

# Inverse Oxygen Model

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## Oxygen process model

Start with a mass balance equation for oxygen concentration  $C$ :

$$\frac{\delta VC}{\delta t} = - \left( \frac{\delta q_x}{\delta x} + \frac{\delta q_y}{\delta y} + \frac{\delta q_z}{\delta z} \right) + S$$

where:  $V$  = volume  $C$  = concentration  $q$  = mass fluxes due to advective and dispersive transport  $S$  = sinks and sources

Assume: Uniform flow/well mixed water column:

$$\begin{aligned} \frac{\delta q_x}{\delta x} + \frac{\delta q_y}{\delta y} &= 0 \\ \frac{\delta VC}{\delta t} &= - \frac{\delta q_z}{\delta z} + S \end{aligned}$$

Integrate with respect to  $z$  and define fluxes per unit ground area  $A$  assuming the channel is rectangular with a depth of  $D$

$$\frac{\delta H \bar{C}}{\delta t} = q(H) - q(0) + \int_0^H S dz$$

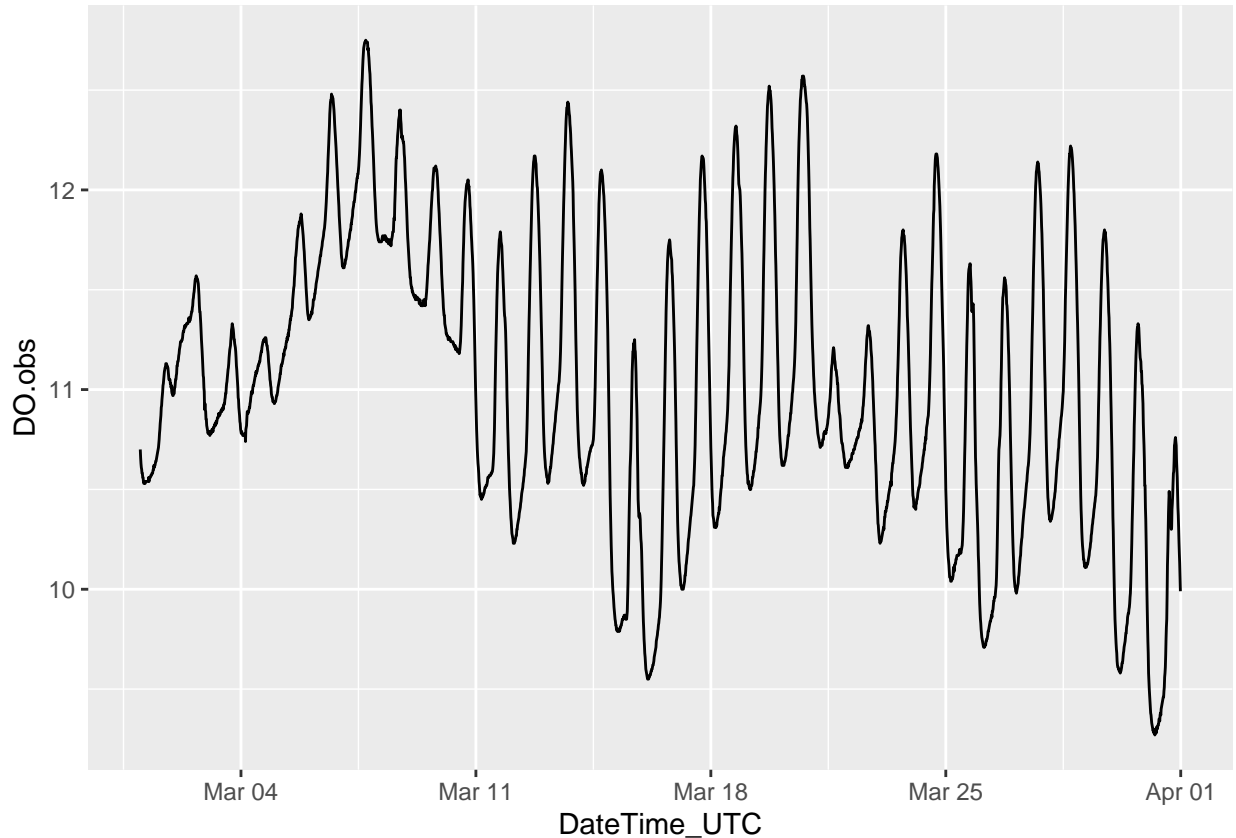
where:  $\bar{C}$  is the depth-averaged concentration,  $H$  is the depth  $q(H) = k(C_{sat} - \bar{C})$  is the exchange of oxygen with the atmosphere  $B(t) = -q(0) + \int_0^D S dz$  represents biological production and consumption of oxygen

So we can describe the biological time series as a function of  $\bar{C}(t)$ , depth, discharge, and riverbed slope using empirical equations:

$$B(t) = \frac{d(H\bar{C})}{dt} - k(C_{sat} - \bar{C})$$

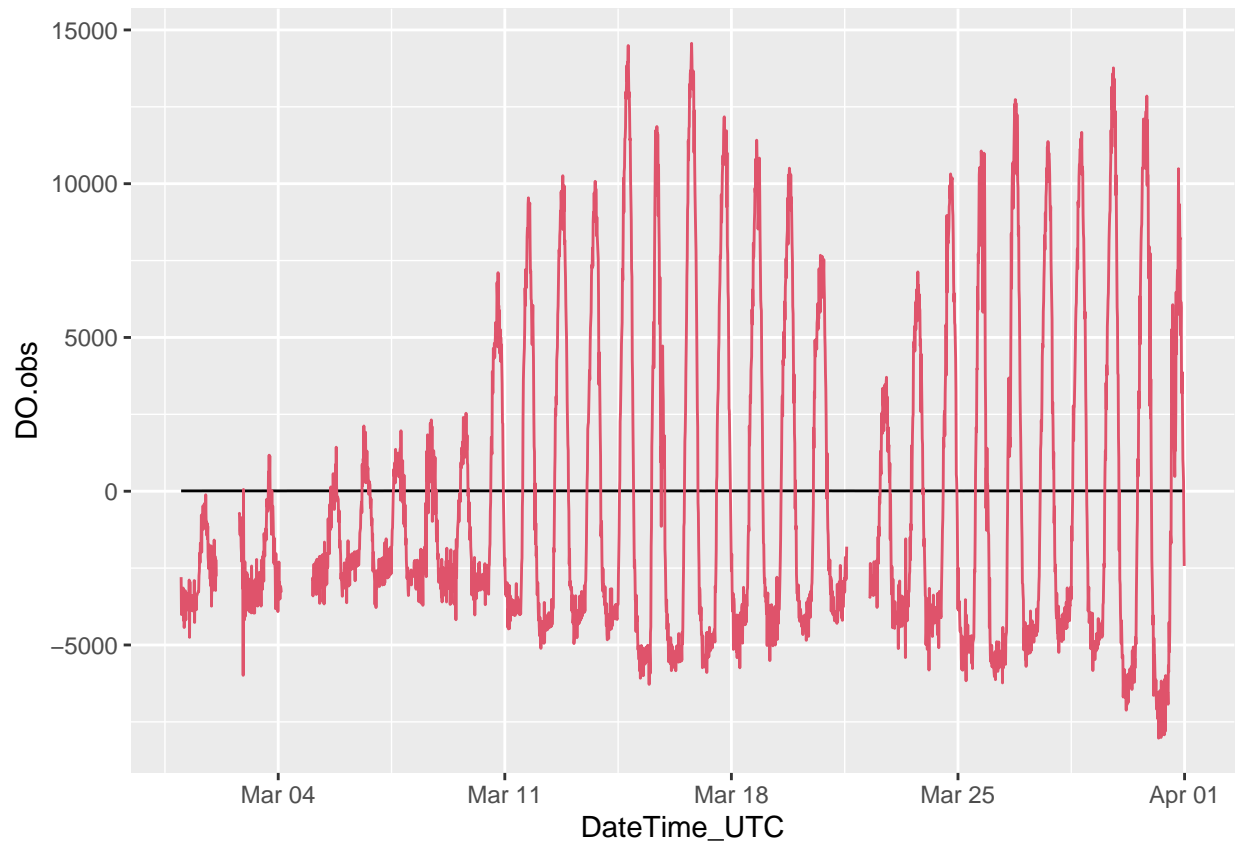
Example Stream:

## New Hope Creek



```
m_dat <- dat %>%
  select(DateTime_UTC, DO.obs, avg_velocity, DO.sat, depth, temp.water) %>%
  mutate(slope = nhc_slope,
         # v_ms = calc_vel_from_disch(discharge),
         k600_mean = calc_k600_raymond5(slope, avg_velocity), # m/d
         # k600_min = calc_k600_raymond5(slope, avg_velocity)[2],
         # k600_max = calc_k600_raymond5(slope, avg_velocity)[3],
         k_O2_mean = kO2fromk600(k600_mean, temp.water),
         q_D = k_O2_mean * (DO.sat - DO.obs)*1000, # m/d * mg/L * 1000 L/m^3 = mg/m2d
         dHCdt = c(NA, diff(dat$DO.obs))*depth/15*60*24*1000, # mg/L/min * 60 min/h * 24 h/d
         B_t = dHCdt - q_D) # biological timeseries mg/m2/d

m_dat %>%
  filter(month(DateTime_UTC) == 3) %>%
  ggplot(aes(DateTime_UTC, DO.obs)) +
  geom_line() +
  geom_line(aes(DateTime_UTC, B_t), col = 2)
```



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When you click the **Knit** button a document will be generated that includes both content as well as the output of any embedded R code chunks within the document. You can embed an R code chunk like this:

```
summary(cars)
```

```
##      speed      dist
##  Min.   : 4.0    Min.   : 2.00
##  1st Qu.:12.0    1st Qu.: 26.00
##  Median :15.0    Median : 36.00
##  Mean   :15.4    Mean   : 42.98
##  3rd Qu.:19.0    3rd Qu.: 56.00
##  Max.   :25.0    Max.   :120.00
```

## Including Plots

You can also embed plots, for example:



Note that the `echo = FALSE` parameter was added to the code chunk to prevent printing of the R code that generated the plot.