

- Q2) - Algorithm that 2 divide b by 2
- use a ~~for~~ ^{while} loop ($b > 0$;) $\{ \dots \}$
 - repeated square only work for powers of 2 (even power)
 - handle odd powers
 - condition for when b is odd
 - need a third variable to pass in as answer
 - any a^n can be written as $a_1 \times \dots \times a_b$
 - can be written using powers of a (a^{2^i})
 - at every odd # multiply ~~answer~~ ^{ans} by a
 - multiply ~~repeated squares~~ ^{ans} by repeated squares

Algo: ~~int~~ $a = \#1$

$b = \#2$

$ans = 1$

while ($b > 0$) {

if b is odd {

multiply ans by a // counts a for repeat \square

$a^2 = a$

$b /= 2$

}

// since it only works for pow of 2

P2) Proof: ~~Induction~~ ^{ans} $f(b) = a^b$

So we have a^b
if everytime we divide b ~~by 2~~ and update it's value
value: $b/2 = 2$, and the num b is even
we multiply a by a and update it's value
so $a *= a$, which is a^2
as we continue to loop, a^2 becomes $(a^2)^2$
and $((a^2)^2)^2 \dots a^{2^i}$, where i is the # of loops
eventually, $b/2$ will equal 1 and at this point,
when b is ~~even~~ ^{odd}, we can multiply ans which = 1, by
the current value of a , which is the answer
we are looking for

Now, when b is ~~even~~ ^{odd} and $b = 1$ we
can apply the same equation used for, $b = 1$
So say that ans is currently = to a^{2^i} and
we have run n loops since then, then

$$a = a^{2^{i+n}}$$

now we update ans , so $ans = a^{2^i} \times a^{2^{i+n}}$

OR $ans = a^{2^i + 2^{i+n}}$ so, essentially $ans = a^{2^i + 2^{i+n}}$

where $i \in \mathbb{Z}$ ~~on the loop~~ ^{on the loop} $ans = a^{2^i + 2^{i+n}}$ which can be written as
~~Since~~ $2^{i+n} = 1$ we can write
 $a^{2^i + 2^{i+n}}$ as any power of a , odd powers
inclusive.

Algo Analysis:

$a = s$
 $b = n$
 $ans = 1$ } $O(1)$

while ($b > 0$) { $O(\log b)$
 if ($b \% 2 == 1$) {
 $ans *= a$ } $O(1)$
 }
 $a += a$; $O(1)$
 $b /= 2$; $O(1)$ // brings us 2x closer to goal
}

$O(1) + O(1) + O(\log b)$
 $O(1) + O(\log b)$
 $O(\log b)$

For an algorithm to run in $O(\log b)$ it's loop has to get twice as close to its goal with each iteration. With $b/2$, b becomes half as large with each iteration, which brings us 2x as close to our goal of $b \leq 0$ (to exit loop)

$\frac{b}{2^i}$ where $i = \#$ of loops

denominator doubles with each iteration which leads to $O(\log b)$