MATH 137 Assignment 1 Due: **11 a.m.**, Friday, September 25

Your assignments come in two parts. The first part is a selection of problems from the textbook that are recommended for you to solve, but are not to be handed in. Solutions to most of the recommended problems are in your Students' Solutions Manual, but it would defeat the purpose if you peeked at the solutions before making an honest attempt to solve the problem on your own. The second part consists of numbered problems for you to hand in. **Questions such as those to be handed in, as well as those that are recommended, will appear on exams.**

To avoid loss of your assignment, be sure to place it in the drop box corresponding to the class section in which you are registered. To help us sort your assignments and keep accurate records, make sure your name and I.D. number at the top of the page are clearly written, and **underline your last name.** To facilitate the marking of your assignment, submit your solutions in the same order as they appear herein.

Feel free to copy and use the assignment templates that appear on UW-ACE.

You will get the most benefit from the assignments if you attempt them on your own. While it is okay to get help if you are stuck, you are expected to acknowledge, at the top of your assignment, the individuals who helped you. However, any outright copying of assignments, in whole or in part, is an act of cheating, and it will be reported.

Your solutions must have legible handwriting, and must be presented in clear, concise and logical steps that fully reveal what you are doing. Use of good English sentences to explain your reasoning is highly encouraged.

The exercises in this first assignment are intended to refresh your memory of things you are likely to have learned in high school, but some are not immediately straight forward to do. Please be sure to start this assignment well before the due date, so that you can have time to refresh your memory and skills, and get help if need be. If you start your assignment only a day or two before the due date, then you could end up feeling frustrated and unsatisfied. **Get working on your assignment as soon as possible.**

The following exercises from the book are **recommended**. Do not hand these in but make sure you know how to solve problems such as these.

- Section 1.3, pp. 43-45 # 27, 28, 35
- Appendix A, pp. A9-A10, # 33, 35, 37, 45, 55, 61, 63

- Section 1.6, pp. 70-72 # 49, 53
- Review, pp. 74-75 # 23, 25
- Problems, p. 81 # 3, 7, 9,13

Hand in your solutions to the following 10 problems.

1. If $-2 \leq x \leq 1$, show that $|2x^3 - 4x^2 + 3x - 1| \leq 39$.

   Hint. Use the very important triangle inequality.

2. The Heaviside function H is the step function given by the formula:

$$H(x) = \begin{cases} 1 \text{ if } x \geq 0 \\ 0 \text{ if } x < 0. \end{cases}$$

   (a) Sketch the function $y = H(x(x-1)(x-2)(x-3))$.

   (b) Prove that $|x| = xH(x) - xH(-x)$ for all x.

   (c) Sketch the function $y = (1-x^2)(H(x+1) - H(x-1))$.

3. Let $f(x) = \dfrac{x^3 + 3x^2 + 3x + 2}{x^2 - 1}$.

   (a) Find the x-intercepts and the locations of the vertical asymptotes of f.

   (b) Determine the sign of f on the intervals between the intercepts and vertical asymptote locations.

   (c) Use long division to explain why f has a slant (oblique) asymptote and to find the slant asymptote.

   (d) For large positive values of x use the result of your long division to explain whether $f(x)$ is above the slant asymptote or below it. Do the same for large negative values of x.

   (e) With the above information, sketch the graph of f. There is no need to worry about derivatives at this time.

4. In the xy-plane, sketch the region of points (x, y) that satisfy

$$H(x+y) + |x+y| \leq 2.$$

   Note: H is the Heaviside function of Problem 2.

5. In the xy-plane, sketch the region of points (x, y) that satisfy $|y^2 - x^2| \leq 1$.

6. Let $f(x) = x^2 - 3x + 2$.

   (a) Sketch the graph of f.

   (b) Prove that f is not one-to-one on $\mathbb{R}$, but is one-to-one on the interval $[3/2, \infty)$.

   Hint. For the second part of the problem it helps to complete the square in the formula for f.

   (c) Sketch the graph of $g(x) = |x^2 - 3x + 2|$.

   (d) Show that g is not one-to-one on the interval $[3/2, \infty)$.

7. Put the numbers $5^{\ln 4}, 2^{10}$ and $4^{\ln 5}$ in increasing order, without using your calculator, and explain your reasoning.

8. Let $f(x) = \dfrac{3}{3 + 2e^{-x}}$.

   (a) As x rises from $-\infty$ to $+\infty$, the function $3 + 2e^{-x}$ decreases from $+\infty$ towards the asymptotic value of 3. What does $f(x)$ do as x rises from $-\infty$ to $+\infty$?

   (b) Using your answer from part (a), sketch the graph of f, and indicate the y-intercept.

   (c) Find a formula for the inverse function of f, and indicate the domain of this inverse function.

9. Let $f(x) = \ln(x + \sqrt{x^2 + 1}\,)$.

   (a) Explain why the domain of f is all of $\mathbb{R}$.

   (b) Prove that $f(-x) = -f(x)$, and thereby that f is an odd function.

   Hint: It's easier to check that $f(x) + f(-x) = 0$ for all x.

   (c) What is the range of f?

   (d) If $s < t$, explain why $f(s) < f(t)$.

   (e) A function with the property in (d) is called an *increasing* function. Increasing functions are one-to-one, and thereby have an inverse function. By solving the equation $y = \ln(x + \sqrt{x^2 + 1})$ for x in terms of y, find a formula for the inverse function of f.

10. The Gateway Arch in St. Louis is reportedly built to conform approximately to the function

$$h(x) = 210 - 10(e^{x/30} + e^{-x/30}),$$

where x is the distance, in meters, from the centre of the arch along the ground, and $h(x)$ is the height of the arch, in meters, at that distance x. Figure 1 is a picture of the function h, but for a picture of the arch itself, see Stewart on page 256.

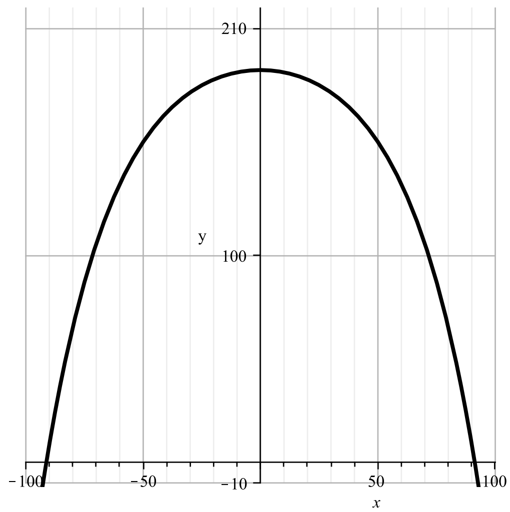

Figure 1: The function h

(a) Verify that h is an even function (which is a good idea if the arch is to be symmetrical).

(b) How high is the arch at its maximum height? Just answer this by inspection. There is no need to use derivatives.

(c) How far apart are the ends of the arch? Round off your answer to one decimal place.