Assignment 2

1 1.6c $\rightarrow \bigcirc \xrightarrow{0} \bigcirc^{1} \xrightarrow{0} \bigcirc \xrightarrow{1} \bigcirc \xrightarrow{0} \bigcirc \xrightarrow{1} \circledcirc^{1}$

1.6f $\rightarrow \bigcirc \xrightarrow{0} \circledcirc^{1,0}$

1.7b $\rightarrow \bigcirc^{1} \xrightarrow{0} \bigcirc \xrightarrow{1} \bigcirc \xrightarrow{0} \bigcirc \xrightarrow{1} \circledcirc^{0}$

1.7c $\rightarrow \bigcirc \xrightarrow{1} \bigcirc^{0} \xrightarrow{1} \circledcirc$
$\searrow \bigcirc \xrightarrow{0} \bigcirc \xrightarrow{0} \bigcirc \xrightarrow{0} \bigcirc$

2

1.16a $\rightarrow \bigcirc \xrightarrow{1} \bigcirc^{1} \xrightarrow{1} \bigcirc^{0} \xrightarrow{0} \circledcirc$

1.16b $\rightarrow \bigcirc \xrightarrow{1} \bigcirc^{0} \xrightarrow{1} \bigcirc^{0} \xrightarrow{1} \bigcirc^{1} \xrightarrow{0} \circledcirc$

3

1.19 a   1.6g + 1.6i

$\rightarrow \bigcirc \xrightarrow{1} \bigcirc \xrightarrow{1} \bigcirc \xrightarrow{1} \bigcirc \xrightarrow{1} \bigcirc \xrightarrow{1} \circledcirc$ — 1.6y

$\rightarrow \bigcirc \xrightarrow{0} \bigcirc \xrightarrow{1} \circledcirc$ — 1.6i – positions start at 0

NFA
$\rightarrow \bigcirc \xrightarrow{1} \bigcirc \xrightarrow{1} \bigcirc \xrightarrow{1} \bigcirc \xrightarrow{1} \bigcirc \xrightarrow{1} \bigcirc \xrightarrow{\varepsilon} \bigcirc \xrightarrow{0} \bigcirc \rightarrow \circledcirc$

.9b 1.6b

$\rightarrow \bigcirc \xrightarrow{1} \bigcirc \xrightarrow{1} \bigcirc \xrightarrow{1} \bigcirc \xrightarrow[1]{0} \bigcirc \xrightarrow{0} \circledcirc$

1.6m

$\rightarrow \bigcirc$

NFA

$\rightarrow \bigcirc \xrightarrow{1} \bigcirc \xrightarrow{1} \bigcirc \xrightarrow{1} \bigcirc \xrightarrow[1]{0} \bigcirc \xrightarrow{0} \bigcirc \xrightarrow{\varepsilon} \bigcirc$

4

.2]a

$\rightarrow \textcircled{1} \, 2a$ GNFA $(a^*b)^+a^*$

$b \uparrow \downarrow b$ $\Rightarrow$ $\rightarrow \textcircled{S} \xrightarrow{\varepsilon} \textcircled{1} \, \circlearrowright a^*$ $\textcircled{S} \xrightarrow{\varepsilon} \textcircled{2}$

$\textcircled{2} \, 2a$ $\downarrow b$ $\Rightarrow$ $\downarrow \varepsilon$

            $\circledcirc \xleftarrow{\varepsilon} \textcircled{2} \, 2a^*$ $\circledcirc$

$\textcircled{S} \xrightarrow{(a^*b)^+a^*} \circledcirc$ RegX : $(a^*b)^+a^*$

5
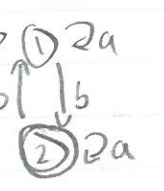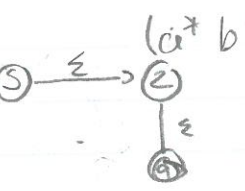
.1 $A = \{a^{n^3} \mid n \geq 0\}$ $a^3 = aaa$

    Assume A is ~~is~~ regular

    Choose $S = a^{p^3}$ where p is the pumping length

    By Pumping Lemma S can be decomposed

    into $S = xyz$ where

    ① $xy^iz \in A$

    ② $|y| > 0$

    ③ $|xy| \leq p$

    By 3, $|xy| \leq p$, so $|y| \leq p$

    Since $|xyz| = p^3$

    $|xy^2z| \leq p^3 + |y| = p^3 + p$

    But $p^3 + p < p^3 + 3p^2 + 3p + 3 = (p+1)^3$

    Since $p^3 < |xy^2z| < (p+1)^3$ as $|y| \geq 1$

    ∴ $xy^2z \notin A$ and contradicts the PL

      So A is irregular $\square$

5.2 $B = \{0^n 1^m 0^n \mid m,n \geq 0\}$

Assume B is regular

Choose $S = 0^p 1 0^p$ where p is the pumping length

By Pumping Lemma S can be decomposed into

$S = xyz$ where

    ① $xy^iz \in B$

    ② $|y| \geq 0$

    ③ $|xy| \leq p$

By ③ $|xy| \leq p$ y contains 0s

So $xyyz \notin B$ because there are more zeroes on
        the left side of the one than the right

∴ B is not Regular due to the contradiction
    in the ① Condition in the Pumping Lemma

Thm 2.9 Any CFL is generated by a CF grammar
in CNF - Cont.