MPC -> QP problem

$$\frac{d}{dt}\begin{bmatrix} x \\ u \end{bmatrix} = \begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ u \end{bmatrix} = D$$

$Where\ A = [13; 13],\ \ B = [13; 12],\ \ x = [13; 1], u = [12; 1]$

$$\begin{bmatrix} x \\ u \end{bmatrix}(t) = \exp(D * t)\begin{bmatrix} x_0 \\ u_0 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ u_1 \end{bmatrix} = \exp(Ddt)\begin{bmatrix} x_0 \\ u_0 \end{bmatrix}$$

$$\dots$$

$$\begin{bmatrix} x_n \\ u_n \end{bmatrix} = \exp(Ddt)\begin{bmatrix} x_{n-1} \\ u_{n-1} \end{bmatrix} = \begin{bmatrix} \hat{A} & \hat{B} \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_{n-1} \\ u_{n-1} \end{bmatrix}$$

$$x_n = \hat{A}x_{n-1} + \hat{B}u_{n-1}$$

$$x_1 = \hat{A}x_0 + \hat{B}u_0$$

$$x_2 = \hat{A}\left(\hat{A}x_0 + \hat{B}u_0\right) + \hat{B}u_1 = \widehat{A^2}x_0 + \hat{A}\hat{B}u_0 + \hat{B}u_1$$

$$X = \begin{bmatrix} \hat{A} \\ \hat{A}^2 \\ \dots \\ \dots \\ \hat{A}^n \end{bmatrix} x_0 + \begin{bmatrix} \hat{B} & 0 & \dots & 0 & 0 \\ \hat{A}\hat{B} & \hat{B} & 0 & \dots & 0 \\ \hat{A}^2\hat{B} & \hat{A}\hat{B} & \hat{B} & 0 & \dots \\ \dots & \hat{A}^2\hat{B} & \hat{A}\hat{B} & \hat{B} & 0 \\ \hat{A}^{n-1}\hat{B} & \dots & \hat{A}^2\hat{B} & \hat{A}\hat{B} & \hat{B} \end{bmatrix}\begin{bmatrix} u_0 \\ u_1 \\ \dots \\ \dots \\ u_n \end{bmatrix} = A_{qp}x_0 + B_{qp}U$$

$Where\ h = horizon,\ \ X\ =\ [13h; 1],\ \ x_0 = [13; 1], U = [12h; 1], A_{qp} = [13h; 13], B_{qp} = [13h; 12h]$

$$J = \left(A_{qp}x0 + B_{qp}U - x_{ref}\right)^T L\left(A_{qp}x0 + B_{qp}U - x_{ref}\right) + U^T KU$$

$$\left(A_{qp}x0 + B_{qp}U - x_{ref}\right)^T L\left(A_{qp}x0 + B_{qp}U - x_{ref}\right)$$
$$= \left(\left(A_{qp}x0\right)^T + \left(B_{qp}U\right)^T - x_{ref}^T\right)\ \ L\left(A_{qp}x0 + B_{qp}U - x_{ref}\right) =$$

$$\begin{aligned} &\left(A_{qp}x0\right)^T L\left(A_{qp}x0 + B_{qp}U - x_{ref}\right) + \\ &\left(B_{qp}U\right)^T L\left(A_{qp}x0 + B_{qp}U - x_{ref}\right) + \\ &-x_{ref}^T L\left(A_{qp}x0 + B_{qp}U - x_{ref}\right) = \end{aligned}$$

We will minimize all that's why omit all constants

$$= \left(A_{qp}x0\right)^T L\, B_{qp}U + \left(B_{qp}U\right)^T LB_{qp}U + \left(B_{qp}U\right)^T L\left(A_{qp}x0 - x_{ref}\right) - x_{ref}^T LB_{qp}U =$$

$$\begin{aligned} &= \left(A_{qp}x0 - x_{ref}\right)^T L\, B_{qp}U + \left(B_{qp}U\right)^T LB_{qp}U + \left(A_{qp}x0 - x_{ref}\right)^T LB_{qp}U = \\ &= 2\left(A_{qp}x0 - x_{ref}\right)^T L\, B_{qp}U + U^T B_{qp}^T LB_{qp}U \end{aligned}$$

$$J = 2\left(A_{qp}x0 - x_{ref}\right)^T L\, B_{qp}U + U^T B_{qp}^T LB_{qp}U + U^T KU =$$

$$= U^T\left(B_{qp}^T LB_{qp} + K\right)U + U^T\left(2B_{qp}^T L\left(A_{qp}x0 - x_{ref}\right)\right) =$$

$$= \frac{1}{2}U^T HU + U^T g$$

$$H = 2\left(B_{qp}^T L B_{qp} + K\right)$$

$$g = 2B_{qp}^T L\left(A_{qp}x0 - x_{ref}\right)$$

Where $x_{ref} = [13h; 1] = X_d\ in\ code, L = [13h; 13h] = S\ in\ code, K = [12h; 12h] = Identity * \alpha\ in\ code,$

Initial Constraint matrix:

$$lbc \le CU \le ubc$$

My constraints:

$$\left|\frac{x_n - x_{refn}}{x_{refn}}\right| < \epsilon$$

$$-\epsilon < \frac{x_n}{x_{refn}} - 1 < \epsilon$$

$$1 - \epsilon < \frac{x_n}{x_{refn}} < 1 + \epsilon$$

$$x_n = \left(A_{qp}x0 + B_{qp}U\right)_n; n = 13 * (horizon - 1) + 1,2, \ldots, 13$$

$$1 - \epsilon < \frac{1}{x_{refn}}\left(A_{qp}x0 + B_{qp}U\right)_n < 1 + \epsilon$$

$$x_{refn}(1-\epsilon) - \left(A_{qp}x0\right)_n < \left(B_{qp}U\right)_n < x_{refn}(1+\epsilon) - \left(A_{qp}x0\right)_n$$

$$lb_n \le \boldsymbol{n}^T B_{qp}U \le ub_n, \qquad where\ n\ are\ projections$$

$$lb_n = x_{refnmin}(1-\epsilon) - \left(A_{qp}x0\right)_n$$

$$ub_n = x_{refnmax}(1+\epsilon) - \left(A_{qp}x0\right)_n$$

$$if\ x_{refn} == 0: x_{refnmin} = -closeZeroDouble, x_{refnmin} = +closeZeroDouble$$

$$else\ x_{refnmin} = x_{refnmax} = x_{refn}$$

Simplest solution is to add this as at the last row to C *in this way so we have:*

$$lbc_{adjusted} \le C_{adjusted}U \le ubc_{adjusted}$$