NVDA and CAKE Summary Statistics

In [...]:
```python
import pandas as pd
from statsmodels.tsa.stattools import acf, pacf
import statsmodels.api as sm
import numpy as np
import matplotlib.pyplot as plt

# trades level schema
df = pd.read_csv('xnas-itch-nvidia-cake.csv', parse_dates=['ts_recv', 'ts_event'])

# mbo schema: for depth-related questions
df_mbo = pd.read_csv('xnas-itch-20240822-mbo.csv', parse_dates=['ts_recv', 'ts_event'])
```

In [...]:
```python
# Split up and prepare dataset

# this is returning nothing
def prepare_bid_ask_data(df):
    """ Helper function to create bid_price, ask_price, bid_depth, and ask_depth columns
        that will be used for later summary statistic calculations."""

    # Create bid and ask price columns based on the 'side' column
    df['bid_price'] = df['price'].where(df['side'] == 'B', None)
    df['ask_price'] = df['price'].where(df['side'] == 'A', None)

    # Forward fill the missing bid and ask prices and depths
    df['bid_price'] = df['bid_price'].ffill()
    df['ask_price'] = df['ask_price'].ffill()

    return df

nvda_df = prepare_bid_ask_data(df[df['symbol'] == 'NVDA'].copy())
cake_df = prepare_bid_ask_data(df[df['symbol'] == 'CAKE'].copy())

nvda_df_mbo = df_mbo[df_mbo['symbol'] == 'NVDA'].copy()
cake_df_mbo = df_mbo[df_mbo['symbol'] == 'CAKE'].copy()
```

In [...]:
```python
nvda_df.columns
```

Out[3]:
```
Index(['ts_recv', 'ts_event', 'rtype', 'publisher_id', 'instrument_id',
       'action', 'side', 'depth', 'price', 'size', 'flags', 'ts_in_delta',
       'sequence', 'symbol', 'bid_price', 'ask_price'],
      dtype='object')
```

Helper functions - contain functionality that calculates results

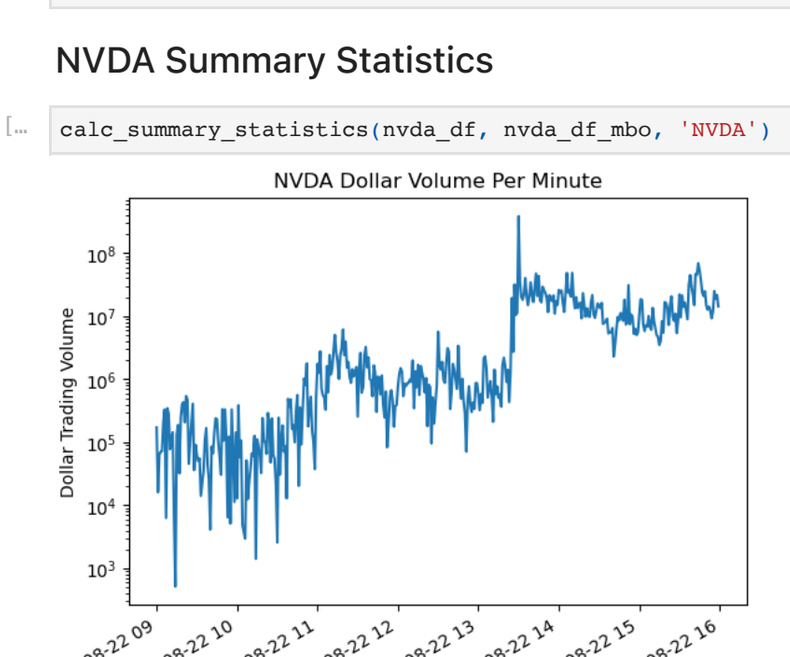
In [...]:
```python
import warnings
from pandas.errors import SettingWithCopyWarning
warnings.simplefilter(action="ignore", category=SettingWithCopyWarning)

def calculate_5s_price_impact(df):
    ''' Helper function that contains logic to calculate 5 second price impact.'''

    # Calculate the midpoint price
    df['midpoint'] = (df['bid_price'] + df['ask_price']) / 2

    # Lag midpoint by 5 seconds
    df['midpoint_lag'] = df['midpoint'].shift(5)

    # Calculate the midpoint return
    df['midpoint_return'] = df['midpoint'] - df['midpoint_lag']

    # Create a trade sign column based on the 'side' (-1 for 'Ask', +1 for 'Bid')
    df['trade_sign'] = df['side'].apply(lambda x: 1 if x == 'B' else -1)

    # Remove NaN values that result from the shift
    clean = df.dropna(subset=['midpoint_return', 'trade_sign'])

    # Perform the regression of 5-second midpoint return on trade sign
    price_impact_model = sm.OLS(pd.to_numeric(clean['midpoint_return']), clean['trade_sign']).fit()

    # Display the summary of the regression model
    return price_impact_model.summary()


def calculate_depth_at_2x_spread(df_mbo):

    # Filter for only relevant actions to speed up book calculations
    df_mbo_filtered = df_mbo[df_mbo['action'].isin(['A', 'C'])].copy()

    # Group data by second
    df_mbo_filtered['second'] = df_mbo_filtered['ts_event'].dt.floor('S')

    # Initialize dictionaries to store order books for bids and asks
    order_book_bids = {}
    order_book_asks = {}

    # Initialize lists to track depth for plotting
    depth_ask_side = []
    depth_bid_side = []

    # Initialize variables to track midquote and spread
    midquotes = []
    spreads = []

    # Iterate by second
    print(len(df_mbo_filtered.groupby('minute')))
    i = 0
    for second, group in df_mbo_filtered.groupby('minute'):
        print(i)
        i+=1

        # Update the order book based on all actions in the group
        for _, row in group.iterrows():
            if row['action'] == 'A':  # Add order
                if row['side'] == 'B':
                    order_book_bids[row['order_id']] = {'price': row['price'], 'size': row['size']}
                elif row['side'] == 'A':
                    order_book_asks[row['order_id']] = {'price': row['price'], 'size': row['size']}
            elif row['action'] == 'C':  # Cancel order
                if row['order_id'] in order_book_bids:
                    del order_book_bids[row['order_id']]
                elif row['order_id'] in order_book_asks:
                    del order_book_asks[row['order_id']]

        # Calculate best bid and ask after processing all orders for the second
        if order_book_bids and order_book_asks:
            best_bid = max([order['price'] for order in order_book_bids.values()])
            best_ask = min([order['price'] for order in order_book_asks.values()])

            # Calculate midquote and spread
            midquote = (best_bid + best_ask) / 2
            spread = best_ask - best_bid

            midquotes.append(midquote)
            spreads.append(spread)

            # Calculate the average spread so far
            avg_spread = sum(spreads) / len(spreads) if spreads else 0

            # Calculate depth at twice the average spread
            depth_ask = sum([order['size'] for order in order_book_asks.values() if order['price'] <= midquote + 2 * avg_spread])
            depth_bid = sum([order['size'] for order in order_book_bids.values() if order['price'] >= midquote - 2 * avg_spread])

            depth_ask_side.append(depth_ask)
            depth_bid_side.append(depth_bid)

    depth_ask_side_series = pd.Series(depth_ask_side)
    depth_bid_side_series = pd.Series(depth_bid_side)

    return depth_ask_side_series, depth_bid_side_series




def calculate_bbo_spread_and_depth_per_minute(df_mbo):
    ''' Calculate the best BBO spread and depth per minute from the provided DataFrame. '''

    # Filter the data to include only bid and ask prices
    bid_data = df_mbo[df_mbo['side'] == 'B']
    ask_data = df_mbo[df_mbo['side'] == 'A']

    # Group by minute
    bid_data.loc[:, 'minute'] = bid_data['ts_recv'].dt.floor('T')
    ask_data.loc[:, 'minute'] = ask_data['ts_recv'].dt.floor('T')

    # Get the best bid and ask for each minute
    best_bid = bid_data.groupby('minute').agg({'price': 'max', 'size': 'sum'}).reset_index()
    best_ask = ask_data.groupby('minute').agg({'price': 'min', 'size': 'sum'}).reset_index()

    # Rescale prices by dividing by 1e-9 to convert to dollars
    best_bid['price'] = best_bid['price'] / 1e9
    best_ask['price'] = best_ask['price'] / 1e9

    # Merge bid and ask data to calculate the spread and depth
    bbo_data = pd.merge(best_bid, best_ask, on='minute', suffixes=('_bid', '_ask'))

    # Calculate BBO Spread (Best Ask - Best Bid)
    bbo_spread = abs(bbo_data['price_ask'] - bbo_data['price_bid'])

    # Calculate BBO Depth (Sum of Sizes at Best Bid and Best Ask)
    bbo_depth_bid = bbo_data['size_bid']
    bbo_depth_ask = bbo_data['size_ask']

    # Set the minute as the index for both series
    bbo_spread.index = bbo_data['minute']
    bbo_depth_bid.index = bbo_data['minute']
    bbo_depth_ask.index = bbo_data['minute']

    return bbo_spread, bbo_depth_bid, bbo_depth_ask


import pandas as pd
import numpy as np
import statsmodels.api as sm
import matplotlib.pyplot as plt

def calculate_5s_price_impact_per_minute(df):
    """
    Helper function to calculate 5-second price impact per minute by regressing the 5-second midpoint quote return
    on the current trade sign (-1 or +1).
    """

    # Calculate the midquote for each row
    df['midquote'] = (df['price'][df['side'] == 'B'] + df['price'][df['side'] == 'A']) / 2

    # Forward-fill the midquote to ensure we have a midquote available at each second
    df['midquote'] = df['midquote'].ffill()

    # Resample the DataFrame to ensure consistent time intervals at the second level
    df_resampled = df.set_index('ts_event').resample('S').last().ffill()

    # Remove any duplicate timestamps from the resampled DataFrame
    df_resampled = df_resampled[~df_resampled.index.duplicated(keep='last')]

    # Filter the DataFrame to only include trades ('T')
    df_trades = df[df['action'] == 'T'].copy()

    # Assign trade sign: +1 for buy, -1 for sell
    df_trades['trade_sign'] = np.where(df_trades['side'] == 'B', +1, -1)

    # Ensure df_resampled has a datetime index for efficient lookups
    df_resampled = df_resampled.sort_index()

    # Debugging step: Ensure that there are no duplicates
    if df_resampled.index.duplicated().any():
        raise ValueError("Duplicates detected in the index after processing.")

    # Get the last available midquote before each trade using asof
    df_trades['prev_midquote'] = df_resampled['midquote'].asof(df_trades['ts_event'])

    # Efficiently compute the midquote 5 seconds after each trade, using the last available midquote
    df_trades['midquote_5sec'] = df_resampled['midquote'].asof(df_trades['ts_event'] + pd.Timedelta(seconds=5))

    # Calculate the 5-second return on the midquote
    df_trades['midquote_return_5sec'] = (df_trades['midquote_5sec'] - df_trades['prev_midquote']) / df_trades['prev_midquote']

    # Drop rows where return or trade sign is NaN
    df_trades = df_trades.dropna(subset=['midquote_return_5sec', 'trade_sign'])

    # Check if there's enough data to run the regression
    if not df_trades.empty:
        # Regress the 5-second midquote return on the trade sign
        X = df_trades['trade_sign']
        X = sm.add_constant(X)  # Adds a constant term to the predictor
        Y = df_trades['midquote_return_5sec']
        model = sm.OLS(Y, X).fit()

        # Print the regression summary
        print(model.summary())

        # Plot the observed vs predicted values
        plt.figure(figsize=(10, 6))
        plt.scatter(df_trades['trade_sign'], df_trades['midquote_return_5sec'], alpha=0.5, label='Observed')
        plt.plot(df_trades['trade_sign'], model.predict(X), color='red', label='Fitted Line')
        plt.xlabel('Trade Sign')
        plt.ylabel('5-Second Midquote Return')
        plt.title('5-Second Midquote Return vs Trade Sign')
        plt.legend()
        plt.show()

        # Plot the residuals
        plt.figure(figsize=(10, 6))
        plt.scatter(model.predict(X), model.resid, alpha=0.5)
        plt.axhline(0, color='red', linestyle='--')
        plt.xlabel('Fitted Values')
        plt.ylabel('Residuals')
        plt.title('Residual Plot')
        plt.show()
    else:
        print("No valid data points available for regression analysis.")


def calculate_midquote_transaction(df):
    ''' Helper function to calculate one-second and one-minute midquote and transaction price series.'''

    # Ensure ts_event is datetime and set it as the index
    df['ts_event'] = pd.to_datetime(df['ts_event'])
    df.set_index('ts_event', inplace=True)

    # Calculate the midpoint price

    # get the highest bid and lowest ask for each minute

    # and then calculate the midpoint based on that
    df['midpoint'] = (df['bid_price'] + df['ask_price']) / 2

    # Resample the data to one-second and one-minute intervals, taking the last value in each interval
    midquote_1sec = df['midpoint'].resample('1S').last()
    midquote_1min = df['midpoint'].resample('1T').last()

    # For transaction prices
    transaction_1sec = df['price'].resample('1S').last()
    transaction_1min = df['price'].resample('1T').last()


    # Drop the first NaN value
    midquote_1sec.dropna(inplace=True)
    midquote_1min.dropna(inplace=True)
    transaction_1sec.dropna(inplace=True)
    transaction_1min.dropna(inplace=True)


    return midquote_1sec, midquote_1min, transaction_1sec, transaction_1min


def calculate_log_returns(midquote_1min, transaction_1min):
    ''' Helper function to calculate log returns.'''

    midquote_log_return_1min = np.log(midquote_1min) - np.log(midquote_1min.shift(1))
    transaction_log_return_1min = np.log(transaction_1min) - np.log(transaction_1min.shift(1))

    # Drop NaN values resulting from the shift
    midquote_log_return_1min = midquote_log_return_1min.dropna()
    transaction_log_return_1min = transaction_log_return_1min.dropna()

    # Display the first few log-returns
    return midquote_log_return_1min, transaction_log_return_1min

def graph_tpm_opm(tpm, opm, name):
    ''' Helper function to break up spike in tpm and opm'''
    if name == 'NVDA':
        # Filter for 9 AM to 1 PM
        opm_morning = opm.between_time('09:00', '13:00')
        tpm_morning = tpm.between_time('09:00', '13:00')


        # Filter for 1 PM to 4 PM
        opm_afternoon = opm.between_time('13:00', '16:00')
        tpm_afternoon = opm.between_time('13:00', '16:00')

        # Plot Number of Trades between 9 AM and 1 PM
        tpm_morning.plot(title=name + ' Number Trades per Minute (9 AM - 1 PM)')
        plt.xlabel('Time (minute)')
        plt.ylabel('Number Trades')
        plt.show()

        # Plot Number of Trades between 1 PM and 4 PM
        tpm_afternoon.plot(title=name + ' Number Trades per Minute (1 PM - 4 PM)')
        plt.xlabel('Time (minute)')
        plt.ylabel('Number Trades')
        plt.show()

        # Plot Number of Orders between 9 AM and 1 PM
        opm_morning.plot(title=name + ' Number Orders per Minute (9 AM - 1 PM)')
        plt.xlabel('Time (minute)')
        plt.ylabel('Number Orders')
        plt.show()

        # Plot Number of Orders between 1 PM and 4 PM
        opm_afternoon.plot(title=name + ' Number Orders per Minute (1 PM - 4 PM)')
        plt.xlabel('Time (minute)')
        plt.ylabel('Number Orders')
        plt.show()

    else: # CAKE - plot as usual
        # Trades per minute
        tpm.plot(title = name + ' Number Trades per Minute')
        plt.xlabel('Time (minute)')
        plt.ylabel('Number Trades')
        plt.show()

        # Orders per minute
        opm.plot(title=name + ' Number Orders per Minute')
        plt.xlabel('Time (minute)')
        plt.ylabel('Number Orders')
        plt.show()

def graph_bbo_depth(bbo_depth_bid, bbo_depth_ask, name):
    ''' Helper function to break up spike in tpm and opm'''
    if name == 'NVDA':
        # Filter for 9 AM to 1 PM
        bid_morning = bbo_depth_bid.between_time('09:00', '13:00')
        ask_morning = bbo_depth_ask.between_time('09:00', '13:00')


        # Filter for 1 PM to 4 PM
        bid_afternoon = bbo_depth_bid.between_time('13:00', '16:00')
        ask_afternoon = bbo_depth_ask.between_time('13:00', '16:00')

        # Plot depth between 9 AM and 1 PM
        bid_morning.plot(title=name + ' BBO Depth (Bid)  (9 AM - 1 PM)')
        plt.xlabel('Time (minute)')
        plt.ylabel('Depth')
        plt.show()

        # Plot depth between 1 PM and 4 PM
        bid_afternoon.plot(title=name + ' BBO Depth (Bid)  (1 PM - 4 PM)')
        plt.xlabel('Time (minute)')
        plt.ylabel('Depth')
        plt.show()

        # Plot depth between 9 AM and 1 PM
        ask_morning.plot(title=name + ' BBO Depth (Ask)  (9 AM - 1 PM)')
        plt.xlabel('Time (minute)')
        plt.ylabel('Depth')
        plt.show()

        # Plot depth between 1 PM and 4 PM
        ask_afternoon.plot(title=name + ' BBO Depth (Ask)  (1 PM - 4 PM)')
        plt.xlabel('Time (minute)')
        plt.ylabel('Depth')
        plt.show()

    else: # CAKE - plot as usual
        # Trades per minute
        tpm.plot(title = name + ' Number Trades per Minute')
        plt.xlabel('Time (minute)')
        plt.ylabel('Number Trades')
        plt.show()

        # Orders per minute
        opm.plot(title=name + ' Number Orders per Minute')
        plt.xlabel('Time (minute)')
        plt.ylabel('Number Orders')
        plt.show()
```

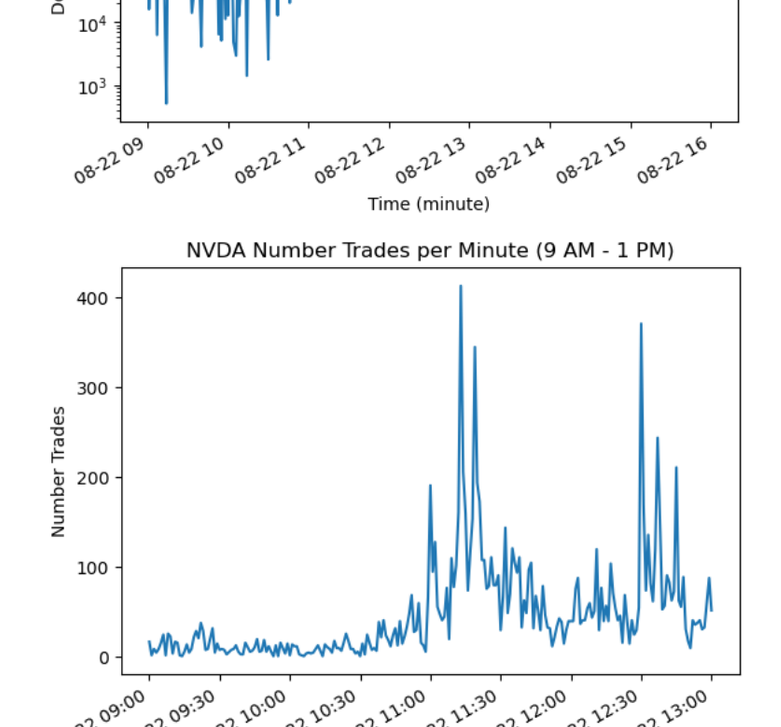
Driver function - calculates all statistics and displays / returns to the user

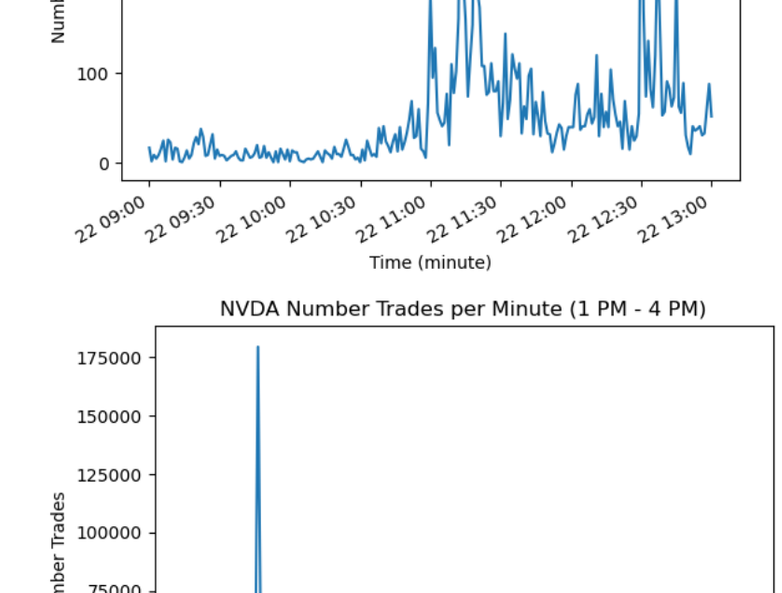
In [...]:
```python
from statsmodels.graphics.tsaplots import plot_acf

def calc_summary_statistics(df, df_bbo, name):
    ''' Driver function for (a) - (j)'''

    # Initialize results table
    results = {}
    results_1s = {}

    # Get the common minute index
    minute_index = pd.to_datetime(df['ts_event']).dt.floor('T').unique()

    # (a) Dollar trading volume per minute
    df['dollar_vol'] = df['price'] * df['size']
    dtv=df.groupby(pd.to_datetime(pd.to_datetime(df['ts_event'])).dt.floor('min'))['dollar_vol'].sum()
    results['dollar_vol'] = dtv
    dtv.plot(title=name + ' Dollar Volume Per Minute', logy=True)
    plt.xlabel('Time (minute)')
    plt.ylabel('Dollar Trading Volume')
    plt.show()


    # (b) Number of trades and number of orders per minute
    df_mbo['minute'] = df_mbo['ts_event'].dt.floor('T')

    # Calculate the number of trades per minute (action 'T')
    tpm = df_mbo[df_mbo['action'] == 'T'].groupby('minute').size()

    # Store the results for trades per minute
    results['number_trades_per_min'] = tpm


    # 'Add' action means act of adding order to order book
    opm = df_mbo[df_mbo['action'] == 'A'].groupby('minute').size()
    results['number_orders_per_min'] = opm

    # NVDA had a huge spike at 1pm, we break up so its easier to read - send to helper
    graph_tpm_opm(tpm, opm, name)


    # (c) Open, close, high, and low prices
    stock_open = df.iloc[0]['price']
    close = df.iloc[-1]['price']
    high = df['price'].max()
    low = df['price'].min()

    print('\n')
    print(name + ' Open: ', stock_open)
    print(name + ' Close: ', close)
    print(name + ' High: ', close)
    print(name + ' Low: ', close, '\n')


    # (d) VWAP per minute
    vwap_per_min = df.groupby(pd.to_datetime(df['ts_event']).dt.floor('T')).apply(
        lambda x: (x['price'] * x['size']).sum() / x['size'].sum())
    results['vwap'] = vwap_per_min.reindex(minute_index, fill_value=0)
    vwap_per_min.plot(title=name + ' VWAP per Minute')
    plt.xlabel('Time (minute)')
    plt.ylabel('VWAP')
    plt.show()

    '''
    # (e) BBO spread and depth per minute
    bbo_spread, bbo_depth_bid, bbo_depth_ask = calculate_bbo_spread_and_depth_per_minute(df_mbo)

    results['best_bbo_spread'] = bbo_spread
    results['best_bbo_depth_bid'] = bbo_depth_bid
    results['best_bbo_depth_ask'] = bbo_depth_ask

    graph_bbo_depth(bbo_depth_bid, bbo_depth_ask, name)

    # (f) Depth at twice that day's average spread
    # takes a long time to run

    ask_depth_series, bid_depth_series = calculate_depth_at_2x_spread(df_mbo)
    results['depth_twice_avg_spread_ask'] = ask_depth_series
    results['depth_twice_avg_spread_bid'] = bid_depth_series

    bid_depth_series.plot(title=name + ' Depth at 2x Avg. Spread (Bid)')
    plt.xlabel('Time (minutes)')
    plt.ylabel('Depth')
    plt.show()

    ask_depth_series.plot(title=name + ' Depth at 2x Avg. Spread (Ask)')
    plt.xlabel('Time (minutes)')
    plt.ylabel('Depth')
    plt.show()


    # (g) 5-second price impact per minute - needs to be fixed
    calculate_5s_price_impact_per_minute(df_mbo.copy())

    # (h) Midquote and transaction price series - needs to be fixed
    midquote_1sec, midquote_1min, transaction_1sec, transaction_1min = calculate_midquote_transaction(df.copy())

    results_1s['midquote_1sec'] = midquote_1sec
    results_1s['transaction_1sec'] = transaction_1sec
    results['midquote_1min'] = midquote_1min.reindex(minute_index, fill_value=0)
    results['transaction_1min'] = transaction_1min.reindex(minute_index, fill_value=0)

    plt.plot(midquote_1sec, label='Midquote 1-sec' )
    plt.plot(transaction_1sec, label='Transaction 1-sec')
    plt.legend()
    plt.title(name + ' Midquote and Transaction 1-sec')
    plt.show()

    plt.plot(midquote_1min, label='Midquote 1-min' )
    plt.plot(transaction_1min, label='Transaction 1-min')
    plt.legend()
    plt.title(name + ' Midquote and Transaction 1-min')
    plt.show()

    # (i) Log returns - needs to be fixed
    midquote_log_return_1min, transaction_log_return_1min = calculate_log_returns(midquote_1min, transaction_1min)
    results['midquote_log_return_1min'] = midquote_log_return_1min.reindex(minute_index, fill_value=0)
    results['transaction_log_return_1min'] = transaction_log_return_1min.reindex(minute_index, fill_value=0)

    plt.plot(results['midquote_log_return_1min'])
    plt.title(name + ' Midquote Log Returns 1-min')
    plt.show()

    plt.plot(results['transaction_log_return_1min'])
    plt.title(name + ' Midquote Transaction Returns 1-min')
    plt.show()

    # (j) Realized variance
    midquote_realized_variance = (midquote_log_return_1min ** 2).sum()
    results['midquote_realized_variance'] = midquote_realized_variance

    # TODO add transaction realized variance

    plt.plot(results['midquote_realized_variance'])
    plt.title(name + ' Midquote Realized Variance')
    plt.show()

    # Convert results to DataFrame

    min_stats = pd.DataFrame(results)
    sec_stats = pd.DataFrame(results_1s)


    # (k) Auto-correlation for midquote and transaction returns
    midquote_pacf = plot_acf(midquote_log_return_1min)
    transaction_pacf = plot_acf(transaction_log_return_1min)
    '''


    # return min_stats, sec_stats, midquote_pacf, transaction_pacf
```

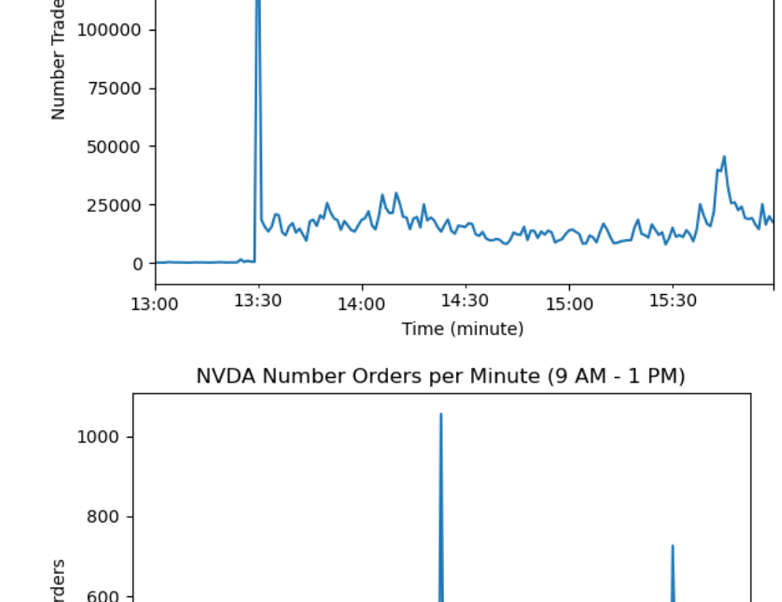
NVDA Summary Statistics

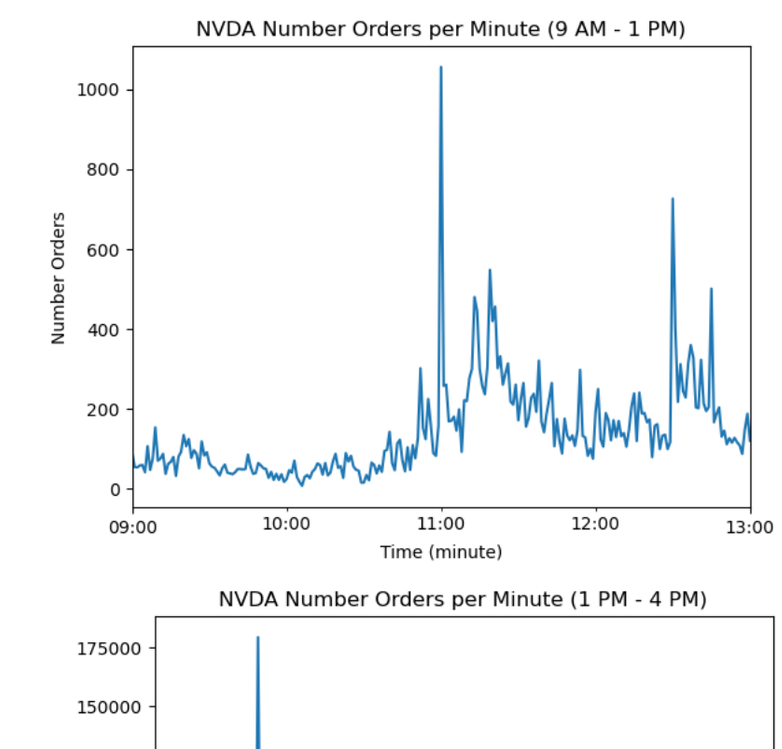
In [...]:
```python
calc_summary_statistics(nvda_df, nvda_df_mbo, 'NVDA')
```



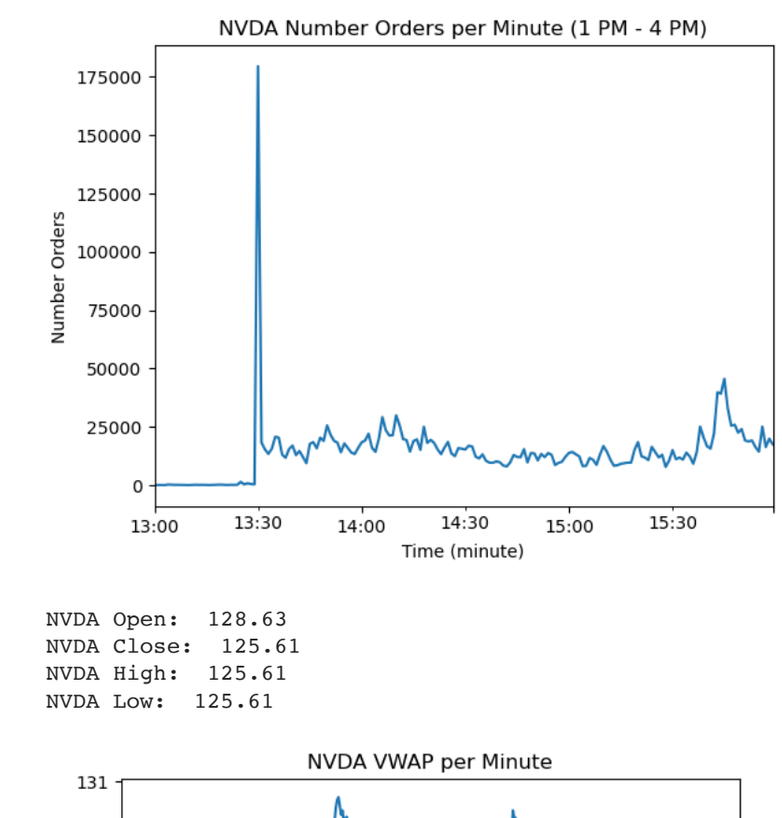






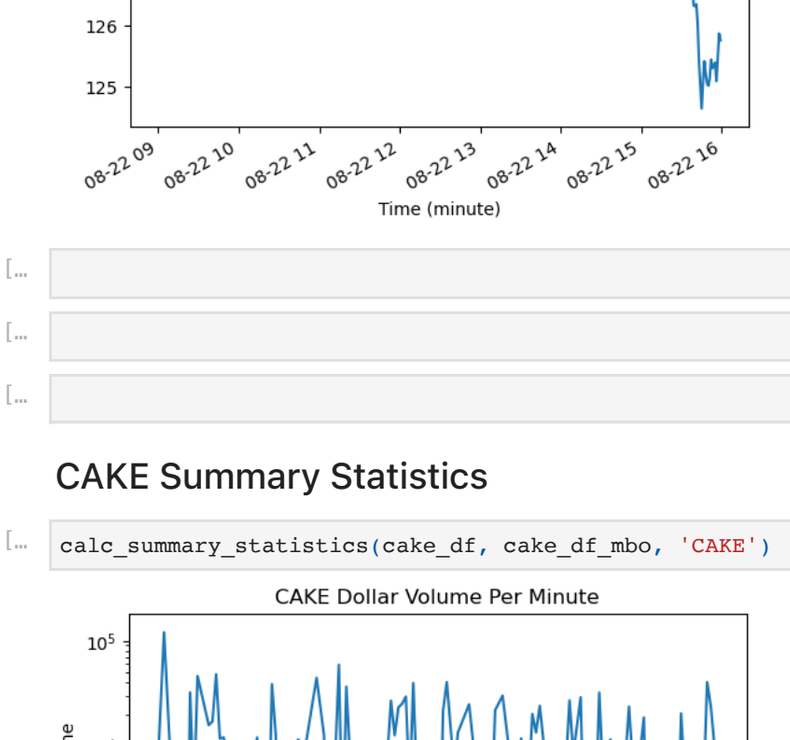


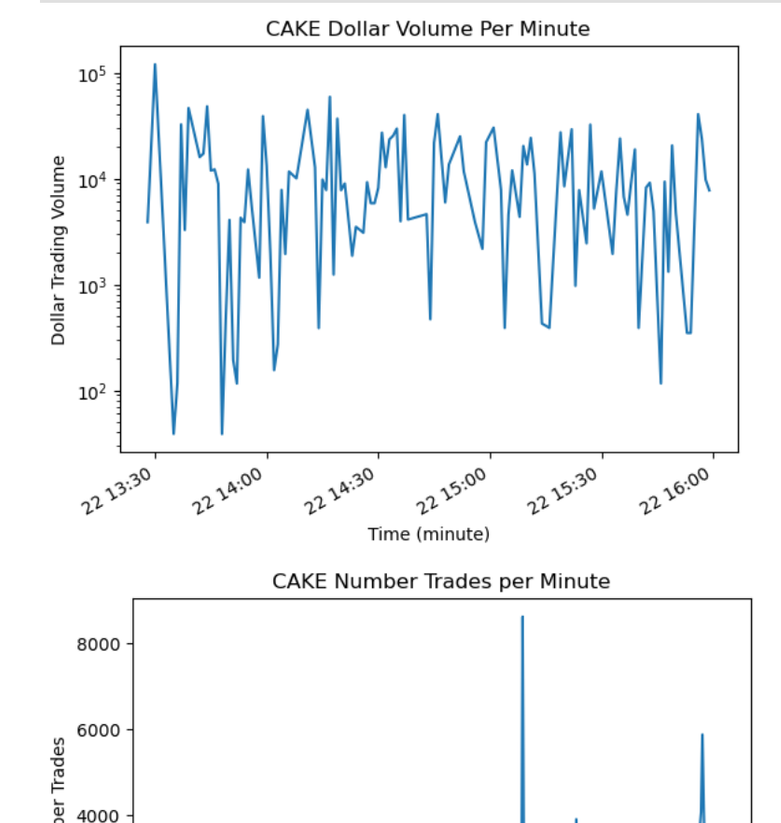

```
NVDA Open:  128.63
NVDA Close:  125.61
NVDA High:  125.61
NVDA Low:  125.61
```

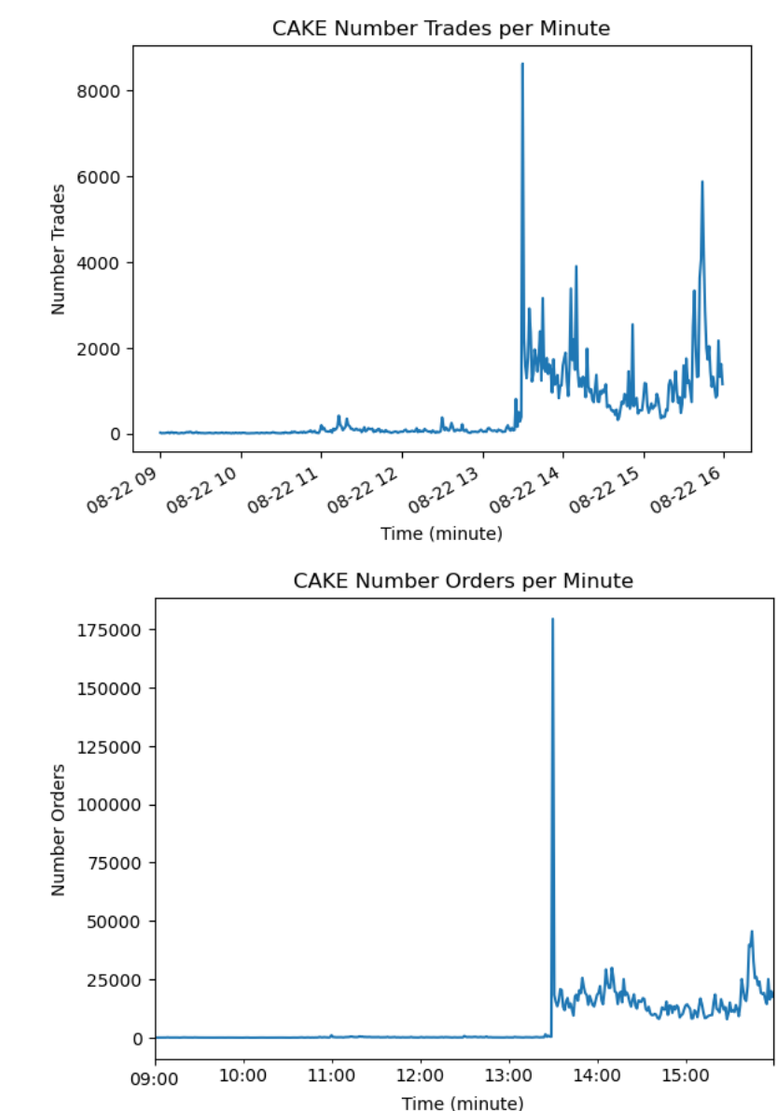


CAKE Summary Statistics

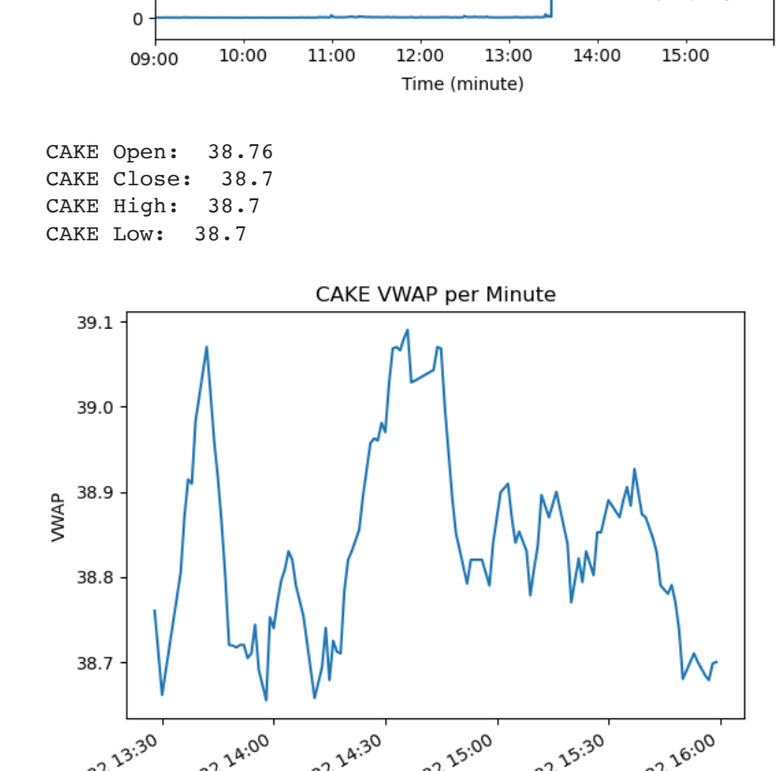
In [...]:
```python
calc_summary_statistics(cake_df, cake_df_mbo, 'CAKE')
```








```
CAKE Open:  38.76
CAKE Close:  38.7
CAKE High:  38.7
CAKE Low:  38.7
```



In [...]:
```python
print(len(nvda_df_mbo))
```