Attitude Determination

- Accelerometers
- Magnetometers
- Attitude Algorithm

NASA Image

Accelerometers

- As long as the vehicle is not moving, an accelerometer measures the three body-axis components of the gravity vector.

  measures in the body axis!

  - The general case of a moving vehicle will be discussed in a later lecture.

body

$$\vec{g}^{B} = T_{L2B}\,\vec{g}^{L}$$

HW 1 Prob 5

local

$$\begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix} = \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ \sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi & \sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi & \sin\phi\cos\theta \\ \cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi & \cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi & \cos\phi\cos\theta \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix}$$

- An accelerometer actually measures applied force, which means the direction of the gravity vector is reversed.

$$\begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix} = \begin{bmatrix} -g\sin\theta \\ g\sin\phi\cos\theta \\ g\cos\phi\cos\theta \end{bmatrix} \Rightarrow \boxed{\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} g\sin\theta \\ -g\sin\phi\cos\theta \\ -g\cos\phi\cos\theta \end{bmatrix}}$$

Roll and Pitch Euler Angles

- Use accelerometer measurements to find roll and pitch Euler angles.

$$\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} g \sin\theta \\ -g \sin\phi \cos\theta \\ -g \cos\phi \cos\theta \end{bmatrix}$$

$$a_x = g \sin\theta$$

$$a_y^2 + a_z^2 = g^2 \sin^2\phi \cos^2\theta + g^2 \cos^2\phi \cos^2\theta = g^2 \cos^2\theta$$

ax, ay, az are real time values from the sensor

$$\boxed{\theta = \tan^{-1}\left[\frac{a_x}{\sqrt{a_y^2 + a_z^2}}\right]}$$

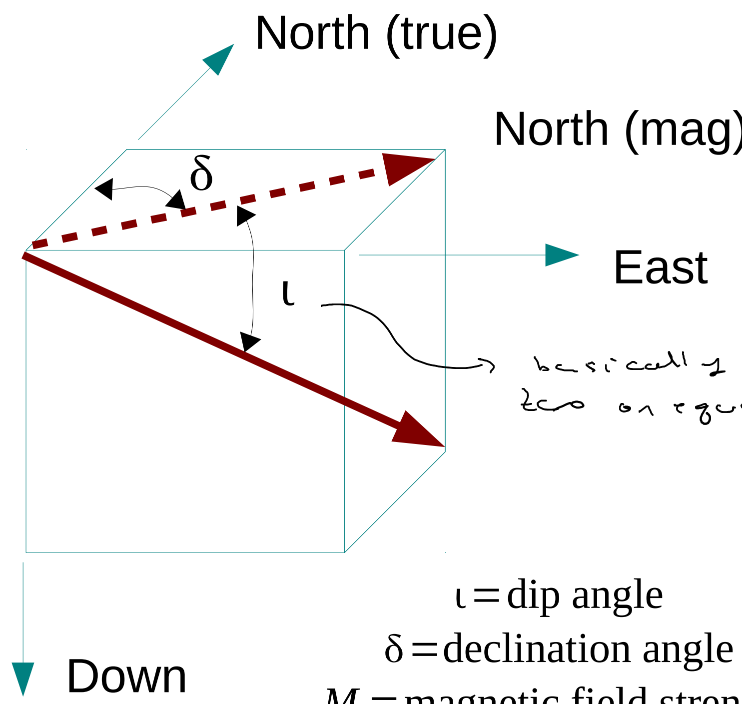
$$\boxed{\phi = \tan^{-1}\left[\frac{-a_y}{-a_z}\right]}$$

↳ draw it out (positive is from the axis)

Magnetometer

- A magnetometer measures three components of the Earth's magnetic field. This data can be used to determine vehicle heading angle.

local csys



basically zero on equator

$$\begin{bmatrix} m_x \\ m_y \\ m_z \end{bmatrix} = T_{L2B} T_{M2L} \begin{bmatrix} M \\ 0 \\ 0 \end{bmatrix}$$

$m_x, m_y, m_z =$ body-axis magnetometer measurements

$\iota =$ dip angle

$\delta =$ declination angle

$M =$ magnetic field strength

can measure heading

uses δ & ι

$$T_{M2L} = T_3(-\delta) T_2(\iota) = \begin{bmatrix} \cos\delta & -\sin\delta & 0 \\ \sin\delta & \cos\delta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\iota & 0 & -\sin\iota \\ 0 & 1 & 0 \\ \sin\iota & 0 & \cos\iota \end{bmatrix}$$

See NGS Magnetic Field Model at magnetic-declination.com

δ is angle between mag N and true N

Solve for Heading Angle

- Break the product of L2B and L2M transformations into two parts.

$$\begin{bmatrix} m_x \\ m_y \\ m_z \end{bmatrix} = T_{L2B} T_{M2L} \begin{bmatrix} M \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} m_x \\ m_y \\ m_z \end{bmatrix} = T_1(\phi) T_2(\theta) T_3(\psi) T_3(-\delta) T_2(\iota) \begin{bmatrix} M \\ 0 \\ 0 \end{bmatrix}$$

$$\Downarrow$$

$$T_2^T(\theta) T_1^T(\phi) \begin{bmatrix} m_x \\ m_y \\ m_z \end{bmatrix} = T_3(\psi) T_3(-\delta) T_2(\iota) \begin{bmatrix} M \\ 0 \\ 0 \end{bmatrix}$$

$$\boxed{\begin{aligned} m_x \cos\theta + m_y \sin\theta \sin\phi + m_z \sin\theta \cos\phi &= M \cos\iota \cos(\psi - \delta) \\ m_y \cos\phi - m_z \sin\phi &= -M \cos\iota \sin(\psi - \delta) \\ -m_x \sin\theta + m_y \cos\theta \sin\phi + m_z \cos\theta \cos\phi &= M \sin\iota \end{aligned}}$$

$x > 0$ $y > 0$ $0 < \theta < 90$ $x < 0$, $y > 0$ $90 < \theta < 180$ (and so on ...)

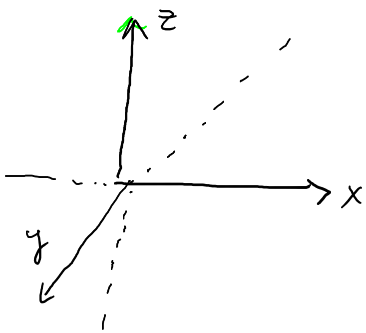
Attitude Determination Algorithm

- Vehicle attitude is determined using the following algorithm.
  - This formulation is sometimes called a "tilt-compensated" compass.

gets correct ψ even w/ tilt

from both sensors, can get Euler angles and look at orientation

Accelerometer measures: ax, ay, az



$$\phi = \tan^{-1}\left[\frac{-a_y}{-a_z}\right]$$

$$\theta = \tan^{-1}\left[\frac{a_x}{\sqrt{a_y^2 + a_z^2}}\right]$$

use atan2 for this algorithm

Magnetometer measures: mx, my, mz

heading (about down, z-axis)

orientation

$$\psi = \delta + \tan^{-1}\left[\frac{m_z \sin\phi - m_y \cos\phi}{m_x \cos\theta + m_y \sin\theta \sin\phi + m_z \sin\theta \cos\phi}\right]$$

heading only (like a compass)

$$\psi = \delta + \tan^{-1}\left(\frac{-m_y}{m_x}\right)$$