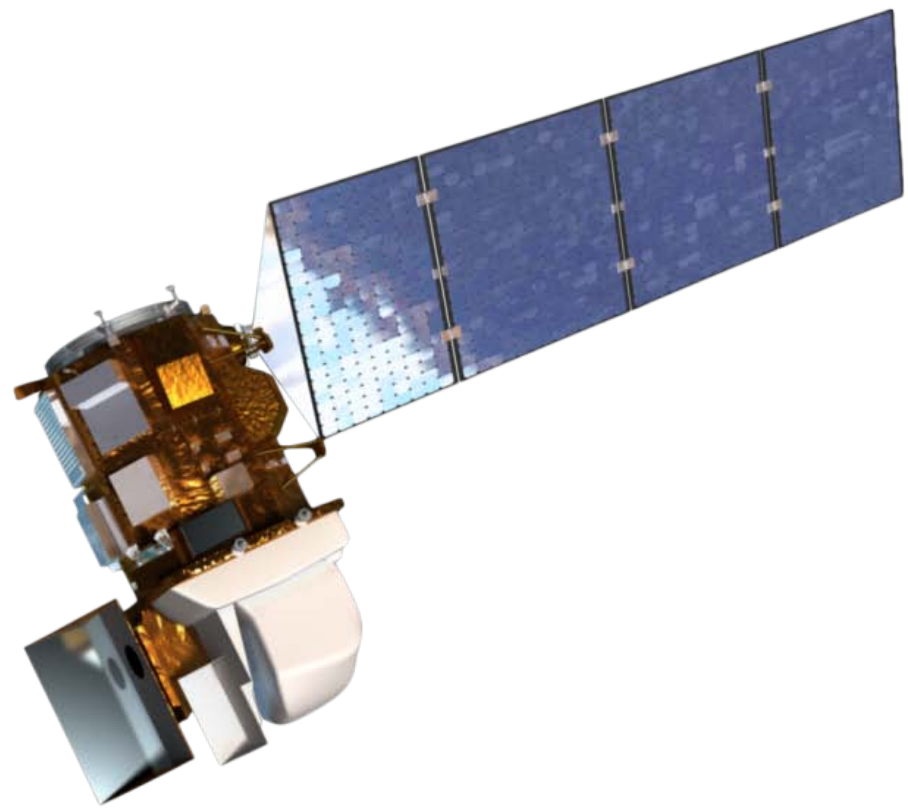
Sliding-window techniques

Anne Denton's Research Group

Traditional Technology: Landsat

- Since 1972
- 30 m resolution
  - Few hundred pixels per field
- Large satellites
- ~ 700 km altitude
- Weighs 5.8k pounds
- Landsat 8 newest



What to use it for?

Agriculture perspective

- Getting information from agricultural fields without going there
  - Recognizing crop disease / storm damage
  - Knowing what is planted where (crop cover)
- Predicting crop quantity / quality
  - Yield prediction (typical machine learning challenge)
  - Knowing where to apply fertilizer
  - Recognizing other chemical deficiencies
  - Understanding water stress
- Understanding soil problems
  - Salinity
  - Erosion

Machine learning perspective

- Mostly classification or regression problems
- Typically lack of training data
- Typically risk of losing access to training data
- Solutions:
  - Predicting land use
  - Use of independent information as proxy

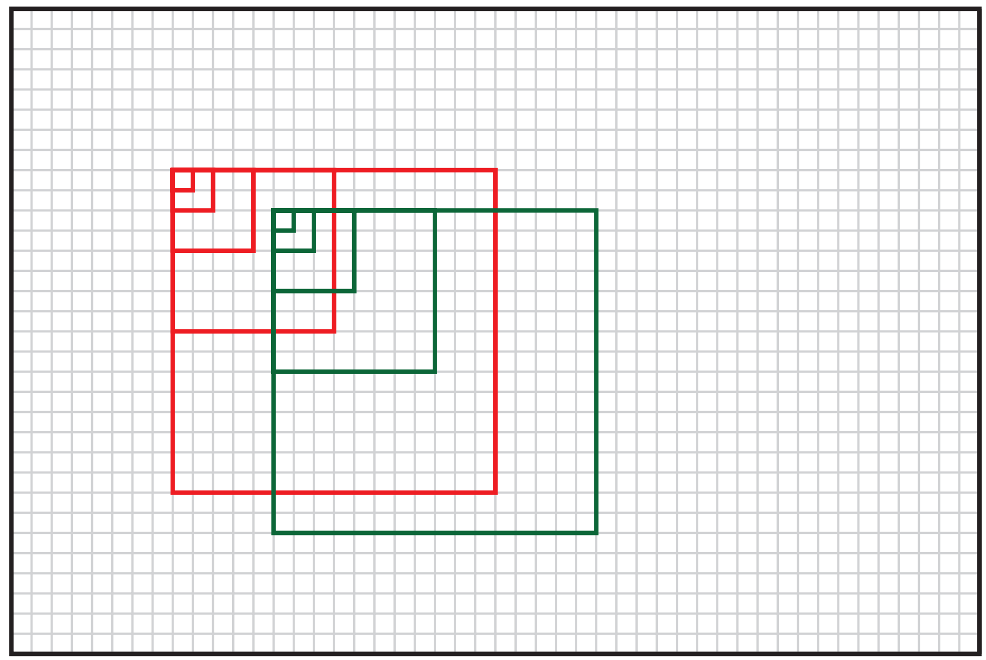
New Technology

- Cube sats
  - 3m resolution
  - 450 km altitude
  - Weigh 9 lbs
  - 88 satellites

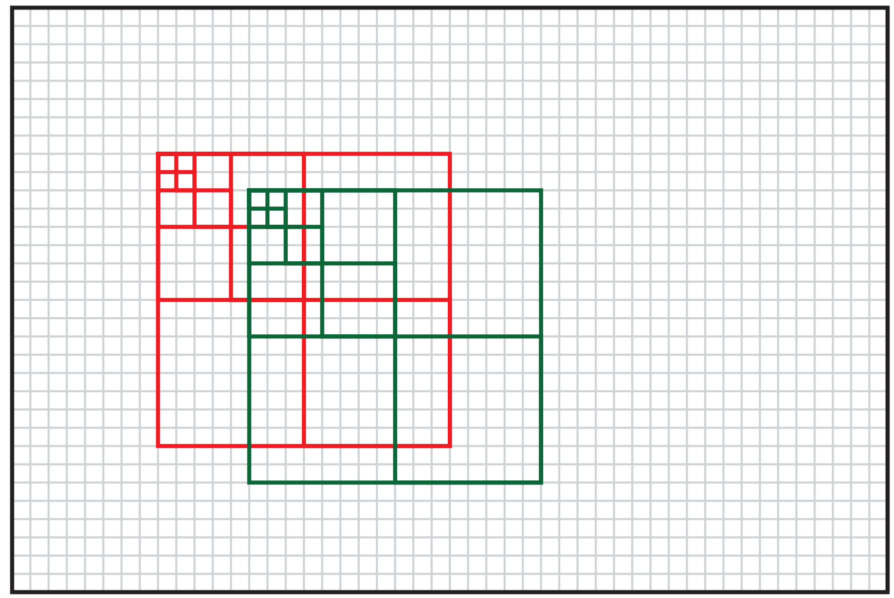
- Drones
  - 1-10 cm resolution

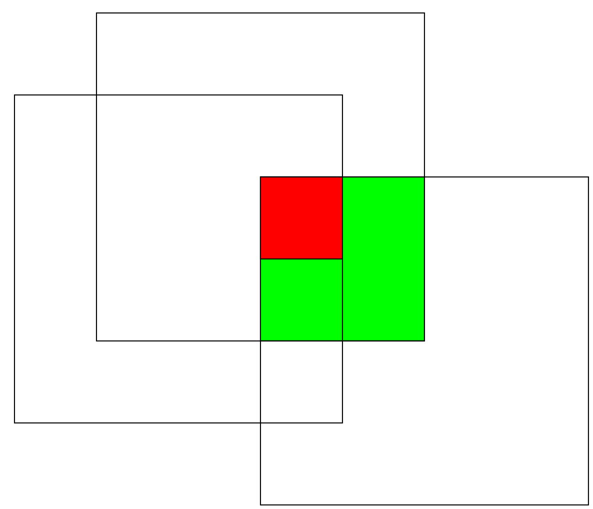
What do you do with that resolution?

- Question: Why do I care about cm resolution if farm implements are 100 feet wide?
- Instead of 1 pixel you get up to 100
- You don't even have to lose resolution



Is it too computationally expensive?





- Efficient algorithm for linear aggregates
  - SUM / COUNT
  - MAX / MIN
  - Sums of squares
  - Sums of higher order terms
- Problems for
  - MEDIAN

Algorithm

- For window size w, image size is size-w+1
- Windows that are aggregated are delta = w/2 apart
- Most aggregates get reused 4 times

Sliding-window-based aggregation in $\log_2(w)$ steps

Window size w = 4
Example: MAX

Step 1: Aggregation over 2x2 windows

Step 2: Aggregation with offset

1	2	3	1	1
4	2	1	1	1
5	1	1	0	1
1	0	0	6	3
1	0	2	7	9

Original raster

4	

Intermediate raster (empty)

4	3	3	1
5	2	1	1
5	1	6	6
1	2	7	9

Intermediate raster (full)

6

Final raster

1	2	3	1	1
4	2	1	1	1
5	1	1	0	1
1	0	0	6	3
1	0	2	7	9

4	3

4	3	3	1
5	2	1	1
5	1	6	6
1	2	7	9

6	6

Values that contribute to element in final raster

1	2	3	1	1
4	2	1	1	1
5	1	1	0	1
1	0	0	6	3
1	0	2	7	9

4	3	3	1
5	2	1	1
5	1	6	6
1	2	7	9

4	3	3	1
5	2	1	1
5	1	6	6
1	2	7	9

6	6
7	9

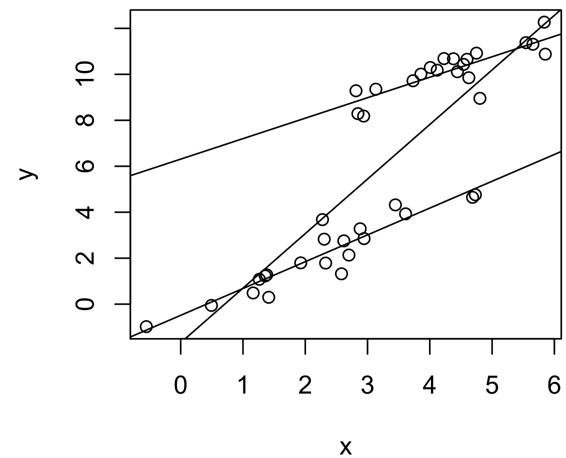
Each aggregation involves 4 raster elements (8 for both steps)
Direct computation would require aggregation of 16 elements

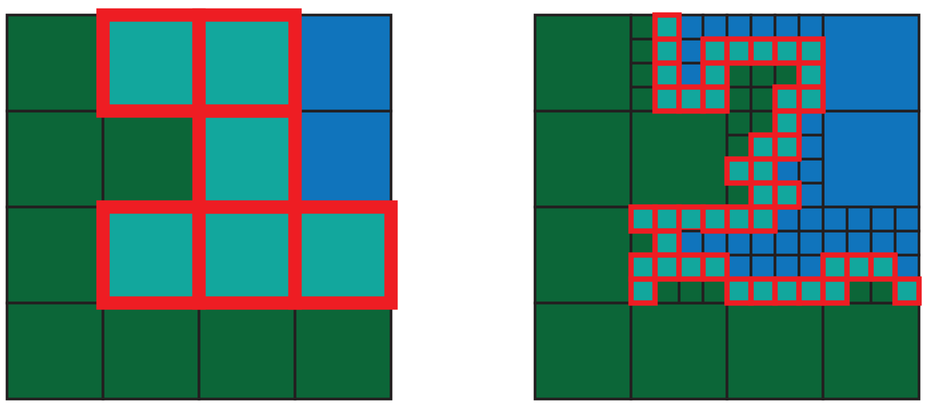
Limitation to “linear” aggregates

- Linear models, but not necessarily linear regression
  - Much of statistics uses linearity
  - Linear models dominate regression
- What does “linear” mean?
  - A fit with a quadratic polynomial is still a linear model
  - It is linear in the parameters
- If you wanted to use polynomial regression across multiple databases, you could locally compute
  - Count, sums, sums of squares, sums of higher order terms
  - Aggregate the aggregates in a central place
- Same holds for iterative aggregation of windows

Some Approaches so far

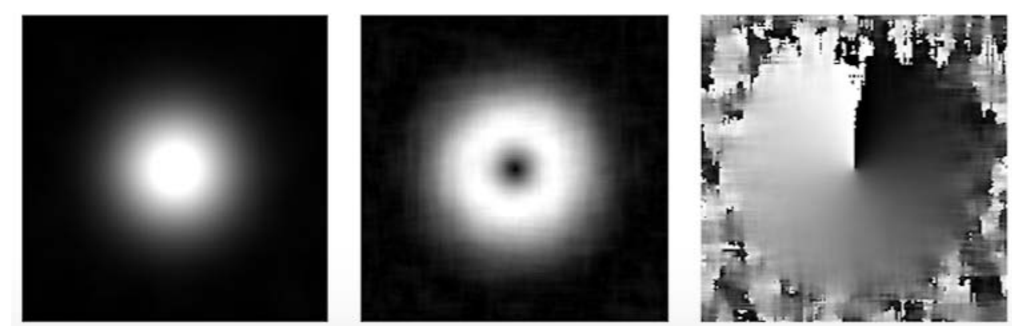
1) Regression





2) Fractal Dimension

3) Topography



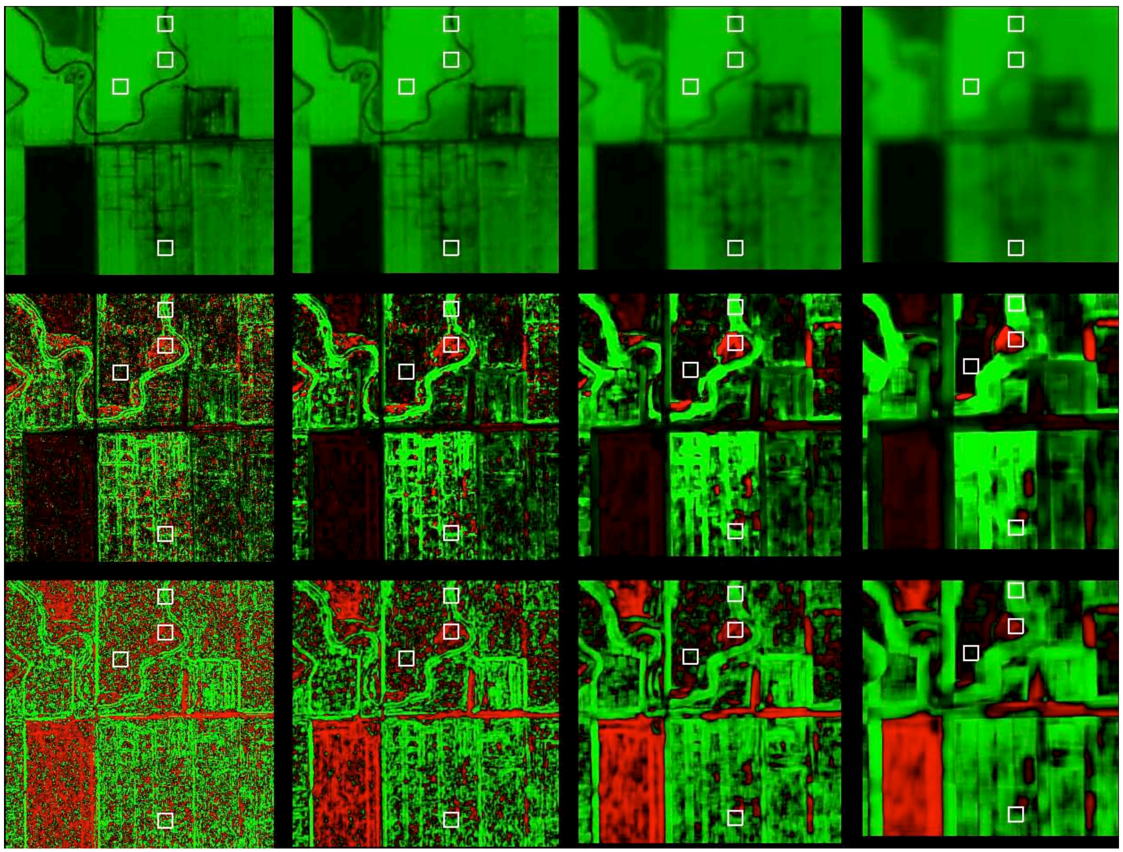
Regression

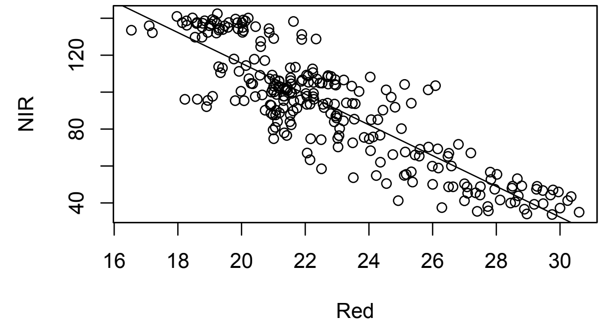
- Linear regression
- Aggregation done for all 5 aggregates
- (Pearson correlation also uses y^2)
- We could use higher order polynomials but haven't (Polynomial regression is still a linear model)

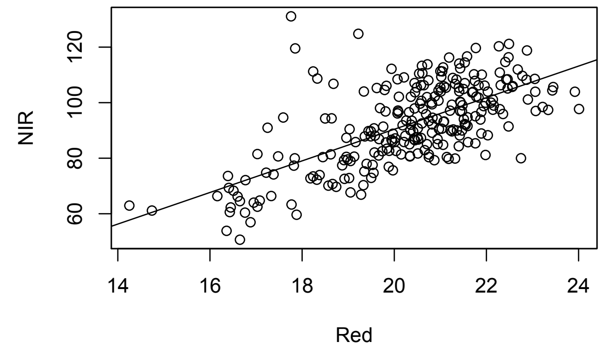
$$slope = \frac{N \sum x_{ij} y_{ij} - \sum x_{ij} \sum y_{ij}}{N \sum x_{ij}^2 - \left(\sum x_{ij}\right)^2}$$

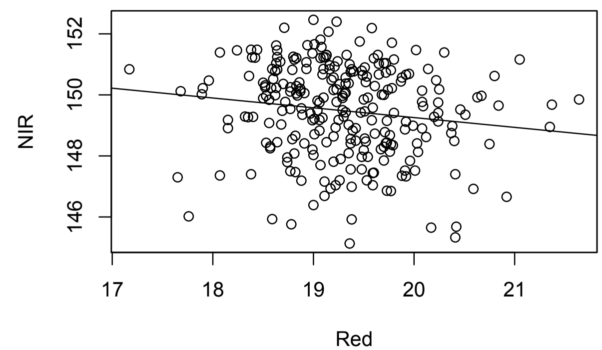
$$z_{ij}^{(base)} = \text{any of} \begin{cases} x_{ij} \\ y_{ij} \\ x_{ij}^2 \\ y_{ij}^2 \\ x_{ij} y_{ij} \end{cases}$$

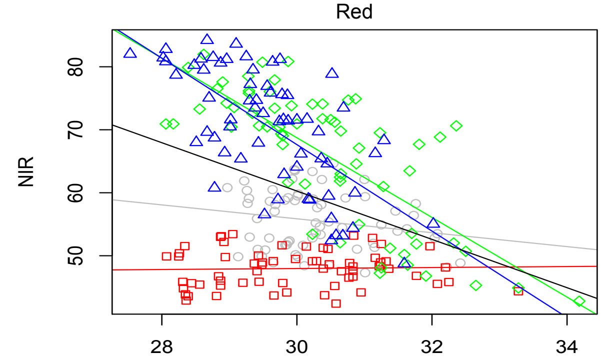
Results of NIR vs. Red Band





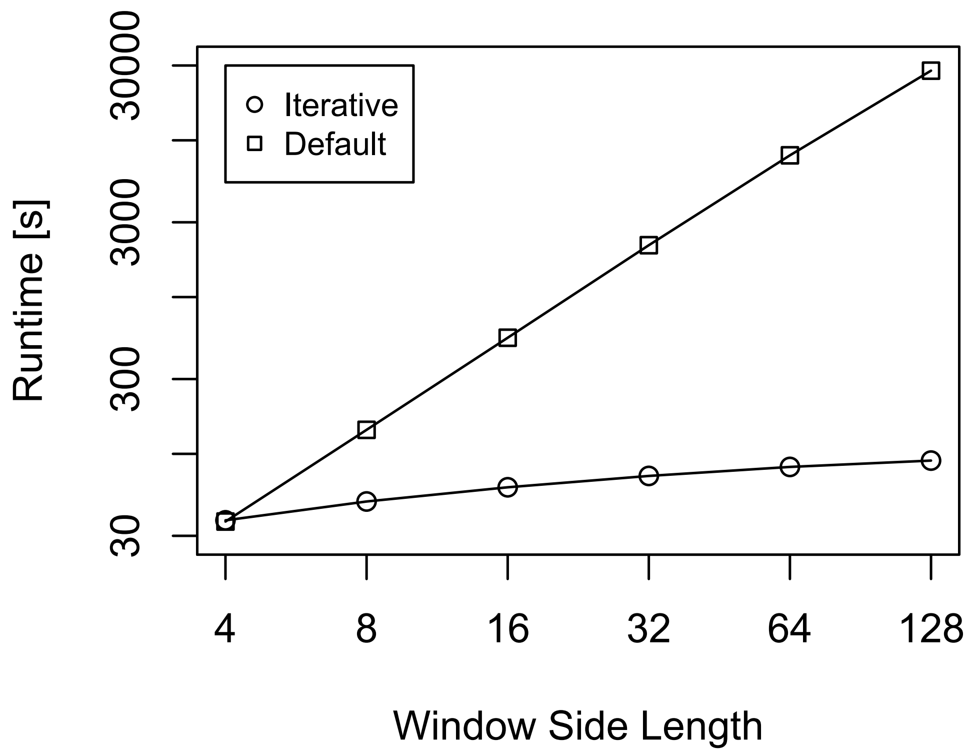






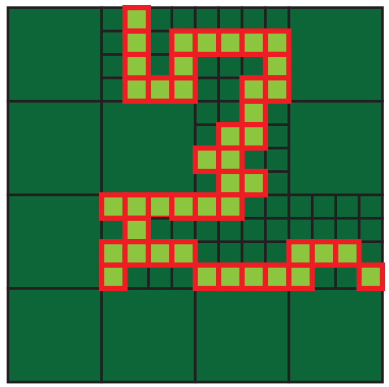
Performance

- Scaling logarithmic in window size
- Dramatic difference for large window sizes

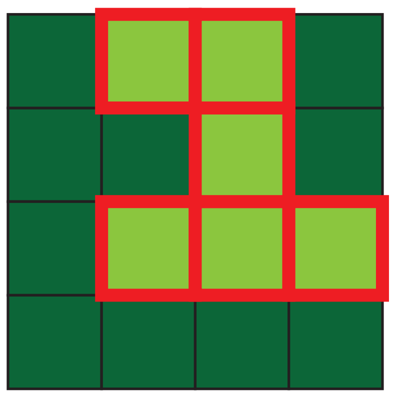


Fractal Dimension

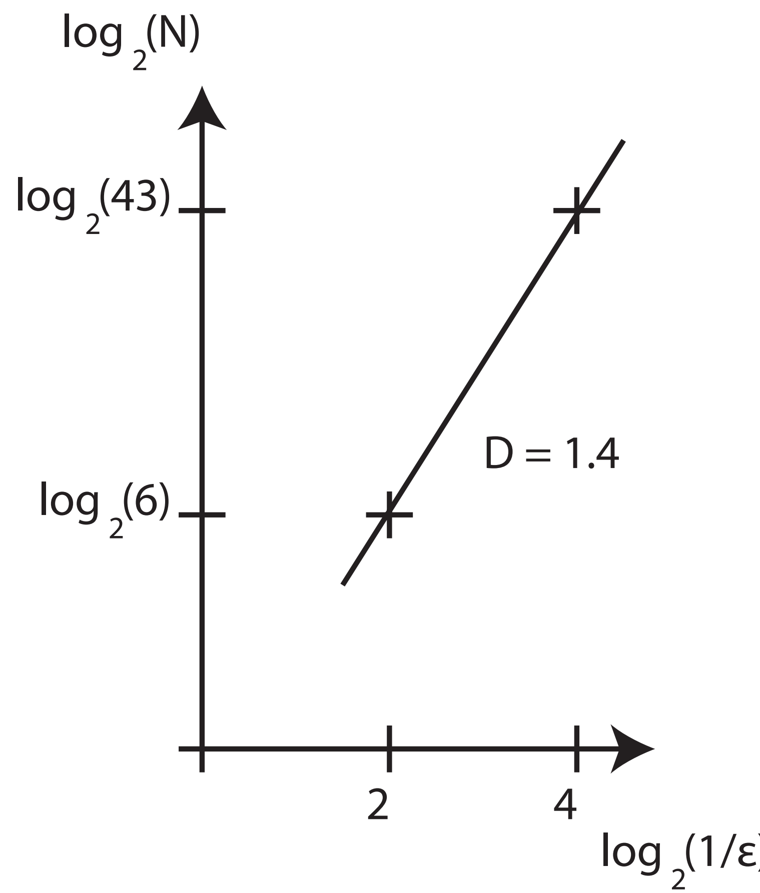
- Motivation
  - Length of shoreline of England
- Fractals proposed by Benoit Mandelbrot
- Captures self-similarity
- Used extensively for past 50 years
  - Used in image processing
  - Including urban growth
- **Only uses sum and max**
- https://en.wikipedia.org/wiki/Koch_snowflake



$N = 43$
$\varepsilon = 1/16$

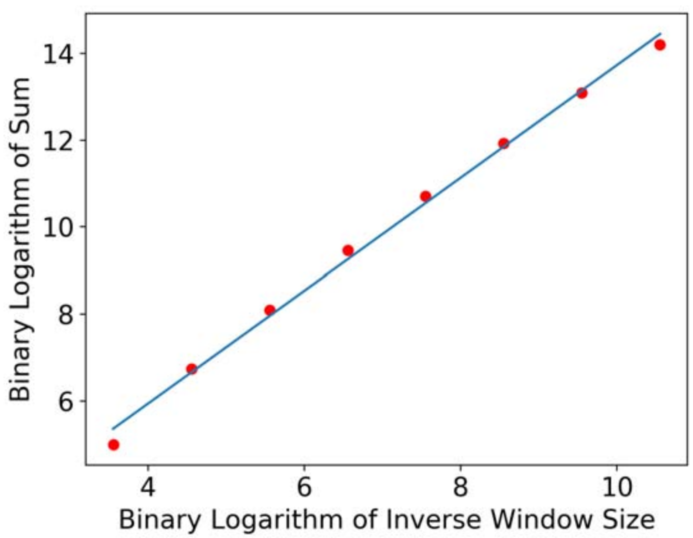


$N = 6$
$\varepsilon = 1/4$



Intermediate Steps

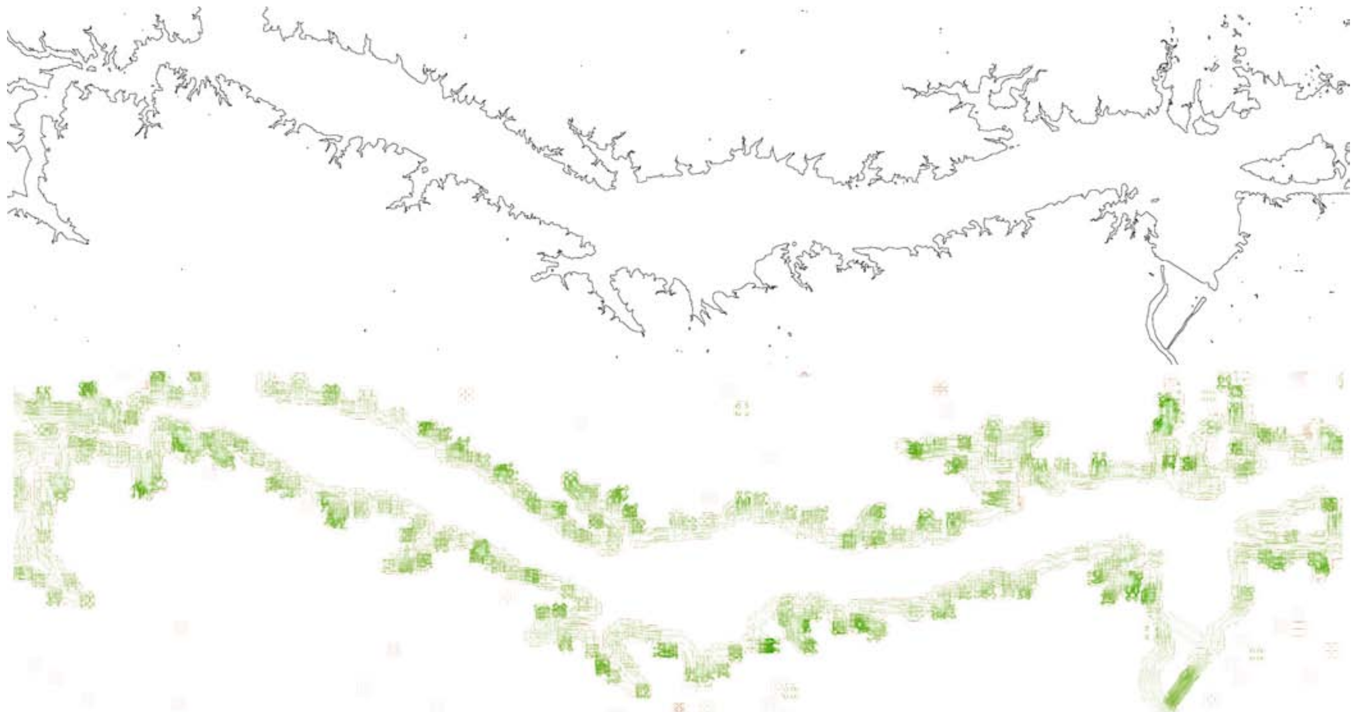
- Right-hand side shows max aggregation
- Sum aggregation for counting them
  - Bridges remaining resolutions
- Only non-overlapping windows used in any one regression
- Different overlapping windows used in multiple shifted windows
- Each level of max aggregation corresponds to one point in regression for each window



Example Lake Sakakawea

- Only non-integral fractal dimensions shown
- Regression values with large margin suppressed

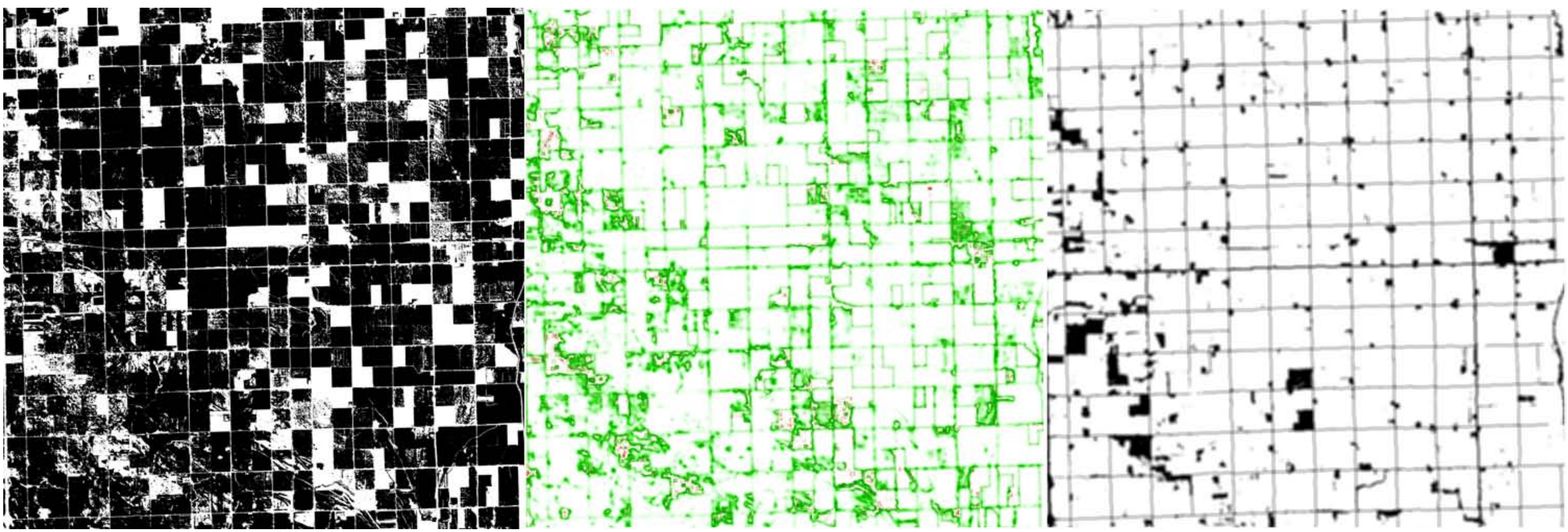
0 1 2



- Dimensions between 1 and 2 green
- Some prairie potholes show up as red

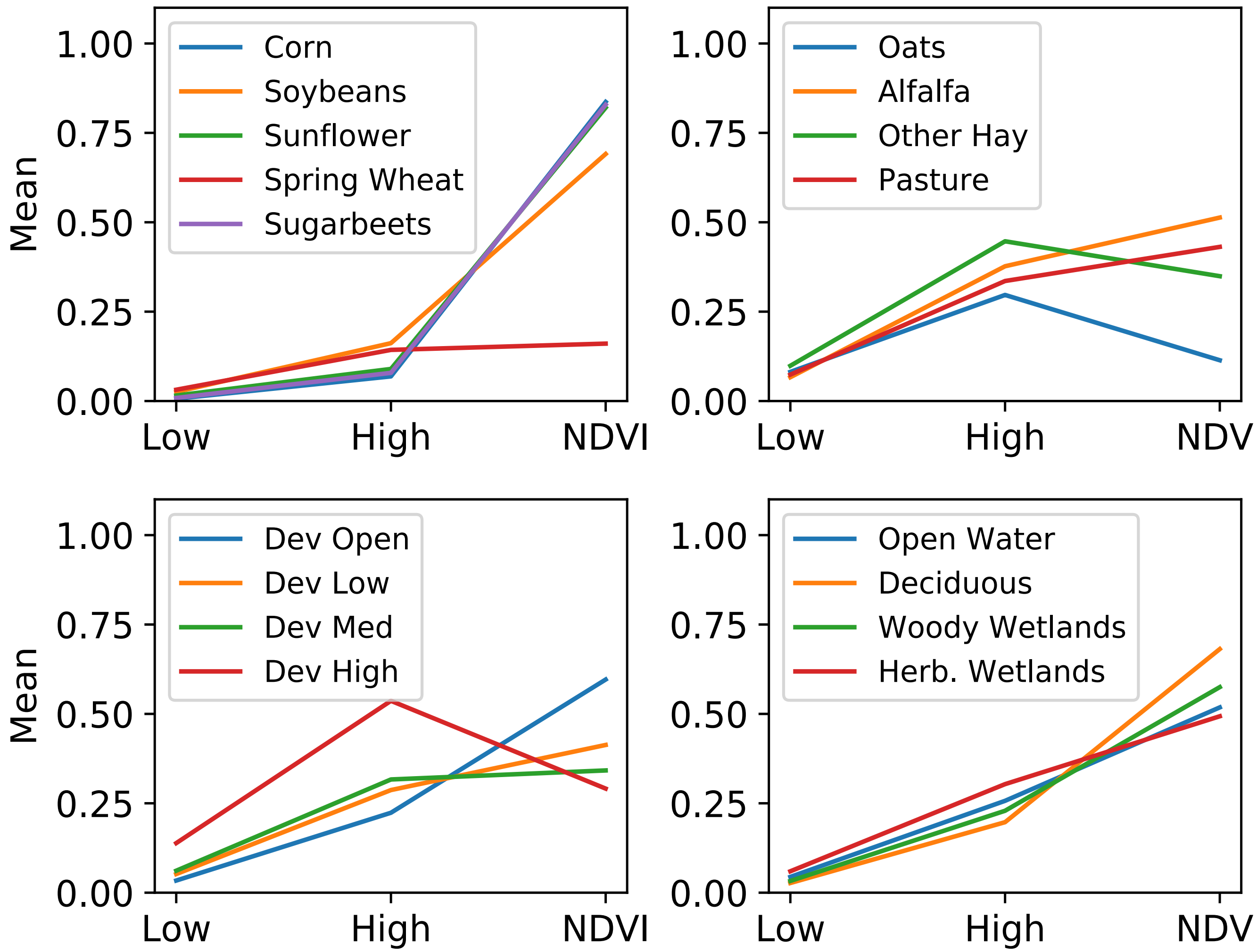
Correlation with Land Use

- Left NDVI with cutoff 0.5
- Middle fractal dimension
- Right non-agricultural land uses



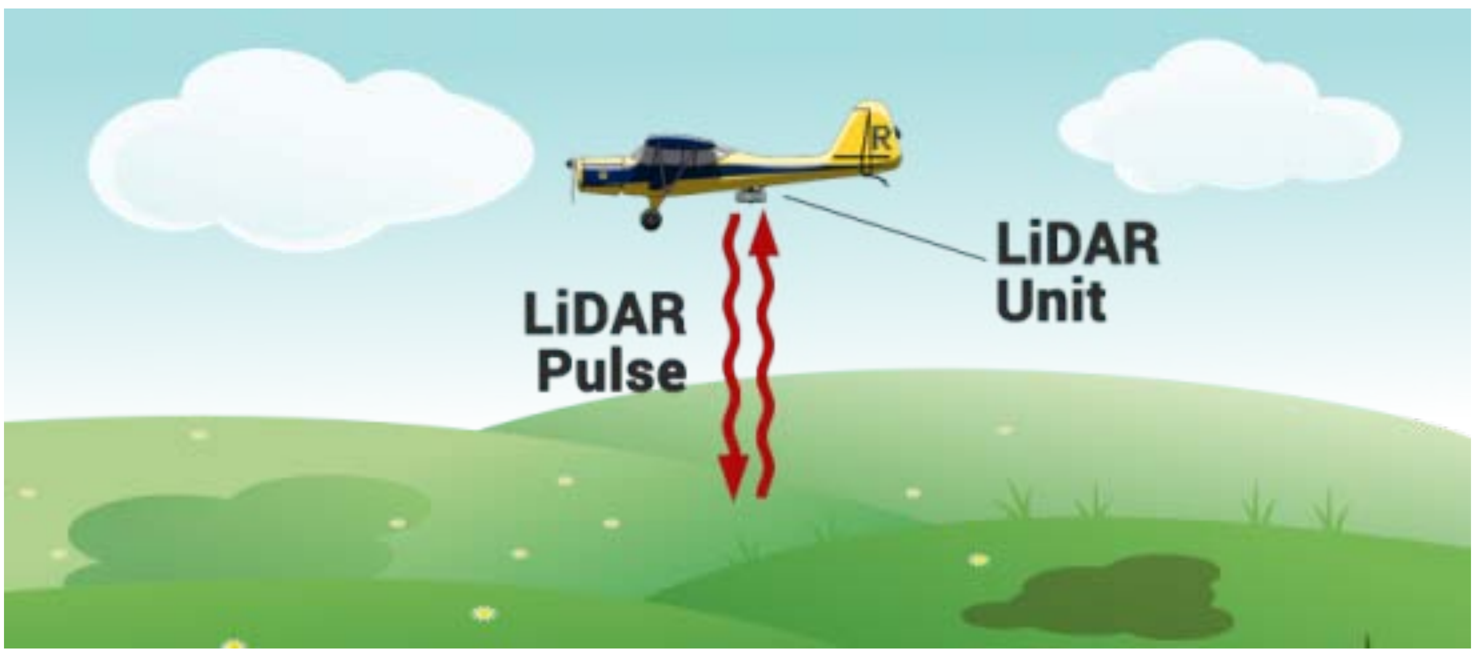
Grouped by Land Use

- Main crop species have low fractality
- Less intense agriculture has higher fractality
- Areas around water high NDVI and fractality between 1 and 2
- Highest fractality in highly developed areas



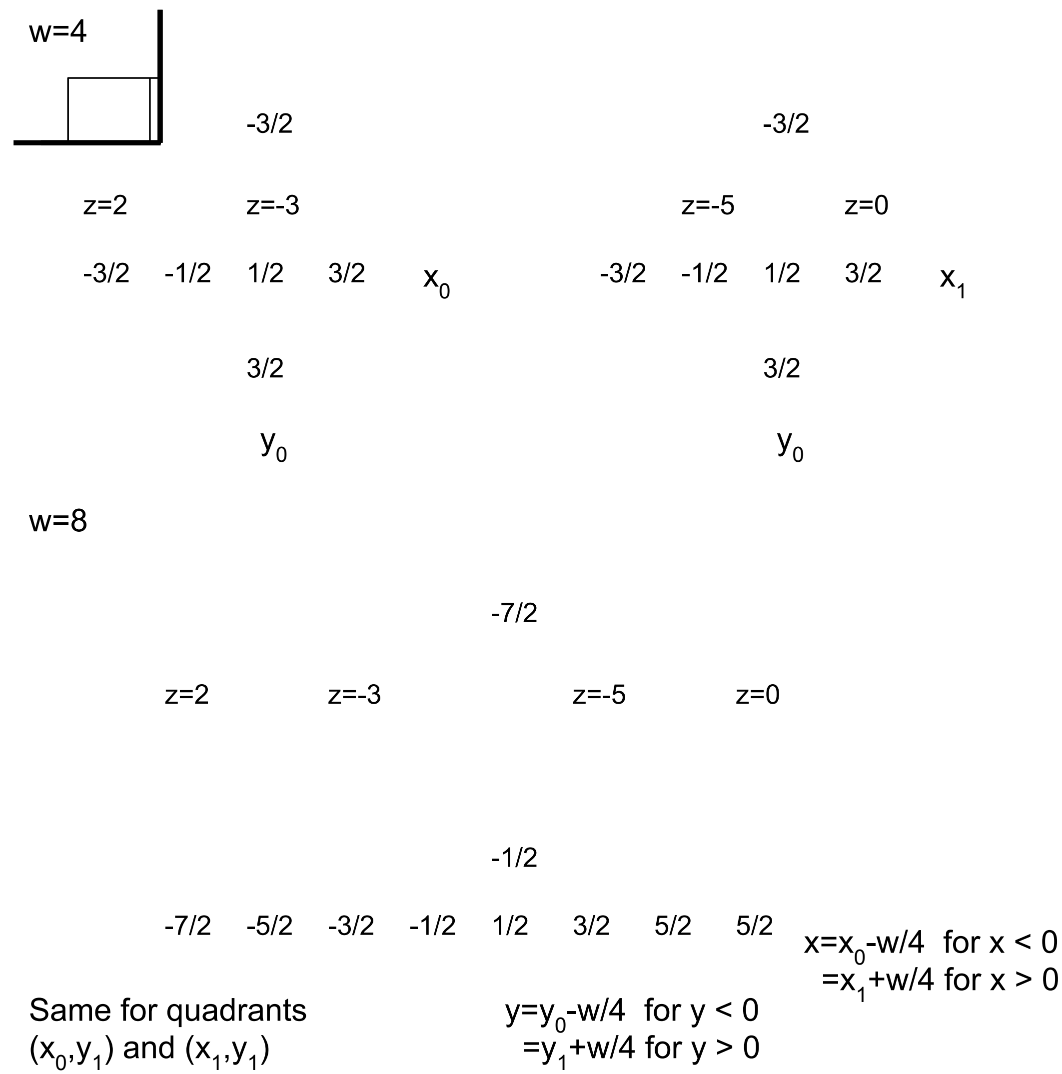
Topography

- Slope and Curvature rely on polynomials of elevation
- 1 m resolution elevation data available from LIDAR measurements
- Existing techniques assume much larger pixels
  - Mostly use 3 x 3 windows
  - Conventionally data rescaled



Additional mathematical challenges

- Now we need aggregates of spatial variables
- Treated as shifting of reference frames
- Curvature also requires higher order aggregates



Coordinate shifts

- Averages over spatial dimension x vanish because of symmetry
- Averages over elevation z can be done as previously
- Averages over xz explicitly involve coordinate shifts

$$
\begin{aligned}
&\langle xz\rangle \\
&= \quad \frac{1}{4}\left(\left\langle\left(x_0-\frac{w}{4}\right) z_{00}\right\rangle+\left\langle\left(x_1+\frac{w}{4}\right) z_{10}\right\rangle\right. \\
&\quad\quad \left.+\left\langle\left(x_0-\frac{w}{4}\right) z_{01}\right\rangle+\left\langle\left(x_1+\frac{w}{4}\right) z_{11}\right\rangle\right)
\end{aligned}
$$

Involves differences between aggregates from previous iteration

$$
\begin{aligned}
&\langle xz\rangle \\
&= \quad \frac{1}{4}\left(\langle xz\rangle_{00}+\langle xz\rangle_{10}+\langle xz\rangle_{01}+\langle xz\rangle_{11}\right. \\
&+ \quad \left.\frac{w}{4}\left(\langle z\rangle_{10}+\langle z\rangle_{11}-\langle z\rangle_{00}-\langle z\rangle_{01}\right)\right)
\end{aligned}
$$

Comparison of elevation, slope, and aspect with ArcGIS results

w=

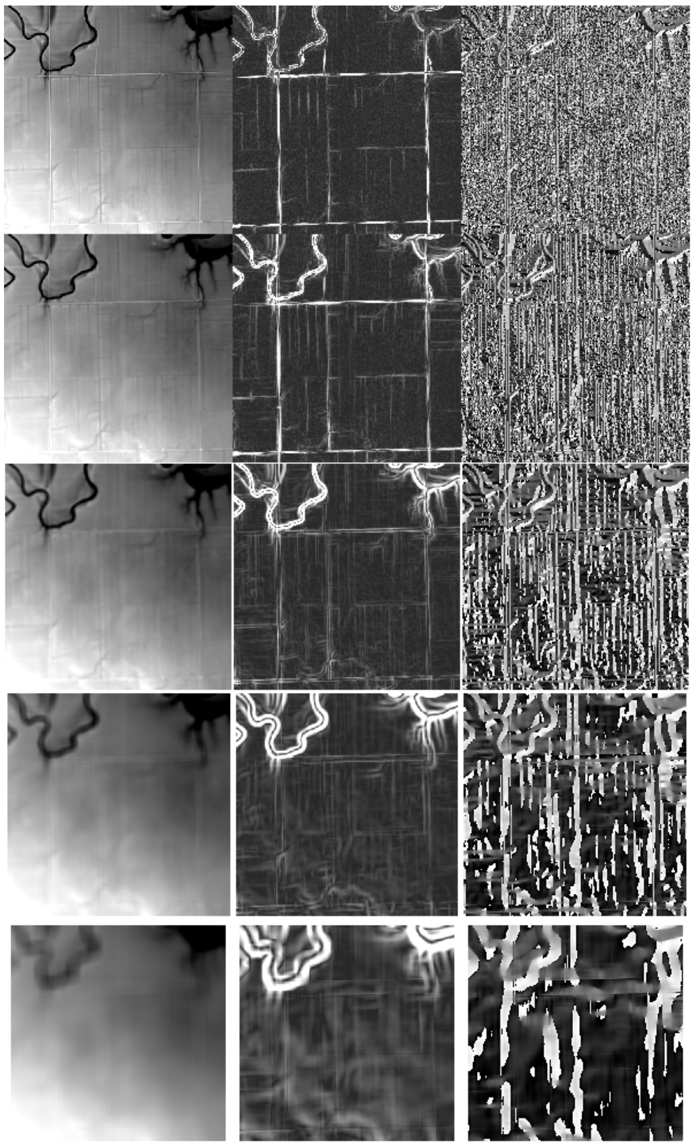
2

4

8

16

32



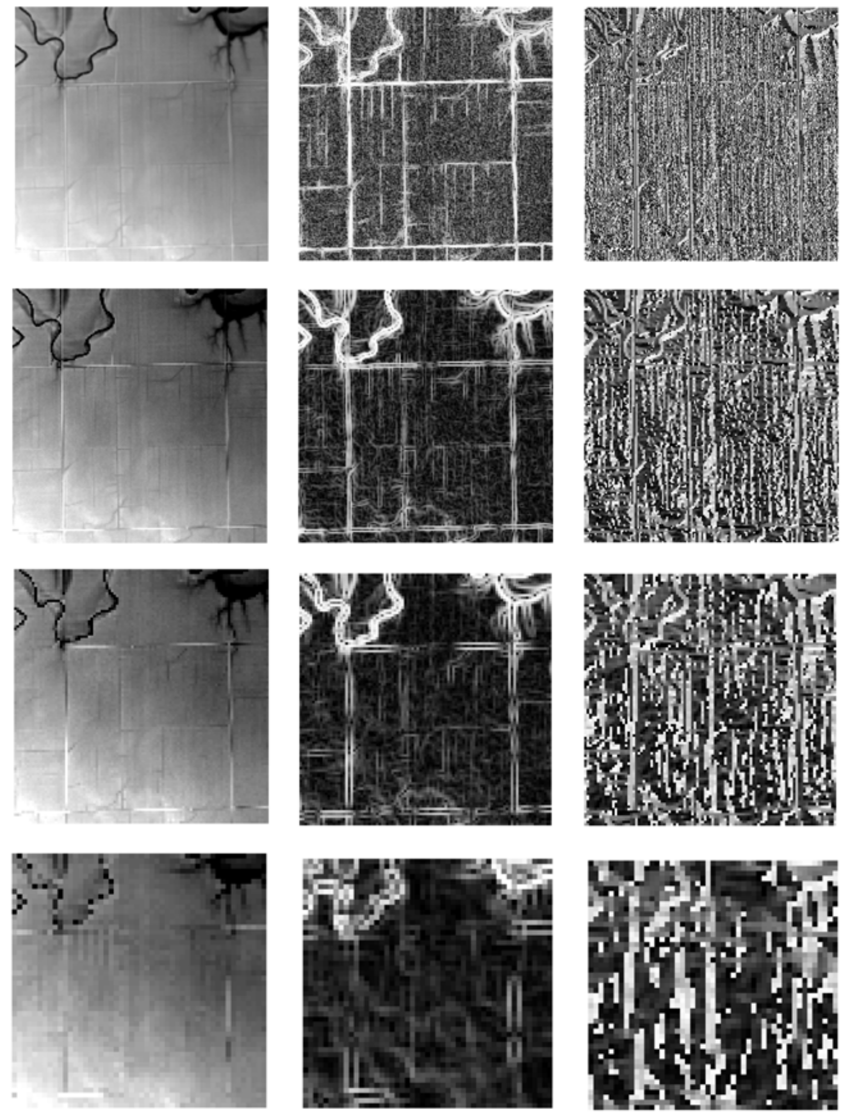
Effec-
tive w

3

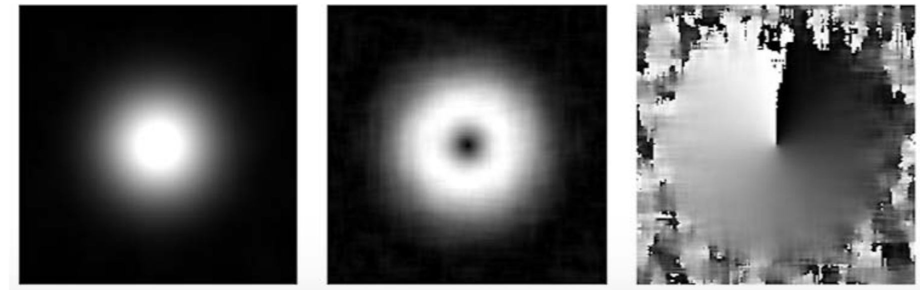
9

15

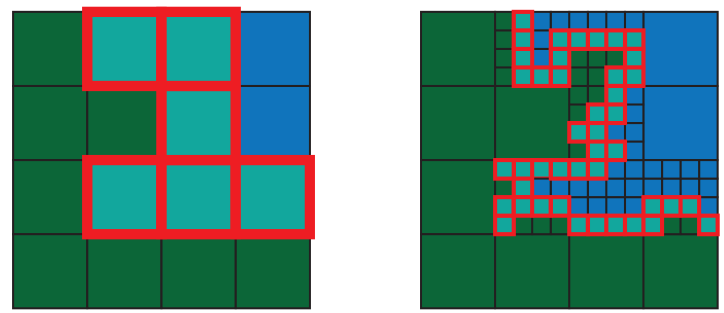
30



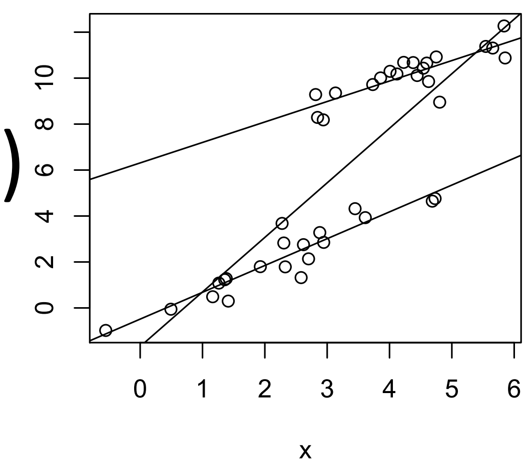
Plenty to do in this …



- Topography (Rahul)
- Histograms (Shuhang)
- Wavelets (Mostofa)



- Fractal Dimension (Nick)
- Bit-level processing (Adam)
- Regression (Dawit)
- Deep learning (David, Riley, Jordan)
- … and other fields (Guy)



Summary

- New remote sensing technology suggest new processing needs / techniques
  - We are only scratching the surface so far
- Linear approaches often go further than expected
- Something that started very practical can created interesting theoretical challenges
  - … but it takes a while to get there