Modeling Extraversion

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1 What is extraversion?

I am obliged to first state the commonly accepted definition of extraversion, in order to reference any earlier work in in psychology. Extraversion is a member of the "Big Five" personality traits, and is typically measured by asking questions such as "whether [they] enjoy the company of others, attend parties frequently, are talkative, outgoing, gregarious, and enthusiastic" [3]. This measure will parallel mine, insofar as they should vary together. I'll make no argument about the nature of causality in such internal phenomena.

The definition I'll use here is that extroversion is *the amount of social time a specific person would like to spend in a given period of time.* Certainly many individuals don't consciously deal with their extraversion, so defined, but still act based on it. There is no direct method to observe extraversion (except possibly the error-prone method of asking them).

2 Measuring extraversion

Extraversion is a rather unobservable concept, and must thus be dealt with via proxies. One measurable consequence of a person's extraversion is *their tendency to increase or decrease the amount of time they spend with other people, given the amount of time they are currently spending socially*, as measured by specific actions, such as choosing to go to social events, seeking social activity, or choosing to begin or end a relationship. This is the outcome of extraversion I will be modeling in this paper. Extroversion might also be measured by the celerity in which individuals seek out friends when they arrive in a new city (where they have no prior relationships). [1]

Observables such as these will have to step in place of my initial conception of extraversion in any concrete sociological study. This is by the fundamentally unobservable nature of the original psychological trait extraversion. Figure 1 has something interesting.

One issue with actually measuring this concept in the wild is that all the observables of the amount of extraversion are confounded by many other causes. For example, individuals may maintain some relationships because they are coerced to do so, e.g. if they feel it is their duty. They may choose to begin or end a relationship because of multifarious reasons, not directly connected to their tendancy towards a specific level of extraversion.

[1]This observable is mitigated by the preponderance these days of non-local means of socializing. A person's use of social media would greatly impact this measure, obfuscating the effect measure of extraversion.

3 Extraversion “in the wild”

Figure 1 shows how much time men spend alone, taken from the American Time Use Survey, as a function of their age [2].

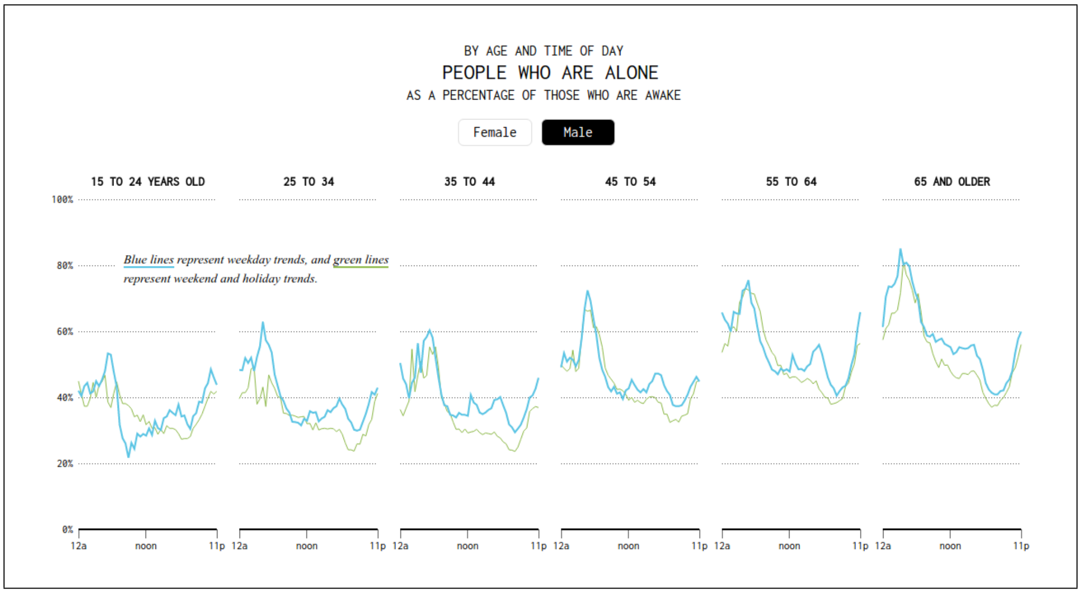


Figure 1: The time men spend alone as a function of their age (ATUS)

There is research indicating that extroverts[3] are happier [6], that they have more friends, they are more central, and their friends are more likely to be strong (with strong closure) [7]. Extraverts also have more extroverted friends than introverts[4] do [2].

There is higher extraversion in European and American compared to Asian and African cultures. Cultures whose members (on average) score high on extraversion have democratic values, an emphasis on individualism and self-expression, higher subjective well-being, have higher rates of obesity, and lower rates of suicide. [8] It’s obvious that the first sentence of this paragraph correlates with the rest of the paragraph, but it was still the most powerful cross-cultural effect reported.

3.1 More ideas for observation

- Observe, via the American Time Use Survey (or the corresponding European one), individuals social habits over time. 1) *Are they relatively consistent* over time? 2) *If they’re not, are there indicators of the reasons why?* What did they do instead of social time? What did they replace with social time?

- What is the actual distribution of extraversion, in various senses, in our and other societies? 1) *How much social time* do people get on average? What’s the spread? 2)

[2]https://flowingdata.com/2017/06/26/alone-time/
[3]In the language of my definition this coincides with relatively more extroverted individuals
[4]In the language of my definition this coincides with relatively less extroverted individuals

How many friends do people have? What's the spread? 3) What's the distribution in *the actual psychological trait* extraversion?

- What do friendship networks look like on a macro scale? What am I actually trying to predict?

4 Survey results

Eight future sociologists answered my survey.

4.1 Introverted vs. Extraverted

Four identified as introverted, four don't identify as either. In general people had many different definitions of what it means to be extraverted or introverted. For example, one respondent said they don't identify as either because they're not shy about social time, but they won't divulge personal information to others they don't know well.

Their lack of specific identity was mostly linked to the vagueness of the concept itself.

4.2 Preferences for the amount of time spent socially

One person in particular indicated that their preference for social time is varying, and what may be ok one week isn't necessarily enough the next week, or may be too much. This breaks any usage of my model in a real-world scenario.

Six of the eight people said they spent enough time socially in the last week, with the other two saying they spent too much time socially. This is also much different than the effects of my initial assumptions, as most people should have too little, or about the right amount of social interaction.

5 Assumptions for modeling extraversion

I'd only like to use simulation if I feel it's representing some aspect of social life which actually exists. This representation doesn't have to be the extent of full reproduction, however I would like to justify that real-world relationship building and this model are both "models," in the logical sense, of a larger and not too general theory of relationship decision-making.

5.1 Scheduling

Individuals very often have relationships which are long-term, and which are built upon an agreement of some sort of recurring social interaction, for example that they will meet every Wednesday at yoga class. In this situation, change will only be exacted over a time-period of weeks or months, as individuals slowly, and sometimes painfully, rearrange their social schedule. This introduces many factors other than extraversion into the process of changing their social activity.

5.2 Random events

Additionally, social interactions *do not* always come about because of decision. Sometimes the beginning or end of a relationship is incited by an event completely out of either person's control, and completely unpredictable given any sensible model parameters.

5.3 Heterogeneity of potential friends

Some pairs of people have a very low chance of friendship, some pairs comparatively higher. As an extreme example, consider individuals who don't speak eachother's language. Less extreme, but still pervasive, is whether they have anything they would both enjoy talking about, whether they have something to offer eachother, etc.

5.4 An excursion on loneliness research

Bradburn (1969) noted that 26% of his U.S. sample surveyed by phone reported feeling very lonely in the past few weeks. In a more recent study on college students, Cutrona (1982) found that 75% ofthe students reported occasional mild feelings of loneliness, and 40% reported severe feelings of loneliness. All of these data indicate that loneliness, like friendlessness, is a familiar aspect of life for many people. [1] *Note: copied and pasted.*

There is also a weak, possibly nonexistent, correlation between the number of friends a person has and their loneliness [1, p. 229], suggesting that people are indeed heterogeneous with respect to their tolerance of such situations.

5.5 Ability to make friends

It's generally agreed among psychologists that persistently lonely individuals are much more likely to have deficits in social skills [10] [1, p. 232]. Many such studies operate by asking the lonely individuals why they think they don't have as many friends as they'd like, but more interestingly others measure verbal behaviors of lonely students, observing that they tend to "speak less, use inappropriately high levels of self-disclosure, and offer advice more than do nonlonely students." [1, p. 232]

5.6 Social activity imposed by others

5.7 Variable susceptibility to new friends

It's also clear that individuals make themselves more or less available to the chance of gaining a relationship, relative to the extent of dissatisfaction they have with their current social status-quo. So a specific social relationship is likely to begin only if both individuals want a relationship.

5.8 Does everyone act in a similar way based on similar discomfort?

This brings to the fore many arguments of assumptions. For example, some individuals will repeatedly report dissatisfaction with how much they socialize, but will remain too shy to take action in any meaningful way.

It's also often the case that restrictions in the amount of free time available to a person restricts their ability to correct their social situation. They simply don't have the time available.

5.9 Restricted free time

5.10 Variability of extraversion, so defined

With extraversion defined as the amount of time one wishes to spend socially, we must concede that this is a variable attribute of any given person through time. One could imagine that some activity not included in this model[5] reduces the need for the individual to interact socially.

There may also be inherent variability in a person's expectation of social interaction. It's been observed that unmarried people tend to feel lonelier when they are around married people, presumably because their expectation of social interaction is higher [1, p. 230].

5.11 Preferences, ulterior motives

5.12 Others modeling extraversion

To my knowledge, all simulation studies of extraversion do not consider an intrinsic threshold for social interaction time. In one case it was simply modeled as a propensity to make friends [5].

6 Analytic description of the "scheduling with extroversion" model, SE1

We will assume that people are comfortable with a certain amount of recurring social activity. We'll also assume that the only social activity individuals engage in is that of a recurring kind. Thus the amount of their time that two individuals spend together change only at discrete moments in time. More formally, assume there are N individuals, labeled $\{1, 2, 3, \ldots N\} \equiv \mathbb{P}$. At each time t, we define an allocation of time $\sigma : \mathbb{P} \times \mathbb{P} \times \mathbb{R}^+ \to [0, 1]$, denoted by $\sigma_t(i, j)$. We assume $\sigma_t(i, i) = 0$ for any $i \in \mathbb{P}$, as alone time is by definition not social time. We will denote by $\boldsymbol{\sigma}_t$ the matrix which corresponds to this function, such that $[\boldsymbol{\sigma}_t]_{i,j} = \sigma_t(i, j)$.

HOW TO FIT THIS IN Additionally we will introduce $\boldsymbol{L}$, a matrix which defines how much each person enjoys specific others. That is, $\sigma_t(i, j) \neq \sigma_t(j, i)$. We would still assume that $\sigma_t(i, j)/\sigma_t(j, i)$ is constant across time if the terms are positive. Specifically, if i and j spend time τ together, i will get $\sigma_t(i, j) = \boldsymbol{L}_{i,j}\tau$ social satisfaction, and j will get $\sigma_t(j, i) = \boldsymbol{L}_{i,j}\tau$ social satisfaction. [6]

Let $\sigma_t(i) = \sum_{j \in \mathbb{P}} \sigma_t(i, j)$ be the total amount of social satisfaction i gets at time t. Define $s(i, t)$ to be the stress of node i at time t:

$$s(i, t) \equiv \mu\left(\frac{\alpha_i - \sigma_t(i)}{\sigma_t(i)}\right) = \mu\left(\frac{\alpha_i}{\sigma_t(i)} - 1\right)$$

where $\mu(x)$ is some bounded weighting function, increasing for $x > 0$, with $\mu(0) = 0$. The argument of μ is the percent individual i needs to change her time socializing to be satisfied. Typically we will take $\mu(x) = |\text{logit}(x) - 1/2| = |\frac{1}{1+e^{-x}} - 1/2|$. Intuitively, $s(i, t)$ is a measure

[5]The most obvious for me is social interaction online, through chat, video games, etc., or watching TV, as is a typical relief of loneliness for the elderly.

[6]By this assumption, we have the requirement $\sigma_t(i, j)\boldsymbol{L}_{j,i} = \sigma_t(j, i)\boldsymbol{L}_{i,j}$.

of how much more or less they'd like to change the amount of time they are spending socially, and corresponds to the inverse of the expected amount of time until they take some action to change their relationship network.

We assume that the amount of time until a person takes some action to change their social network is memoriless[7]. Then $T(i,t)$, the amount of time until i takes some action to change her social situation, has the distribution $\exp\left[\frac{1}{s(i,t)}\right]$. By this definition σ_t is piecewise constant with exponential jump times.

At time $t = 0$, the first change in decision will happen in time $T(t) \sim \min_i[T(i,t)] \sim \exp\left[\sum_{i\in\mathbb{P}} s(i,t)^{-1}\right]$[8]. An important descriptive function of this network for the forgoing analysis is related to the Bonacich centrality measure: $\mathbf{B}(p) = \sum_{k=1}^{\infty}(p\boldsymbol{\sigma})^k = [\mathbf{I} - p\boldsymbol{\sigma}]^{-1}$. Intuitively $\mathbf{B}_{ij}(p)$ represents how likely i is to run into j, modeled as a contagion (possibly of information) flowing through the network.

We will assume that at the time of decision the individual looks at her situation and tries to find a better social situation. She can cut or add to a single relationship[9]. Adding a new tie must be consensual. This means that a person who doesn't want more social interaction is unlikely to acquire new relationships. Breaking a tie has no such constraint, as either party can choose to stop a relationship.

If the individual who is deciding is over-socialized ($\sigma_t(i) > \alpha_i$), they will seek to break off a relationship. To choose the relationship, they will take into account their connectivity to the individual, i.e. $\mathbf{B}(p)_{ij}$. In particular, we assume that the more connected a person j is with the actor's social network, the less likely i is to breaking j out of their life, because it'd have more social consequences.

$$P(i \text{ break connection with j }) \sim I[\sigma_t(i,j) > 0]\left(\frac{1}{\mathbf{B}(p)_{ij}} + \epsilon_1\right)$$

This probability distribution will be normalized, such that

$$P(i \text{ breaks a connection with anyone}) = 1$$

.

If the individual is under-socialized ($\sigma_t(i) < \alpha_i$), they will seek to add a new recurring tie. This tie is influenced largely by the structure of their social network, via $\mathbf{B}(p)$, with some small chance of running into any person in the network, regardless of network structure. Analytically, we say that

$$P(i \text{ alters tie with } j) \sim (\epsilon_2 + s(j,t)I[\sigma_t(j) < \alpha_j])(\epsilon_3 + \mathbf{B}(p)_{ij})$$

normalizing such that $\sum P(i \text{ alters tie with } j) = 1$. Note that there is no restriction that i doesn't know j, so this decision may strengthen a relationship the focal actor already has.

If i alters their tie with j, we will set their tie to $U([\alpha_i - s(i,t), \alpha_j - s(j,t)])$, simulating a sort of compromise between what they both want. If $\alpha_i - s(i,t) < 0$ and $\alpha_j - s(j,t) < 0$ (there's only a small probability of this), then we set their tie equal to the average ties of them and their friends (**this is a strange condition. please fix!**).

[7]This property is deep and intrinsic to many social systems. It expresses only that the probability for change in any time period depends only on the stress $s(i,t)$ of the individual at that time, and does not depend on the value of this stress at any time in the past.

[8]This is a standard property of the exponential distribution

[9]The fact that they can only make one change reflects the effort involved in changing the nature of an existing social relationship.

7 Analytic properties

One easy analytic consequence of this model is that long term best friends (those who have had no other friends, and have interacted without changing structure for a long time), must like eachother's company the same amount, and furthermore have the same extroversion.

8 Simulation methodology

8.1 General comments

8.2 Extroversion

The methodology for simulation is relatively straightforward. First, initialize α_i and σ_0. In each example below we initialize σ_0, the relationship network, to $\mathbf{0}$, so the individuals do not know eachother, or anyone else for that matter. We then compute the first time of any relationship change, drawing each node i the time of next change from $\exp\left[\frac{1}{s(i,t)}\right]$ and finding the minimum. The individual $i*$ which corresponds to this minimum will now make a change to their network. Then, following the analytic description outlined above, alter the ties between $i*$ and their chosen j. Then wash, rinse and repeat.[10] [11]

Given a population with extroversion α_i, there exists a configuration that minimizes $\sum_i s(i,t)$, and some, maybe different configuration σ which minimizes the effective stress of the network, $[\sum_{i\in\mathbb{P}} s(i,t)^{-1}]^{-1}$. The minimum discomfort, analytically, is not an easy thing to come up with, and depends on the chosen μ. Based on simulation there's reason to believe that there are local minima in this system which trap the network in an uncomfortable state.

8.2.1 More random noise

Another obvious generalization is to allow random disturbances to modify network structure, outside of the relationships among individuals.

9 Simulation results

9.1 Summary

From modeling the above system we saw, somewhat surprisingly, no extraversion bias, as observed in [2].

[10] Note that when calculating $\mathbf{B}(p)$ we always use $p = \frac{1}{2\lambda_1}$, where λ_1 is the largest eigenvector of $\boldsymbol{\sigma}$. This gives us some effects social contagion, while ensuring that $\mathbf{B}(p)$ actually converges (we need $p < \frac{1}{\lambda_1}$).

[11] Because the process of changing ties is memoryless, it's not problematic to compute these first changes over and over again for the updated networks.

9.2 Balance

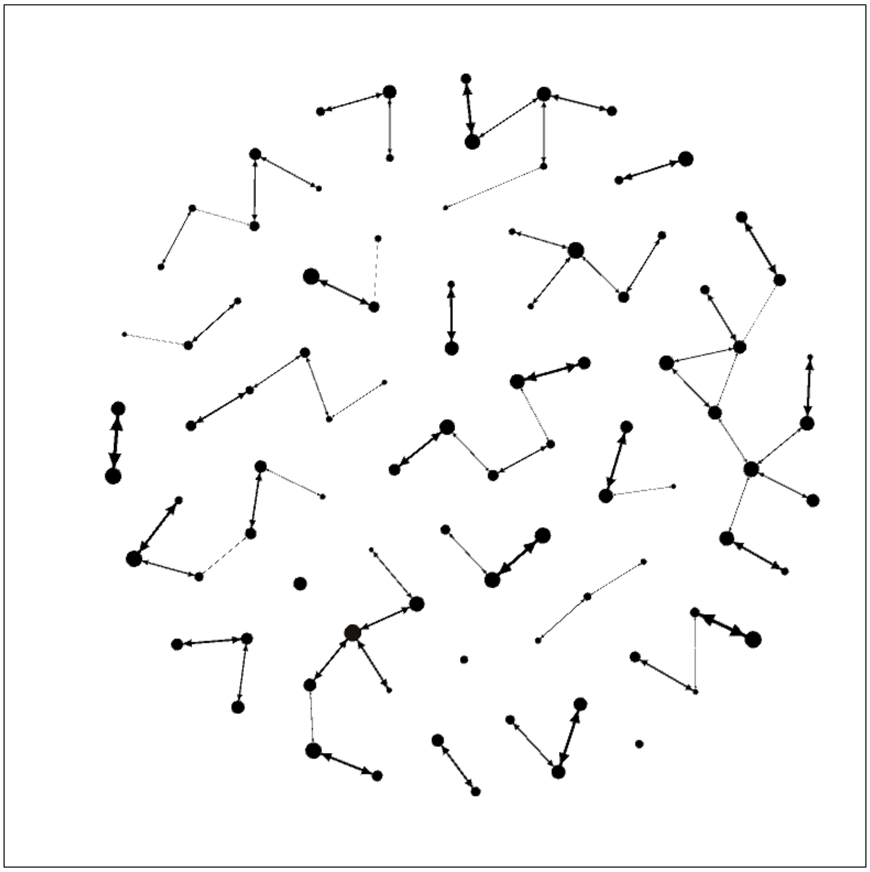


Figure 2: This figure represents a network of individuals with uniformly distributed extraversion ($\alpha_1 = 0$, $\alpha_1 = \frac{1}{N}$, ... , $\alpha_{N-1} = \frac{N-1}{N}$, $\alpha_N = 1$). This figure is a snapshot σ_{50} at $t = 50$. The size of nodes represents the extraversion of the individual, and the tie width the extent of their social obligation.

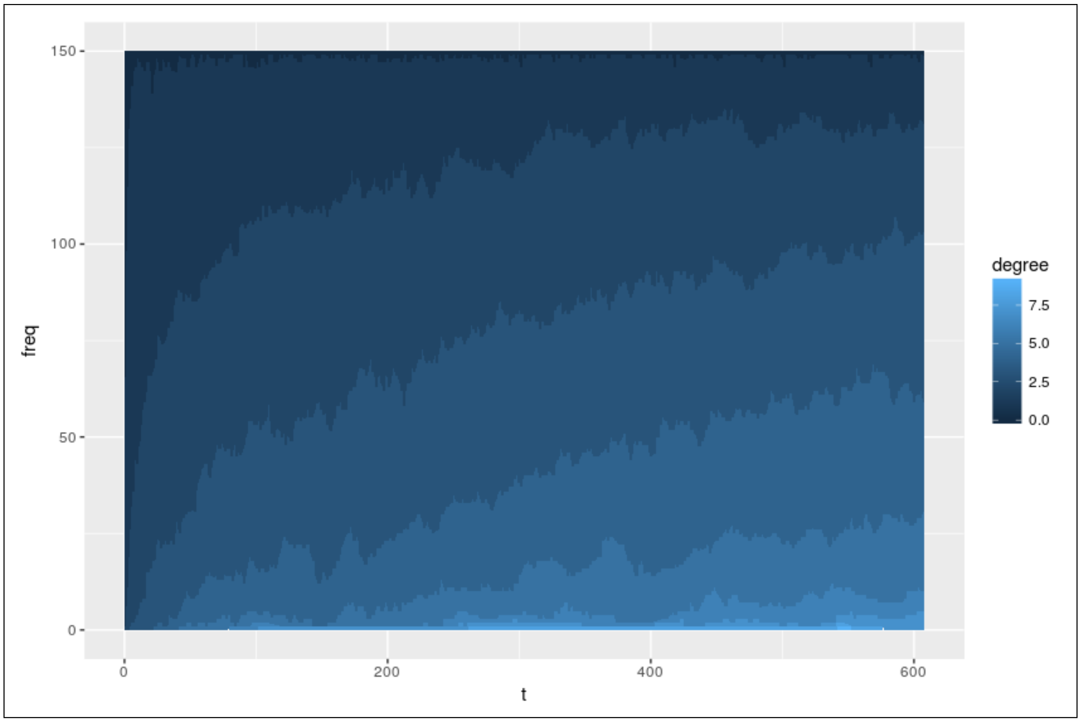
9.3 Dynamics of degree distribution

Figures 3a, 3b, and 4 show how the degree distribution of a network develops over time under this model. In figure 3a we show long-term dynamics of a system of 100 individuals with $\alpha = \{0, 0.1, 0.2, ..., 9.9\}$. In figure 3b we add 50 new members to a relatively stable cohort to see what happens. We can also simulate a natural disaster, an army draft, the graduation of a subset of a school's population, a disease[12], etc. by removing some number of individuals randomly from the population. We can then assess the effect on socializing (see figure 4).

To get some idea of how the equilibrium network's efficiency changes with the distribution $\{\alpha_i\}$, I've compared $s \in [0, 10]$, and $\alpha_i \in [10 - s, 10]$, uniformly. I ran each spread 10 times

[12]It's likely that a disease would spread across social networks, having different implications in these models than random selection, as in a disaster.

until $t = 50$ to get some notion of the uncertainty associated with each $\{\alpha_i\}$ distribution. The results are shown in figure 5.



(a) Degree distribution evolving over time in a network of 100 individuals whose extraversion is spread evenly between $\alpha_1 = 0$ and $\alpha_{100} = 1$

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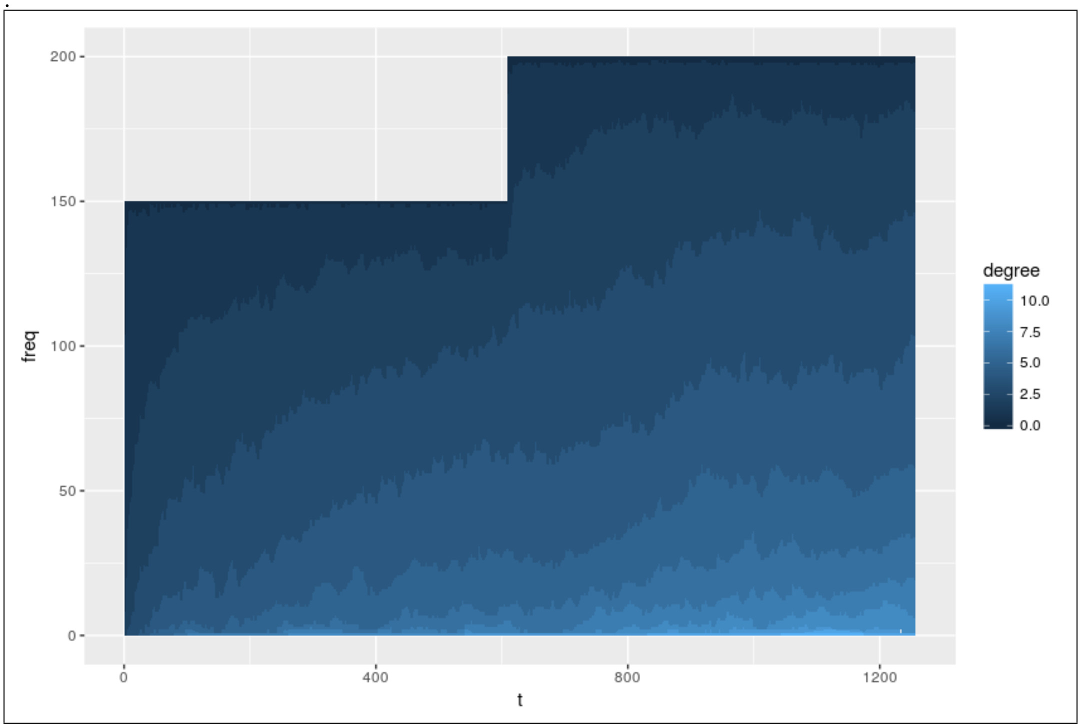

(b) Shown here is the effect of adding 50 new members to a relatively stable social collective of 150.

Figure 3

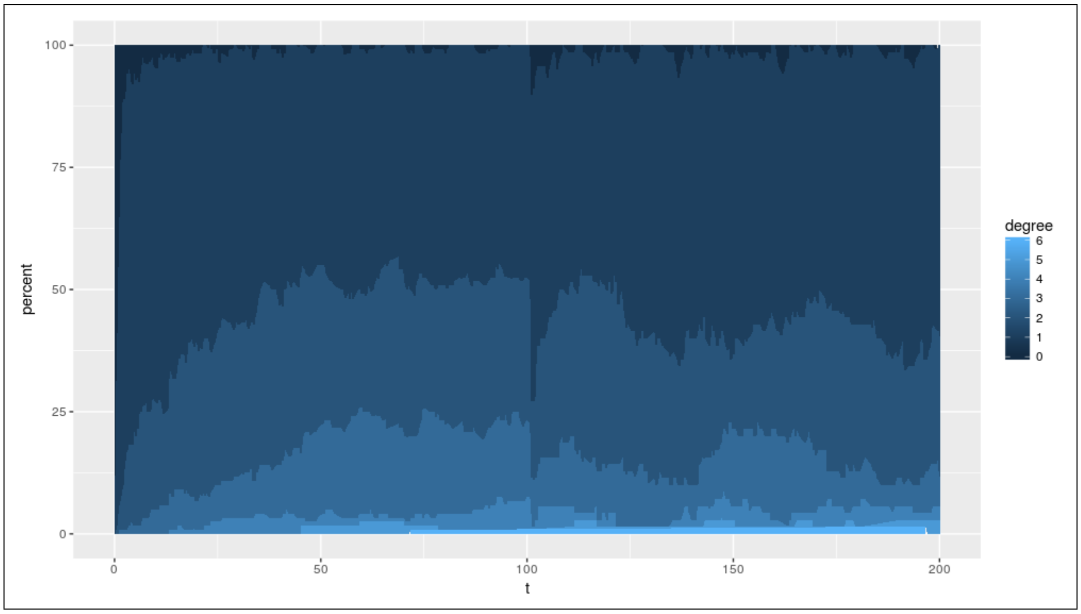

Figure 4: The effect of a disaster on a relatively stable cohort. 50 individuals were killed randomly out of 120 total at $t = 100$. Note that here $\alpha = 1, \ldots, 1, 5, \ldots, 5, 10, \ldots, 10$. The y-axis is normalized to a sum of 100. The disaster happens at $t = 100$.

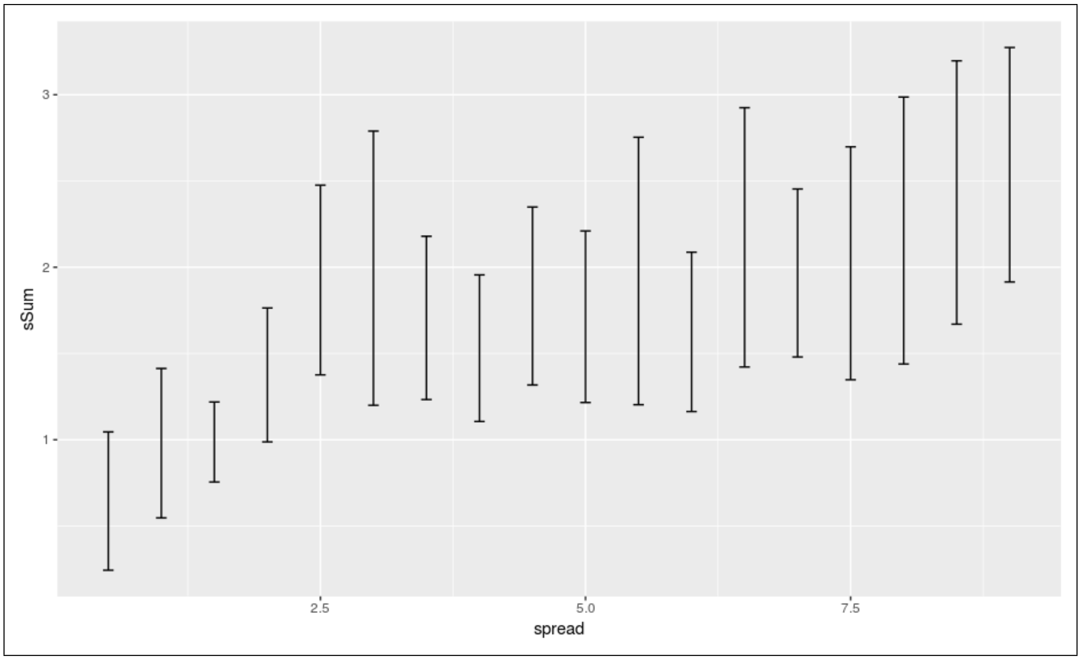

Figure 5: The sum of unhappiness ($\sum_j s(i, 50)$) over the population at a given time $t = 50$ for different values of spread, s. In each simulation there are 50 individuals with α uniformly distributed over $[10 - s, 10]$. The error bars in the figure represent standard deviations over ten experiments at that spread.

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