CSC 236 Tutorial 11

Harry Sha

August 2, 2023

Regular

Dropout

Regular?

Show the following languages are regular

Use any means to show that the following languages are regular.
Hint: using closure properties often give you quicker proofs.

1. $\{w : w$ ends with 01 and doesn't contain 111 as a substring$\}$
2. $\{w : w$ never has more than five 1s in a row$\}$
3. $\{w :$
   w has length divisible by 4 and the number of 1s is divisible by 3$\}$
4. $\{w :$
   w has an even number of 1s or an even number of 0s but not both$\}$

Solution (sketch)

1. Write the language as

   $$\{w : w \text{ ends with } 01\} \cap \overline{\{w : w \text{ contains } 111 \text{ as a substring}\}}.$$

   The first language is matched by the regex Σ^*01, and the second is matched by $\Sigma^*111\Sigma^*$. Then, since regular languages are closed under intersection and negation, the original language is regular.
2. $\Sigma^*111111\Sigma^*$ is a regular expression for *has more five 1s in a row*. Regular languages are closed under negation, so done.
3. $(\Sigma\Sigma\Sigma\Sigma)^*$ is w has length divisible by 4, and $((0+\epsilon)^*1(0+\epsilon)^*1(0+\epsilon)^*1(0+\epsilon)^*)^*$ is regex for the number of 1s is divisible by 3. Regular languages are closed under intersections.
4. We showed previously has an even number of 1s, and even number of 0s are both regular. The language in the question is the symmetric difference of those two (and we showed that the symmetric difference of regular languages is regular.)

Regular

Dropout

Regular?

Dropout[1]

Let A be any language, define $\mathrm{Dropout}(A)$ to be the language consisting of all strings that can be obtained by removing one symbol from A.

Show that the regular languages are closed under $\mathrm{Dropout}$. I.e., if A is regular, then so is $\mathrm{Dropout}(A)$.

A correct construction of a DFA/NFA/Regex is good enough - no need to formally prove that the language of your construction is equal to $\mathrm{Dropout}(A)$.

[1] Sipser, exercise 1.43

Solution

Let M be a DFA for A. We'll construct an NFA, N for $\mathrm{Dropout}(A)$.

N will have two copies of M, M_1 and M_2. We will start by running the string on M_1 until we decide to skip a letter and continue running the DFA on M_2.

A little more detail:

- Since we must skip a letter, make all the states in M_1 rejecting.
- For each transition $(q, \sigma) \to q'$, add an ϵ transition for M_1's copy of q to M_2's copy of q'. This allows us to choose to skip a letter and forces us to move to M_2 after the skip.

Regular

Dropout

Regular?

Practice

For each of the languages below, guess whether or not the language is regular using the intuition that regular languages require a finite amount of memory. If the language is not regular, prove formally that the language is not regular using both methods we learned in class today.

- $\{w : w \text{ contains } 001 \text{ as a substring}\}$ only need to store the last 2 letters read and whether or not you have already seen 001 or not
- $\{w : w \text{ has more than one hundred 0s}\}$ only need to store a counter which takes value at most 100.
- $\{w : w \text{ has more 0s than 1s}\}$ need to store number of 0s which can be an arbitrarily large counter
- $\{w : w \text{ has more 0s mod 10 than 1s mod 10}\}$ need to store two counters, each taking value at most 10.
- $\{w : w \text{ has twice as many 0s as 1s}\}$ need to store number of 0s which can be an arbitrarily large counter.

$A = \{w : w \text{ has more 0s than 1s}\}$ is not regular (MNT)

By the Myhill-Nerode Theorem, it suffices to find an infinite set of strings that are pairwise distinguishable relative to A.

Let $S = \{0^{n+1} : n \in \mathbb{N}\}$ note that S is infinite since it has at least one element for every natural number.

Now we'll show that S is pairwise distinguishable. Let $x, y \in S$ with $x \neq y$. Since $x \neq y$, we have that $x = 0^p, y = 0^q$ for some $p, q \in \mathbb{N}$ with $1 < p, 1 < q$, and $p \neq q$. WLOG, suppose $p < q$.

Then $x1^p = 0^p1^p \notin A$, since the number of 0 and 1s are equal, but $y1^p = 0^q1^p \in A$, since $p < q$. Thus, x and y are distinguishable as required.

$A = \{w : w \text{ has more 0s than 1s}\}$ is not regular (Pumping Lemma)

By contradiction, suppose A is regular. Then there exists a pumping length k. Note that $w = 0^k 1^{k-1} \in A$, and has length $\geq k$. Thus, we can write $w = xyz$ satisfying the conditions of The Pumping Lemma.

By conditions 1 and 2, we have that $y = 0^i$ for some $i \in \mathbb{N}$ with $i > 0$. By condition 3, we have $xz \in A$, however, $xz = 0^{k-i}1^{k-1}$ which does not have more 0s than 1s (since $i \geq 1$) so we've reached a contradiction.

The twice as many $0s$ as $1s$ one is similar.