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pyADCG

A preliminary implementation of a new parallel solver for nonconvex MINLPs in Pyomo/Python

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Introduction

Branch & Cut, Column Generation and ADCG

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ADCG (Alternating Direction Column Generation Method):

- exact parallel MINLP decomposition method
- combines AD and CG without using branching

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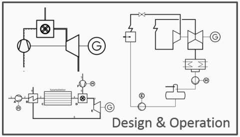
Planning (energy systems)



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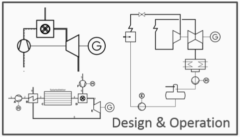

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Transport optimization (crew scheduling)

sub-problems: constrained shortest path

The quasi-separable MINLP

Nonconvex quasi-separable MINLP:

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where G_k defines the feasible set of a (MINLP) sub-problem:

$$G_k := \{y \in [\underline{x}_{I_k}, \overline{x}_{I_k}] : y_i \in \{0,1\},\ i \in I_k^{\text{int}},\ g_j(y) \leq 0,\ j \in J_k\}$$

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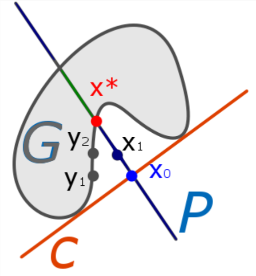
(MINLPs can be transformed into quasi-separable MINLPs by adding new variables and constraints)

Assumption: MINLP sub-problems can be solved quickly

The Alternating Direction Method

Basic steps of an ADM for solving $\min\{c^T x : x \in P \cap \bigtimes_{k\in K} G_k\}$:

1. $y^{i+1} \leftarrow$ G-project $x^i \in P$ regarding $c + A^T\lambda^i$
2. $x^{i+1} \leftarrow$ P-project $y^{i+1} \in G$ regarding $c - A^T\lambda^i$
3. $\lambda^{i+1} \leftarrow$ update λ^i



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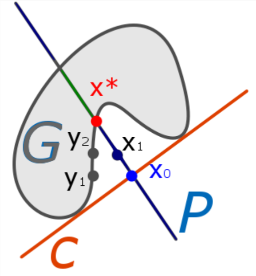

where

- G-project by (parallel) solving MINLP sub-problems
- P-project by solving a (large) QP master-problem
- the starting point $x^0 \in P$ is the solution of a convex relaxation

Traditional ADMs do not converge always towards the solution point x^*.

The Alternating Direction Column Generation Method

(Global optimization without branching)

Penalty function, full-projection problem, global convergence

Motivation: Houska, Frasch and Diehl, *An Augmented Lagrangian based Algorithm for Distributed Non-Convex Optimization* (2014), **ALADIN**:

- Check quality of x^i using the penalty function
  $PenF(x^i) := c^T x^i + \gamma \cdot Viol(x^i, P \cap G)$
  ($\rightarrow$ min $PenF(x) \Leftrightarrow$ MINLP)

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- If $PenF(x^i) - PenF(x^{i+1})$ is not large enough:
  solve approximately (via a dual line search)
  a full-projection problem ($x^i \in P \rightarrow P \cap G$)

$$
\begin{aligned}
x^{i+1} = \text{ approx argmin} \quad & c^T x + \rho \sum_{k \in K} \|x_{I_k} - x^i_{I_k}\|^2 \\
s.t. \quad & x \in G \cap P
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Theorem: If ρ is large, then the full-projection problem has a zero duality gap $\rightarrow$ global convergence of ALADIN

ADCG: target constraint and CG-based full-projection

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  CG generates sample points $S_k \subseteq G_k$ (inner approximation)
  $\to$ efficient warm-start possible
- ADCG performs CG-steps, until
  (i) $PenF$ is reduced sufficiently or
  (ii) $s + redCost(\lambda) > 0 \qquad \Rightarrow P' \cap G = \emptyset \Leftrightarrow tarVal < v^*$

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Theorem: x^i converges towards a global solution of the MINLP (without branching)

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For details of ADCG see:

1. Nowak. *Column Generation based Alternating Direction Methods for solving MINLPs.* 2015 (Optimization Online)

2. Nowak, Breitfeld, *The Alternating Direction Column Generation Method (Global Optimization without Branching)* (in preparation)

Preliminary results with pyADCG

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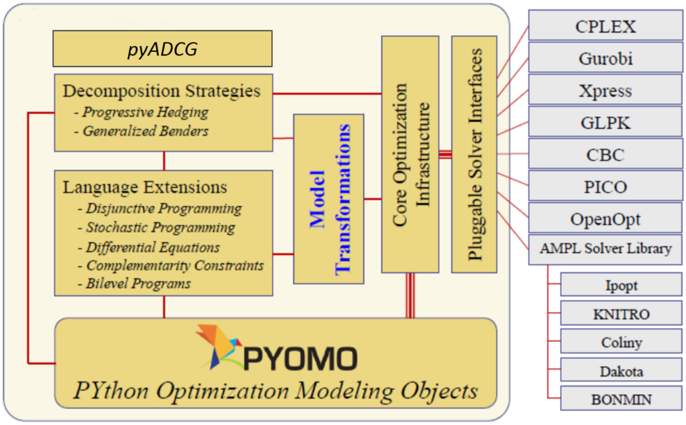
pyADCG is a meta-solver consisting of the sub-solvers:

- IpOpt for master-problems and local optimization
- SCIP for MINLP sub-problems

What is Pyomo?



Pyomo provides a modeling language and supports the development of meta-solvers based on sub-solvers via Python ($\rightarrow$ fast development)

Current status

Preliminary numerical results with random QQPs
(without parallelization):

- very few ADCG iterations (maybe because duality gap is small)
- time for solving master-problems 1%
- time for solving sub-problems 94%

$\Rightarrow$ significant potential for parallel solving the sub problems

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Planned:

- solving MINLPlib2 models (translated to Pyomo) in development
- automatic decomposition and other enhancements
- application to energy conversion systems

Questions?