Compressed Sensing MRI

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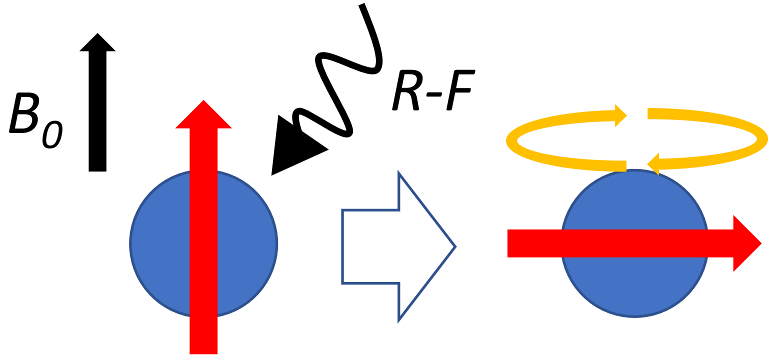
How MRI works



Spins are in external magnetic field B_0
R-F pulse flips spins which precess at $\omega=\gamma B$

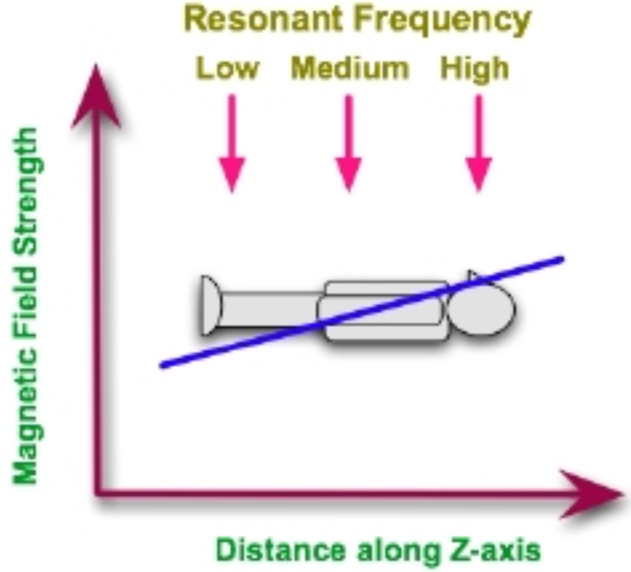
Gradient Fields applied in X , Y and Z directions

Ex: $B= B_0+G\cdot x$

2-D acquisitions:

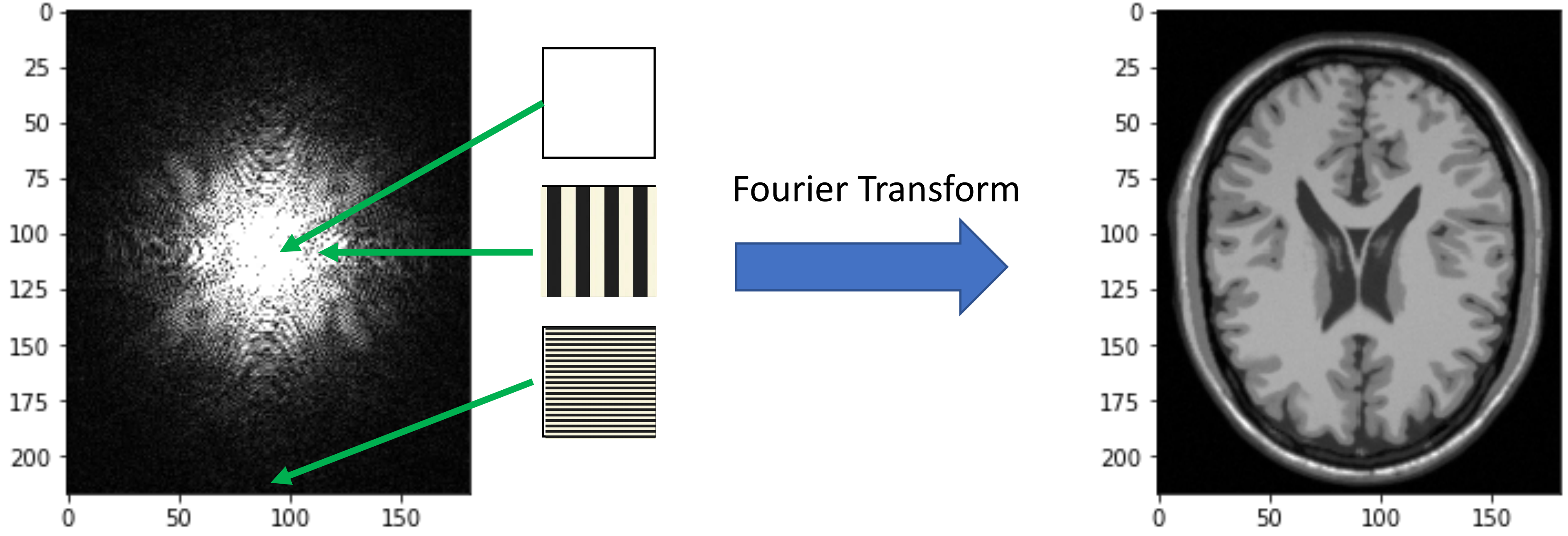
- Slice selection gradient
- Phase and Readout gradient move through 2-D k-space
- NxN k-space matrix Fourier transformed to NxN image

Illustration of z-gradient



The Problem

- Larger Gradients -> Higher Freq. in K-space -> Higher Resolution
  - Theoretically MRI resolution is dependent on Gradient Strength



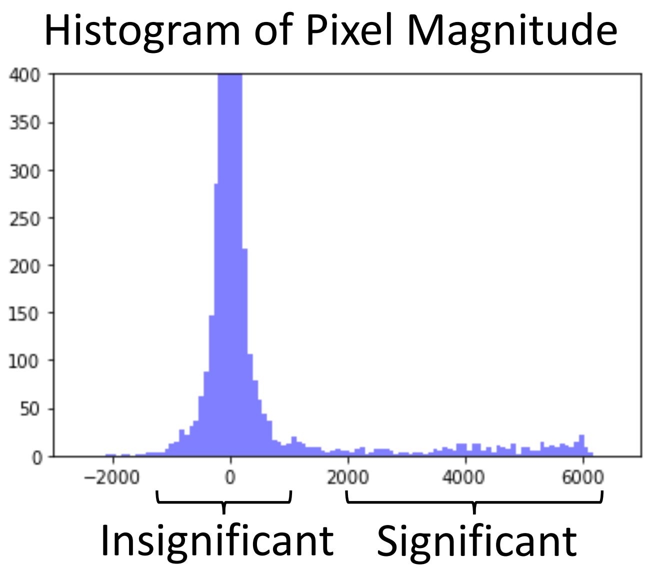
- In practice we image living things

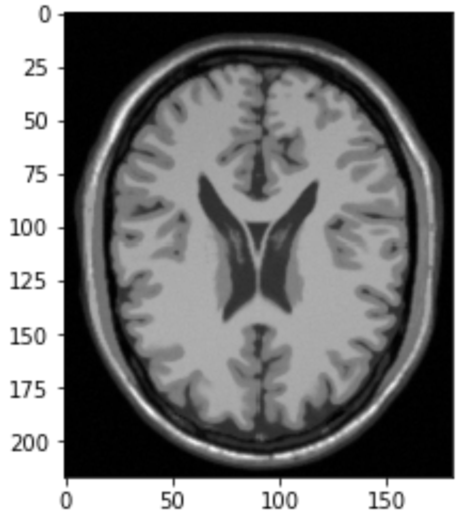
->Practically, scan time limits resolution

How do we reduce K-space points but not resolution?

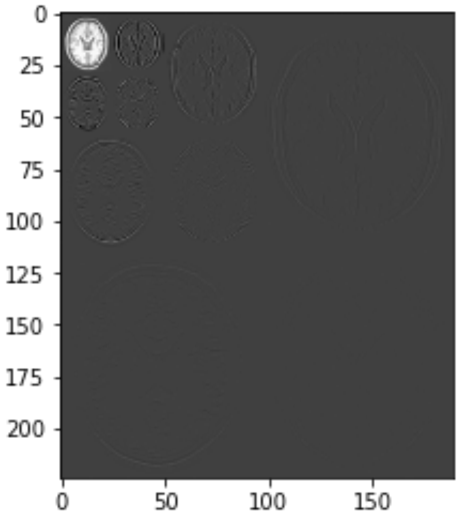
What prior knowledge do we have about MRI k-space?

1. Symmetrical
   - Partial Fourier reconstruction
   - Projection Onto Convex Sets (POCS)
2. Compressible
   - Under wavelet or other transform, MRI images are sparse



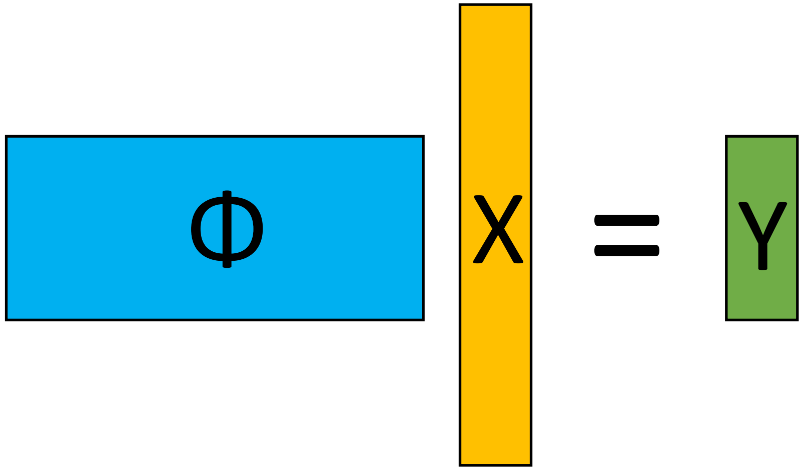


Discrete Wavelet Transform



It would be great if we could acquire in DWT space and only acquire the interesting points ……. But we can’t

Compressed Sensing



If we know X is sparse,
can we invert Φ to get X from Y?

Theorem: If Φ satisfies Restricted Isometry Property (RIP), then x is unique, sparse solution to $\Phi x=y$

Meaning: If Φ is nearly an orthogonal matrix, we can recover x by solving: $\boxed{\min |x|_0 \text{ such that } \Phi x=y}$

L_p norms: $|x|_p=(\Sigma|x|^p)^{1/p}$ where $|x|_0$ is the number of non-zero entries

Application to MRI

Minimizing the L_0 norm is computationally very expensive

L_1 norm is convex and much simpler to minimize

->Theorem: L_1 min = L_0 min if Φ satisfies RIP to a more stringent limit

Applying to MRI:

Fourier transforming k-space "f" to get image "u" that we sparsify with wavelet transform "W" and minimize

$$\min|Wu|_0 = \text{ such that } \mathcal{F}u=f$$

In practice due to noise we don't enforce $\mathcal{F}u=f$

$$\min|Wu|_0 = \text{ such that } |\mathcal{F}u-f|_2<\boldsymbol{\sigma}$$

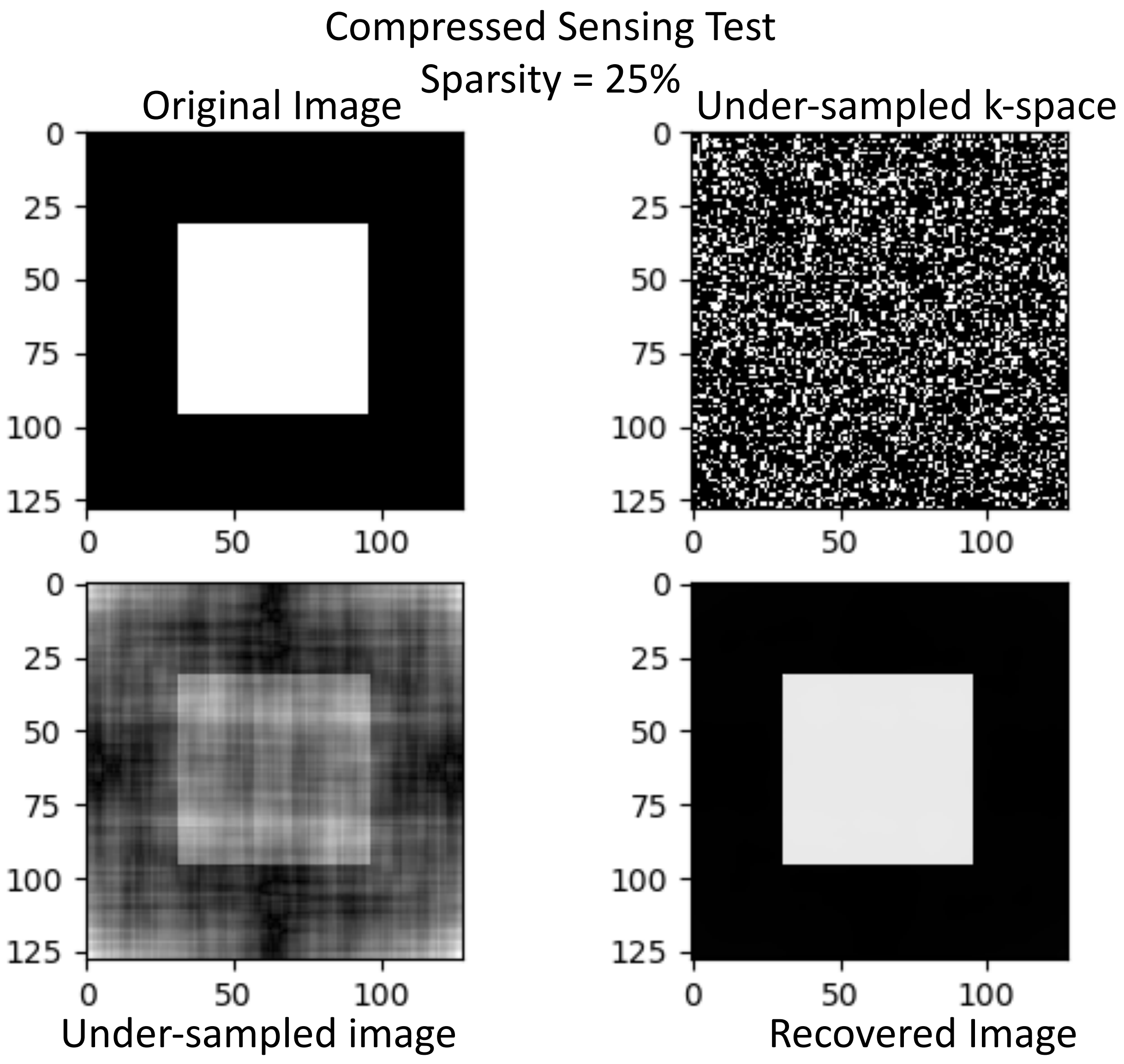
Minimization

If we randomly under sample our k-space, satisfies RIP

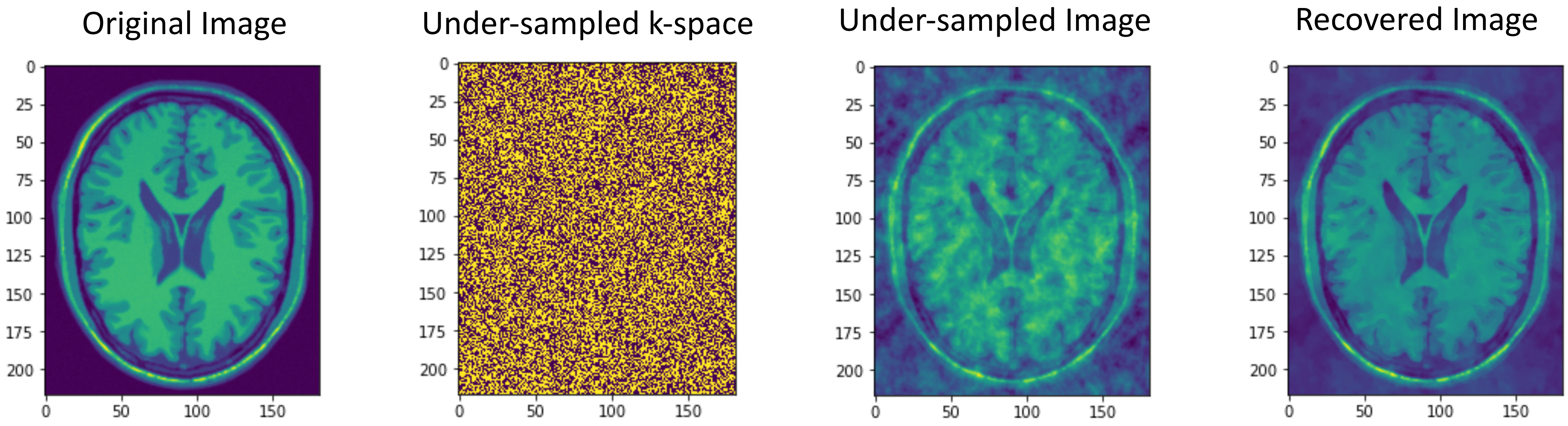
Split-Bregman Method

- Modified Bregman-Iteration method
  - commonly used for L_1 minimization

Used Haar wavelet for DWT



Brain image with Sparsity = 50%



Future Directions

Minimization performance depends on a length-scale gamma

-Characterize parameter space for relevant image types

Implement compressed sensing for 3-D k-space images

Compressed Sensing

Allows for recovery of data that is sparse in a wavelet basis from a randomly under-sampled "sensing" measurement basis

Applications far beyond MRI

1. Sensor Networks and data acquisition
   - Reduction of power consumption and reduced sensor processing
2. Astronomy
   - Reconstruction of signal from incomplete and noisy data
3. Linear coding
   - Reconstruction of corrupted transmissions

Good short intro for those interested:
Terrence Tao's blog on single pixel camera's