Homework 4

1.

7.1

7.1

$$\frac{dN}{dt} = r_1 N_1 (1 - \alpha_{11} N_1 - a_{12} N_2 - mN_i)$$

if $a_{11} = a_{22} = 1$ for isocline

for species 1:

$$0 = r_1 N_1 (1 - N_1 - a_{12} N_2 - mN_i)$$

$$0 = r_1 N_1 - r_1 N_1^2 - r_1 N_1 a_{12} N_2 - r_1 N_1 m N_i$$

$$\cancel{r_1 N_1} a_{12} N_2 = \cancel{r_1} N_1 - \cancel{r_1} N_1^2 - r_1 N_1 m N_i$$

$$a_{12} N_2 = N_1 - mN_i$$

$$N_2 = \frac{N_1}{a_{12}} - \frac{mN_i}{a_{12}}$$

b intercept $= \frac{N_1}{\alpha_{12}}$

$$Y = b + mx$$

x intercept $= mN_i$

$$0 = \frac{N_1}{a_{12}} - \frac{mN_1}{a_{12}}$$

$$N_1 = mN_i$$

for species 2:

$$0 = r_2 N_2 (1 - N_2 - a_{21} N_1 - mN_i)$$

$$1 - N_2 - a_{21} N_1 - mN_i = 0$$

$$N_2 = \underbrace{1 - a_{21} N_1}_{b} \underbrace{- mN_i}_{mx}$$

$b = 1 - a_{21} N_1$ y-intercept

$$0 = 1 - a_{21} N_1 - mN_i$$

$$a_{21} N_1 = 1 - mN_i$$

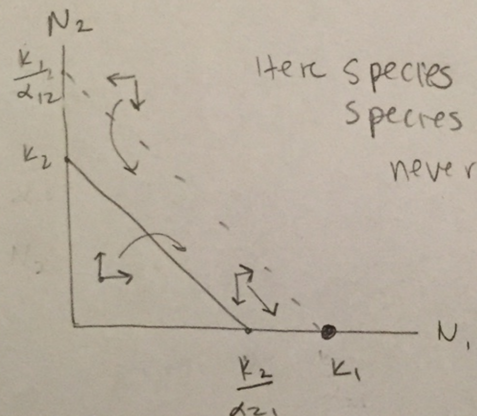
$$N_1 = \frac{1 - mN_i}{a_{21}}$$

$$\frac{dN_1}{dt} = r_1 N_1 (1 - a_{11} N_1 - a_{12} N_2)$$

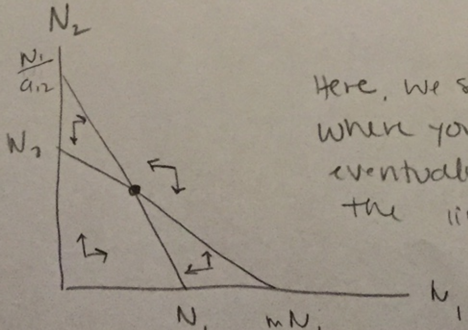
$$\frac{dN_2}{dt} = r_2 N_2 (1 - a_{22} N_1 - a_{21} N_1)$$

$N_2 = \frac{k_1 - N_1}{\alpha_1}$ $\rightarrow$ y interacept $= \frac{k_1}{\alpha_{12}}$

$N_2 = k_2 - \alpha_{21} N_1$ $\rightarrow$ interept k_2



Here species 1 always outcompetes species 2, so coexistance is never possible.



Here, we see that no matter where you are, the populatons eventually go to where the lines interecect, and the two populatons coexist, at a stable equilibrum.

7.2

7.2

a. $m_1 P_1(1-P_1) - eP_1 = 0$

$m_1 P_1 - m_1 P_1^2 - eP_1 = 0$

$P_1(m_1 - m_1 P_1 - e) = 0$

$m_1 - m_1 P_1 - e = 0$

$m_1 - e = m_1 P_1$

$\frac{m_1 - e}{m_1} = P_1$

$1 - \frac{e_1}{m_1} = P_1$ $\quad 1 - \frac{e_1}{m_1} > 0 \rightarrow 1 > \frac{e}{m_1} \rightarrow m_1 > e$

For this to be positive, colonizaton must be greater than extincton.

b. $m_2 P_2(1 - P_1 - P_2) - m_1 P_1 P_2 - eP_2 = 0$

$m_2 P_2(1 - 1 + \frac{e}{m_1} - P_2) - m_1(1 - \frac{e}{m_1})P_2 - eP_2 = 0$

$m_2 P_2(\frac{e}{m_1} - P_2) - m_1(1 - \frac{e}{m_1})P_2 - eP_2 = 0$

$P_2(m_2(\frac{e}{m_1} - P_2) - m(1 - \frac{e}{m_1}) - e) = 0$

$m_2(\frac{e}{m_1} - P_2) = m_1(1 - \frac{e}{m_1}) + e$

$m_2(\frac{e}{m_1} - P_2) = m_1$

$\frac{e}{m_1} - P_2 = \frac{m_1}{m_2}$

$P_2 = \frac{e}{m_1} - \frac{m_1}{m_2}$

The colonizaton of rate m_2 must be greater than m_1^2 in order for bothe species to survive. This maces P_2 at equilibrium greater than zero.

$0 < \frac{e}{m_1} - \frac{m_1}{m_2} \rightarrow \frac{m_1}{m_2} < \frac{e}{m_1} \rightarrow \frac{m_1^2}{e} < m_2$

(c.) Species one would be eliminated first at equilibrium if the extincton rate was slowely increased. This is because P_1 at nonzero equillbrium is $P_1 = 1 - \frac{e_1}{m_1}$, this means as e incrases, P_1 actually decreases. Also, because $P_2 = \frac{e}{m_1} - \frac{m_1}{m_2}$, we know that an increase in extincton rate would cause P_2 to increase. For species 1, the equilibrum will become negative at higher vales of e, because of what I said before.

(d) As I said above with regards to the relatonships with extincton rate, we know as extincton rate increases, P_1 decreases, this then allows room for species 2, and correlates with the increase in P_2 as extincton increases. therefore, when extincton increases, P_2 equilibrum will increase to a higher abundance, because there is more room of species 2, as species 1 abundance decreases.

7.4

a. In field experiments competition is found quite often, more than half the time. There is conflicting news on whether or not you can find competition as a function of trophic level. In some experiments, when one animal was removed, the other increased dramatically in density. In the laboratory and nature, all possible outcomes, coexistence or elimination of once species or the other, and parameters are difficult to measure. One way to test the models in experiments is to look at fixed environment, where the two species growth in isolation together. Some problems with field experiments is controls. Another partial problem is with population densities to be used in the treatments. Another problem is deciding whether the experimental results can be applied to natural populations, such as with the addition of enclosures.

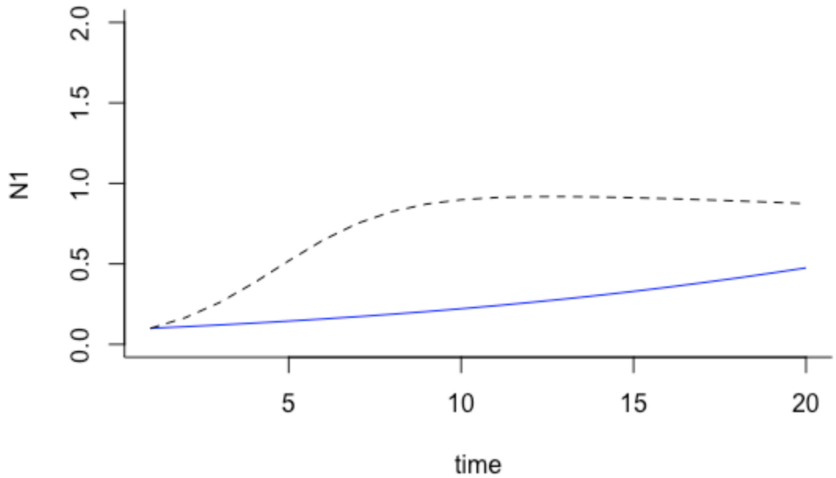
b. We can look at isolated systems where species are grown in a fixed environment. This could be a garden plot for plants or a bottle for small insects. The Lotka-Volterra competition model can make strong predictions about the relationship among population numbers in these cases, if they can coexist. In isolation, species 1 will be at the population level K1 , while species 2 will be at the population level K2. It the species coexist, we can find the equilibrium point between the two species. This was seen with Drosophila pseudoobscura and Drosophila serrate. Also seen using two pieces of Paramecium. The phase plane of the coexistence equilibrium contradicts the Lota-Volterra theory for one of the two experiments preformed. This results in a curved isocline. Where frequency-dependent competition could make it curved. Energetic consideration could make it curved, or age dependence, where larval and adult competition are described by different models, each with linear isoclines, but would be lead to a curved isocline for the whole system,

c. In the early mathematical model looking at resource use to competitive exclusion, the competition coefficient could be computed as a measure of ecological overlap. The equation or aij uses the fraction of total resources used by species I from resource k, where k be a food or habitat. It seems that ecological overlap does not indicate great competition, but results from interspecific tolerance, whereas low overlap may result from aggressive exclusion. Many experiments have seen that low ecological overlap was associated with low competition. Improvements to this equation results when a certain consumer-resource system reaches equilibrium. Field experiments can pose inconsistencies with this equation as specific species could be dominant at particular times during the year, despite no obvious change in food overlap. Lastly a non linear Lotka-Volterra competition model may be necessary. Non linear models can be explicitly models of food overlap. The inconsistencies with both equations does not imply that ecological overlap is irrelevant, nor that resource competition is not occurring.

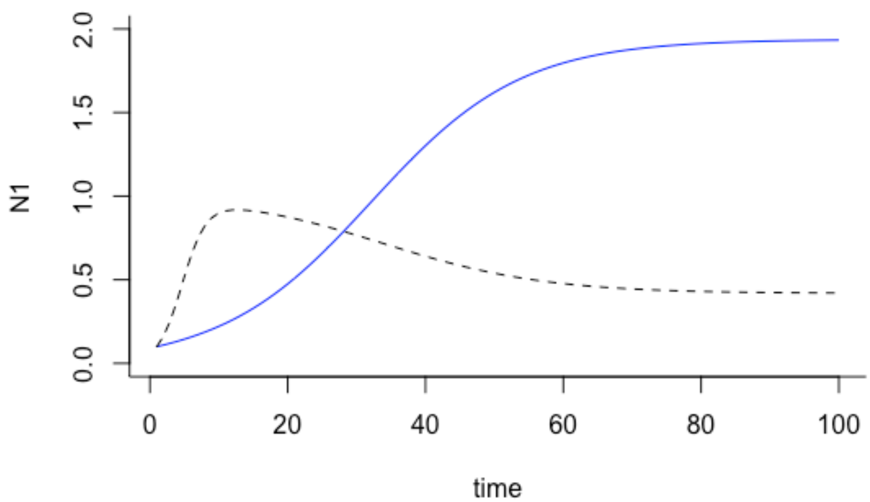
d. Manipulations of field populations can vary greatly. In some cases, they may just remove or introduce species to an area, which this in itself can range from competitors, mutualistic species, parasitic species for example. Other times, they may use cages to change the immediate environment around a certain population. Sometimes experiments are done in somewhat arterial conditions, such as in crops, orchards, artificial ponds, settlement panels placed in conditions different from natural ones. This can cause a wide range of issue, including how relevant this data is to a natural population.

2.
You would conclude that species 2 is a better competitor, and the system approaches an equilibrium here species 2 has a higher population than species 1. I would not come to the same conclusion. With this graph, it is clear that species 2 outcompetes species 1 at the beginning, but then they reach equilibrium, where species 1 is actually has a higher population than species 2. This is completely opposite of the original conclusion. It is important to interoperate short and long term experiments differently, because we can clearly see that depending on the conditions the interaction between two species can drastically change. This seems like an example of a disturbance. Where a primary species thrives at the beginning, but when the climax community individuals come in after time, they end up doing better and have higher population levels in the long run.

```
library(deSolve)

# gives function
comp <- function(t, y, p) {
  N1 <- y[1]
  N2 <- y[2]
  with(as.list(p), {
    dN1.dt <- (r1*N1/K1)*(K1-N1-a12*N2)
    dN2.dt <- (r2*N2/K2)*(K2-N2-a21*N1)
    return(list(c(dN1.dt, dN2.dt)))
  })
}

#define model imput (Speices 1 has higher K but lower competition efefct)
t <- 1:100
y0 <- c('N1' = 0.1, 'N2' = 0.1)
p <- c('r1' = 0.1, 'r2' = 0.6,
       'K1' = 2, 'K2' = 1,
       'a12' = 0.15, 'a21' = 0.3)
sim <- ode(y = y0, times = t, func = comp, parms = p, method = 'lsoda')
sim <- as.data.frame(sim)

# grahps values
plot(N1 ~ time, type = 'l', col = 'blue', bty = 'l', data = sim, ylim = c(0,2))
points(N2 ~ time, type = 'l', lty = 2, data = sim)
```



N1
0.0
0.5
1.0
1.5
2.0
5
10
15
20
time



N1
0.0
0.5
1.0
1.5
2.0
0
20
40
60
80
100
time

3.
Zika Virus!!