UW Math Readiness Test

Solutions Guide

1) $4x^4 + 15x^2 - 4 = 0$
Trick: → $4(x^2)^2 + 15(x^2) - 4 = 0$
By the quadratic equation: $x^2 = \frac{-15 \pm \sqrt{15^2-4(4)(-4)}}{2(4)}$
Solve for x^2, and dispose of negative x^2 values, then solve x.
$x^2 = 0.25$ or $x^2 = -4$ (which is infeasible)
Hence $x = (0.25)^{1/2} = \pm 0.5$

2) $|2x + 1| = x + 3$ → $|2x + 1| >= 0$ for $x \geq -1/2$, thus we have the intervals $(-\infty, -1/2)$ and $[-1/2, \infty)$ to consider.

On the first interval, $2x+1 < 0$, so we solve the following
$-(2x + 1) = x + 3$ → $-2x - 1 = x + 3$ -> $-3x = 4$ → $x = -4/3 < -1/2$ so this is a valid solution

On the second interval, $2x + 1 >= 0$, so we have the following
$2x + 1 = x + 3$ → $x = 2 > -1/2$, a valid solution.

So the solutions are $x=-4/3$ and 2

3) $\frac{x^3 - 3x^2 + 2x}{x^4 - 8x} = \frac{x(x^2 - 3x^1 + 2)}{x(x^3 - 8)} = \frac{x^2 - 3x^1 + 2}{x^3 - 8} = \frac{(x-1)(x-2)}{x^3 - 2^3} = \frac{(x-1)(x-2)}{(x-2)(x^2+2x+2^2)} = \frac{(x-1)}{(x^2+2x+2^2)}$
Note that this uses the formula for factoring the difference of cubes. Make sure to state restrictions (where the denominator = 0), i.e. $x \neq 0$.

4) The midpoint of (-1,4) and (3,2) is calculated by for formula:
$(x_m, y_m) = (\frac{x_2 - x_1}{2}, \frac{y_2 - y_1}{2}) = (1, 3)$
We wish to find the equation of the line joining this point with (2,3)

Slope is rise over run → (0/1), this implies that b= 3 and $y = 0x + 3 = 3$.

5) First find the equation of the line that goes through (-1,2), with slope 1/2.
$y = 1/2x + b \rightarrow$ at $x = -1$ $y = 2$, implying that $b = 2+1/2 = 5/2$
So $y = 1/2x + 5/2$
Rearrange to find $x = 2y - 5$ (1)
The equation of the circle is of the form $x^2 + y^2 = 5^2$ (2)
Substitute (1) in (2)
$(2y-5)^2 + y^2 = 25$
So $y(5y-20) = 0$
So $y = 0$ or $y = 4$
Substitute y=0 and y=4 into (1) to get coordinates (-5,0) and (3,4) as points of intersection.

6) $4kx^2 + 3x + k = 0$, using the quadratic equation we find
$x = \frac{-3+\sqrt{3^2-4(4k)(k)}}{2(4k)}$ and $\frac{-3-\sqrt{3^2-4(4k)(k)}}{2(4k)}$
Since we want the roots to be real, we distinctly require that $3^2 - 4(4k)(k) \geq 0$
Note that this is called the discriminant: the term under the square-root

$9 - 16k^2 \geq 0 \rightarrow 16k^2 \leq 9 \rightarrow k^2 \leq 9/16 \rightarrow |k| \leq 3/4$

7) $4^{7x} = 32^{3x-1} \rightarrow (2^2)^{7x} = (2^5)^{3x-1} \rightarrow 2^{14x} = 2^{15x-5} \rightarrow \log_2(2^{14x}) = \log_2(2^{15x-5})$
$14x = 15x - 5 \rightarrow x = 5$

8) $\log_4(x + 4) - 2\log_4(x + 1) = ½$, $x > 0$

$\log_4(x + 4) - \log_4(x + 1)^2 = ½$, $x > 0$

$\log_4\frac{x+4}{(x+1)^2} = ½$

$\frac{x+4}{(x+1)^2} = 4^{1/2}$

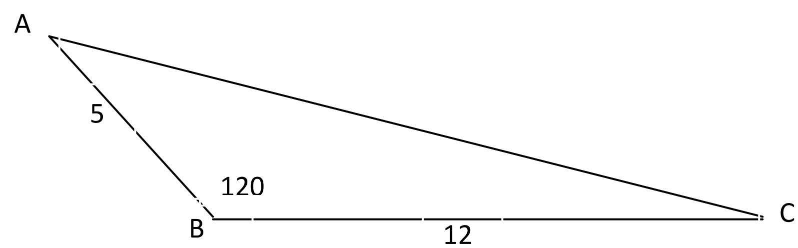
$x + 4 = 2(x+1)^2$

$2x^2 + 3x - 2 = 0$
$x = ½$ or $x = -2$ (but since $x > 0$, $x = -2$ is not feasible) so $x = ½$

9) $e^{3x-2} = 4$ → use ln on both sides, then solve for x. See question 7 as an example
$3x - 2 = \ln 4$
$x = \frac{2+ln4}{3}$

10) The trick is to realize that the two triangles are similar. You can find the base of the smaller triangle using Pythagoras (call this value b), then use similar triangles to realize that r/b = x/t
Let θ be the angle between t and x.
We know that $\sqrt{r^2 - h^2}$ is the adjacent side of θ for the smaller triangle.
Note that $\cos\theta = \frac{t}{x}$ → $x = \frac{t}{cos\theta}$
Since $\cos\theta = \frac{\sqrt{r^2-h^2}}{r}$ from smaller triangle, you get $x = \frac{rt}{\sqrt{r^2-h^2}}$

11)

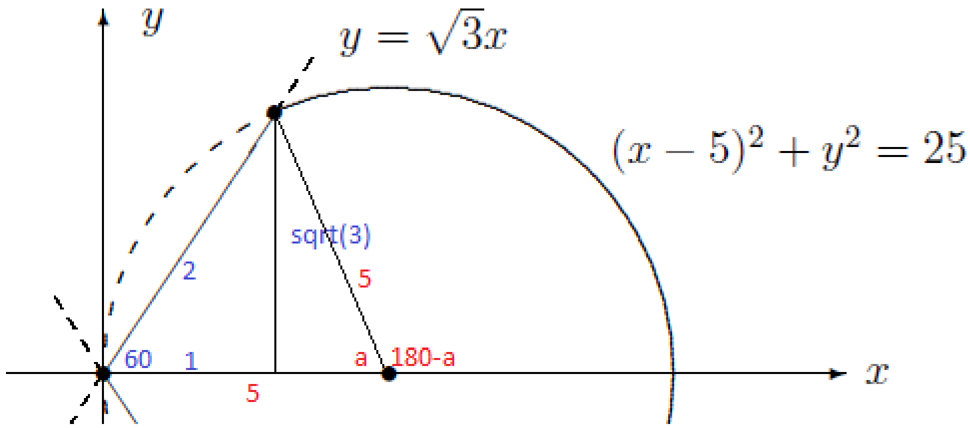


Use the cosine law: $AC^2 = AB^2 + BC^2 + 2(AB)(BC)\cos(b)$
Thus, $AC^2 = 5^2 + 12^2 + 2(5)(12)\cos(120)$ → Solve for AC = 16.33

12) The first step is to find the perimeter of the circular part of the figure. We will do this by finding the circumference of the entire circle, then determining what percentage of the circle's circumference counts towards the required figure's perimeter

Notice that the radius of the circle is 5 (the square-root of 25), and that we can draw a vertical line from the point at (1, sqrt(3)) to the circle's radius.

Using special triangles, we can find that the angle near the origin is 60-degrees, and that the hypotenuse of the triangle is 2.



Then use the Sine Law to determine the value of angle a: sin(a)/2 = sin(60)/5
Solve to get a = 20.2679

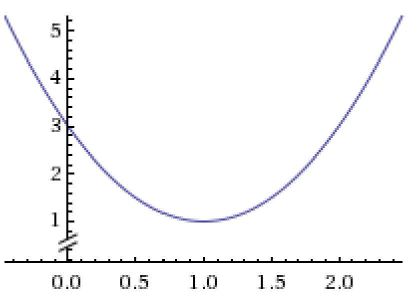
The total angle of the circular part of the figure is therefore 360-2*a = 319.4642

Therefore, the length of the rounded end of the figure is:
2*pi*5*(319.4642)/360 = 27.87851

Add 2+2=4 to this value to determine the total perimeter of the figure = 31.87851

13) Notice the following before sketching:

- The function is a parabola that opens upwards
- The y-intercept is (0, 3)
- To find the vertex of the parabola:
  - Differentiate → f'(x) = 4x – 4
  - Set to 0 to find the vertex: x = 1, so y = 1.
  - The vertex is (1, 1)
- To find the roots, use the quadratic formula: No solution



(x from –0.5 to 2.5)

14) $\cos^2 x - \cos x = \sin^2 x \rightarrow \cos^2 x - \sin^2 x - \cos x = 0 \rightarrow \cos(2x) - \cos x = 0 \rightarrow \cos(2x) = \cos(x)$

This occurs at $x = 0, 2\pi$, the rest from the unit circle are $x = 2\pi/3$ and $4\pi/3$

15) A(x) = area of square + area of circle

A(square) = $[(36-x)/4]^2 = (9 - \frac{x}{4})^2$

$x = 2\pi r$, which is the perimeter, the area is πr^2, where $r = x/(2\pi)$

A(circle) = $\frac{x^2}{4\pi}$

Total area = A(square) + A(circle) = $(9 - \frac{x}{4})^2 + \frac{x^2}{4\pi}$