PHYSICS 20323/60323: Fall 2024 - LaTeX Example

- 1. An electron is found to be in a spin state (in the z-basis): $\chi = A \begin{pmatrix} 3i \\ 4 \end{pmatrix}$
 - (a) (5 points) Determine the possible values of A such that the state is normalized.
 - (b) (5 points) Find the expectation values of the operators S_x , S_y , S_z , and \vec{S}^2 .

The matrix representations in the z-basis for the components of electron spin operators are given by:

$$\mathbf{S}_{\mathbf{x}} = \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \qquad \mathbf{S}_{\mathbf{y}} = \frac{\hbar}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}; \qquad \mathbf{S}_{\mathbf{z}} = \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix};$$

2. The average electrostatic field in the earth's atmosphere in fair weather is approximately given:

$$\vec{E} = E_0 (Ae^{-\alpha z} + Be^{-\beta z})\hat{z},\tag{1}$$

where A, B, α , β are positive constants and z is the height above the (locally flat) earth surface.

- (a) (5 points) Find the average charge density in the atmosphere as a function of height
- (b) (5 points) Find the electric potential as a function of height above the earth.

3. The following questions refer to the stars in the Table below.

Note: There may be multiple answers.

Name	Mass	Luminosity	Lifetime	Temperature	Radius
β Cyg.	1.3 <i>M</i> ⊙	$3.5L_{\odot}$			1 R ⊙
α Cen.	$1.0 M_{\odot}$				
η Car.	60. <i>M</i> ⊙	$10^6 L_{\odot}$	8.0×10^5 years		
ε Eri.	$6.0M_{\odot}$	$10^3 L_{\odot}$		20,000 K	
δ Scu.	$2.0M_{\odot}$		5.0×10^8 years		$2R_{\odot}$
γ Del.	$0.7 M_{\odot}$		$4.5 \times 10^{10} \text{ years}$	5000 K	

- (a) (4 points) Which of these stars will produce a planetary nebula.
- (b) (4 points) Elements heavier than *Carbon* will be produced in which stars.