

CSC165H1 – Problem Set 0 (SAMPLE SOLUTION)

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My Courses

- CSC165H1, Mathematical Expression and Reasoning, G. Baum
- CSC148H1, Introduction to Computer Science, D. Horton
- PHL206H1, Later Medieval Philosophy, D. Black
- USA300H1, Theories and Methods in American Studies, A. Rahr
- WGS370H1, Utopian Visions, Activist Realities, J. Taylor

Set notation

$$S_1 \cap S_2 = \{0, 1, 4, 5, 6, 9, 10, 11, 14\}$$

Note that $15 \in S_1$ but $15 \notin S_2$, since 15 is not less than 15.

A truth table

| p | q | r | $(p \vee q) \Rightarrow (p \Leftrightarrow r)$ |
|-------|-------|-------|--|
| False | False | False | True |
| False | False | True | True |
| False | True | False | True |
| False | True | True | False |
| True | False | False | False |
| True | False | True | True |
| True | True | False | False |
| True | True | True | True |

A calculation

First we simplify the summation using the given formula, and substituting $d = 2$ and $k = 3$:

$$\begin{aligned}\sum_{i=0}^{n-1} (2i + 3) &= 3n + \frac{2n(n-1)}{2} \\ &= 3n + n^2 - n \\ &= n^2 + 2n\end{aligned}$$

Next, we need to solve this inequality:

$$\begin{aligned}\sum_{i=0}^{n-1} (2i + 3) &> 165 \\ n^2 + 2n &> 165 \\ n^2 + 2n - 165 &> 0\end{aligned}$$

Using the quadratic formula, the zeros of the polynomial $n^2 + 2n - 165$ are $\frac{-2 \pm \sqrt{4+660}}{2}$, or roughly $n_1 = -13.884 \dots$ and $n_2 = 11.884 \dots$

Noting that this parabola “points up”, its values are > 0 when $n < -13.884 \dots$ or $n > 11.884 \dots$

So the smallest positive integer value of n that makes $\sum_{i=0}^{n-1} (2i + 3)$ greater than 165 is 12.