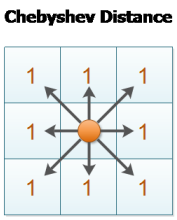
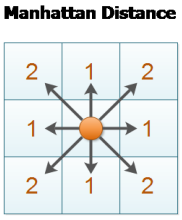
```
**********
Playing Matches
**********

Match #  Opponent    AB_Improved   AB_Custom   AB_Custom_2  AB_Custom_3
                  Won | Lost   Won | Lost   Won | Lost   Won | Lost
   1      Random      10 |  0    10 |  0     9 |  1     9 |  1
   2      MM_Open      9 |  1     6 |  4     6 |  4     6 |  4
   3     MM_Center     8 |  2    10 |  0     7 |  3     9 |  1
   4    MM_Improved    6 |  4     5 |  5     8 |  2     8 |  2
   5      AB_Open      4 |  6     5 |  5     4 |  6     3 |  7
   6     AB_Center     6 |  4     4 |  6     5 |  5     5 |  5
   7    AB_Improved    5 |  5     6 |  4     5 |  5     4 |  6
--------------------------------------------------------------------------
         Win Rate:      68.6%       65.7%       62.9%       62.9%
```

Three heuristics were considered for this project. Isolation allows for moves in eight directions, which means simple manhattan distance isn't enough. There are two variations of diagonal distance, Chebyshev and octile distance. Chebyshev distance is used here since it doesn't reward diagonal moves as octile distance would. Manhattan distance is used as another heuristic, but is expected to perform worse. The same is true for Euclidean distance, which accounts for angles in addition to grid directions.





$\max(|x_1 - x_2|, |y_1 - y_2|)$ $|x_1 - x_2| + |y_1 - y_2|$

Source: https://lyfat.wordpress.com/2012/05/22/euclidean-vs-chebyshev-vs-manhattan-distance/

Of these three heuristics, Chebyshev distance performs best, slightly better than AB_Improved. This is because Chebyshev distance values diagonal, horizontal, and vertical moves equally. On the contrary, Manhattan distance gives a higher value to diagonal moves. Given the nature of isolation, that all directions are equally important, Chebyshev distance is expected to perform the best.

Out of the three I developed, Chebyshev distance is the recommended heuristic to use. This is because of the following:

1. Highest win rate (65.5% vs 62.9% and 62.9%)
2. Computational efficiency, there's no arithmetic beyond multiplication, operated in O(1) time. On my machine, every calculation takes 1.66 * 10^-5 seconds.
3. Equal cost for all eight directions