Sprites (Fig. 22-10a) are huge flashes that occur far above a large thunderstorm. They were seen for decades by pilots flying at night, but they were so brief and dim that most pilots figured they were just illusions. Then in the 1990s sprites were captured on video. They are still not well understood but are believed to be produced when especially powerful lightning occurs between the ground and storm clouds, particularly when the lightning transfers a huge amount of negative charge $-q$ from the ground to the base of the clouds (Fig. 22-10b).

Just after such a transfer, the ground has a complicated distribution of positive charge. However, we can model the electric field due to the charges in the clouds and the ground by assuming a vertical electric dipole that has charge $-q$ at cloud height h and charge $+q$ at below-ground depth h (Fig. 22-10c). If $q = 200$ C and $h = 6.0$ km, what is the magnitude of the dipole's electric field at altitude $z_1 = 30$ km somewhat above the clouds and altitude $z_2 = 60$ km somewhat above the stratosphere?

KEY IDEA

We can approximate the magnitude E of an electric dipole's electric field on the dipole axis with Eq. 22-8.

Calculations: We write that equation as

$$E = \frac{1}{2\pi\varepsilon_0}\frac{q(2h)}{z^3},$$

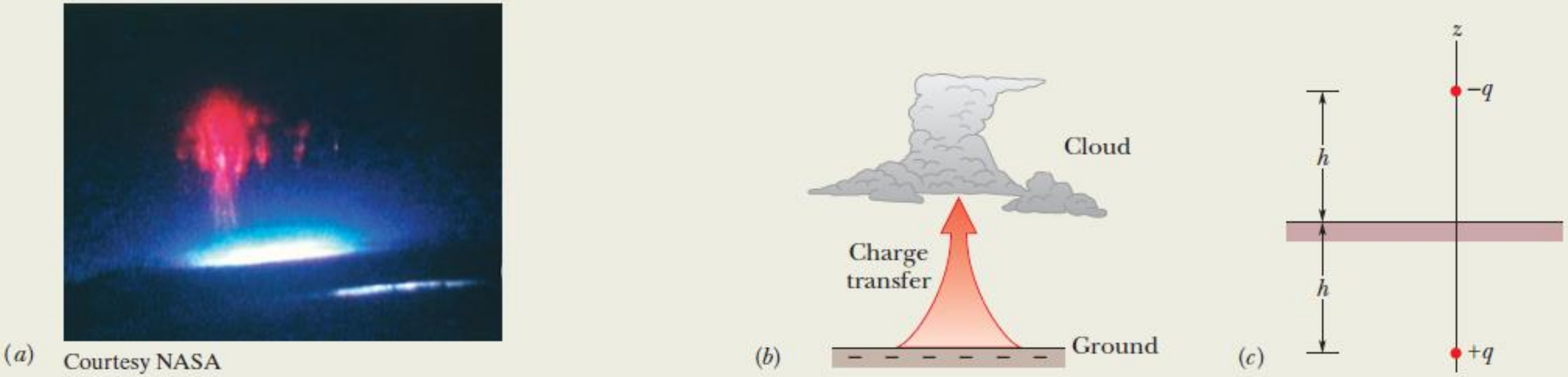
where $2h$ is the separation between $-q$ and $+q$ in Fig. 22-10c. For the electric field at altitude $z_1 = 30$ km, we find

$$E = \frac{1}{2\pi\varepsilon_0}\frac{(200\text{ C})(2)(6.0\times10^3\text{ m})}{(30\times10^3\text{ m})^3}$$
$$= 1.6\times10^3\text{ N/C.} \qquad \text{(Answer)}$$

Similarly, for altitude $z_2 = 60$ km, we find

$$E = 2.0\times10^2\text{ N/C.} \qquad \text{(Answer)}$$

As we discuss in Module 22-6, when the magnitude of



83 **SSM** An electric dipole with dipole moment

$$\vec{p} = (3.00\hat{i} + 4.00\hat{j})(1.24 \times 10^{-30}\ \mathrm{C \cdot m})$$

is in an electric field $\vec{E} = (4000\ \mathrm{N/C})\hat{i}$. (a) What is the potential energy of the electric dipole? (b) What is the torque acting on it? (c) If an external agent turns the dipole until its electric dipole moment is

$$\vec{p} = (-4.00\hat{i} + 3.00\hat{j})(1.24 \times 10^{-30}\ \mathrm{C \cdot m}),$$

how much work is done by the agent?

a) $U = -\bar{p} \cdot \bar{E}$

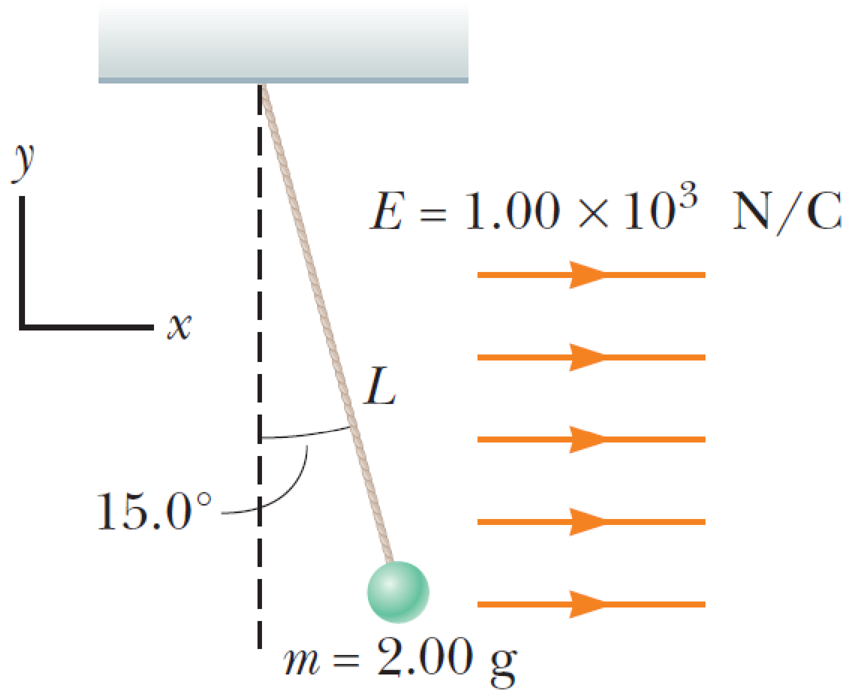
$= -\left[(3\hat{i}+4\hat{j})(1.24\times10^{-30}\,Cm)\right] \cdot \left[(4000\ N/C)\hat{i}\right]$

$= -1.49\times10^{-26}\ J$

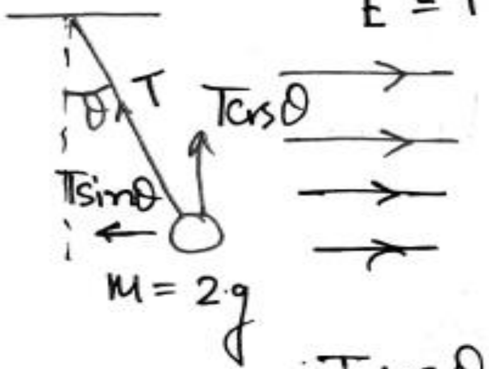
b) $\bar{\tau} = \bar{p}\times\bar{E}$ = Do the calculation.

c) Home Work.

A small, 2.00-g plastic ball is suspended by a 20.0-cm-long string in a uniform electric field as shown in Figure P23.33. If the ball is in equilibrium when the string makes a 15.0° angle with the vertical, what is the net charge on the ball?



$E = 1 \times 10^{3}\ N/C$



$$T\cos\theta = mg$$

$$T\cos 15^\circ = 1.96 \times 10^{-2}\,N$$

$$\text{so, } T = 2.03 \times 10^{-2}\,N$$

from $\sum F_x = 0$, so $qE = T\sin 15^\circ$

$$\Rightarrow q = \frac{T\sin 15^\circ}{E}$$

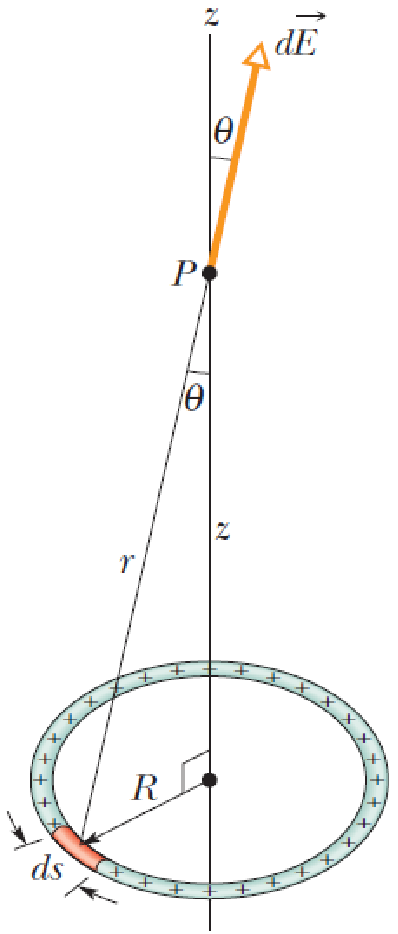
$$= \frac{(2.03 \times 10^{-2}\,N)\sin 15^\circ}{1 \times 10^{3}\,N/C}$$

$$\boxed{q = 5.25\,\mu C}$$

72 An electron is constrained to the central axis of the ring of charge of radius R in Fig. 22-11, with $z \ll R$. Show that the electrostatic force on the electron can cause it to oscillate through the ring center with an angular frequency

$$\omega = \sqrt{\frac{eq}{4\pi\varepsilon_0 mR^3}},$$

where q is the ring's charge and m is the electron's mass.



Electric field of a uniformly charge ring:

$$E = \frac{q z}{4\pi\epsilon_0 (z^2+R^2)^{3/2}}$$

The force acting on the electron:

$$F = -\frac{e q z}{4\pi\epsilon_0 (z^2+R^2)^{3/2}}$$

For small amplitude oscillations $z << R$ and z can be neglected, thus:

$$F = -\frac{e q z}{4\pi\epsilon_0 R^3}$$

$$F = -K z$$

$$k = \frac{eq}{4\pi\epsilon_0 R^3}$$

The electron moves in simple harmonic motion with an angular frequency given by:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{eq}{4\pi\epsilon_0 m R^3}}$$

••54 GO In Fig. 22-61, an electron is shot at an initial speed of $v_0 = 2.00 \times 10^6$ m/s, at angle $\theta_0 = 40.0°$ from an x axis. It moves through a uniform electric field $\vec{E} = (5.00\ \text{N/C})\hat{\text{j}}$. A screen for detecting electrons is positioned parallel to the y axis, at distance $x = 3.00$ m. In unit-vector notation, what is the velocity of the electron when it hits the screen?

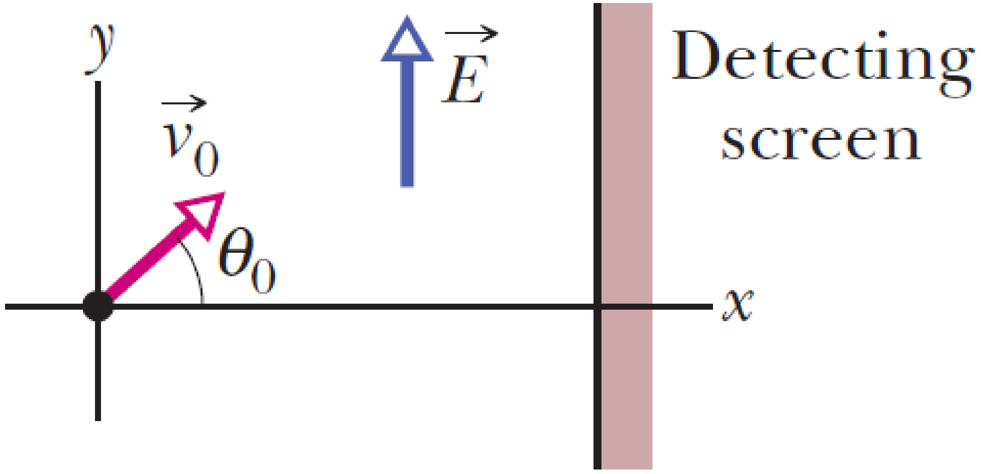


Figure 22-61 Problem 54.

This is exactly a projectile motion problem

so, $a = \frac{eE}{m} = 8.78 \times 10^{11}\ m/s^2$

from equation of motion: $t = \frac{x}{v_0 \cos\theta_0}$

$= \frac{3}{(2 \times 10^6\ m/s) \cos 40^\circ} = 1.96 \times 10^{-6}\ s$

$v_y = v_0 \sin\theta_0 - at$
$= -4.34 \times 10^5\ m/s$

$\bar{v} = (1.53 \times 10^6\ m/s)\hat{i} - (4.34 \times 10^5\ m/s)\hat{j}$