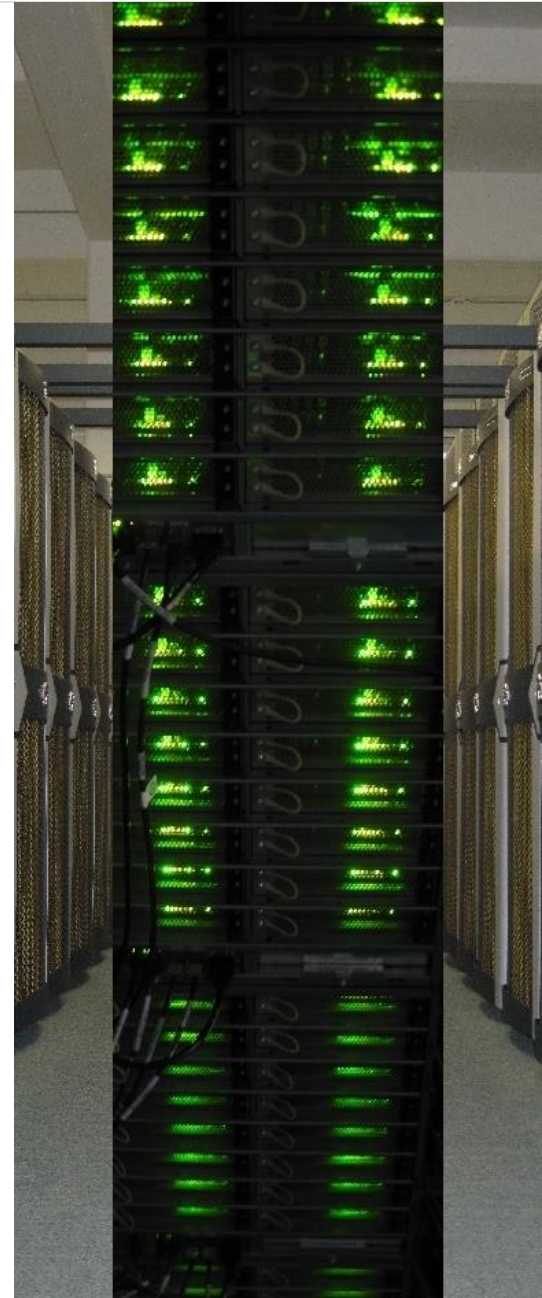
- Two 10-core sockets
- 256 GB of RAM

## My smartphone

- One 4-core socket
- 2 GB of RAM

## (My) HRLN III compute node

- Two 12-core sockets
- 64 GB of RAM





# Parallel technology: Distributed memory

## Supercomputers (Top 500)

#	Site	Manufact.	Processor	Cores	Total cores
1	Wuxi, China	Sunway	ShenWei 26010	256	10,649,600
2	Guangzhou, China	NUDT	Intel Xeon E5-2692v2	12	3,120,000
3	Livermore, USA	Cray	Power BQC	16	1,572,864
96	Berlin, Germany	Cray	Intel Xeon E5-2695v2	12	44,928
500	Nizhni Novgorod, Russia	Niagara	Intel Xeon E5-2660v2	10	10,800

June 2016



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Following presentation by Yuji Shinano

June 2016



Shared memory parallelization  
is needed in any case!

# Parallelism in Xpress 7.9

## Parallelization of branch-and-bound algorithm

### Parallel threads

Individual node-divides from active nodes of the branch-and-bound tree.

### Work pool

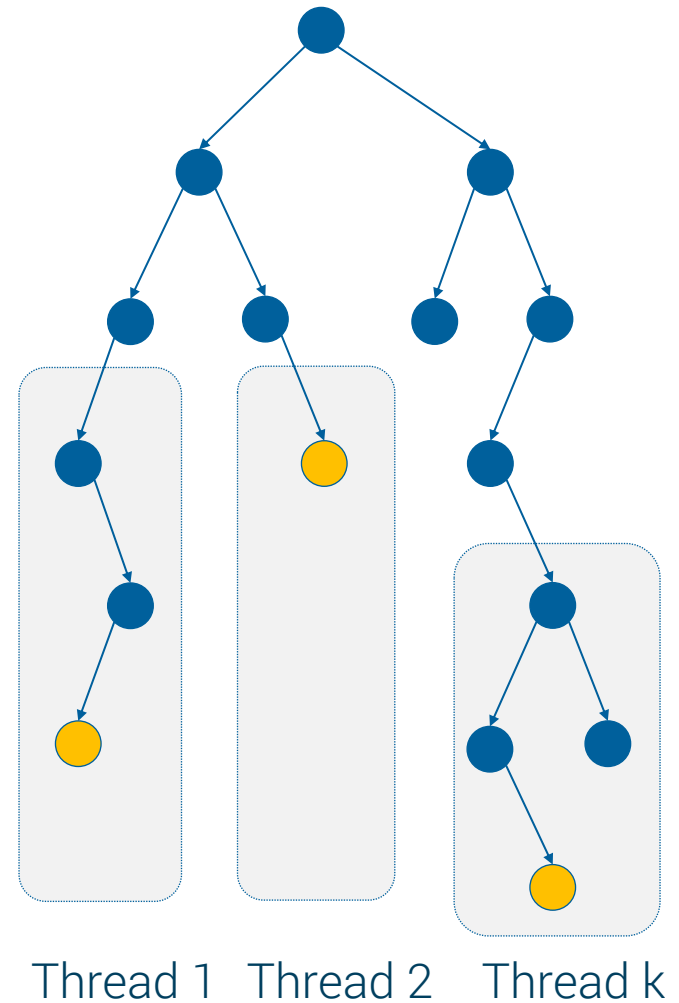
Central pool of all active nodes.

### Synchronization

Exchange collected search data (nodes, solutions, statistics, ...).

### Deterministic Timer

Effort estimates that roughly track elapsed time.





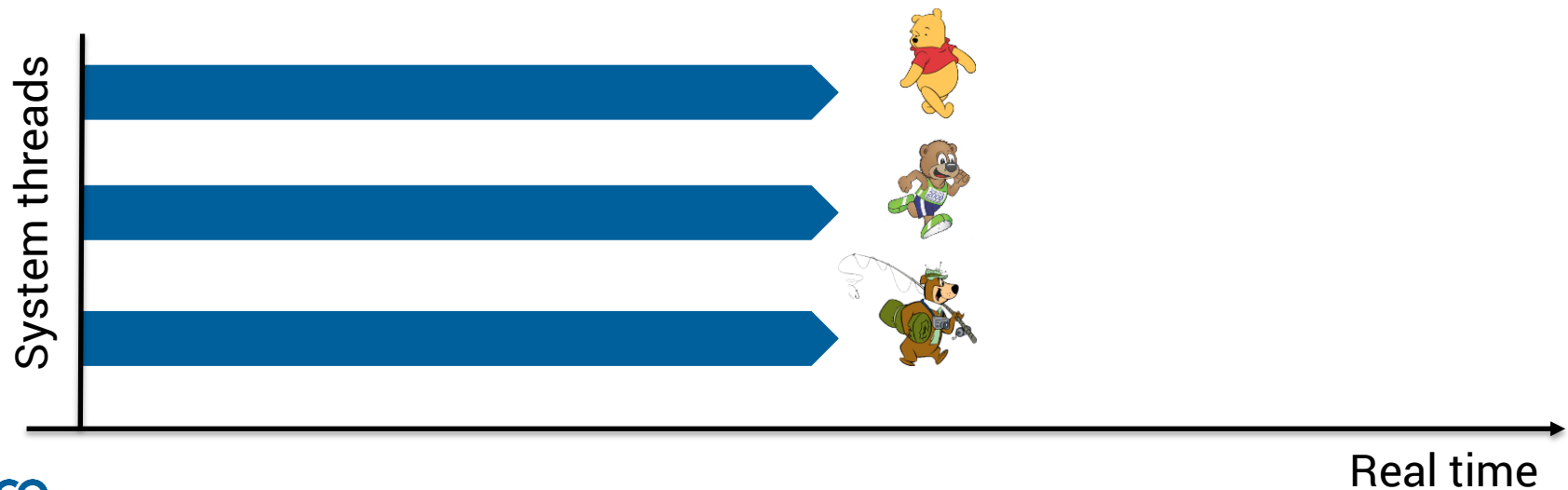
## Parallelization bottlenecks

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- Access to shared data
- Up-to-dateness of shared data
- Accuracy of effort counts

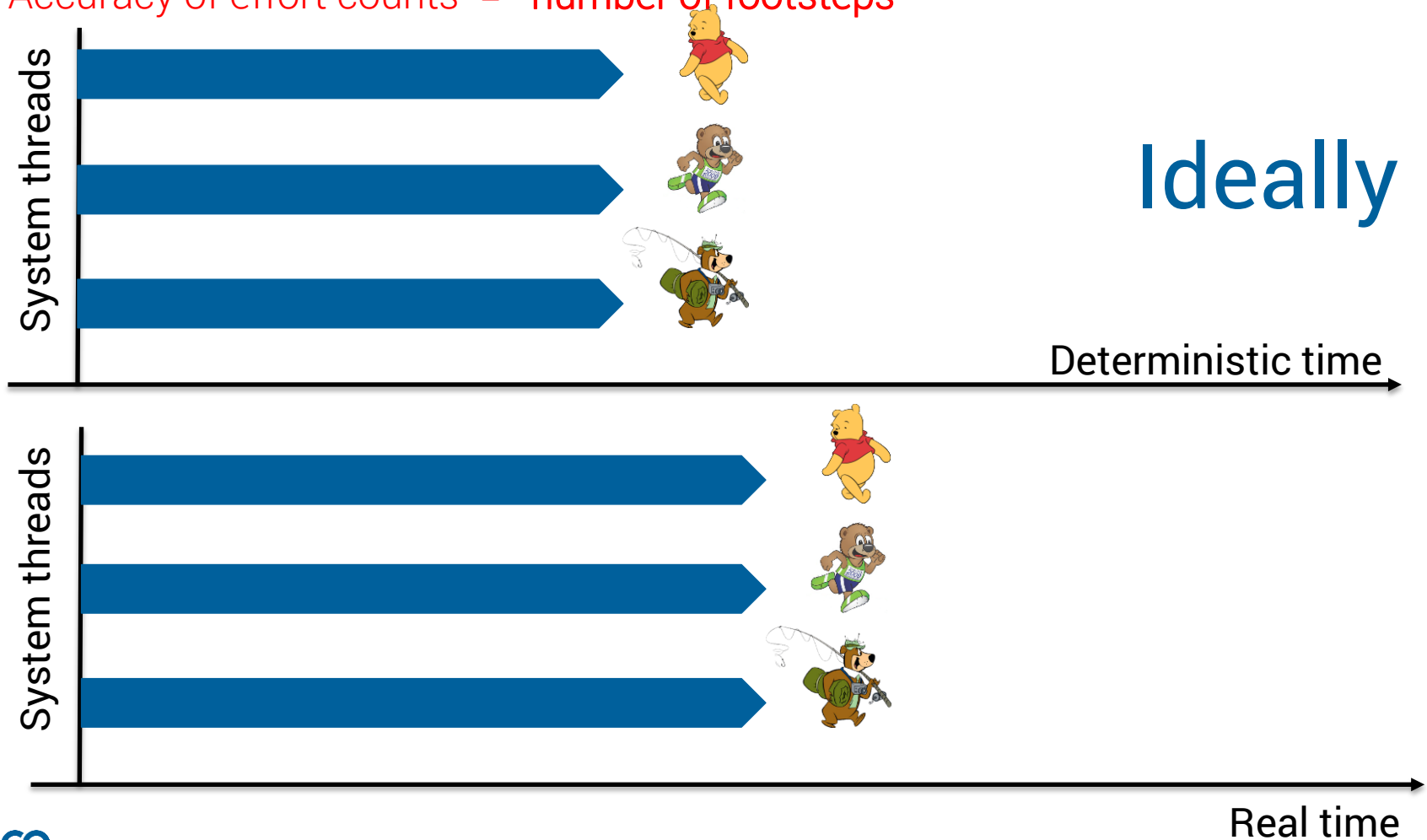
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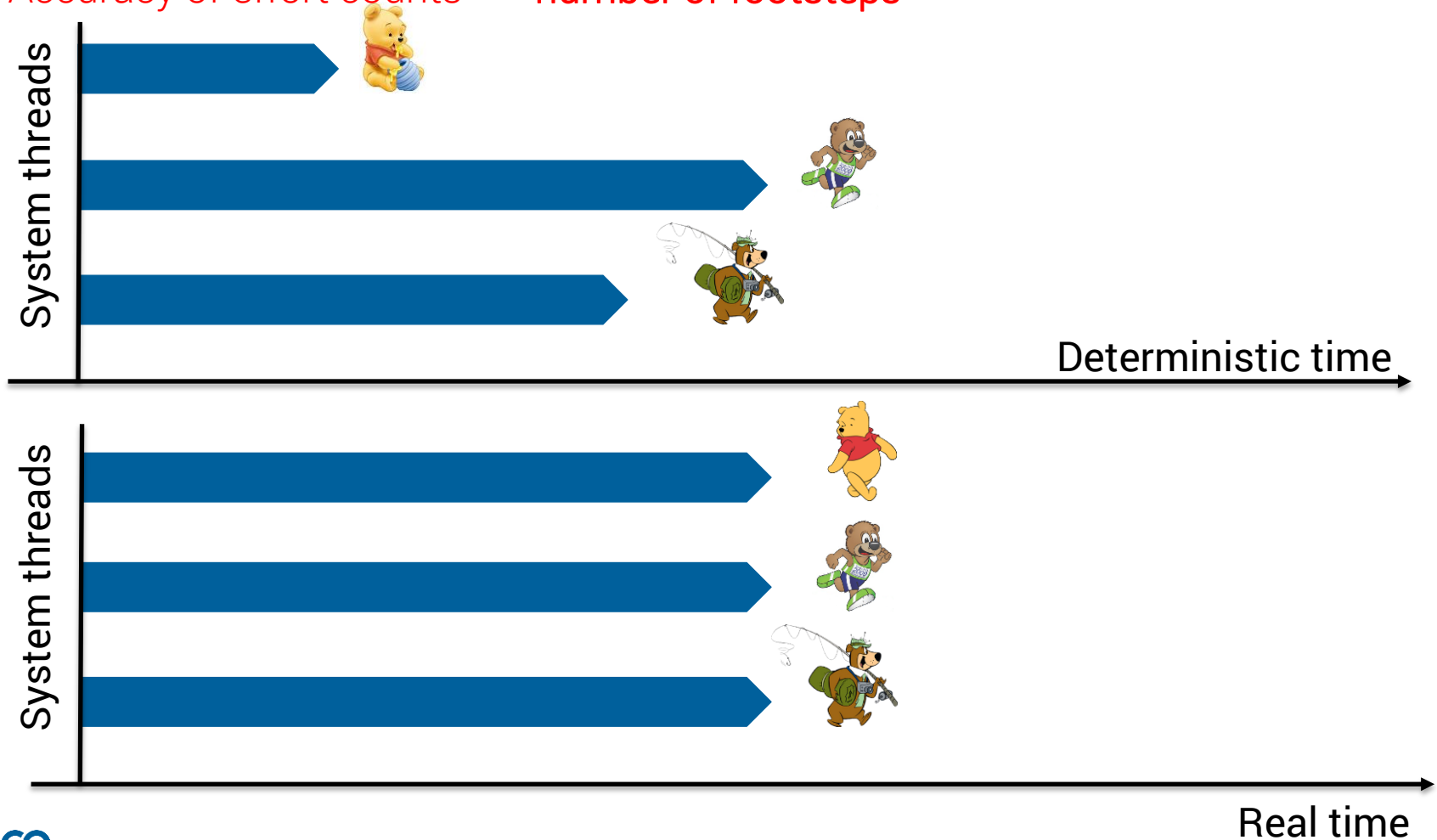
# Parallelization bottlenecks

- Access to shared data
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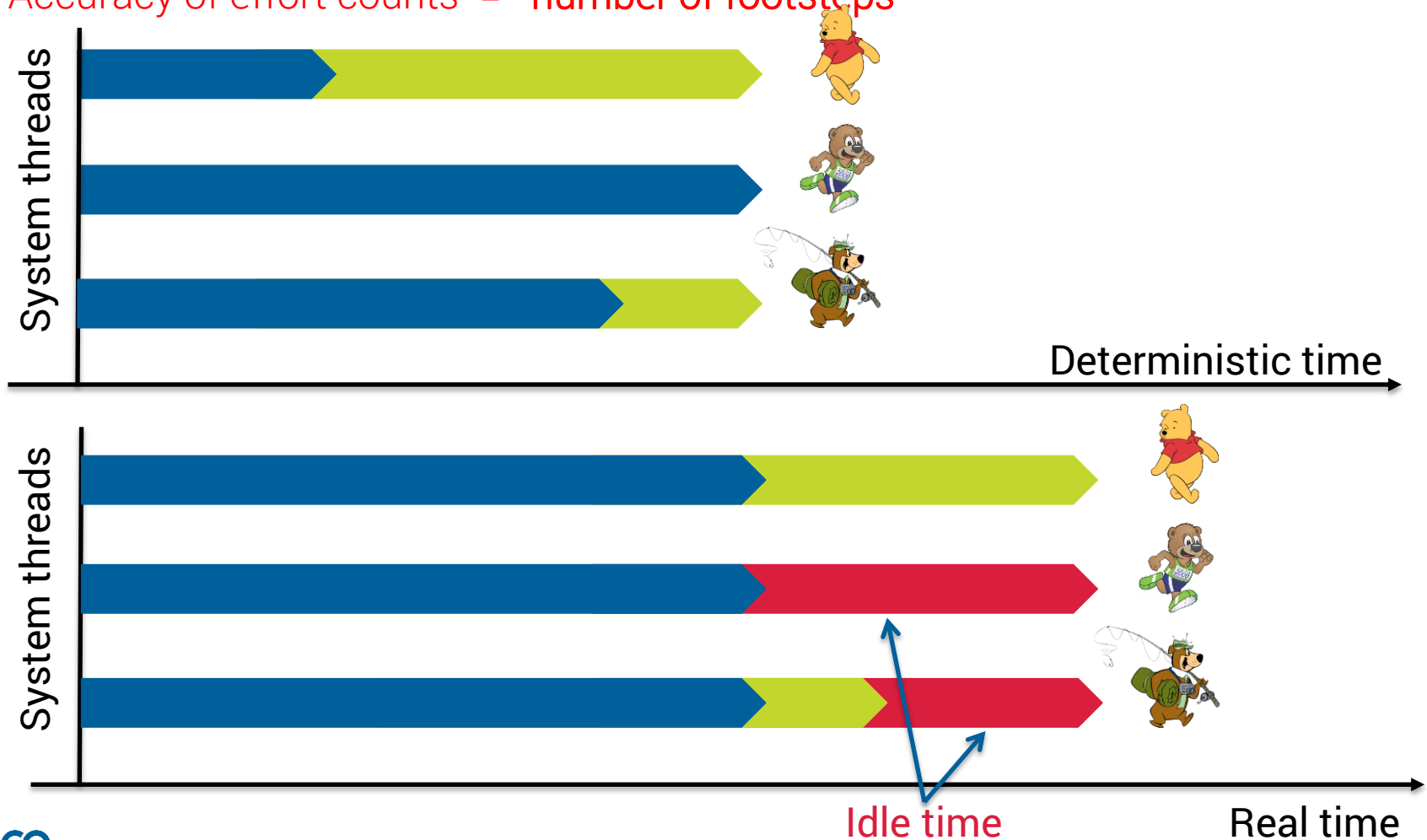
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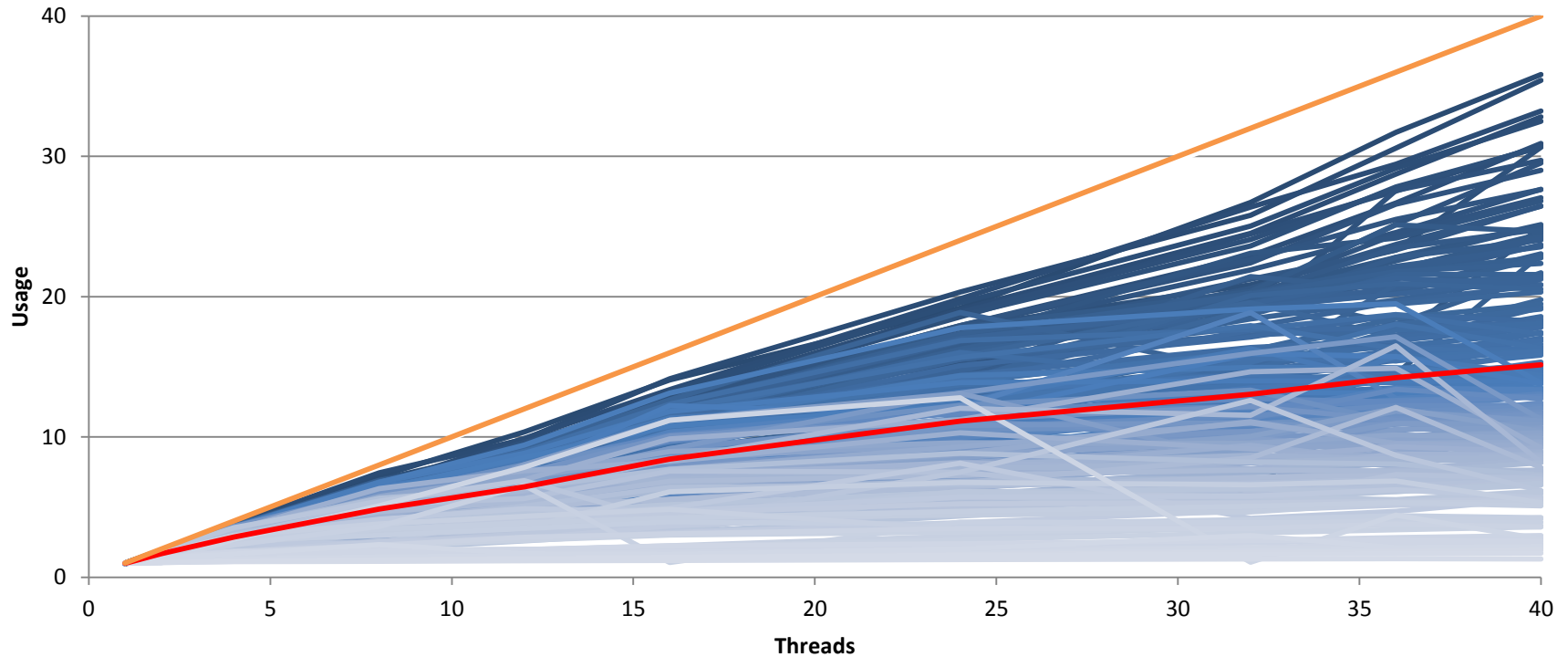




## Parallelism in Xpress 7.9

Workload MIPLIB 2010 Easy/Solvable (205 Instances)

Usage = Total Thread Time / Elapsed Time

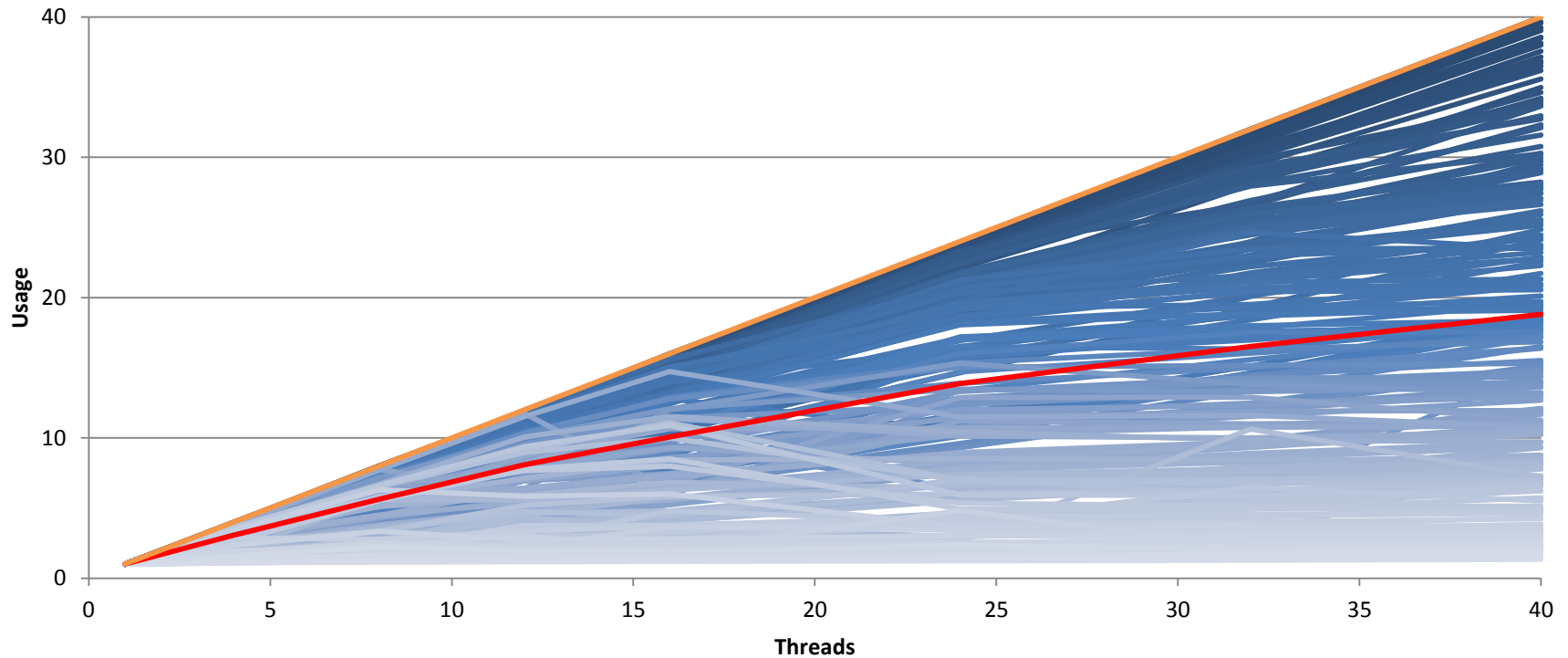


Threads	1	2	4	8	12	16	24	32	36	40
Usage	0.99	1.67	2.87	4.85	6.45	8.42	11.12	13.04	14.23	15.13

# Parallelism in Xpress 8.0

Workload MIPLIB 2010 Easy/Solvable (205 Instances)

Usage = Total Thread Time / Elapsed Time

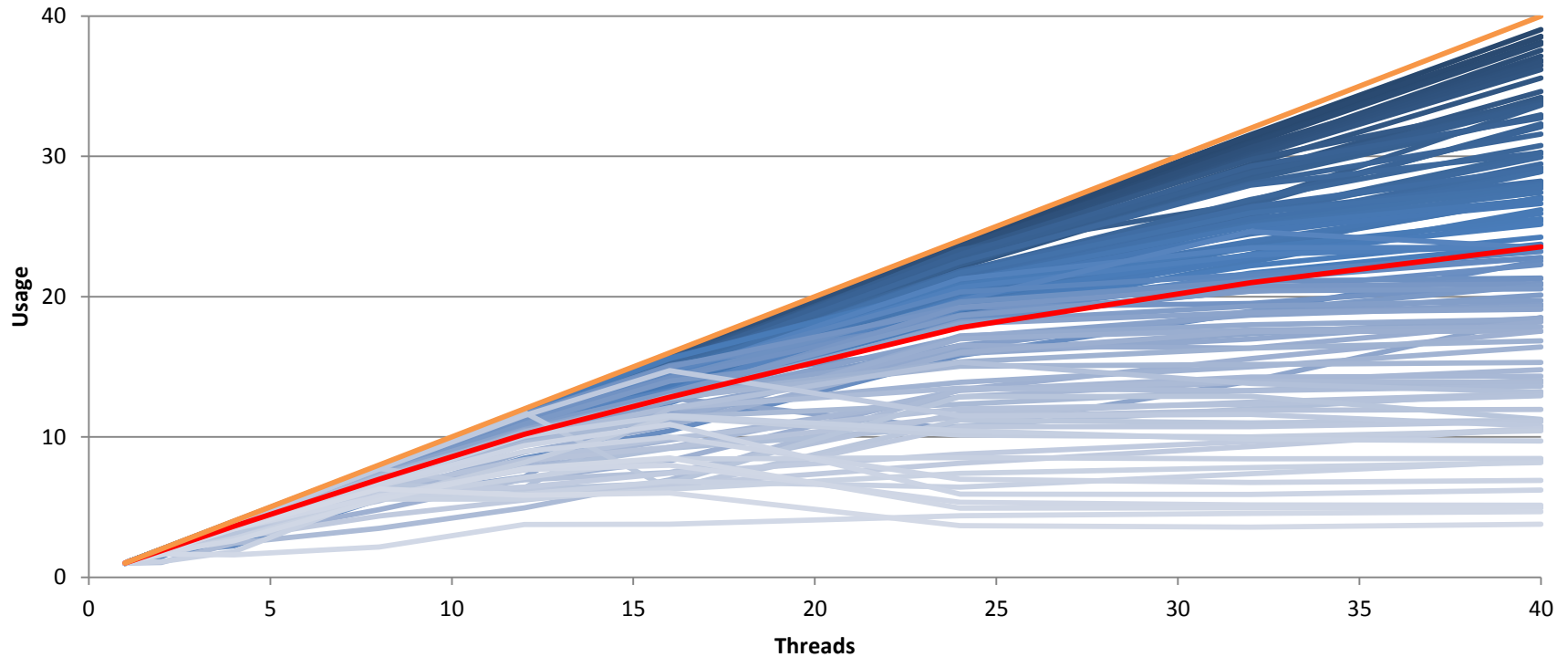


Threads	1	2	4	8	12	16	24	32	36	40
7.9	0.99	1.67	2.87	4.85	6.45	8.42	11.12	13.04	14.23	15.13
8.0	0.99	1.70	3.07	5.52	8.02	10.07	13.86	16.51	--	18.81

## Parallelism in Xpress 8.0

Workload MIPLIB 2010 Easy/Solvable **at least 10000 nodes** (117 instances)

Usage = Total Thread Time / Elapsed Time



Threads	1	2	4	8	12	16	24	32	36	40
7.9	0.99	1.76	3.15	5.50	7.24	9.50	12.54	14.50	15.65	16.51
8.0	0.99	1.82	3.63	6.99	10.10	12.82	17.78	20.98	--	23.55

## Parallelism in Xpress 8.0

Main goal: Decrease dependence on effort counters

1. Always have work available, produce more tasks than (system) threads
2. To decrease waiting time, all tasks access only partial information

Uses a general purpose **Job Scheduler**:

- Specifically designed for deterministic, parallel execution of jobs.
- Intended for any multi-threaded application with dynamic jobs and data exchanges.
- Automatic scheduling and execution of jobs.
- Handles communication and ensures data delivery is deterministic.
- Ensures events and read barriers are deterministic.
- Frees application developer from worrying about determinism.
- Scalable.

Parallel MIP built on top of the Job Scheduler

- Jobs = parallel heuristics or node solves or ..

## Parallelism in Xpress 8.0 (cntd)

Main goal: Decrease dependence on effort counters

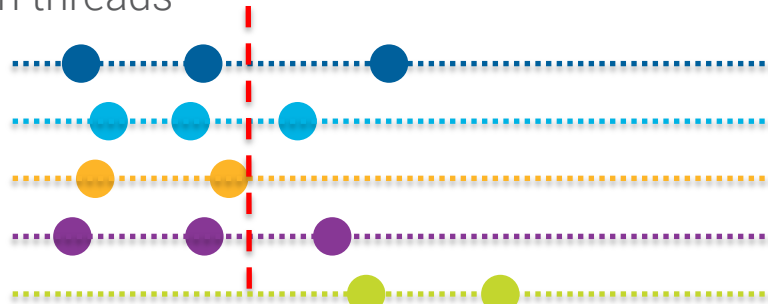
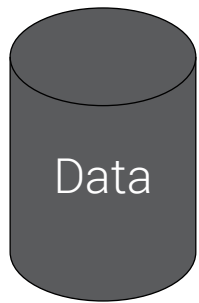
1. Always have work available, produce more tasks than (system) threads
  2. To decrease waiting time, all tasks access only partial information
- **Job Scheduler** can be told to run in opportunistic mode.
    - Relaxes deterministic ordering of events.
    - No special handling required in the application.
  - Sequential run = single-threaded opportunistic.
  - Rough proportionality of effort counter with elapsed time sufficient.  
±50% easily good enough.

**Scheduler ensures scalability and proper CPU utilization.  
Application only needs to provide sufficiently many jobs.**

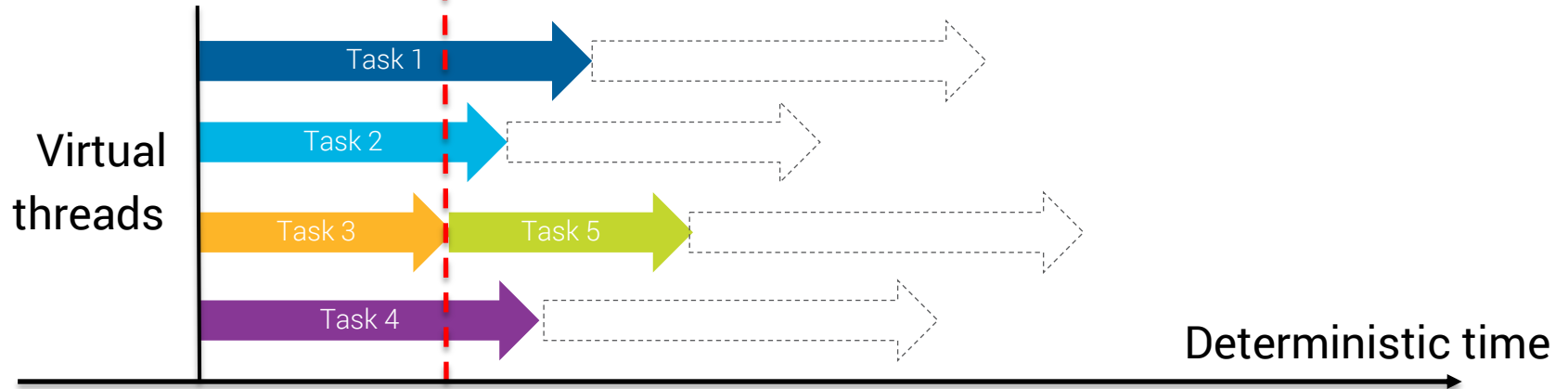


# Parallelism visualization

More tasks than threads

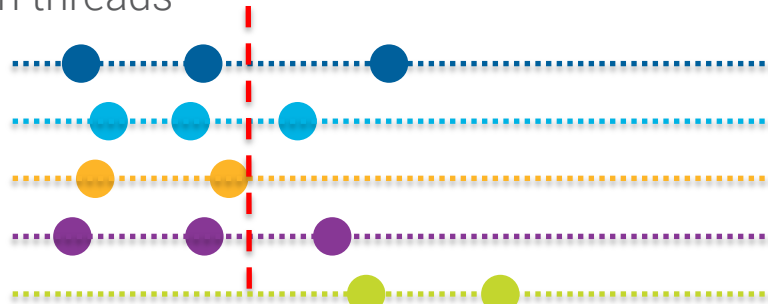
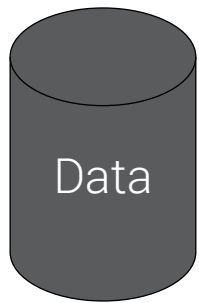


Data updates

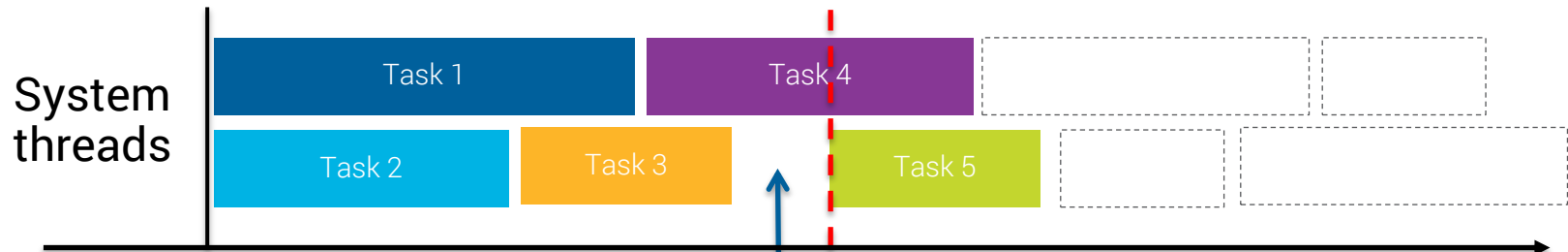
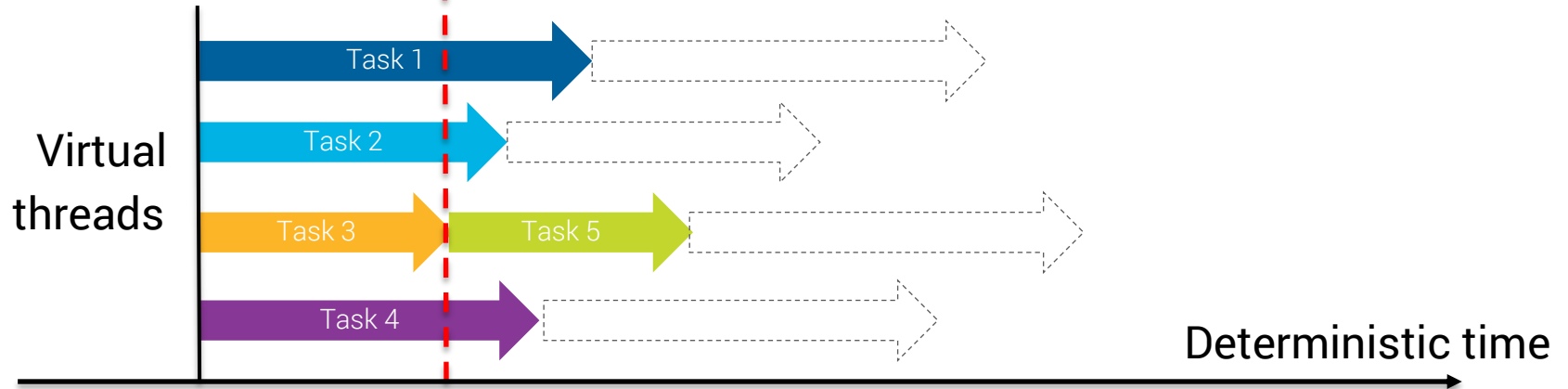


# Parallelism visualization

More tasks than threads



Data updates

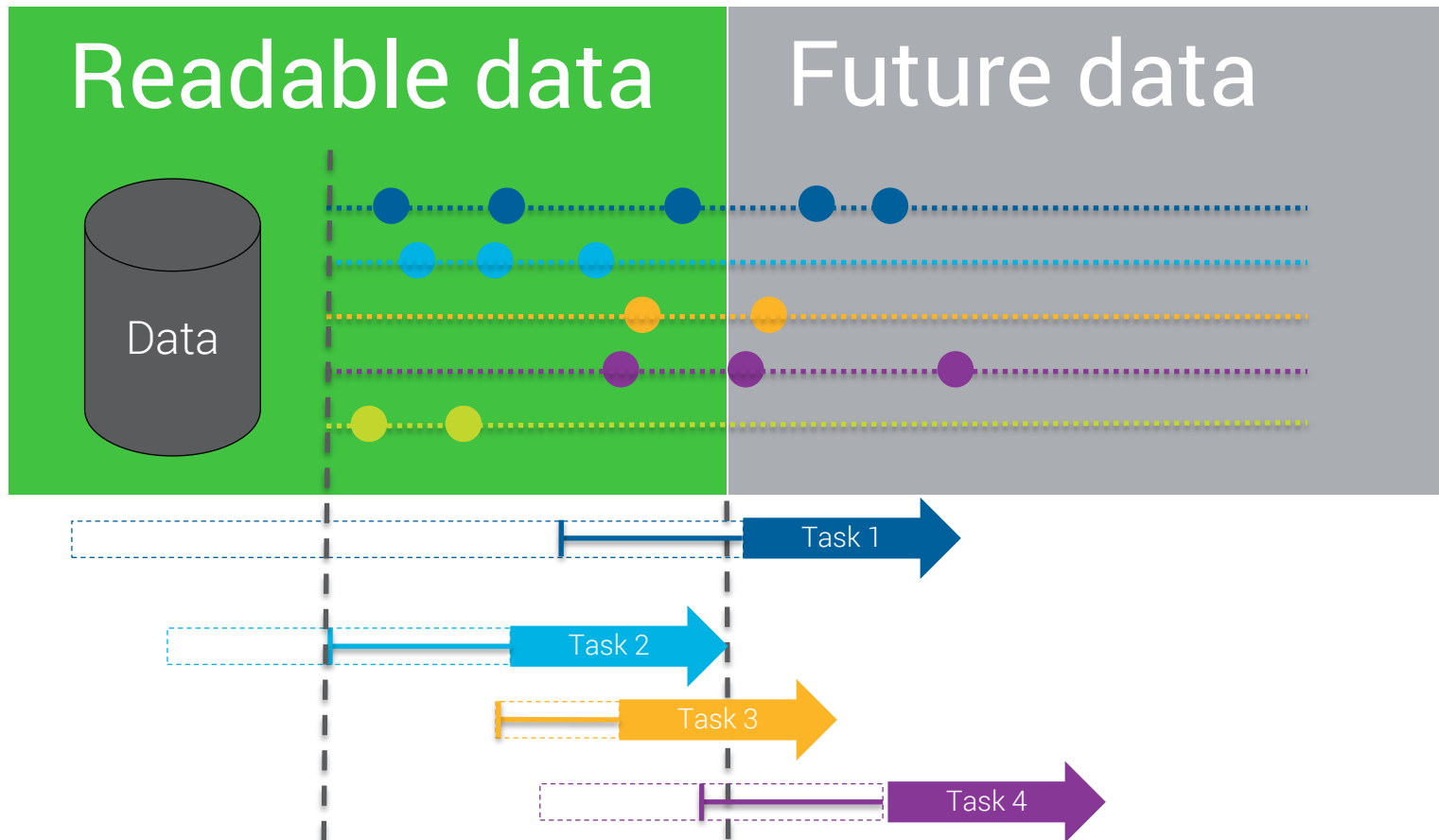
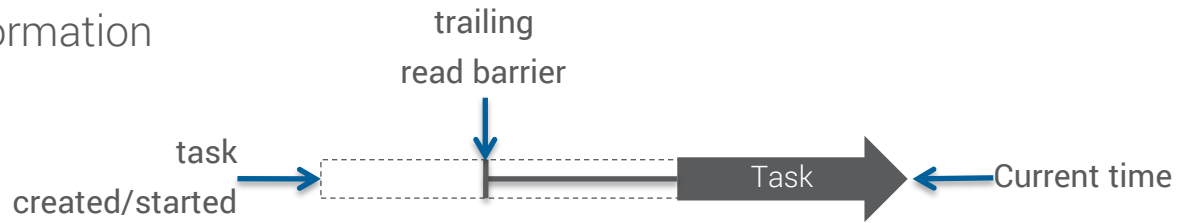


Idle time

Elapsed time

# Parallelism visualization

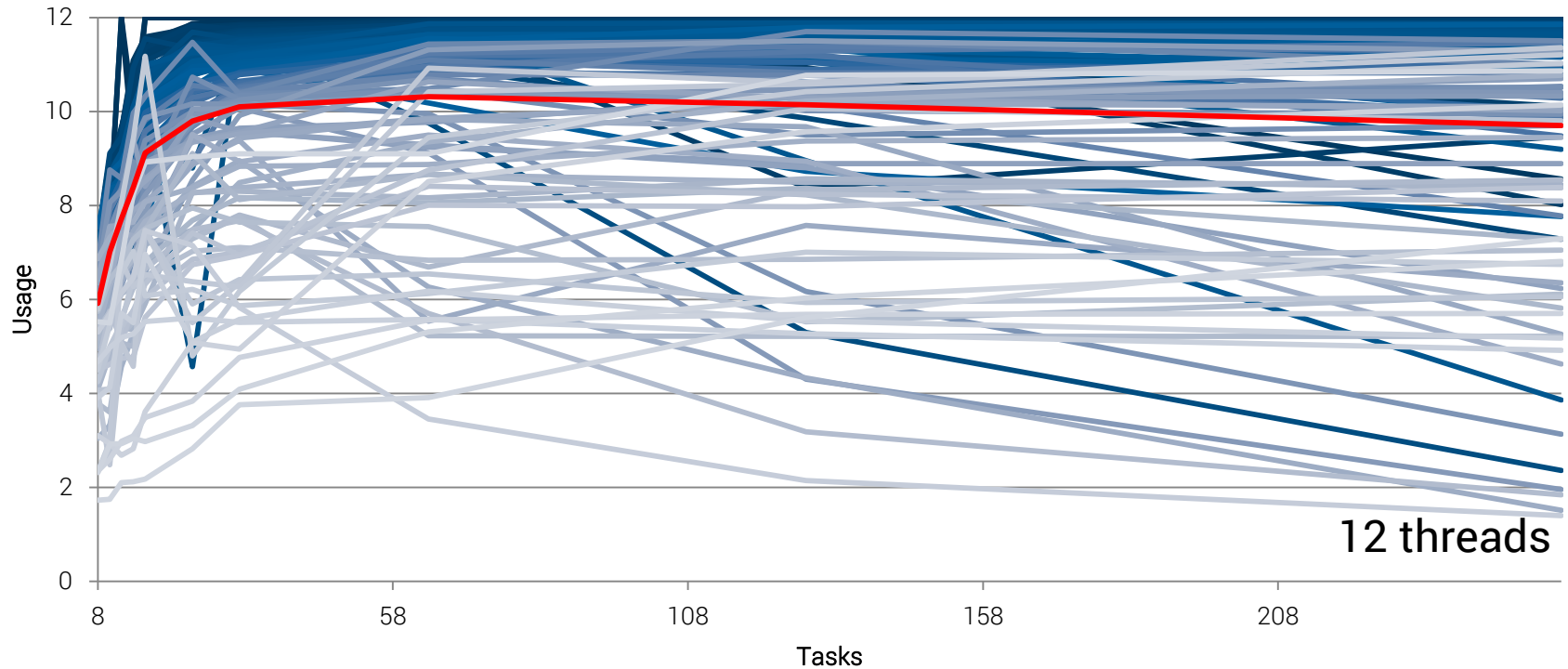
Partial information



## Parallelism in Xpress 8.0

Workload MIPLIB 2010 Easy/Solvable **at least 10000 nodes** (123 instances)

Usage = Total Thread Time / Elapsed Time

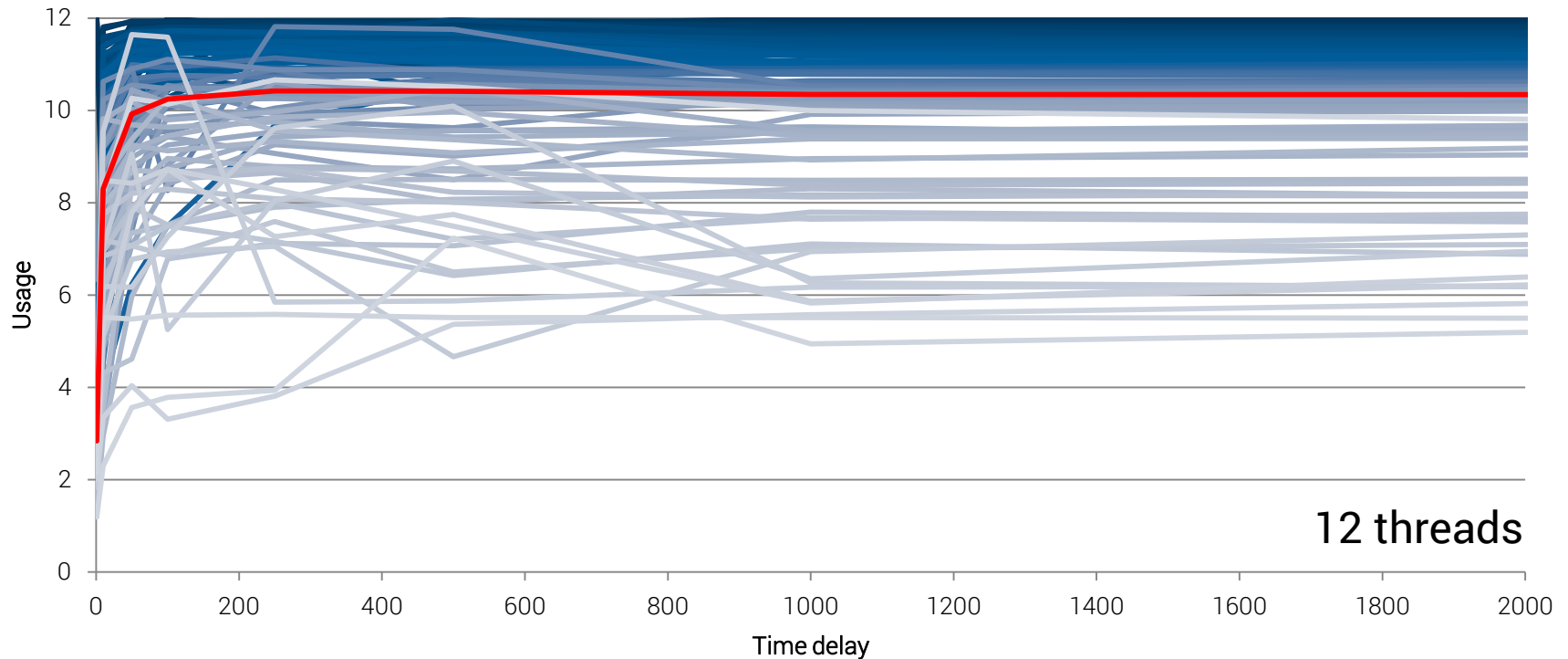


Tasks	8	10	12	14	16	24	32	64	128	256
Usage	5.92	7.00	7.71	8.38	9.12	9.80	10.10	10.31	10.14	9.70
Speed	1.00	1.16	1.27	1.16	1.33	1.41	1.39	1.41	1.28	1.09

## Parallelism in Xpress 8.0

Workload MIPLIB 2010 Easy/Solvable **at least 10000 nodes** (117 instances)

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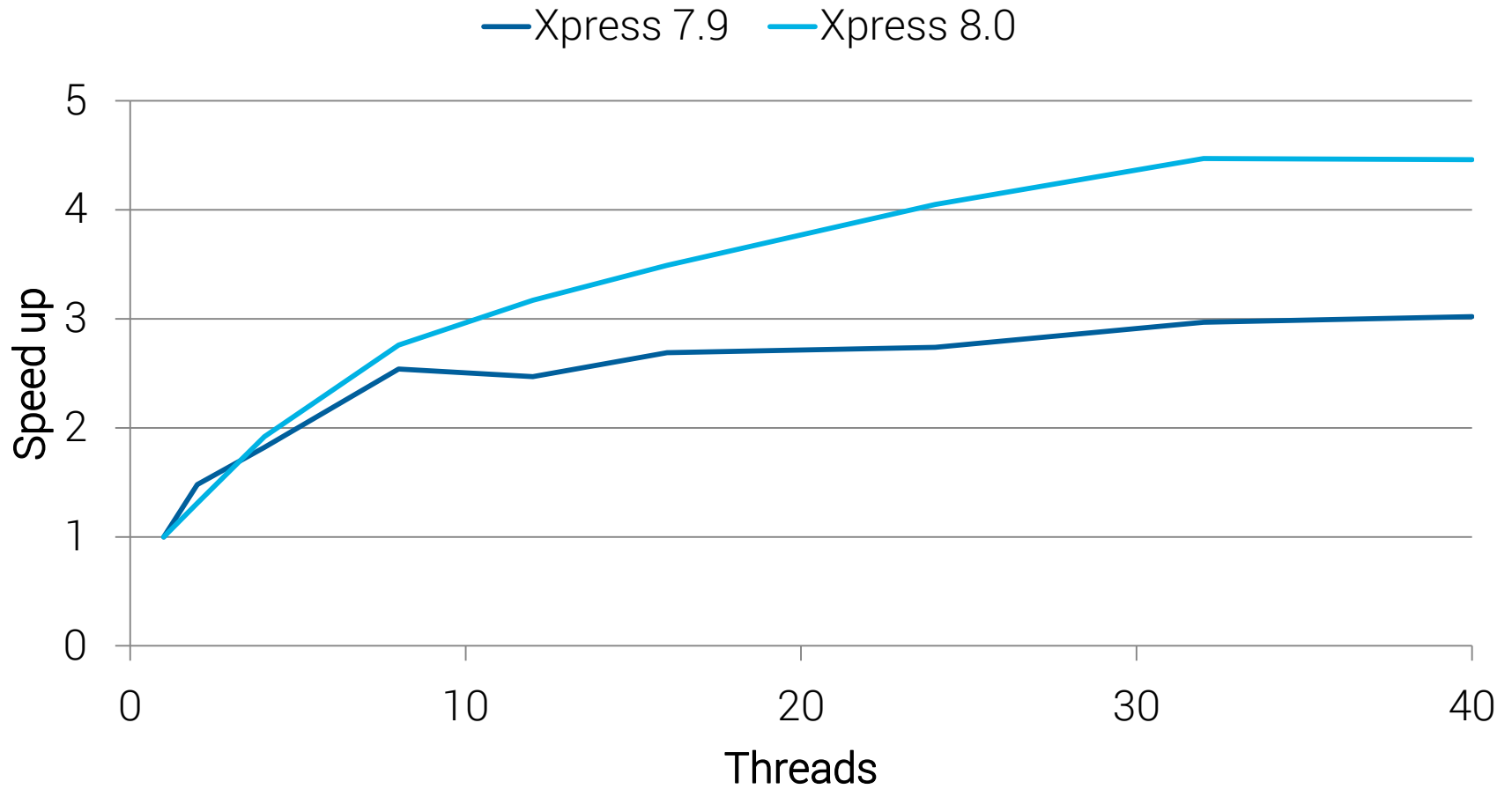


Delay	1	10	50	100	250	500	1000	5000	10000	Inf
Usage	2.84	8.30	9.92	10.24	10.42	10.41	10.34	10.32	10.34	10.36
Speed	1.00	1.60	1.89	2.33	2.04	2.11	2.31	2.36	2.30	2.29



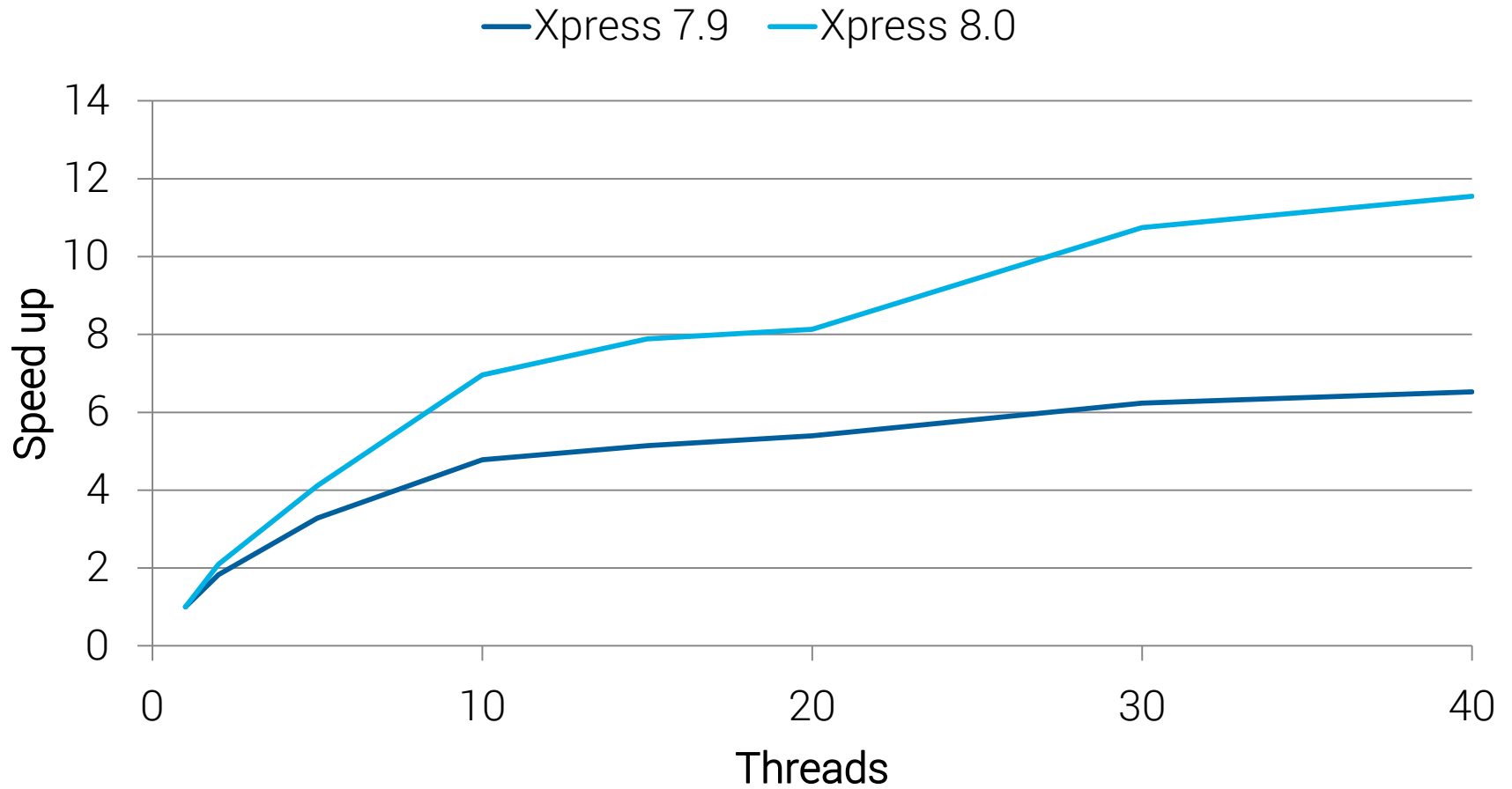
## Parallel speed up Xpress 7.9 versus 8.0

MIPLIB 2010 Easy/Solvable **at least 10000 nodes**



## Parallel speed up Xpress 7.9 versus 8.0

Internal parallel test set (152 instances)



## Conclusion

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### New Parallel Job Scheduler

- Weak dependence on effort counters
- Use a trailing read barrier for each task → partial information
- Create more tasks than threads

### Solution path is

- Platform independent
- **System threads independent** (depends on the number of tasks)

### Reproducibility and Reliability

**Any multi-threaded run can be reproduced as a single-threaded run!**

Thank You!

**Timo Berthold**  
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