SUPPRESSION OF ROTATIONAL MOTION ARTIFACTS IN MRI USING A FUZZY DATA MODEL AND POCS WITH SOFT CONSTRAINTS

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Abstract-Rotational motion artifacts in magnetic resonance (MR) images can have a significantly adverse effect on the diagnostic value of the image. Post-processing techniques for the suppression of such artifacts often involve re-gridding of the acquired k-space data. This technique create significant data void regions in the corrected k-space. Projection onto convex sets (POCS) is proved to be successful in estimating the missing k-space values. However, the available data are often subjected to noise and interpolation errors which cause the k-space constraints to form an ill defined convex set, inconsistent with the spatial constraints. Inconsistent constraints often lead to divergence of POCS from the desired solution, within a finite number of iterations. In this paper, we propose a fuzzy model for representing the available k-space values to avoid such divergence. We also present a set of fuzzy rules, to impose soft k-space constraints. Simulation results indicate that the proposed algorithm avoids divergence, producing a superior quality final image.

Key words: MRI, image reconstruction, rotational motion artifacts, POCS, Fuzzy models.

1 Introduction

The presence of patient motion during MR data acquisition causes visible artifacts in the reconstructed image which obscure vital anatomical detail. In standard two dimensional Fourier transform (2DFT) imaging, the artifacts appear as blurring or ghost repetitions of the moving structures in the phase encoded direction. The image needs to be corrected or refocussed prior to its application as a diagnostic tool.

Re-acquisition of motion corrupted data can significantly elongate the scanning time. Therefore, post processing techniques continue to gain interest of the research community. Software solutions rarely require modification to system hardware or the pulse sequences used for imaging.

Several recent publications have proposed post processing methods for the suppression of artifacts caused by in plane rigid rotational motion [1]-[5]. It has been shown that, such rotations around arbitrary points in the field of view (FOV) can be modelled by separate in plane 2D translations and rotations around the origin [2]. Translational motion tends to corrupt the phase of k-space data while rotations tend to rotate the locations of the spatial frequency components around the origin of the k-space. A popular correction method is to use bilinear interpolation in the spatial domain for rotation compensation followed by super-position on to the corrected k-space [2]. However, this method is shown to create significant regions of data voids, causing degradation of the final image quality, which becomes pronounced when the rotation angles are large [1].

In our previous work [1], a POCS based algorithm was proposed to estimate the missing spatial frequencies in the corrected k-space. However, it was observed that the reconstructed image diverges from the desired image within finite number of iterations. The cause for such divergence is found to be the inconsistent k-space constraints imposed on the POCS algorithm.

It can be shown that the convex set defined by the available k-space data form a linear variety in the Hilbert space [6]. A linear variety is a translation of a subspace by a fixed vector. This fixed vector is determined by the available k-space information [7]. Therefore, unreliable information leads to an erroneous fixed vector, resulting in an inconsistency of constraints [8][9].

If the possibility spread [10] is known, the k-space values can be fuzzified using a simple membership function, in order to produce soft k-space constraints, consistent with the spatial constraints, avoiding the problem of divergence.

2 Fuzzy Data Model

It is possible to define a fuzzy set so that its membership function can be seen as a possibility distribution. The information from a specialist, expressed by " about $\alpha \approx A$", is replaced by $\mu_A(x)$, where this new expression is taken as the information that expresses possibility of A being in the vicinity of α [10]. Therefore, a fuzzy number A is expressed as $A = (\alpha, c)$, where α expresses the center of the region of possibility and c its spread. The membership

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function of A can be given by

$$\mu_A(x) = L\left(\frac{x-\alpha}{c}\right) \qquad c > 0 \tag{1}$$

where $L(x)$ is called the reference function [10] and posses the following properties:

- $L(x) = L(-x)$
- $L(0) = 1$
- $L(x)$ is a strictly decreasing function for $[0, \infty)$.

Examples of $L(x)$ for $p > 0$ are functions such as $L_1(x) = max(0, 1 - |x|^p)$, $L_2(x) = e^{-|x|^p}$ etc. If we consider $L_1(x)$ for $p = 1$, the result is a triangular fuzzy number as depicted in Figure 1.

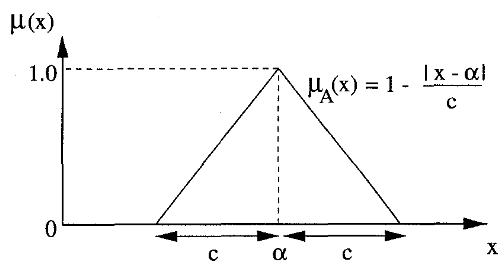

Figure 1: $\mu_A(x)$ of a triangular fuzzy number.

The major obstacle to the fuzzification of k-space data is the estimation of the spread (c) of the possibility function. Since each spatial frequency value is subjected to differing measurement errors, interpolation errors and noise, the overall effect of such errors is highly complex and difficult to accurately model as a general rule. However, it is possible to identify pairs of complex conjugate k-space points, since $S(k_x, k_y) \approx S^*(N - k_x, N - k_y)$, where $S^*(\cdot)$ indicate the complex conjugate and N is the number of phase encoding (k_y) or frequency encoding (k_x) steps. Although the phase of these alleged complex conjugate values can be corrupted due to acquisition timing, the magnitudes should generally be matched within an error limit.

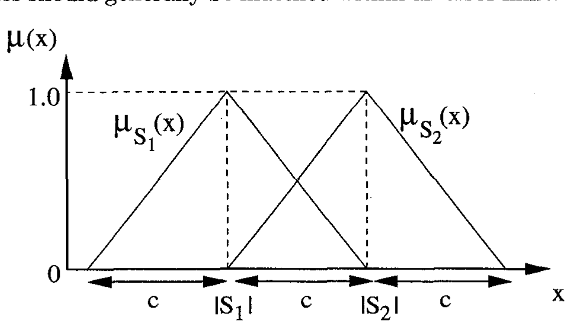

Figure 2: Membership functions of a conjugate pair.

If the re-gridded k-space provides $p(< N(\frac{N}{2} - 1))$ conjugate pairs of interpolated data, it is possible to determine the spread of the possibility function based on the difference of the magnitudes of the conjugate pairs, leading to triangular fuzzy models for each element S_1 and S_2, as shown in Figure 2.

The separation between $|S_1|$ and $|S_2|$ in fact represent the spread of the possibility function, since the "closeness" of the values $|S_1|$ and $|S_2|$ indicate a higher possibility of both $|S_1|$ and $|S_2|$ being "closer" to the uncorrupted k-space value. The noise corruption and interpolation errors force $|S_1|$ and $|S_2|$ to be different and this difference approximately indicate the reliability of the data. Higher the difference, the less reliable the data will be. However, it is possible that one of the numbers in the conjugate pair to be "closer" to the uncorrupted value than the other. This information will not be apparent in the possibility spread. Therefore, the fuzzy model describes the composite reliability of the conjugate pair rather than the reliability of individual elements. Hence, we combine the membership functions of S_1 and S_2 to provide a composite membership function ($\mu_S(x)$) for the conjugate pair, as given in Equation 2.

$$\mu_S(x) = min(1, \mu_{S_1}(x) + \mu_{S_2}(x)) \tag{2}$$

3 Fuzzy Rules

The reason for fuzzification of k-space data is primarily to provide soft k-space constraints during the POCS iterations. Hence, the algorithm takes the form of a simple fuzzy dynamic program, in which the "decision points" include varying degrees of fuzziness or ambiguity [10]. Since POCS iterations do not converge to the k-space elements at the same rate, we propose fuzzy rules for the "decision points" based on the convergence rate of each k-space element and the overall convergence rate of POCS.

- Let the k-space values for a fuzzy conjugate pair at the n^{th} iteration be S_n and $S^*{}_n$. Let ϵ_0 be a "very small" value. If $\frac{||S_n| - |S_{n-1}||}{|S|} < \epsilon_0$ and $\mu_S(|S_n|) = 0$, then the corresponding values are removed from the k-space constraints due to high degree of unreliability. The vacant k-space points are added to the set of values to be estimated during consequent POCS iterations.

- If $||S_n| - |S|| < \rho$, the corresponding values are preserved as k-space constraints due to high degree of reliability. ρ is a small fuzzy limit with its membership function mapped from the overall convergence rate of the POCS algorithm. Let the error outside the region of interest (ROI) for the reconstructed image at the n^{th} iteration be E_n. We define a measure of the convergence rate ζ_n as follows:

$$\zeta_n = log_{10}(E_{n-1} - E_n) \tag{3}$$

Note that $E_{n-1} > E_n$ for all n, since the POCS algorithm is converging. Therefore, ζ_n exists for all n. The membership function of ρ is given by

$$\mu_\rho(\zeta_n) = \begin{cases} 1.0 & \text{for } \zeta_n \geq 0.0 \\ e^{-\frac{|\zeta_n|^2}{4}} & \text{for } \zeta_n < 0.0 \end{cases} \tag{4}$$

Hence, $\rho = \rho_0 \mu_\rho(\zeta_n)$ where ρ_0 is a "small" number.

- If $\frac{||S_n|-|S_{n-1}||}{|S|} < \epsilon_0$, $||S_n| - |S|| > \rho$ and $\mu_S(|S_n|) > 0$, then the original k-space values are updated to the new values at the n^{th} iteration, and preserved as k-space constraints.

After each iteration of POCS, the average $\mu_S(x)$ value for the entire k-space, is computed. If there is a large decrease in the average membership, it is an indication that POCS algorithm is significantly diverging from the initial k-space values, often due to excessive fuzzification. This is also a fuzzy measure of the compatibility of the final image to the interpolated k-space data.

4 Simulations and Discussion

In order to test the effectiveness of the proposed fuzzy algorithm, we performed several simulation studies on rotational motion affected data sets. This section presents some of the important results to support our hypothesis.

(a) *(b)*

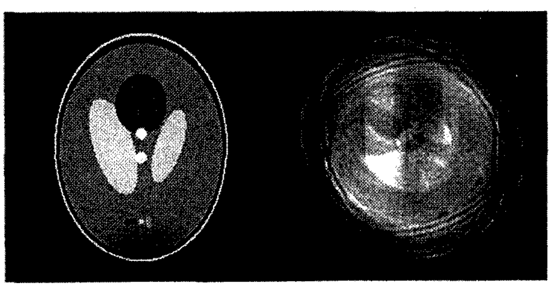

Figure 3: (a) Shepp & Logan phantom. (b) Computed artifactual image (MSE =2180.4)

The rotational motion artifact was simulated by rotating the Shepp & Logan phantom [11] shown in Figure 3(a) by an angle θ prior to obtaining the spatial frequency information for each view. The corrupted N samples of data for the ${k_y}^{th}$ view is given by

$$S'(k_x, k_y) = \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} m(\bar{x}, \bar{y}) e^{[-i\frac{2\pi}{N}(xk_x + yk_y)]} \tag{5}$$

where $\bar{x} = x\cos\theta - y\sin\theta$ and $\bar{y} = x\sin\theta + y\cos\theta$. k_x is the sample number in the frequency encoded direction, $m(x, y)$ is the original Shepp and Logan phantom and θ is the angle of rotation for the ${k_y}^{th}$ view.

Figure 4 shows the view numbers and corresponding angles of rotation. The motion involved stepping rotations with maximum angular span of $\pm 70^o$. The artifactual image is shown in Figure 3(b).

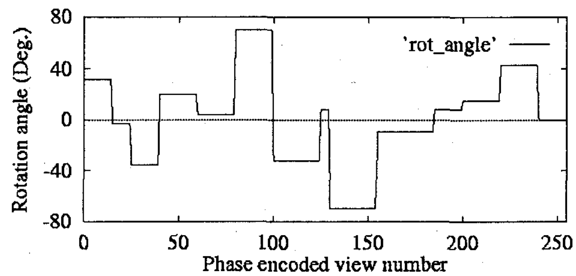

Figure 4: Rotational motion parameters for each phase encode.

As a comparison, conventional POCS with crisp sets was used to improve the quality of the image. The proposed fuzzy data model and POCS with soft k-space constraints was then used for the same purpose. The parameters used for iterations are $\epsilon_0 = 0.005$ and $\rho_0 = 0.2$.

A relative measure on the quality of the reconstructed image was obtained using the mean squared error (MSE), which is defined by

$$MSE = \frac{1}{N^2} \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} [m_{2j}(x, y) - m(x, y)]^2 \tag{6}$$

where $m_{2j}(x, y)$ is the reconstructed image at the j^{th} iteration and $m(x, y)$ represent the original image. It should be noted that in practice, $m(x, y)$ is unknown and hence MSE value is unavailable.

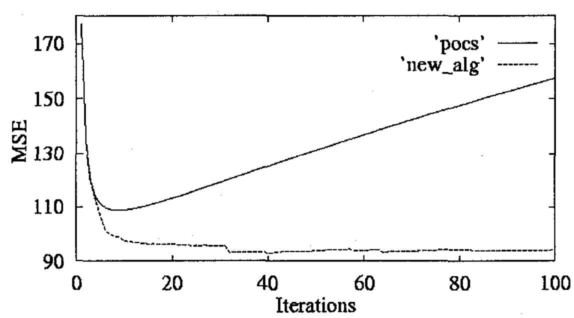

Figure 5: Variation of MSE for 100 iterations.

Figure 5 shows the relative performance of the proposed method compared to conventional POCS. It can be clearly seen that with conventional POCS, the reconstructed image diverges from the desired image after finite number of iterations. Since MSE measure is unavailable, this method has to rely on computationally complex and highly time consuming cost functions [1] in order to find the optimum point to terminate the iterations. With the proposed algorithm, iterations can be terminated when E_n reaches a desired limit without compromising the quality of the image due to the avoidance of divergence in MSE.

Figure 6(a) shows the reconstructed image using conventional POCS after 100 iterations, whereas Figure 6(b) shows the result using the proposed algorithm after same number of iterations.

(a) *(b)*

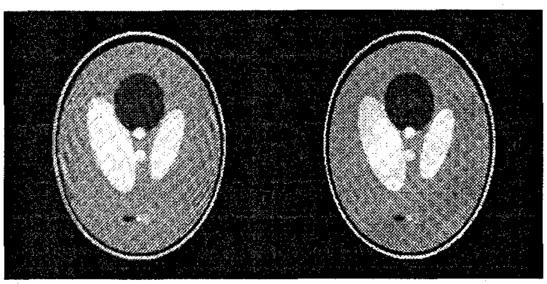

Figure 6: (a) Reconstructed image using conventional POCS for 100 iterations (MSE=152.8). (b) Reconstructed image using proposed algorithm for 100 iterations (MSE=94.1).

Convergence properties of the algorithm was examined subjected to noisy data at 10dB SNR with the same MRI signal. The comparative results are shown in Figure 7.

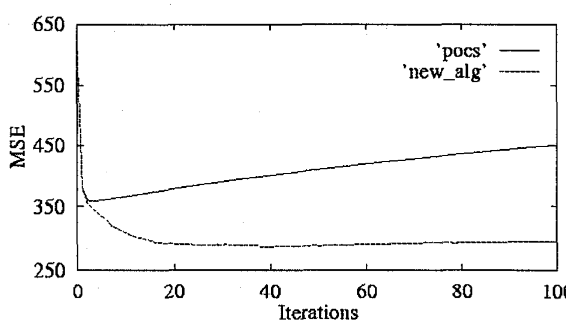

Figure 7: Variation of MSE for 100 iterations.

It should be noted that the choice of ϵ_0 and ρ_0 were not optimized for the above simulations. It is assumed that proper optimization of these parameters will produce higher quality solutions. Further research is being carried out to investigate the effect of these parameters on the overall fuzzy algorithm.

5 Conclusion

The original contributions of this paper are the introduction of a fuzzy data model to represent the possibility function of the interpolated k-space data and development of fuzzy rules based on POCS convergence rates to regulate the image reconstruction algorithm. The simulation results indicate that the proposed algorithm is capable of avoiding the divergence of the reconstructed image from the desired image. An additional advantage of the proposed algorithm is that it also avoids the necessity of computing a complicated and highly time consuming cost function to determine the point of termination of iterations.

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