Solo Project Derivations

Solving the equation of radiative transfer

This solution follows that in Introduction to Stellar Astrophysics (E. Böhm-Vitnese) pages 30-35 (Reference 7 in bibliography)

$$\frac{1}{\varkappa_\lambda}\frac{dI_\lambda}{ds_\lambda} = -I_\lambda + S_\lambda$$

$d\tau_\lambda = \varkappa_\lambda ds$

$$\Rightarrow \frac{dI_\lambda}{d\tau_\lambda} = -I_\lambda + S_\lambda$$

Multiplying by integrating factor e^{τ_λ}

$$\Rightarrow \frac{dI_\lambda}{d\tau_\lambda}e^{\tau_\lambda} + I_\lambda e^{\tau_\lambda} = S_\lambda e^{\tau_\lambda}$$

$$\Rightarrow \frac{d}{d\tau_\lambda}\left(I_\lambda e^{\tau_\lambda}\right) = S_\lambda e^{\tau_\lambda}$$

$$\Rightarrow \int_0^{\tau_\lambda} \frac{d}{d\tau_\lambda}\left(I_\lambda e^{\tau_\lambda}\right) d\tau_\lambda = \int_0^{\tau_\lambda} S_\lambda e^{\tau_\lambda}\, d\tau_\lambda$$

$$\Rightarrow \left[I_\lambda e^{\tau_\lambda}\right]_0^{\tau_\lambda} = \left[S_\lambda e^{\tau_\lambda}\right]_0^{\tau_\lambda}$$ assuming S_λ constant along s.

$I_\lambda e^0 = I_{\lambda 0}$

$$\Rightarrow I_\lambda e^{\tau_\lambda} - I_{\lambda 0} = S_\lambda e^{\tau_\lambda} - S_\lambda$$

$$\Rightarrow I_\lambda - I_{\lambda 0} e^{-\tau_\lambda} = S_\lambda\left(1 - e^{-\tau_\lambda}\right)$$

$$\Rightarrow I_\lambda = I_{\lambda 0} e^{-\tau_\lambda} + S_\lambda\left(1 - e^{-\tau_\lambda}\right)$$

If $\tau_\lambda \ll 1$, $e^{-\tau_\lambda} \simeq 1 - \tau_\lambda$ (can expand the exponential in a Taylor series).

$$\Rightarrow I_\lambda = I_{\lambda 0}(1-\tau_\lambda) + S_\lambda(1-(1-\tau_\lambda))$$
$$= I_{\lambda 0} - I_{\lambda 0}\tau_\lambda + S_\lambda - S_\lambda + S_\lambda \tau_\lambda$$
$$= I_{\lambda 0} + \tau_\lambda (S_\lambda - I_{\lambda 0})$$

If $\tau_\lambda >> 1$, $e^{-\tau_\lambda} \to 0$

$$\Rightarrow I_\lambda = S_\lambda$$

Finding the wavelength of light emitted at the peak of the Sun's spectrum

By Wien's Law (Reference 12 in bibliography)

$$\lambda_{peak} = \frac{2.898 \times 10^{-3} m \cancel{K}}{6000 \cancel{K}}$$

$$\lambda_{peak} = 4.83 \times 10^{-7} \cancel{m} \times \frac{1 cm}{0.01 \cancel{m}}$$

$$= 4.83 \times 10^{-5} cm$$

$$= 483.3 \times 10^{-7} cm$$

(as shown in the report).