Lindenmayer Systems

Alasdair McAndrew

College of Engineering and Science
Victoria University,
Melbourne, Australia

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What is a Lindenmayer system?

- Designed to explore organic growth
- Creates complex shapes from simple rules
- For example, with the rules:

$$0 \to 1, \qquad 1 \to 01$$

  we have this string transformation:

$$01101 = 0\ 1\ 1\ 0\ 1 \longrightarrow 1\ 01\ 01\ 1\ 01 = 10101101$$

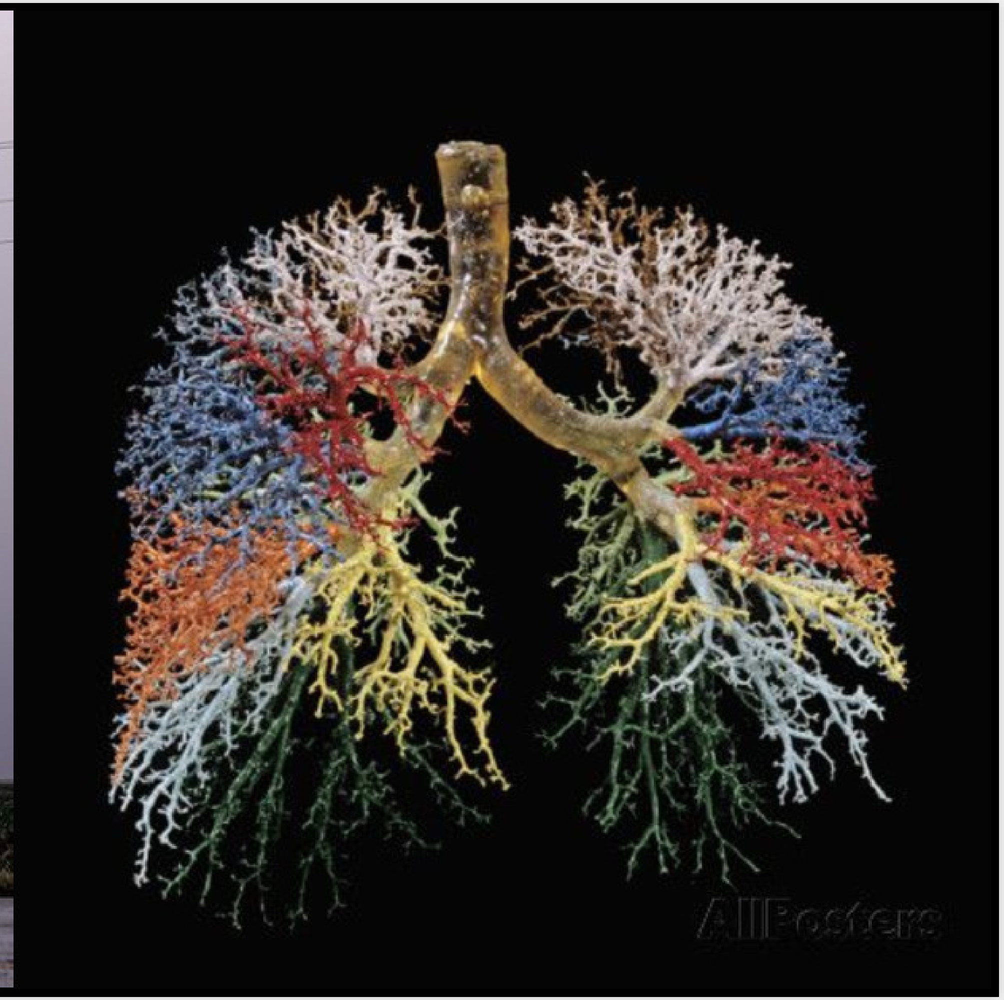
- All operations take place *simultaneously*
- L-systems are *term-rewriting systems*

Fractals: real...

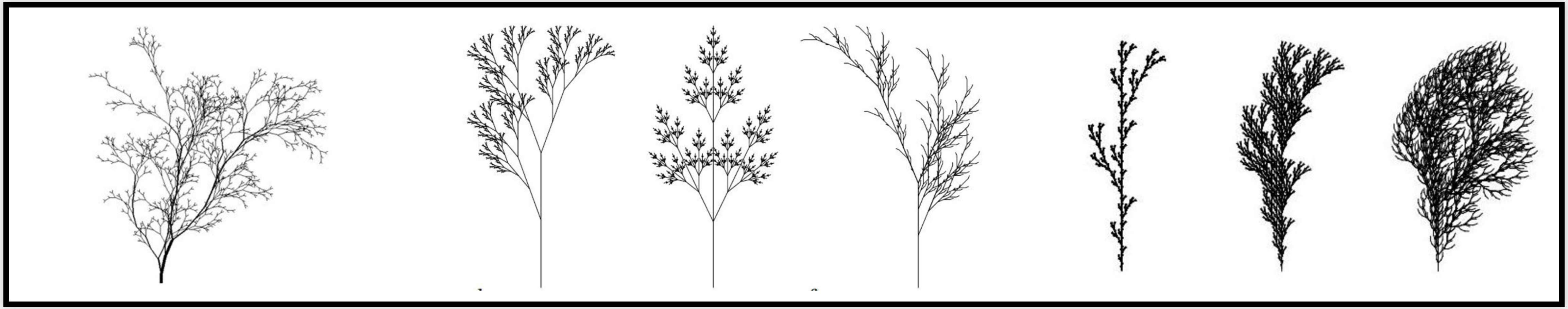
All these lovely fractal trees in the city of Melbourne, Australia, in the winter.

Fractals are everywhere

Once you start looking, you can't stop seeing them



Fractals: manufactured...



These are all examples from *The Algorithmic Beauty of Plants* by Aristid Lindenmayer and Przemysław Prusinkiewicz, available at http://algorithmicbotany.org/papers/abop/abop.pdf

Turning strings of symbols into pictures

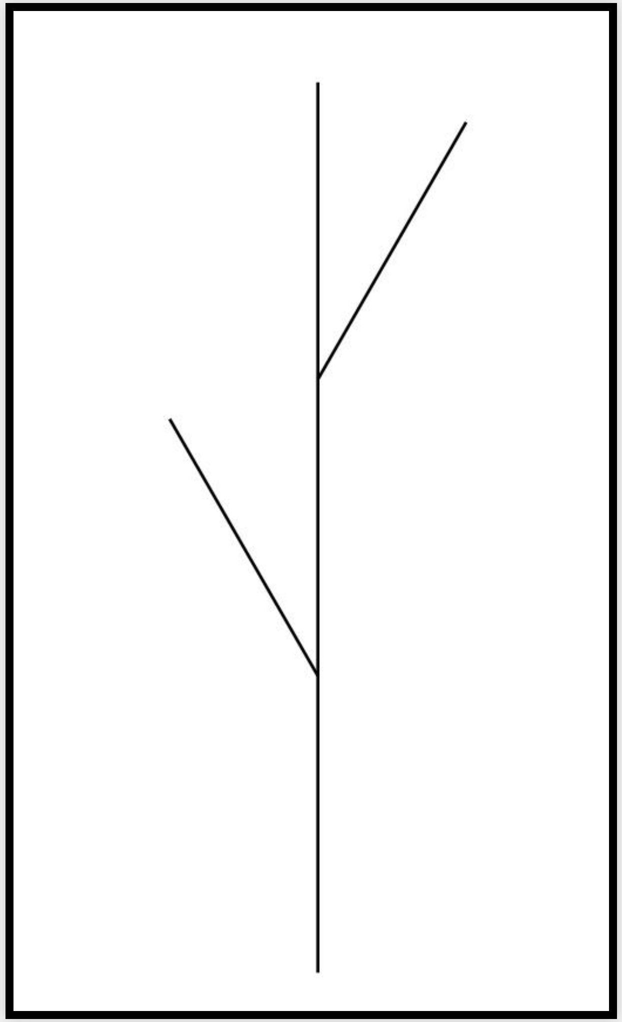
- Sets of rules describe how one string of symbols will be expanded to a new string
- Each symbol corresponds to a *turtle graphics* instruction:
  - F: Move forward
  - -: Turn left
  - +: Turn right
  - [: Memorize current position and heading
  -]: Move to most recently memorized position and heading

An example

For example, this sequence of symbols:

F[+F]F[-F]F

has this output:

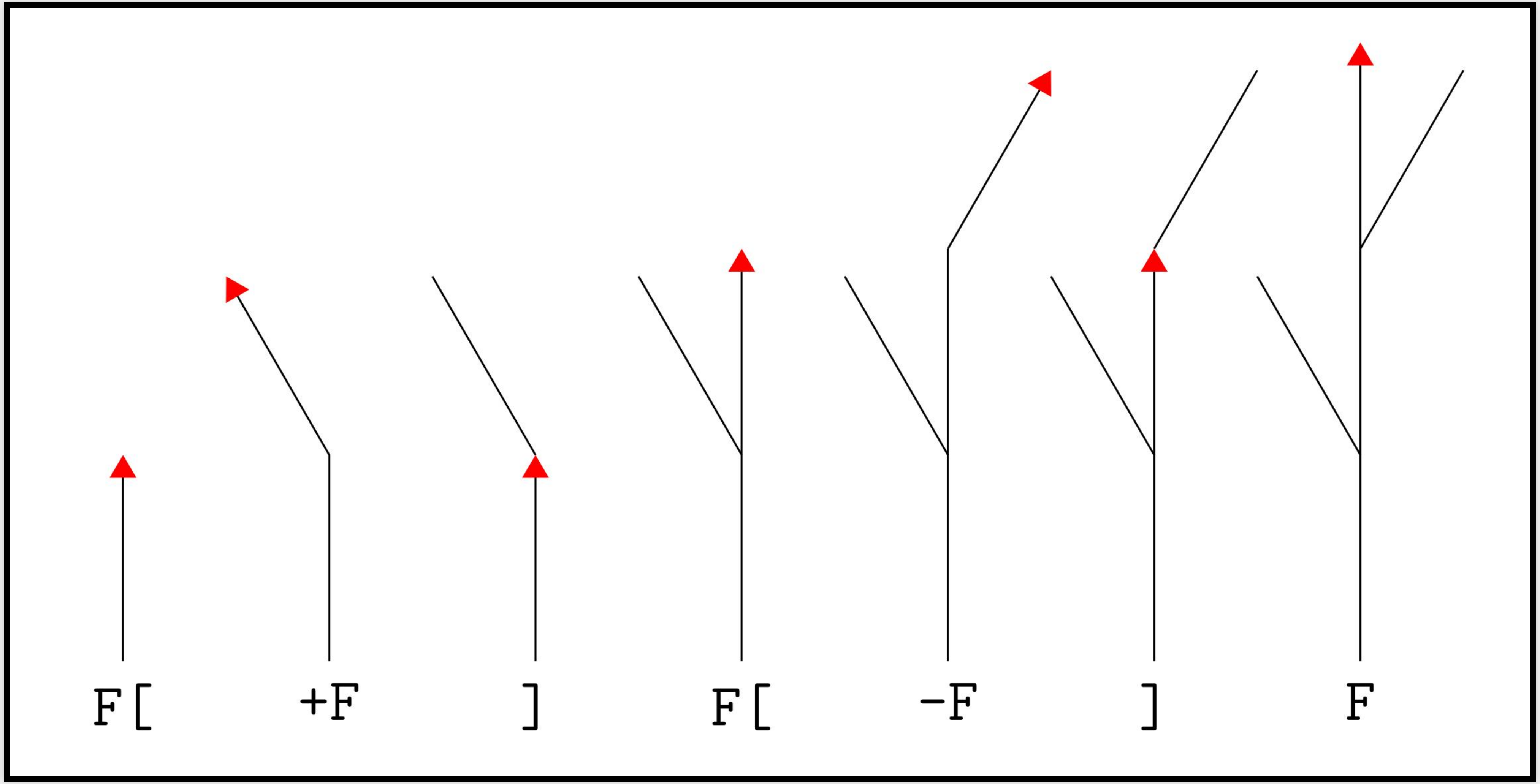


We can clearly alter the output by changing the angle of the turns, and the length of the move forward.

In this example, the angle is 26°

How turtle graphics works

This shows how the turtle draws a path with branches:



More on turtle graphics

It's all done from the point of view of the turtle. A side of *Koch's snowflake* can be computed by the rules:

- Start: F
- Modify: F $\longrightarrow$ F+F--F+F (with turns of 60°)
- At every further step, each F is replaced by the string F+F--F+F
- The second iteration produces

  F+F--F+F+F+F--F+F--F+F--F+F+F+F--F+F

- The third iteration produces:

  F+F--F+F+F+F--F+F--F+F--F+F+F+F--F+F+F+F--F+F+F+F--F+F--F+F--F+F+F+F--F+F--F+F--F+F+F+F--F+F--F+F--F+F+F+F--F+F+F+F--F+F+F+F--F+F--F+F--F+F+F+F--F+F

- and so on...

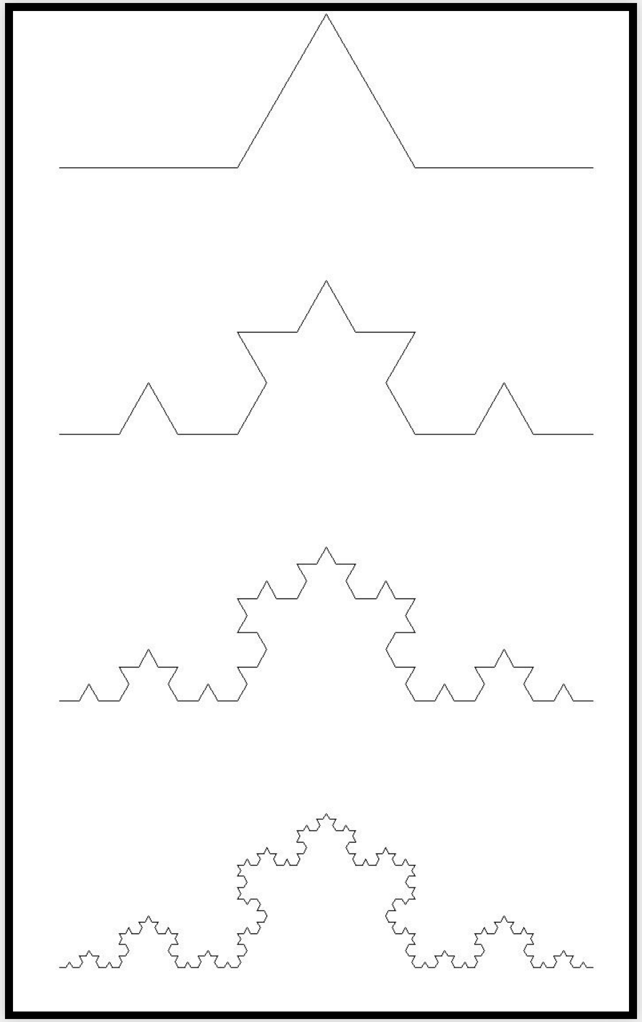
Turtle graphics *with pictures!*

First iteration:

Second iteration:

Third iteration:

Fourth iteration:



Some mathematics

Remember the F $\longrightarrow$ F+F- -F+F iteration? How many symbols are in the n^{th} string?

Let f_n be the number of F's, and k_n be the number of other symbols in the n^{th} string. We have:

$$
\begin{aligned}
f_{n+1} &= 4f_n, \quad f_1 = 4 \\
k_{n+1} &= 4f_n + k_{n-1}, \quad k_1 = 4
\end{aligned}
$$

It follows immediately that

$$f_n = 4^n \text{ and } k_n = 4 + 4^2 + 4^3 + \cdots + 4^n = \frac{4}{3}(4^n - 1).$$

The total length is thus

$$f_n + k_n = 4^n + \frac{4}{3}(4^n - 1) = \frac{1}{3}(7(4^n) - 4).$$

The fractal plant in modern languages: Python

```
import turtle as t  # "turtle" is a turtle graphics module

# Lindenmayer system (a) from ABOP figure 1.24(a), p 25
def edgetree(level, size, angle):
    if (level==0):
        t.fd(size)
    else:
        edgetree(level-1, size/3, angle)
        t.lt(angle)
        edgetree(level-1, size/3, angle)
        t.bk(size/3)
        t.rt(angle)
        edgetree(level-1, size/3, angle)
        t.rt(angle)
        edgetree(level-1, size/3, angle)
        t.bk(size/3)
        t.lt(angle)
        edgetree(level-1, size/3, angle)
```

The fractal plant in modern languages: Racket

Racket is a modern lisp; descended from Scheme.

```
;;   F -> F[+F]F[-F]F

(require furtle) ;; furtle is a simple but fast turtle graphics library
(: ltree_b (-> Real Real Real TurtleF))  ;; typed Racket so must declare types
(define (ltree level size angle)
  (if (= level 0)
      (turtles (forward size))
      (turtles (ltree (- level 1) (/ size 3) angle)                    ; F
               (save)                                                   ; [
               (left angle) (ltree (- level 1) (/ size 3) angle)       ; +F
               (restore)                                                ; ]
               (ltree (- level 1) (/ size 3) angle)                    ; F
               (save)                                                   ; [
               (right angle) (ltree (- level 1) (/ size 3) angle)      ; -F
               (restore)                                                ; ]
               (ltree (- level 1) (/ size 3) angle))))                 ; F
```

Some more mathematics

Fractal dimension can be defined by the "box-counting measure":

Suppose our picture is subdivided into boxes of size b, and $N(b)$ boxes are needed to cover the shape. Its dimension can be defined as

$$\lim_{b\to 0}\frac{\log(N(b))}{\log(1/b)}.$$

For example, take a curve of length k. As $b \to 0$, we would find that

$$N(b) \to \frac{k}{b}.$$

Thus

$$\lim_{b\to 0}\frac{\log(N(b))}{\log(1/b)} = \lim_{b\to 0}\frac{\log(k/b)}{\log(1/b)} = \lim_{b\to 0} 1 - \frac{\log(k)}{\log(b)} = 1.$$

In general a fractal will have a non-integer dimension between 1 and 2.

The end

Thanks, folks!