

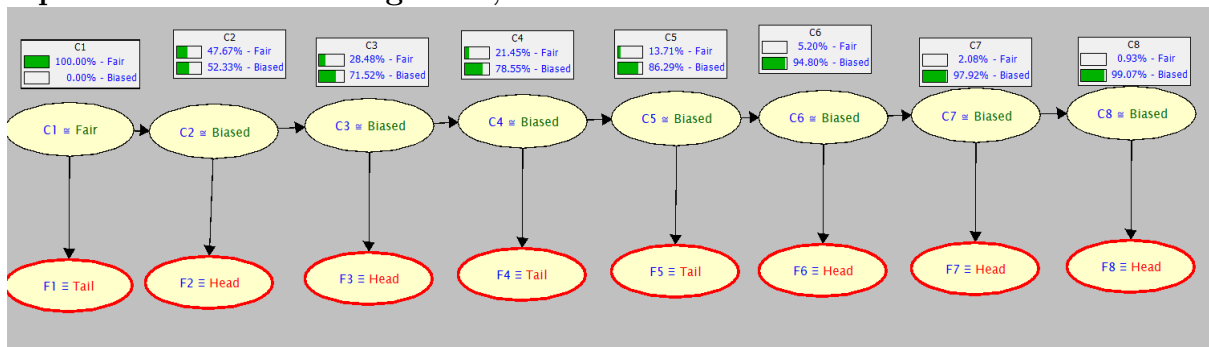
Project 3: Reasoning with Bayesian Networks

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Bayesian Network Construction

Below is my Bayesian network. The "F" nodes store the evidence, or the outcome of the coin toss, as given by the pdf specification. The "C" nodes represent the coin being used, whether it be the biased or unbiased one.



Below are the CPT tables for all of the nodes:

For node C1 (first coin toss, we know the coin used is 100 percent the fair coin:

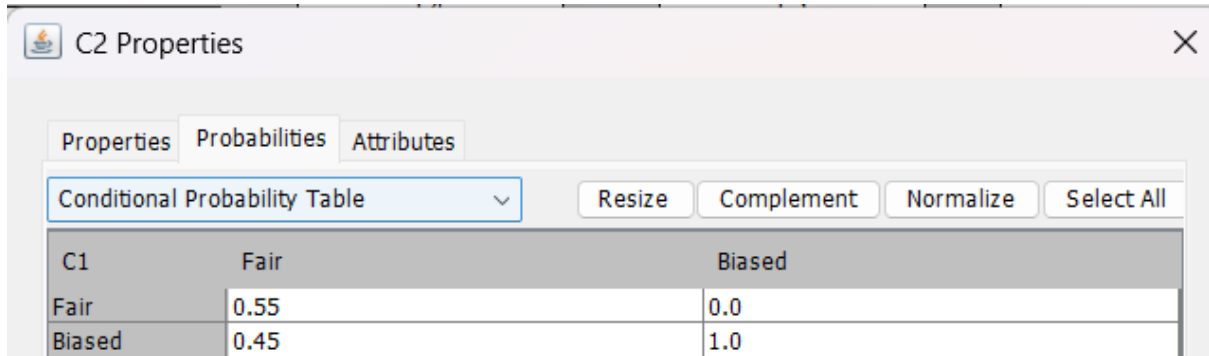
C1 Properties

Properties Probabilities Attributes

Conditional Probability Table

Fair	1.0
Biased	0.0

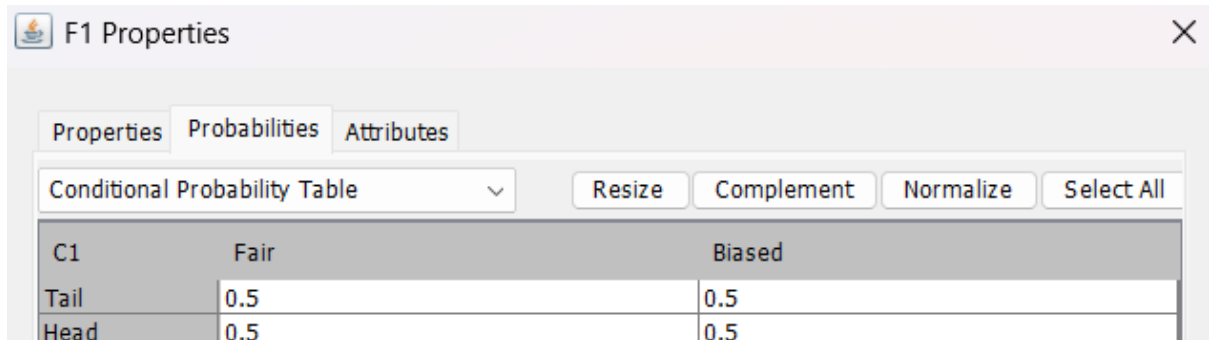
For nodes C2-C8 (2nd to 8th coin toss, we know if the coin is fair, there is a 45 percent chance that a switch to the biased coin occurs (per the assignment specifications), if the coin is biased, it will stay biased):



The image shows the 'C2 Properties' dialog box with the 'Probabilities' tab selected. It displays a 'Conditional Probability Table' for node C2. The table has two columns: 'Fair' and 'Biased'. The rows are 'Fair' and 'Biased'. The values are: Fair/Fair = 0.55, Fair/Biased = 0.0, Biased/Fair = 0.45, and Biased/Biased = 1.0.

C1	Fair	Biased
Fair	0.55	0.0
Biased	0.45	1.0

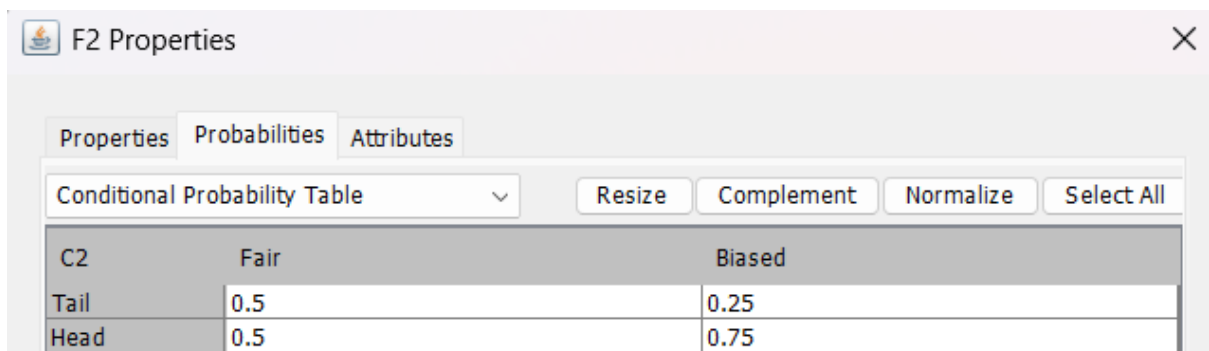
For node F1 (coin is fair so 50/50 chance here (biased probabilities are unused, so they can remain as placeholder values of 50/50)):



The image shows the 'F1 Properties' dialog box with the 'Probabilities' tab selected. It displays a 'Conditional Probability Table' for node F1. The table has two columns: 'Fair' and 'Biased'. The rows are 'Tail' and 'Head'. The values are: Tail/Fair = 0.5, Tail/Biased = 0.5, Head/Fair = 0.5, and Head/Biased = 0.5.

C1	Fair	Biased
Tail	0.5	0.5
Head	0.5	0.5

And for nodes F2-F8 (if the coin is fair, it is 50/50 for head/tail result; if the coin is biased, it is 75 percent heads, 25 percent tails, per the assignment specifications):



The image shows the 'F2 Properties' dialog box with the 'Probabilities' tab selected. It displays a 'Conditional Probability Table' for node F2. The table has two columns: 'Fair' and 'Biased'. The rows are 'Tail' and 'Head'. The values are: Tail/Fair = 0.5, Tail/Biased = 0.25, Head/Fair = 0.5, and Head/Biased = 0.75.

C2	Fair	Biased
Tail	0.5	0.25
Head	0.5	0.75

Questions

Suppose that we were able to observe the outcomes of her eight coin flips sequentially as tail, head, head, tail, tail, head, head, head. Now, we want to find out:

a) whether Lisa managed to perform a coin switch, and

Given the outcome of the queries in my Bayesian network, Lisa did perform a coin switch. This can be seen as after the last coin toss, the probability that the coin is biased is 99.07 percent. Thus, we can deduce that Lisa indeed did perform a coin switch.

b) when the switch took place.

Using my Bayesian network, the switch took place after the 1st coin toss, or before the 2nd coin toss. This means that the first coin toss was done with the fair/non-biased coin, but the second coin toss was done with the biased coin. I deduced this by seeing that $P(C_2 = \text{biased}) > 50$ percent, and that for all C_i from C_2 and beyond, the probability of the coin being the biased one increased, and never decreased.

Essentially, since for all C_i where $i \geq 2$, $P(C_i = \text{biased}) > 50$ percent, and for all C_i where $i \geq 2$, $P(C_i = \text{biased}) < P(C_i + 1 = \text{biased})$, it can be seen that the coin switch occurred before the second flip, and therefore from the second coin toss onwards, the biased coin was being used.

In summary, my answers are:

a) Lisa did perform a coin switch.

b) The coin switch occurred between the first and second coin tosses (the 1st toss was using the fair coin, but the 2nd toss was using the biased coin).