Petroleum Reservoir Simulation with Python

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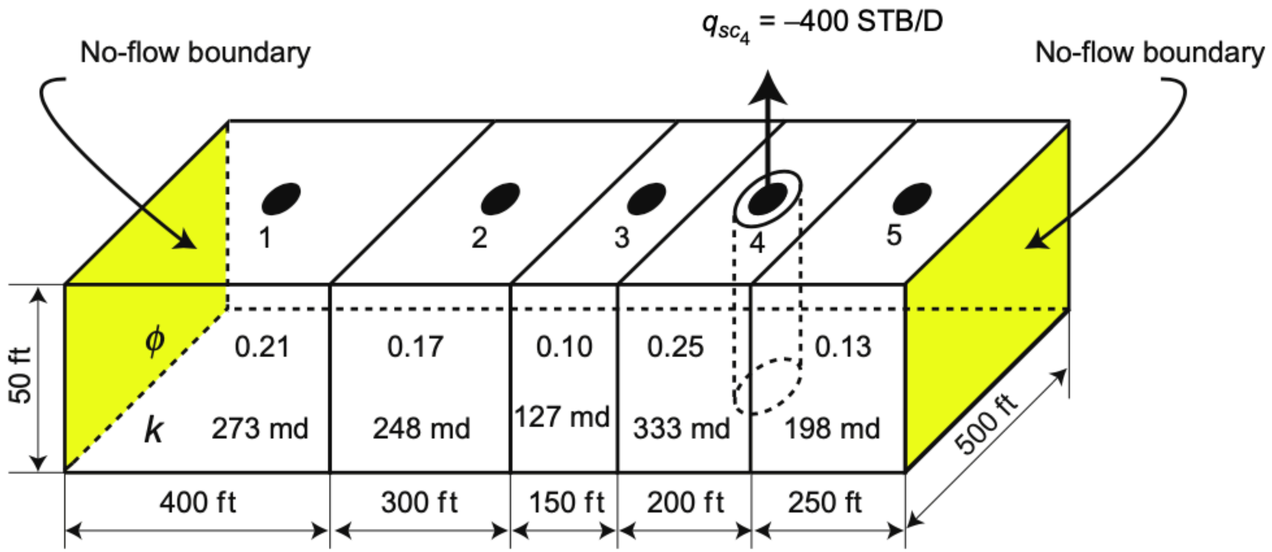


Contents

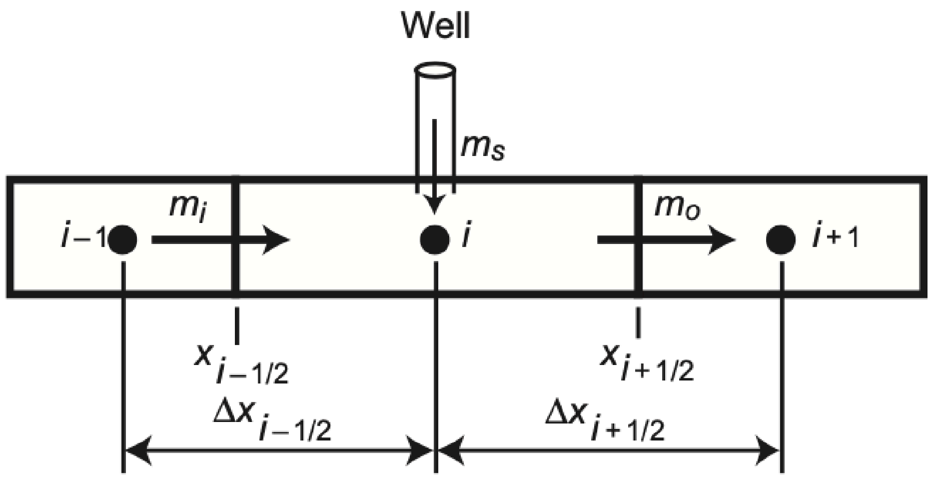
Basic principles

- Mass conservation:

  $m_i - m_o + m_s = m_a$

- Fluid flow rate (Darcy's law):

  $u_x = q_x / A_x = -\frac{k_x}{\mu} \frac{\partial p}{\partial x}$



Derivation of PDE

- Mass rate, $w = m/t = \rho u A$
- Density, $\rho = m/V_p$
- Pore volume, $V_p = V_b \phi$
- Volumetric rate, $q = q_s/\rho$

$$
\begin{aligned}
m_i - m_o + m_s &= m_a \\
w_i \Delta t - w_o \Delta t + q_s \Delta t &= V_b \Delta_t(\phi\rho) \\
(\rho u_x A_x)_i \Delta t - (\rho u_x A_x)_o \Delta t + \rho q \Delta t &= V_b \Delta_t(\phi\rho) \\
\frac{(\rho u_x A_x)_i - (\rho u_x A_x)_o + \rho q}{V_b} &= \frac{\Delta_t(\phi\rho)}{\Delta t} \\
\frac{(\rho u_x)_i - (\rho u_x)_o}{\Delta x} + \frac{\rho q}{V_b} &= \frac{\Delta_t(\phi\rho)}{\Delta t}
\end{aligned}
$$

Derivation of PDE

$$\frac{-[(\rho u_x)_o - (\rho u_x)_i]}{\Delta x} + \frac{\rho q}{V_b} = \frac{\Delta_t(\phi\rho)}{\Delta t}$$

$$\lim_{\Delta x \to 0} \frac{-[(\rho u_x)_o - (\rho u_x)_i]}{\Delta x} + \frac{\rho q}{V_b} = \lim_{\Delta t \to 0} \frac{\Delta_t(\phi\rho)}{\Delta t}$$

$$-\frac{\partial(\rho u_x)}{\partial x} + \frac{\rho q}{V_b} = \frac{\partial(\phi\rho)}{\partial t}$$

$$\frac{\partial}{\partial x}\left(\frac{k_x}{\mu}\frac{\partial p}{\partial x}\right) + \frac{\rho q}{V_b} = \frac{\partial(\phi\rho)}{\partial t}$$

- Formation volume factor, $B = \rho_{sc}/\rho$

$$\frac{\partial}{\partial x}\left(\frac{k_x}{\mu B}\frac{\partial p}{\partial x}\right) + \frac{q_{sc}}{V_b} = \frac{\partial}{\partial t}\left(\frac{\phi}{B}\right)$$

Discretization of PDE in space and time

- Volume, $V = A\Delta x$
- For gridblock i,

$$\frac{\partial}{\partial x}\left(\frac{A_x k_x}{\mu B}\frac{\partial p}{\partial x}\right)_i \Delta x_i + q_{sc_i} = V_{b_i}\frac{\partial}{\partial t}\left(\frac{\phi}{B}\right)_i$$

$$\frac{\partial}{\partial x}\left(\frac{A_x k_x}{\mu B}\frac{\partial p}{\partial x}\right)_i \approx \left[\left(\frac{A_x k_x}{\mu B}\frac{\partial p}{\partial x}\right)_{i+1/2} - \left(\frac{A_x k_x}{\mu B}\frac{\partial p}{\partial x}\right)_{i-1/2}\right]/\Delta x$$

$$\frac{\partial}{\partial x}\left(\frac{A_x k_x}{\mu B}\frac{\partial p}{\partial x}\right)_i \Delta x_i \approx \left[\left(\frac{A_x k_x}{\mu B \Delta x}\right)_{i+1/2}(p_{i+1} - p_i) - \left(\frac{A_x k_x}{\mu B \Delta x}\right)_{i-1/2}(p_i - p_{i-1})\right]$$

$$\frac{\partial}{\partial x}\left(\frac{A_x k_x}{\mu B}\frac{\partial p}{\partial x}\right)_i \Delta x_i \approx T_{i+1/2}(p_{i+1} - p_i) - T_{i-1/2}(p_i - p_{i-1})$$

Discretization of PDE in space and time

$$T_{i+1/2}(p_{i+1} - p_i) - T_{i-1/2}(p_i - p_{i-1}) + q_{sc_i} \approx V_{b_i} \frac{\partial}{\partial t}\left(\frac{\phi}{B}\right)_i$$

$$T_{i-1/2}(p_{i-1} - p_i) + T_{i+1/2}(p_{i+1} - p_i) + q_{sc_i} \approx V_{b_i} \frac{\partial}{\partial t}\left(\frac{\phi}{B}\right)_i$$

$$\frac{\partial}{\partial t}\left(\frac{\phi}{B}\right)_i \approx \frac{1}{\Delta t}\left[\left(\frac{\phi}{B}\right)_i^{n+1} - \left(\frac{\phi}{B}\right)_i^{n}\right]$$

Discretization of PDE in space and time

- Forward-difference discretization (Explicit)

$$T^n_{i-1/2}(p^n_{i-1} - p^n_i) + T^n_{i+1/2}(p^n_{i+1} - p^n_i) + q^n_{sc_i} \approx \frac{V_{b_i}}{\Delta t}\left[\left(\frac{\phi}{B}\right)^{n+1}_i - \left(\frac{\phi}{B}\right)^n_i\right]$$

$$T^n_{i-1/2}(p^n_{i-1} - p^n_i) + T^n_{i+1/2}(p^n_{i+1} - p^n_i) + q^n_{sc_i} \approx \frac{V_{b_i}}{\Delta t}\left(\frac{\phi}{B}\right)'_i \left[p^{n+1}_i - p^n_i\right]$$

$$\left(\frac{\phi}{B}\right)'_i = \left[\left(\frac{\phi}{B}\right)^{n+1}_i - \left(\frac{\phi}{B}\right)^n_i\right] / \left[p^{n+1}_i - p^n_i\right]$$

Discretization of PDE in space and time

- Backward-difference discretization (Implicit)

$$T_{i-1/2}^{n+1}(p_{i-1}^{n+1} - p_i^{n+1}) + T_{i+1/2}^{n+1}(p_{i+1}^{n+1} - p_i^{n+1}) + q_{sc_i}^{n+1} \approx \frac{V_{b_i}}{\Delta t}\left(\frac{\phi}{B}\right)_i^{'}\left[p_i^{n+1} - p_i^n\right]$$

Initial and boundary conditions

- Specified boundary pressure, p_b

$$\left[\frac{k_x A_x}{\mu B(\Delta x)/2}\right]_{bB}(p_b - p_i)$$

- Specified boundary pressure-gradient, $\partial p/\partial x$

$$\mp\left[\frac{k_x A_x}{\mu B}\right]_{bB}\frac{\partial p}{\partial x}$$

Well representation

- Specified well pressure gradient, $\partial p / \partial r$

$$q_{sc_i} = -\frac{2\pi r_w kh}{B\mu}\frac{\partial p}{\partial r}$$

- Specified well FBHP, p_{wf}

$$q_{sc_i} = -\frac{2\pi kh}{B\mu[\log_e(r_{eq}/r_w) + s]}(p_i - p_{wf})$$

$$q_{sc_i} = -G(p_i - p_{wf})$$

$$r_{eq} = 0.14[(\Delta x)^2 + (\Delta y)^2]^{0.5}$$

Single phase flow

- Incompressible flow, $c_\phi = 0$

$$\phi = \phi^{\cdot}[1 + c_\phi(p - p^{\cdot})]$$

$$T_{i-1/2}(p_{i-1} - p_i) + T_{i+1/2}(p_{i+1} - p_i) + q_{sc_i} \approx \frac{V_{b_i}}{\Delta t}\left[\left(\frac{\phi}{B}\right)_i^{n+1} - \left(\frac{\phi}{B}\right)_i^{n}\right]$$

$$\frac{V_{b_i}}{\Delta t}\left[\left(\frac{\phi}{B}\right)_i^{n+1} - \left(\frac{\phi}{B}\right)_i^{n}\right] = \frac{V_{b_i}\phi_i^{\cdot}c_\phi}{B\Delta t}\left[p_i^{n+1} - p_i^{n}\right] = 0$$

$$T_{i-1/2}(p_{i-1} - p_i) + T_{i+1/2}(p_{i+1} - p_i) + q_{sc_i} \approx 0$$

Single phase flow

- Slightly compressible flow,

$$\frac{V_{b_i}}{\Delta t}\left[\left(\frac{\phi}{B}\right)_i^{n+1}-\left(\frac{\phi}{B}\right)_i^{n}\right]\approx\frac{V_{b_i}\phi_i^{\circ}(c_\phi+c)}{B\Delta t}\left[p_i^{n+1}-p_i^n\right]$$

$$T_{i-1/2}(p_{i-1}-p_i)+T_{i+1/2}(p_{i+1}-p_i)+q_{sc_i}\approx\frac{V_{b_i}\phi_i^{\circ}(c_\phi+c)}{B\Delta t}\left[p_i^{n+1}-p_i^n\right]$$

Transmissibility

$$T_{i,i\pm1} = T_{i\mp1/2} = \frac{1}{\mu B} \times \frac{2}{\Delta x_i/(A_{x_i}k_{x_i}) + \Delta x_{i\mp1}/(A_{x_{i\mp1}}k_{x_{i\mp1}})}$$

Methods of solution of linear equations (Thomas' algorithm)

- Incompressible flow

$$
\begin{aligned}
T_{i-1/2}(p_{i-1}-p_i)+T_{i+1/2}(p_{i+1}-p_i)+q_{sc_i} &= 0\\
T_{i-1/2}p_{i-1}-T_{i-1/2}p_i+T_{i+1/2}p_{i+1}-T_{i+1/2}p_i &= -q_{sc_i}\\
T_{i-1/2}p_{i-1}-(T_{i-1/2}p_i+T_{i+1/2}p_i)+T_{i+1/2}p_{i+1} &= -q_{sc_i}\\
T_{i-1/2}p_{i-1}-(T_{i-1/2}+T_{i+1/2})p_i+T_{i+1/2}p_{i+1} &= -q_{sc_i}
\end{aligned}
$$

- Sligthly compressibile flow

$$
T_{i-1/2}(p_{i-1}^{n+1}-p_i^{n+1})+T_{i+1/2}(p_{i+1}^{n+1}-p_i^{n+1})+q_{sc_i}^{n+1} \approx \frac{V_{b_i}\phi_i^{\cdot}(c_\phi+c)}{B\Delta t}\left[p_i^{n+1}-p_i^{n}\right]
$$

$$
T_{i-1/2}p_{i-1}^{n+1}-\left(T_{i-1/2}+T_{i+1/2}+\frac{V_{b_i}\phi_i^{\cdot}(c_\phi+c)}{B\Delta t}\right)p_i^{n+1}+T_{i+1/2}p_{i+1}^{n+1}=-q_{sc_i}^{n+1}-\frac{V_{b_i}\phi_i^{\cdot}(c_\phi+c)}{B\Delta t}p_i^{n}
$$

Methods of solution of linear equations (Thomas' algorithm)

- The equation for block, i

$$w_i x_{i-1} + c_i x_i + e_i x_{i+1} = d_i$$

- For example, for incompressible flow

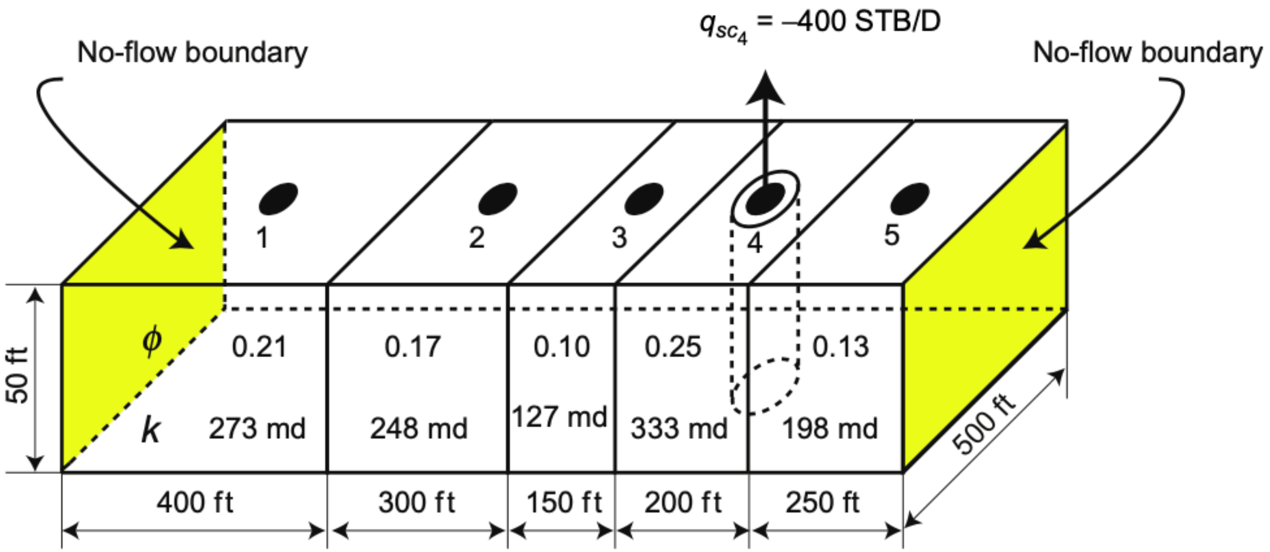
$$w_i = T_{i-1/2}, c_i = -(T_{i-1/2} + T_{i+1/2}), e_i = T_{i+1/2}, d_i = -q_{sc_i}$$

Methods of solution of linear equations (Thomas' algorithm)

- Alogrithm
  i. Set $u_1 = e_1/c_1$ and $g_1 = d_1/c_1$
  ii. For $i = 2, 3, \ldots, N-1$,
  $u_i = e_i/(c_i - w_i u_{i-1})$ and
  For $i = 2, 3, \ldots, N$,
  $g_i = (d_i - w_i g_{i-1})/(c_i - w_i u_{i-1})$
  iii. Set $x_N = g_N$
  iv. For $i = N-1, N-2, \ldots, 3, 2, 1$,
  $x_i = g_i - u_i x_{i+1}$

Case study

Reservoir fluid properties are $B = 1RB/STB$, $\mu = 1.5cp$, and $c = 2.5 \times 10^{-5} psi^{-1}$. Initially, reservoir pressure is $3000psia$. A $6in$ vertical well was drilled at the center of gridblock 4. The well is switched to a constant FBHP of $1500psia$ if the reservoir cannot sustain the specified production rate. Find the pressure distribution in the reservoir after 10 days using the implicit formulation.



Case study: Single phase 1D slightly compressible implicit reservoir simulation

$$T_{i-1/2}(p_{i-1}^{n+1} - p_i^{n+1}) + T_{i+1/2}(p_{i+1}^{n+1} - p_i^{n+1}) + q_{sc_i}^{n+1} \approx \frac{V_{b_i}\phi_i^{\cdot}(c_\phi + c)}{B\Delta t}\left[p_i^{n+1} - p_i^n\right]$$

$$T_{i-1/2}p_{i-1}^{n+1} - \left(T_{i-1/2} + T_{i+1/2} + \frac{V_{b_i}\phi_i^{\cdot}(c_\phi + c)}{B\Delta t}\right)p_i^{n+1} + T_{i+1/2}p_{i+1}^{n+1} = -q_{sc_i}^{n+1} - \frac{V_{b_i}\phi_i^{\cdot}(c_\phi + c)}{B\Delta t}p_i^n$$

$$T_{i-1/2}p_{i-1}^{n+1} - \left(T_{i-1/2} + T_{i+1/2} + \frac{V_{b_i}\phi_i^{\cdot}(c_\phi + c)}{B\Delta t}\right)p_i^{n+1} + T_{i+1/2}p_{i+1}^{n+1} = G(p_i^{n+1} - p_{wf}) - \frac{V_{b_i}\phi_i^{\cdot}(c_\phi + c)}{B\Delta t}p_i^n$$

$$T_{i-1/2}p_{i-1}^{n+1} - \left(T_{i-1/2} + T_{i+1/2} + \frac{V_{b_i}\phi_i^{\cdot}(c_\phi + c)}{B\Delta t} + G\right)p_i^{n+1} + T_{i+1/2}p_{i+1}^{n+1} = -Gp_{wf} - \frac{V_{b_i}\phi_i^{\cdot}(c_\phi + c)}{B\Delta t}p_i^n$$

$$w_i p_{i-1}^{n+1} + c_i p_i^{n+1} + e_i p_{i+1}^{n+1} = d_i$$

Case study: Single phase 1D slightly compressible implicit reservoir simulation with Python

- Inputs

```
k = np.array([273, 248, 127, 333, 198]) # permeability (mD)
phi = np.array([0.21, 0.17, 0.10, 0.25, 0.13]) # porosity (%)
x = np.array([400, 300, 150, 200, 250]) # length (ft)
n = len(x) # number of blocks
t = 5 # time step (days)
t_end = 135//t # after (135) days
p = np.zeros((t_end, 5)) # pressure (psia)
p[0]= 3000 # initial pressure (psia)
h = 50 # height (ft)
w = 500 # width (ft)
B = 1 # FVF (RB/STB)
mu = 1.5 # viscosity (cp)
cp = 2.5e-5 # compressibility (1/psi)
```

Case study: Single phase 1D slightly compressible implicit reservoir simulation with Python

- Inputs

```
rw = 6/2 * 0.08333 # interal radius (ft)
re = 0.14 * (x[3]**2 + w**2)**0.5 # external radius (ft)
A = h * w # area (ft2)
Bc = 0.001127 # conversion factor to stb
ac = 5.614583 # rb to stb
t = 5 # time step (days)
qsc = np.zeros(5) # well flow rates (STB/D)
qsc[3] = -400 # well flow rate at block 4 (STB/D)
qw = 0 # west boundary flow rate (STB/D)
qe = 0 # east boundary flow rate (STB/D)
G = ((2*np.pi*Bc*k[3]*h)/(B*mu*np.log(re/rw)))
```

Case study: Single phase 1D slightly compressible implicit reservoir simulation with Python

- Transmissibility

```
T = (1/(mu*B)) * (2*Bc) / ((x[:-1]/(A*k[:-1])) + (x[1:]/(A*k[1:])))
```

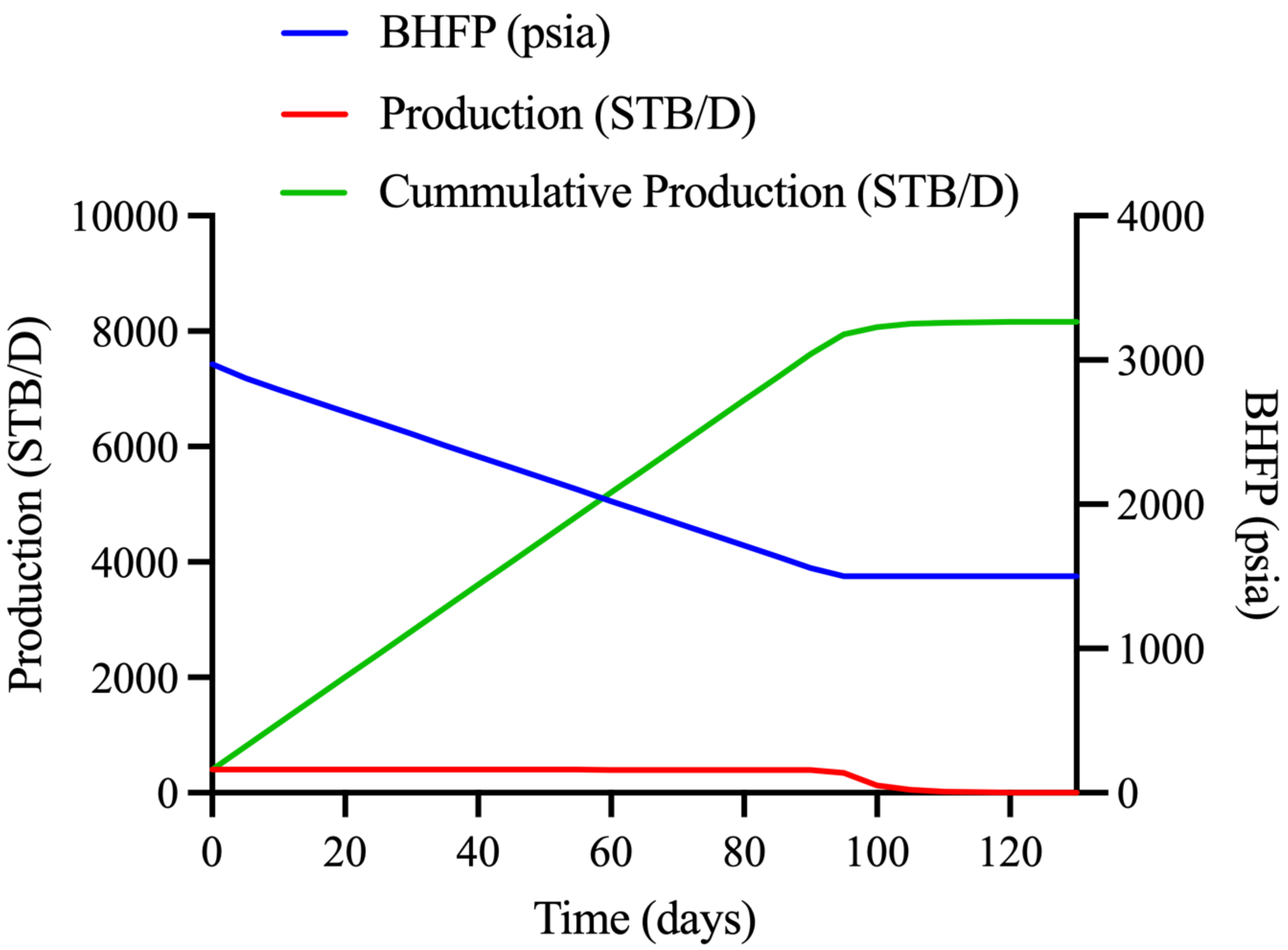
- Accumulation

```
ma = (A * x * phi * cp) / (ac * B * t)
```

Case study: Single phase 1D slightly compressible implicit reservoir simulation with Python

- Thomas' algorithm

```
for t_idx in range(1, t_end):
    c = np.zeros((n, 7))
    for i in range(n):
        if i == 0:
            c[i][:4] = [0, -(qw + T[0] + ma[0]), T[0], -qsc[0] - ma[0] * p[t_idx - 1][0]]
            c[i][4:] = [c[i][2] / c[i][1], c[i][3] / c[i][1], 0]
        elif i == n - 1:
            c[i][:4] = [T[-1], -(qe + T[-1] + ma[-1]), 0, -qsc[-1] - ma[-1] * p[t_idx - 1][-1]]
            c[i][4:] = [0, (c[i][3] - (c[i][0] * c[i - 1][5])) / (c[i][1] - (c[i][0] * c[i - 1][4])), 0]
        else:
            c[i][:4] = [T[i - 1], -(T[i - 1] + T[i] + ma[i]), T[i], -qsc[i] - ma[i] * p[t_idx - 1][i]]
            c[i][4:] = [c[i][2] / (c[i][1] - (c[i][0] * c[i - 1][4])),
                        (c[i][3] - (c[i][0] * c[i - 1][5])) / (c[i][1] - (c[i][0] * c[i - 1][4])), 0]
    c[:, -1][-1] = c[:, -2][-1]
    for i in range(0, n - 1)[::-1]:
        c[i, -1] = c[i, -2] - (c[i, -3] * c[i + 1, -1])
    p[t_idx] = c[:, -1]
```


BHFP (psia)
Production (STB/D)
Cummulative Production (STB/D)
10000
8000
6000
4000
2000
0
Production (STB/D)
4000
3000
2000
1000
0
BHFP (psia)
0
20
40
60
80
100
120
Time (days)

References

1. Petroleum Reservoir Simulation: The Engineering Approach
   https://www.geokniga.org/bookfiles/geokniga-petroleum-reservoir-simulation.pdf