HW2

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1

Sorry this got a little out of order, but I numbered everything.

4.1 a $\frac{dN}{dt} = F(N)$

set

$\frac{dW}{dt} = 0$ and get $\hat{N}$ $0 = F(\hat{N})$

$0 = r\hat{N}\left(1 - \left(\frac{\hat{N}}{K}\right)^{\theta}\right)$

equilibria $\hat{N} = 0$

$\hat{N} = K$

Stability

$\frac{dN}{dt} = rN$

$\frac{dN}{N} = r\,dt$

$\int \frac{dN}{N} = \int r\,dt$

$\ln(N) = rt + C$ e^{C} in this case $= N(0)$

$N = e^{rt} N(0)$

$F(N) = rN\left(1 - \left(\frac{N}{K}\right)^{\theta}\right)$

$\frac{dF}{dN} = \frac{d}{dN}\left[rN - \frac{rN(N^{\theta})}{K^{\theta}}\right]$

$= \frac{d}{dN}\left[rN - \frac{rN^{1+\theta}}{K^{\theta}}\right]$

$\frac{dF}{dN} = \frac{d}{dN} rn - \frac{d}{dN} \frac{rN^{1+\theta}}{K^{\theta}}$

$= r - \frac{r}{K^{\theta}}\left[\frac{d}{dN} N^{1+\theta}\right]$

$= r - \frac{r}{K^{\theta}}(1+\theta)N^{\theta}$

$\frac{dn}{dt} \approx n \frac{dF}{dN}\Big|_{\hat{N}=0}$

$\frac{dn}{dt} \approx n\left(r - \frac{r}{K^{\theta}}(1+\theta)N^{\theta}\right)\Big|_{N=0}$

$\frac{dn}{dt} = nr$

$\frac{dn}{dt} \approx n\left(r - \frac{r}{K^{\theta}}(1+\theta)N^{\theta}\right)\Big|_{N=K}$

$= n(r - r(1+\theta))$

$= n(r - [r + r\theta])$

$= n(r - r - r\theta)$

$= n(-r\theta)$

$= -nr\theta$

$\frac{dn}{dt} = \lambda n$

for $N=0$ $\lambda = r$ unstable

for $N=K$ $\lambda = -r\theta$ stable

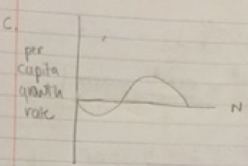
4.3 a $\frac{dW}{dt} = rN(N-a)\left[1 - \frac{N}{K}\right]$

$\hat{N} = K$

$\hat{N} = 0$

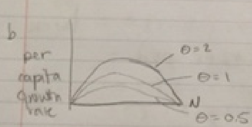
$\hat{N} = a$

4.3 c.



4.3 d. with this model, we see 3 times when $N=0$ while simple logistic models have 2 times. This we see because of the Allee effect, when the population declines when its too low. This model is a little more dynamic than the simple logistic model.

4.1 b



As theta increases, the per capita growth rate decreases for the same N. This means a population with a lower θ would have a lower growth rate.

c. The theta model takes into account how dependent the growth rate is on population size as the population grows. Logistic models assume that the growth rate is equally dependent on population size for all values of N. It shows how growth rate of a population slows as the abundance increases. Theta effects the max population growth rate per capita. Large theta could be for smaller

b. $F(N) = rN(N-a)\left[1-\frac{N}{k}\right]$

$$\frac{dF}{dN} = \frac{d}{dN}\left[rN^2 - \frac{rN^3}{k} + \frac{arN^2}{k} - arN\right]$$

$$= 2rN - \frac{3rN^2}{k} + \frac{2arN}{k} - ar$$

$$\frac{dn}{dt} \approx n\left(2rN - \frac{3rN^2}{k} + \frac{2arN}{k} - ar\right)\Big|_{N=0}$$

$$\frac{dn}{dt} = -arn$$

$$\frac{dn}{dt} \approx \Big|_{N=k}$$

$$= n(2rk - 3rk + 2ar - ar)$$

$$= n(-rk + ar)$$

$$\frac{dn}{dt} = \Big|_{N=a}$$

$$= n\left(2ra - \frac{3ra^2}{k} + \frac{2a^2r}{k} - ar\right)$$

$$= n\left(ra - \frac{ra^2}{k}\right)$$

for $N=0$ $\lambda = -ar$ stable
$N=k$ $\lambda = -rk+ar$ stable
$N=a$ $\lambda = ra\left(1-\frac{a}{k}\right)$ unstable.

animals like insects. middle theta values for larger animals like birds, and small theta for big animals like elephants.

Extra credit.

$$\frac{dN}{dt} = rN\left(1-\frac{N}{k}\right)$$

$$\frac{dN}{N\left(1-\frac{N}{k}\right)} = \frac{dN}{N-\frac{N^2}{k}} = \int \frac{1}{N}\,dN + \int \frac{dN}{k-N} = \int r\,dt$$

$$\left(\frac{1}{N-\frac{N^2}{k}} = \frac{1}{N} + \frac{1}{k-N}\right)$$

$$rt + c = \ln(N) - \ln(k-N)$$

$$e^{rt}e^{c} = e^{\ln\left(\frac{N}{k-N}\right)} = \frac{N}{k-N}$$

$$(k-N)(e^{c}e^{rt}) = N$$

$$ke^{c}e^{rt} - Ne^{c}e^{rt} = N$$

$$ke^{c}e^{rt} = N + Ne^{c}e^{rt}$$

$$= N(1+e^{c}e^{rt})$$

$$\frac{ke^{c}e^{rt}}{1+e^{c}e^{rt}} = N$$

Sub $N(0) = ke^{c}$

$$\frac{N(0)}{k} = e^{c}$$

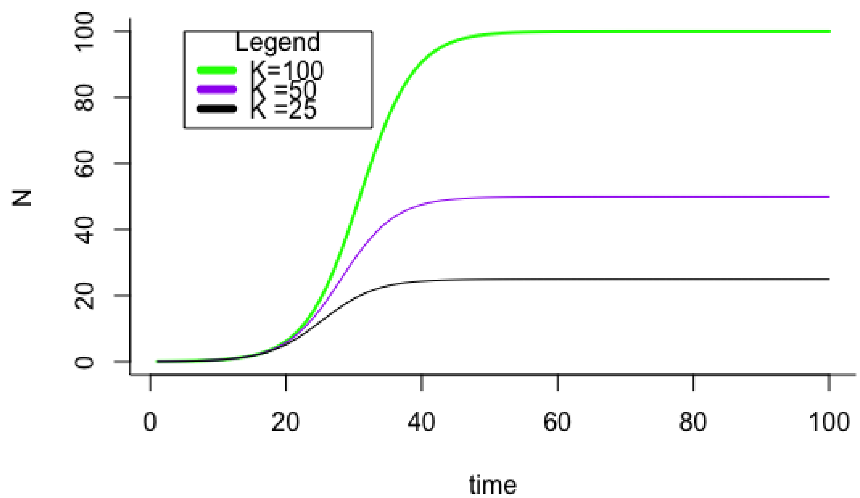
$$\frac{N(0)e^{rt}}{1+\frac{N(0)}{k}e^{rt}} = N$$

2

Logistic Growth
a. Code:

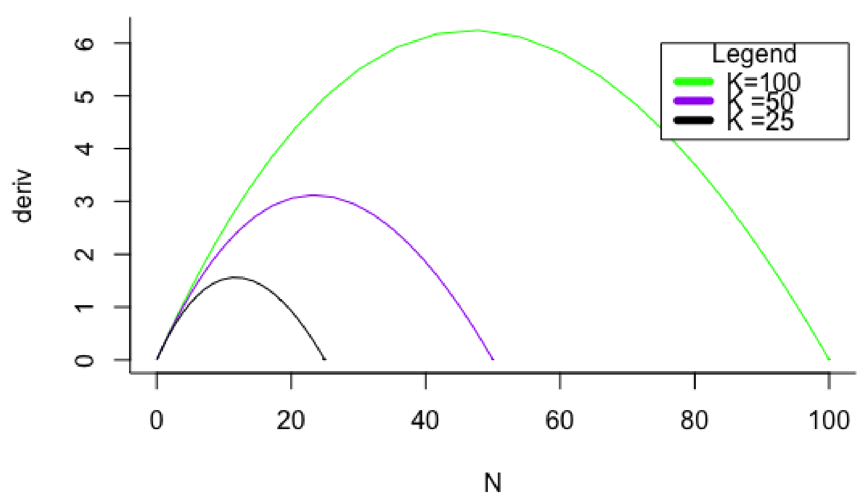
```
# gives dsSolve control over the script
library(deSolve)

# logistic growth function
log.growth <- function(t, y, p) {
  N <- y[1]
  with(as.list(p), {
    dN.dt <- r * N * (1 - (N/K))
    return(list(dN.dt))
  })
}

# gives vales for function
p <- c('r' = 0.25, 'K' = 100)
y0 <- c('N' = runif(1, min = 0.01, max = 0.1))
t <- 1:100

#runs and stores solution data for the ode
sim <- ode(y = y0, time = t, func = log.growth, parms = p
    , method = 'lsoda')
sim <- as.data.frame(sim)

# plot my simulation
plot(N ~ time, data = sim, type = 'l', lwd = 2, bty = 'l'
    , col = 'green')

# defines new values for function and plots it
p.2 <- c('r' = 0.25, 'K' = 50)
sim.2 <- ode(y = y0, time = t, func = log.growth, parms =
     p.2, method = 'lsoda')
sim.2 <- as.data.frame(sim.2)
points(N ~ time, data = sim.2, type = 'l', col = 'purple'
    )

# defines new values for function and plots it
p.3 <- c('r' = 0.25, 'K' = 25)
sim.3 <- ode(y = y0, time = t, func = log.growth, parms =
     p.3, method = 'lsoda')
sim.3 <- as.data.frame(sim.3)
points(N ~ time, data = sim.3, type = 'l')
```

```r
#to create legend
v <- strwidth('green K = 100')
legend(5,100,c("K=100", "K =50", "K =25"), text.width = v
    , lty=c(1,1,1), title = "Legend",lwd = c(5.0, 5.0,
    5.0), col = c("green", "purple", "black") )
```



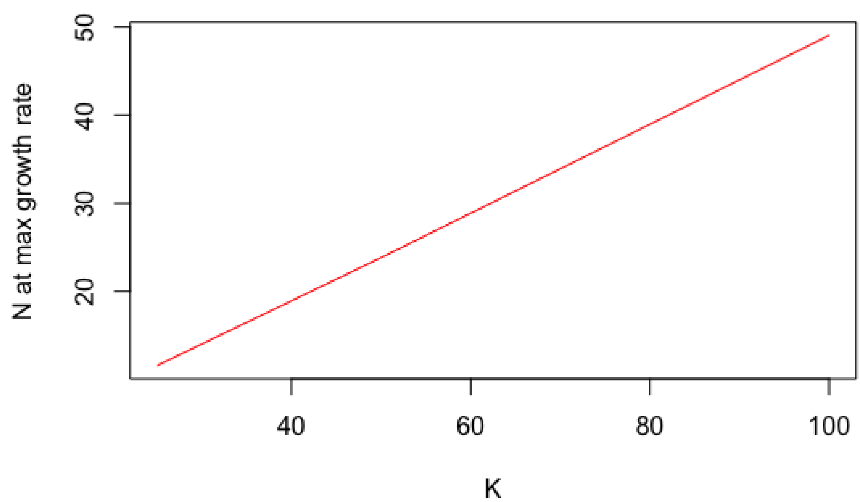
b. Code:

```
# computes derivative
sim$deriv <- c(diff(sim$N), NA)
sim.2$deriv <- c(diff(sim.2$N), NA)
sim.3$deriv <- c(diff(sim.3$N), NA)

# plots pop level growth rate vs pop abundance
plot(deriv ~ N, data = sim, type = 'l', col = 'green',
    bty = 'l')
points(deriv ~ N, data = sim.2, type = 'l', col = 'purple
    ', bty = 'l')
points(deriv ~ N, data = sim.3, type = 'l', bty = 'l')

# to create legend
legend(75,7,c("K=100", "K =50", "K =25"), text.width = v,
     lty=c(1,1,1), title = "Legend",lwd = c(5.0, 5.0, 5.0)
    , col = c("green", "purple", "black") )
```



c. Code:

```
#c. finds pop abundance that yeilds max growth rate
a1 <- sim$N[which(sim$deriv == max(sim$deriv, na.rm =
    TRUE))]
a2 <- sim.2$N[which(sim.2$deriv == max(sim.2$deriv, na.rm
    = TRUE))]
a3 <- sim.3$N[which(sim.3$deriv == max(sim.3$deriv, na.rm
    = TRUE))]

#defines K and N at max growth rate values
d1 <- c(25, 50, 100)
d2 <- c(a3, a2, a1)

# plots carrying capacity vs pop abundance at max growth
    rate
plot(d2 ~ d1, xlab = 'K', ylab = 'N at max growth rate',
    type = 'l', col = 'red')
```



3

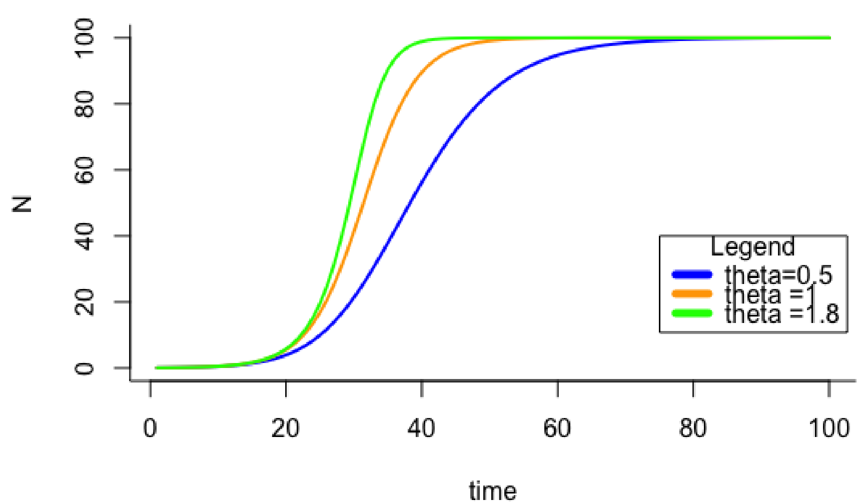
Fishery Population Growth

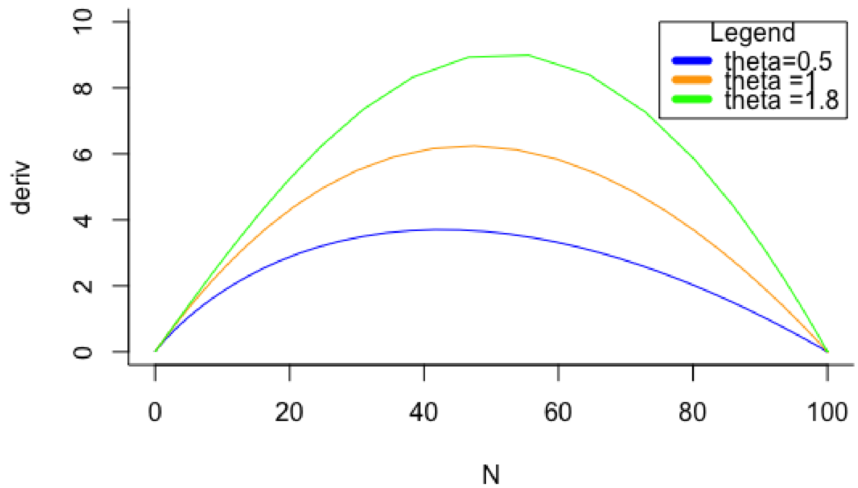
Species C will be maintained at the highest abundance because it has the highest population (N) at max the growth
$N(C) \approx 55.55$ compared to $N(A) \approx 41.60$ and $N(B) \approx 47.49$

Code:

```
library(deSolve)

# logistic growth function
log.growth <- function(t, y, p) {
  N <- y[1]
  with(as.list(p), {
    dN.dt <- r * N * (1 - (N / K)^theta)
    return(list(dN.dt))
  })
}

# gives vales for function
p <- c('r' = 0.25, 'K' = 100, 'theta' = 0.5)
y0 <- c('N' = 0.05)
t <- 1:100

#runs and stores solution data for the ode
sim <- ode(y = y0, times = t, func = log.growth, parms =
    p, method = 'lsoda')
sim <- as.data.frame(sim)

# plot my simulation
plot(N ~ time, data = sim, type = 'l', lwd = 2, bty = 'l'
    , col = 'blue')

# defines new values for function and plots it
p.2 <- c('r' = 0.25, 'K' = 100, 'theta' = 1)
sim.2 <- ode(y = y0, times = t, func = log.growth, parms
    = p.2, method ='lsoda')
sim.2 <- as.data.frame(sim.2)
points(N ~ time, data = sim.2, type = 'l', lwd = 2, bty =
     'l', col = 'orange')

# defines new values for function and plots it
p.3 <- c('r' = 0.25, 'K' = 100, 'theta' = 1.8)
sim.3 <- ode(y = y0, times = t, func = log.growth, parms
```

```
    = p.3, method = 'lsoda')
sim.3 <- as.data.frame(sim.3)
points(N ~ time, data = sim.3, type = 'l', lwd = 2, bty =
     'l', col = 'green')

# computes derivative
sim$deriv <- c(diff(sim$N), NA)
sim.2$deriv <- c(diff(sim.2$N), NA)
sim.3$deriv <- c(diff(sim.3$N), NA)

# plots pop level growth rate vs pop abundance
plot(deriv ~ N, data = sim, type = 'l', col = 'green',
    bty = 'l', ylim = c(0,10))
points(deriv ~ N, data = sim.2, type = 'l', col = 'purple
    ', bty = 'l')
points(deriv ~ N, data = sim.3, type = 'l', bty = 'l')

#c. finds pop abundance that yeilds max growth rate
a1 <- sim$N[which(sim$deriv == max(sim$deriv, na.rm =
    TRUE))]
a2 <- sim.2$N[which(sim.2$deriv == max(sim.2$deriv, na.rm
     = TRUE))]
a3 <- sim.3$N[which(sim.3$deriv == max(sim.3$deriv, na.rm
     = TRUE))]
```




Legend
theta=0.5
theta =1
theta =1.8
deriv
N
0
2
4
6
8
10
0
20
40
60
80
100