Assignment 1 Aaron McCarthy

1 $1^2+3^2+5^2+\ldots (2n-1)^2 = 1/3\, n(4n^2-1)$

Base Case $1^2 = 1/3 \cdot 1(4 \cdot 1^2 - 1)$

$1 = 1/3(4-1)$

$1 = 1$

Ind Hyp Assume Conc is true for $i \le k$

Ind Step $\sum_{i=1}^{k+1}(2i-1)^2 = 1^2+3^2+\ldots(2k-1)^2+(2(k+1)-1)^2$ // by Def

$= 1/3\,k(4k^2-1)+(2(k+1)-1)^2$ // by IH

$= 4/3 k^3 - 1/3 k + (2k+1)^2$

$= 4/3 k^3 - 1/3 k + 4k^2 + 4k + 1$

$= 4/3 k^3 - 4k^2 + 11/3 k + 1$ $= 1/3 (k+1)(4(k+1)^2-1)$

$= 1/3 (4k^3 - 12k^2 + 11k + 3)$

$= 1/3 (k+1)(4k^2+8k+3)$

$= 1/3 (k+1)(4(k^2+2k+1)-1)$

$= 1/3 (k+1)(4(k+1)^2-1)$ ∎

$-x \ge 2$ $x \le -2$

$-x$

2 $P = (V, E)$

Euler's Formula $|V| + |F| - |E| = 2$

$|V| = n$

one face has min 3 edges, one edge has max 2 faces

$\therefore 2e \ge 3f \Rightarrow \frac{3}{2}f \le e$

$|V| + |F| = 2 + |E|$

$n + |F| \ge 2 + 3/2|F|$

$n \ge 2 + 1/2|F|$ ex: All faces triangular

$2n \ge 4 + |F|$ All but 2 edges have 2 faces

$2n > |F|$

∎

$2n = 4 + |F|$

3 Simple Grapsh are undirected
Since all vertexes are connected
You can get from any vertex
to any other vertex
∴ there exists a path from
any vertex of odd degree to any
other odd degree vertex

4 Full Binary Tree (FBT)
n int. Nodes 2n+1 total nodes
Suppose There Exists an FBT with
n internal Nodes and 2n+2 total
Nodes. In this case one internal
Node would have to have
3 leafs. This would make the
Tree not a binary tree ∎

5 Let $D(G) = (a,b)$

let start node $= s$

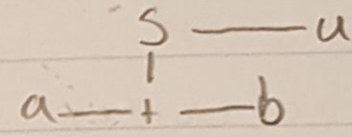
Let $bfs(s) = u$

Let $bfs(u) = v$

Let $p(a,b) =$ path from A to B

Let t be the closest node to s on $p(a,b)$

Case 1



$p(a,b)$ and $p(s,u)$ share no edges

Therefore $p(t,u)$ includes s

$\delta(t,u) = \delta(s,u) + \delta(t,s)$

$\therefore \delta(t,u) \geq \delta(s,u)$

$\delta(a,s) \leq \delta(s,u)$

$\therefore \delta(t,u) \geq (s,a)$

$\delta(s,a) \geq \delta(t,a)$

$\therefore (t,u) \geq (t,a)$

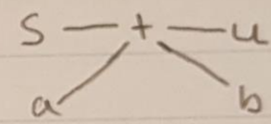
Since (a,b) is a diameter of G

$(a,b) = (u,b)$

Therefore the algorithm will always return a diameter

Case 2

$p(s,u)$ and $p(a,b)$ do share edges
Therefore $t \in p(s,u)$
Since u was the last vert. found by BFS
$d(t,u) \geq d(t,a)$
Since $p(a,b)$ is the longest path
$d(t,a) \geq d(t,u)$
$\therefore d(t,a) = d(t,u)$
and $d(u,b) = d(a,b)$



Therefore BFS(u) would return b or another node equally far away. Because $d(u,b) = d(a,b)$ and $d(a,b)$ is a diameter the algorithm will always return a diameter