MATH 137 Assignment 2 Due by: **11 a.m.**, Friday, October 2

Place your assignment in the drop box corresponding to the class section in which you are registered. Make sure your name and I.D. number at the top of the page are clearly written, and **underline your last name.** Submit your solutions in the same order as that of the questions appearing herein. Feel free to copy and use the assignment templates that appear on UW-ACE.

Attempt the problems on your own, and while it is okay to get help if you are stuck, you are expected to acknowledge, at the top of your assignment, the names of those who helped you. However, outright copying of assignments will be reported.

Your solutions must have legible handwriting, and must be presented in clear, concise and logical steps that fully reveal what you are doing. Use of good English sentences to explain your reasoning is highly encouraged.

Questions such as those to be handed in, as well as those that are recommended, may appear on exams. To avoid frustration and disappointment, *get started on your assignment early*.

The following problems from the book are **recommended**. Make sure you know how to solve problems such as these.

- Take the true-false quiz on page 73.
- Appendix D, pp. A32-A33 # 51, 67, 71, 73, 75, 79
- Section 2.4, pp. 117-118 # 1, 3, 4, 13, 17, 41

Hand in your solutions to the following 10 problems.

1. Recall the double angle formula for cosine: $\cos(2x) = 1 - 2\sin^2 x$.
   Put $x = \pi/12$ into this formula to show that $\sin(\pi/12) = \frac{1}{2}\sqrt{2-\sqrt{3}}$.

   Using the alternate formula $\cos(2x) = 2\cos^2 x - 1$ with $x = \pi/12$, find a formula for $\cos(\pi/12)$ similar to the one for $\sin(\pi/12)$.

   (Just in case you trust technology more, verify these formulas on your calculator for your own peace of mind. Don't forget to first put your calculator in radian mode.)

2. List the numbers $\arctan(\sin(n\pi/2))$ in simplified form, for $n = 1, 2, \ldots, 8$.

3. Using Figure 1 to help you, show that

$$\sin(\arctan x) = \frac{x}{\sqrt{1+x^2}} \text{ and that } \cos(\arctan x) = \frac{1}{\sqrt{1+x^2}}.$$

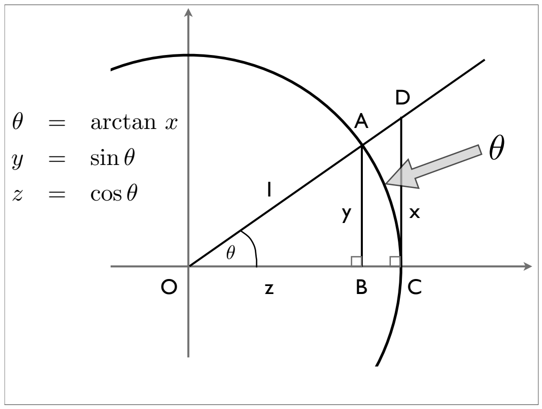

Figure 1: Circle diagram for $\arctan x$

4. Take your calculator and put it in radian mode. With $x = .5$, evaluate $\arctan x$. Then evaluate $\arcsin\left(x/\sqrt{1+x^2}\right)$. Repeat these two calculations with $x = 3$, and then with $x = 10$. Write a relationship involving arcsin and arctan as suggested by your empirical discoveries. Explain why your relationship must hold.

   Hint. A look at Figure 1 might help.

5. Sketch the following functions over their appropriate domains.

$$f(x) = \sin(\arcsin x) \text{ and } g(x) = \arcsin(\sin x).$$

   Hint. For the second function, note that it is 2π-periodic. So, if you can figure it out for x in the interval $[-\pi, \pi]$, then you can just replicate the graph from the piece you have. Just a reminder: if $-1 \leq t \leq 1$, then $\arcsin t$ is the one and only number θ in $[-\pi/2, \pi/2]$ for which $\sin\theta = t$.

6. Give a formal and logically coherent proof that $\lim_{x\to 1}(5x-2) = 3$.
   Make sure that your formal proof clearly specifies the roles that the ϵ's and δ's play. Without this kind of careful reasoning, painful as it may be, limits will always remain a mystery.

7. Give a formal and logically coherent proof that $x^5 \to 0$ as $x \to 0$.

8. Let $f(x) = \dfrac{1}{1+x^2}$.
We can sense that $f(x)$ will be as close to 1 as we like, provided we keep x close enough to 0. Today we would like to have $|f(x) - 1| < 1/10$. Prove that this happens if we make $|x - 0| < 1/3$.
Hint. You have $|x| < 1/3$, and you want $\left|\dfrac{1}{1+x^2} - 1\right| < 1/10$. Now keep reformulating what you want, until you discover that you actually have what you want.
Note. It was this process of reformulating the problem that also led to the discovery of $1/3$ as a suitable constraint on $|x|$.

9. Let $f(x) = x^3 - 2x + 6$. In order to prove that $f(x) \to 10$ as $x \to 2$, complete the following steps.

   (a) Show that $f(x) - 10 = (x-2)(x^2 + 2x + 2)$.

   (b) If $|x - 2| < 1$, prove that $|x^2 + 2x + 2| < 17$.

   (c) If $|x - 2| < 1$, prove that $|f(x) - 10| < 17|x - 2|$.

   (d) Now let ϵ be any positive number. For this ϵ, take δ to be the number $\min\{1, \epsilon/17\}$. In other words take δ to be either $\epsilon/17$ or 1, whichever is less. Prove that if $|x - 2| < \delta$, then $|f(x) - 10| < \epsilon$.

10. Let $f(x) = x^2 - 5$. We can sense that $f(x) \to \infty$ as $x \to \infty$. Precisely, this means that we can make $f(x)$ greater than any huge positive amount by taking x sufficiently large. Today we would like our huge positive amount to be 10^9. Find a sufficient value K such that $f(x) > 10^9$ when $x > K$.