034IN - FONDAMENTI DI AUTOMATICA - FUNDAMENTALS OF AUTOMATIC CONTROL A.Y. 2023-2024

Part I: Motivations, Concepts, Examples

Gianfranco Fenu, Thomas Parisini

Department of Engineering and Architecture

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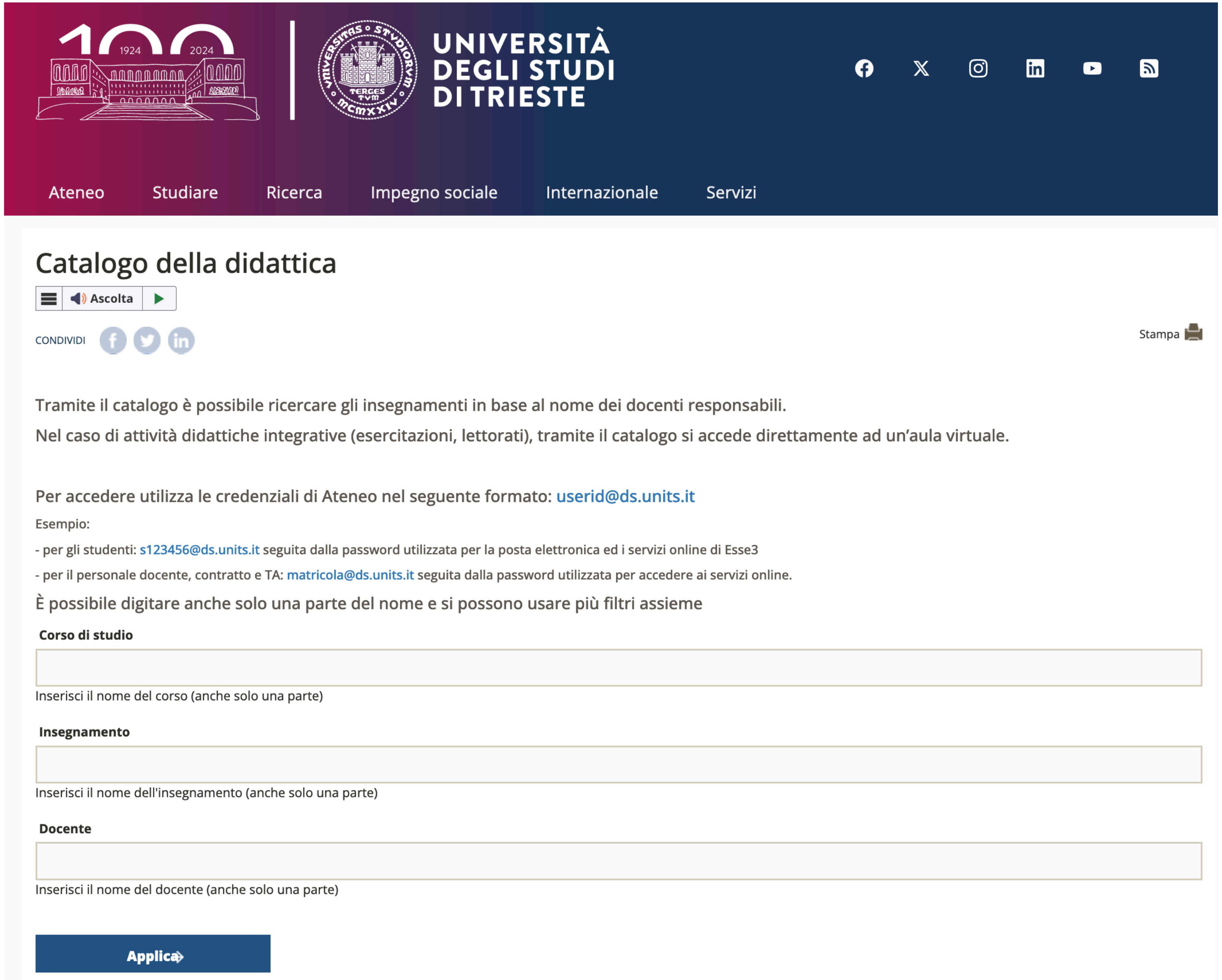
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Corso di studio

Inserisci il nome del corso (anche solo una parte)

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Insegnamento

fondamenti di automatica

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Docente

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INGEGNERIA INDUSTRIALE (IN03)

ANNO DI OFFERTA: 2023/2024

INSEGNAMENTO	CODICE TEAMS	PERIODO	DOCENTI
FONDAMENTI DI AUTOMATICA (034IN - 2023 - [IN03+1+ - ORD. 2014] ENERGIA ELETTRICA E DEI SISTEMI - AC 2)	1yyq8ko	S2	PARISINI THOMAS FENU GIANFRANCO
FONDAMENTI DI AUTOMATICA (034IN - 2023 - [IN03+3+ - ORD. 2014] MECCANICA - AC 2)	1yyq8ko	S2	PARISINI THOMAS FENU GIANFRANCO
FONDAMENTI DI AUTOMATICA (034IN - 2023 - [IN03+5+ - ORD. 2014] GESTIONALE - AC 2)	1yyq8ko	S2	PARISINI THOMAS FENU GIANFRANCO

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INGEGNERIA NAVALE (IN04)

ANNO DI OFFERTA: 2023/2024

INSEGNAMENTO	CODICE TEAMS	PERIODO	DOCENTI
FONDAMENTI DI AUTOMATICA (034IN - 2023 - [PDS0-2020 - ORD. 2020] COMUNE - AC 3)	1yyq8ko	S2	PARISINI THOMAS FENU GIANFRANCO

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INGEGNERIA ELETTRONICA E INFORMATICA (IN05)

ANNO DI OFFERTA: 2023/2024

INSEGNAMENTO	CODICE TEAMS	PERIODO	DOCENTI
FONDAMENTI DI AUTOMATICA (034IN - 2023 - [PDS0-2016 - ORD. 2016] COMUNE - AC 2)	1yyq8ko	S2	PARISINI THOMAS FENU GIANFRANCO

Course Content & Material

- **The Syllabus of the course in 2023/2024 is slightly different from the one in 2022/2023**
- Lecture Handouts
- Matlab/Simulink examples on Live-scripts
- **Book on the "theory"**:
  - ✦ Bolzern, Scattolini, Schiavoni, Fondamenti di Controlli automatici, McGraw-Hill (available in the Library)
- **Book on solved exercises**:
  - ✦ Papadopoulos, Prandini, Fondamenti di Automatica - Esercizi, Pearson Italia
- **Books on Matlab/Simulink**:
  - ✦ MATLAB® by Example: Programming Basics, Gdeisat, Munther; Lilley, Francis. Elsevier. ISBN: 978-0-12-405212-3, 978-0-12-405853-8, 978-1-283-93776-4. Science (available in the Library as e-book).
  - ✦ Online MATLAB Courses: https://it.mathworks.com/support/learn-with-matlab-tutorials.html

Structure and Requirements

- Two components:
  - ✦ **Lectures (classroom)**
  - ✦ **Practical Exercises Classes (using computers and Matlab/Simulink)**
    - **Classroom (please bring your computers/tablets)**
    - **"Aula informatizzata Nettuno" (available Thursday/Friday afternoon)**
- **Compulsory** ("propedeuticità") earlier courses:
  - ✦ "Analisi 1"
  - ✦ "Geometria"
- Desirable ("consigliato") earlier courses:
  - ✦ "Analisi 2"
- Material from previous courses **assumed to be known** by Students:
  - ✦ Linear differential equations
  - ✦ Complex numbers
  - ✦ Vector and matrix algebra

Examination: Intermediate Tests

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Partial tests (highly recommended)

- **Two** partial tests are carried out to evaluate the competences acquired by the students enrolled in the course in academic year 2023/2024
- Students enrolled in previous academic years will not be allowed to take the intermediate tests and will take the examination with the content and format of the academic year 2022/2023
- The partial tests have an **open-book** format and are carried out by the students during the course timetable.
- The duration of each partial test is of **one hour and will be carried out in "Aula Nettuno" equipped with computers with Matlab/Simulink installed**
- The partial tests are strictly personal and group work is not allowed.
- Each partial evaluation will consist in the solution of a specific problem via the creation of a Matlab/Simulink solution code and an explanation of the method used.
- The solution will have to be uploaded online in the cloud using Matlab Grader

Examination: Intermediate Tests

<u>Partial tests & final grade</u>

- If final cumulative mark **Ptot** of the two intermediate tests is >=18, the following options are available at the student discretion:
  - Register in Esse3 the final mark **Ptot**
  - Carry out an additional (open book) partial written examination with a maximum mark equal to 5 points and a duration of one hour. The registration of the cumulative mark is at the student discretion
  - Carry out the full examination
- **<u>Expiration</u>**: using the partial tests cumulative mark **Ptot** is allowed during the current academic year (for the academic year 2023/2024 until the end of the examination session in February 2025)
- When the exam sessions of the current academic year are over, the cumulative mark **Ptot** **<u>expires</u>**.

Examination: Full Test

<u>Full test</u>

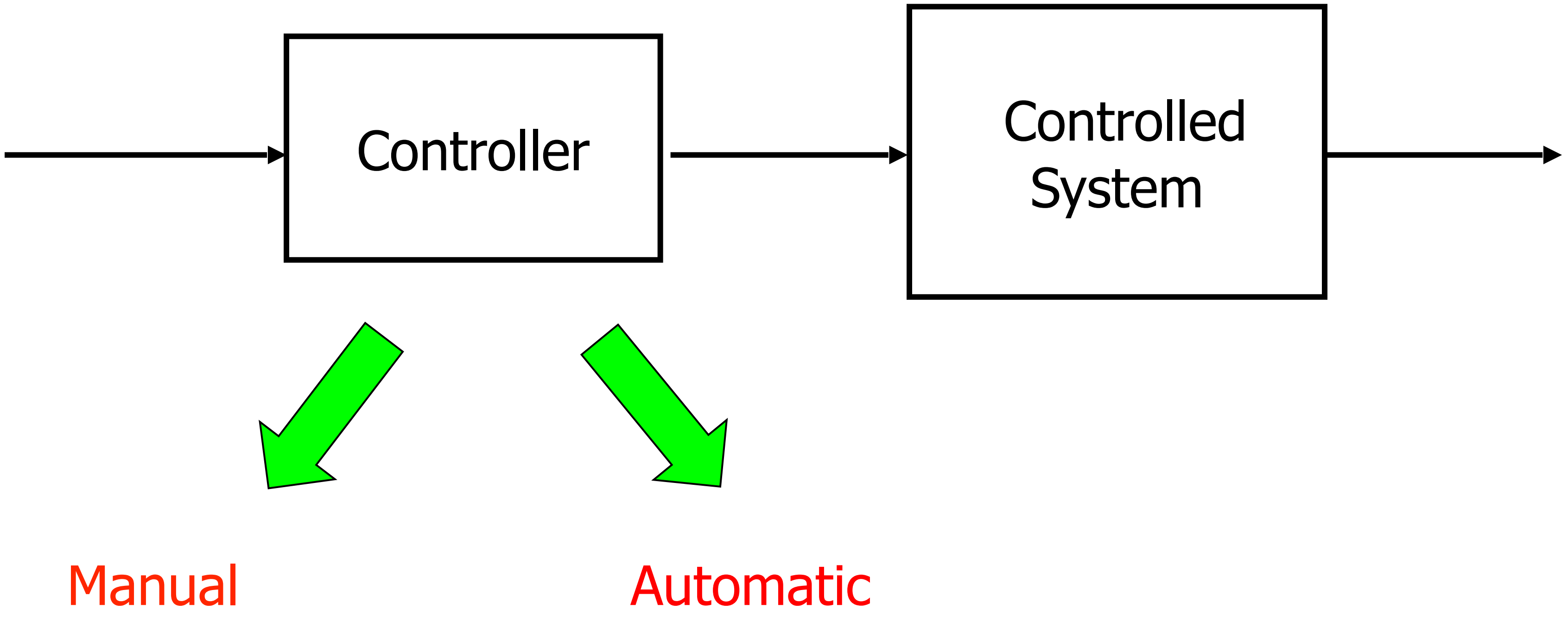
- **Seven** (two in June/July, two in late August/September, three in January/February) full tests will be carried out to evaluate the competences acquired by the students enrolled in the course in academic year 2023/2024
- Each full test evaluation will consist in the solution of a specific problem via the creation of a Matlab/Simulink solution code and an explanation of the method used. Moreover, some additional questions will require a hand-written solution using the forms provided on the day
- The duration of each full test is of **<u>two hours and will be carried out in person in "Aula Nettuno" equipped with computers</u>**
- The full tests are strictly personal and group work is not allowed.
- The solution has to be uploaded online in the cloud using Matlab Grader

Examination: Full Test

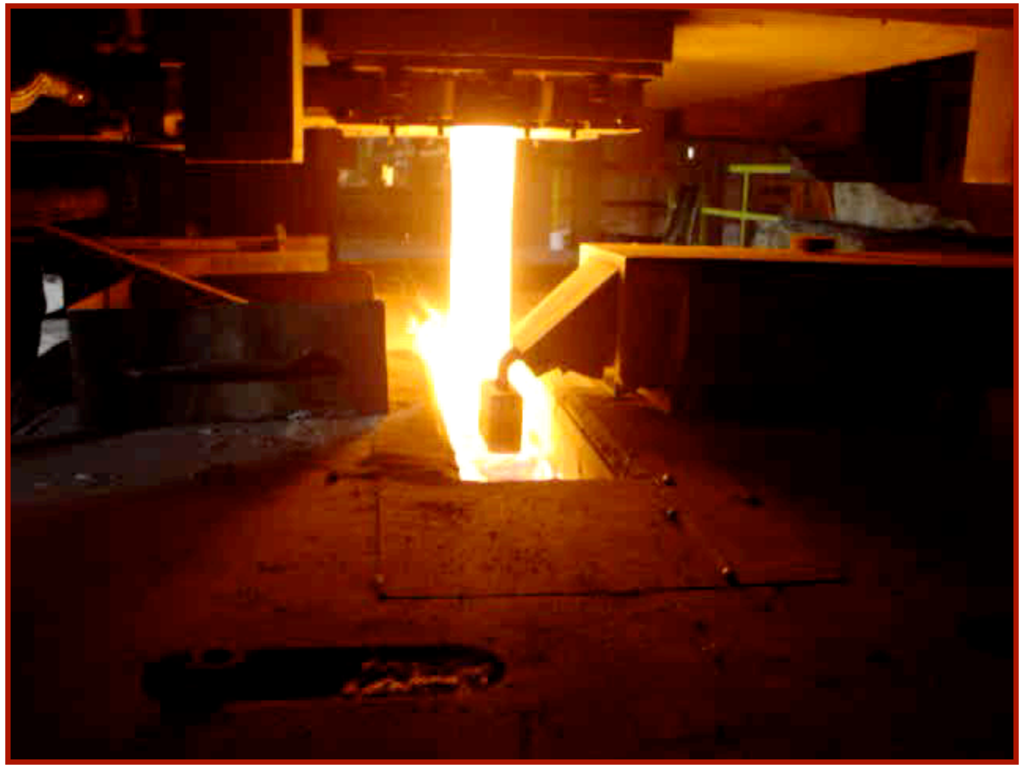
Remarks

- The full examination in the format introduced in 2023/2024 is based on the Syllabus of 2023/2024 which is different from the one in the previous academic years
- Students enrolled in previous academic years have to choose among two options:
  - **Option 1**: carrying out the full written examination in the format used in 2022/2023 with topics referring to the 2022/2023 Syllabus.
  - **Option 2**: carrying out the full examination in the new format used in 2023/2024 with topics referring to the 2023/2024 Syllabus and following the instructions described in the previous slide. **In case Option 2 is chosen, it will not be allowed any more to carry out the full examination in the format used in 2022/2023 with topics referring to the 2022/2023 Syllabus**

Content of the Module? → Methods and Tools to analyse and design automatic control systems of practical relevance



Example: Industrial Robot



Automatic Control: Key Enabling Technology in Industry

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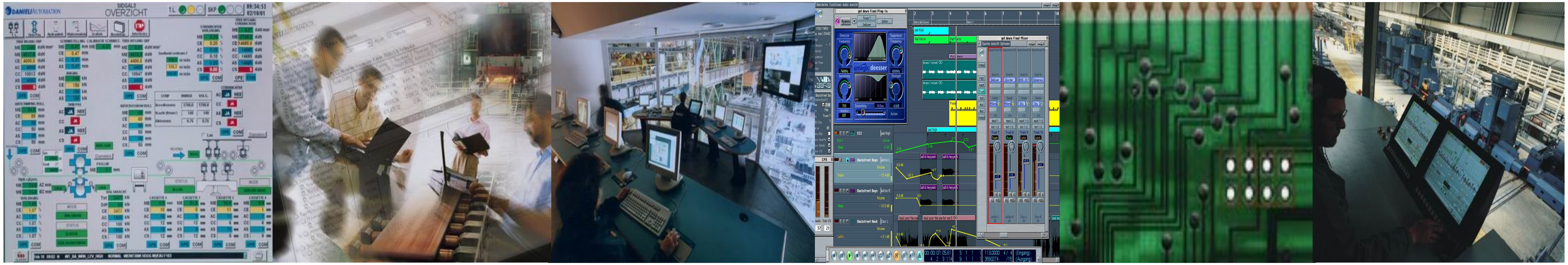
… Control is Ubiquitous …



… BUT …

Automatic Control: Key Enabling Technology in Industry

… Control is noticeable/perceived when … it does not work ....

Control Theory Brief Tour (Youtube)

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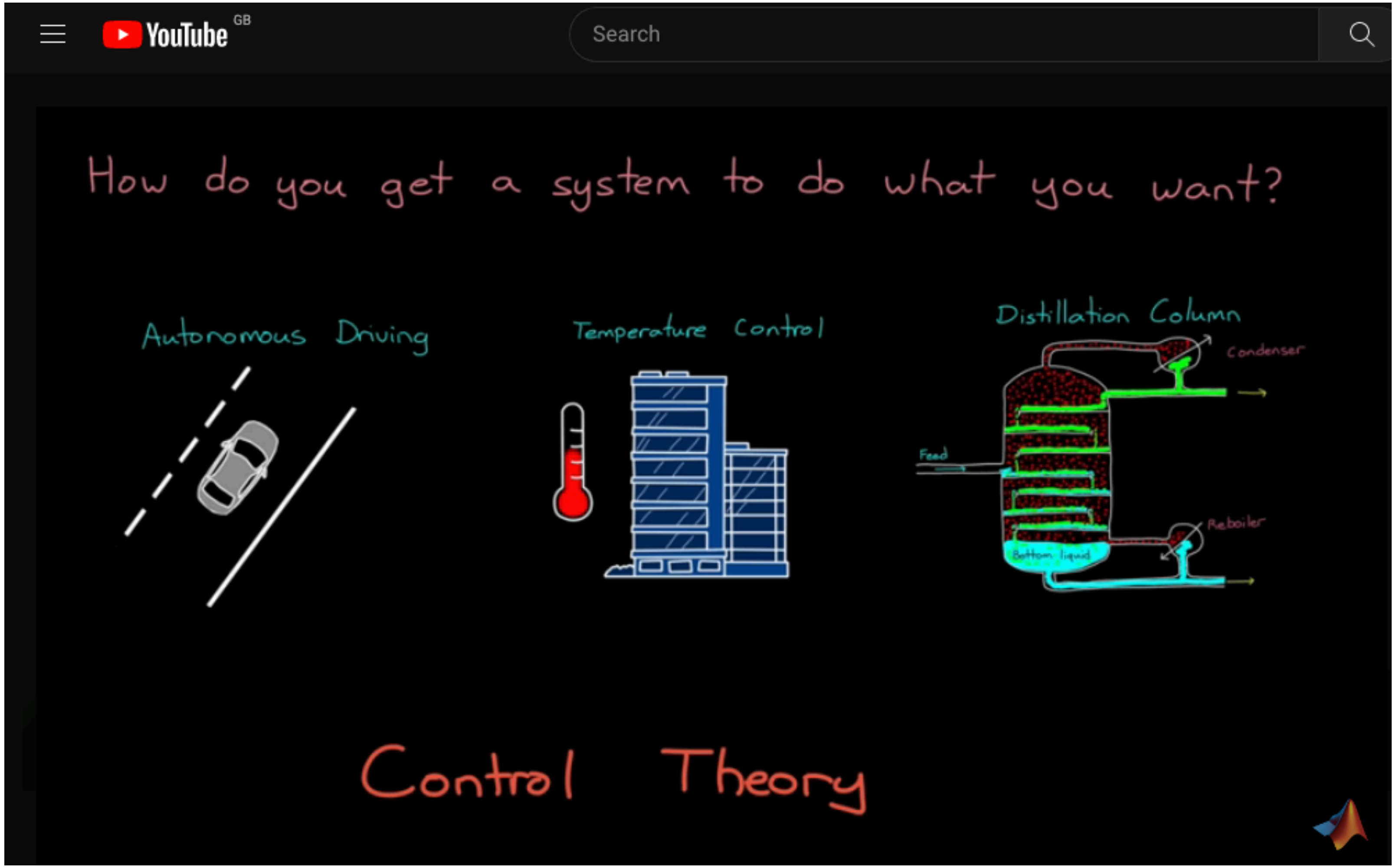
https://www.youtube.com/watch?v=lBC1nEq0_nk



Control Systems: Elements in Common?

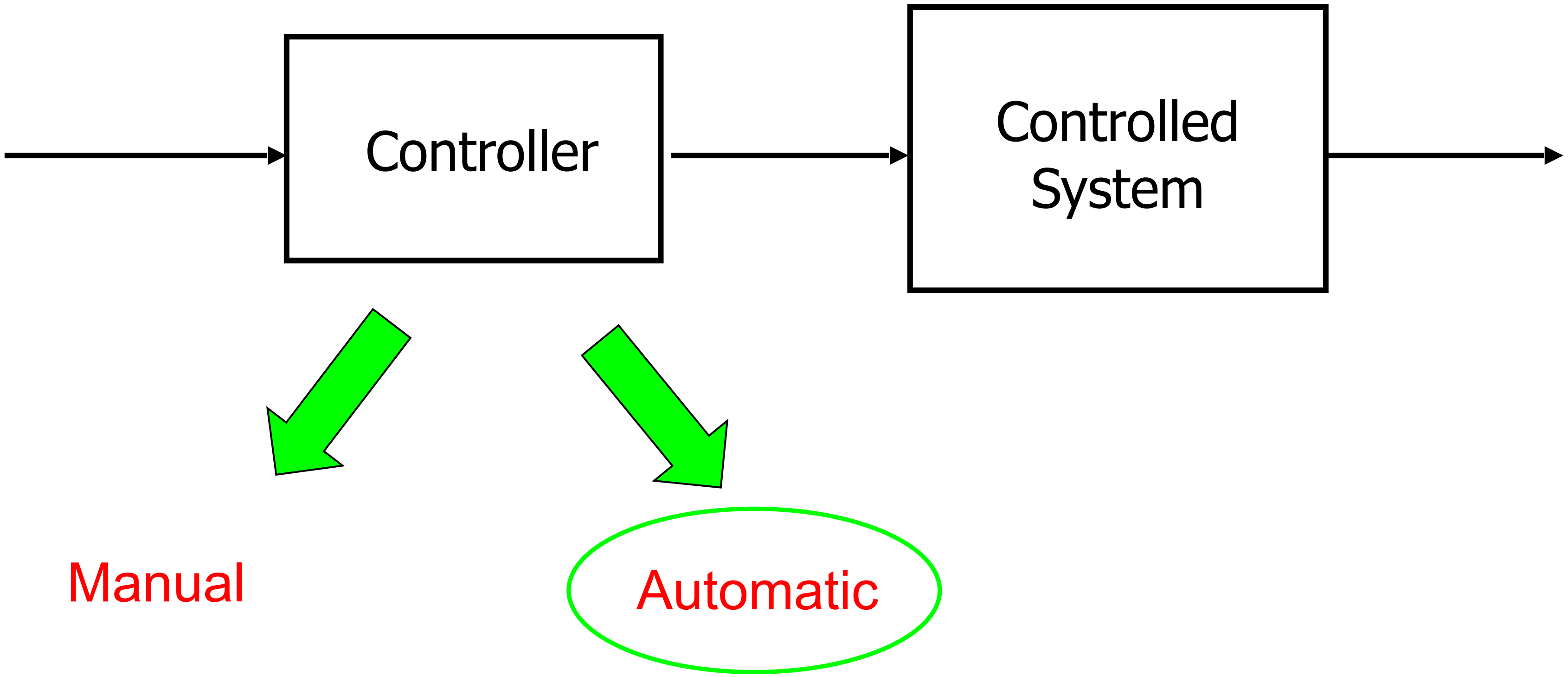
Mathematical Models

Conceptual Working Principle

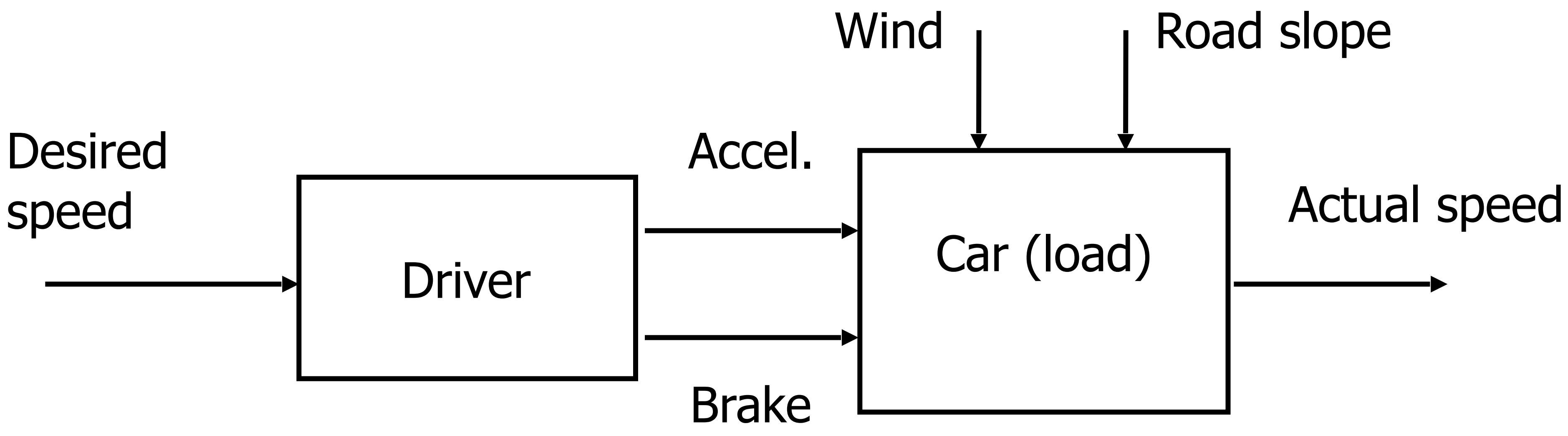
Control Theory

The Control Problem

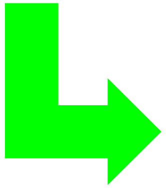
To impose a given time-trajectory to one or more specific variables of an engineering system by acting on other variables that influence the behaviour of the system itself



Example: Speed Control



“open-loop” strategy



Not effective in the presence of uncertainty

Remarks:

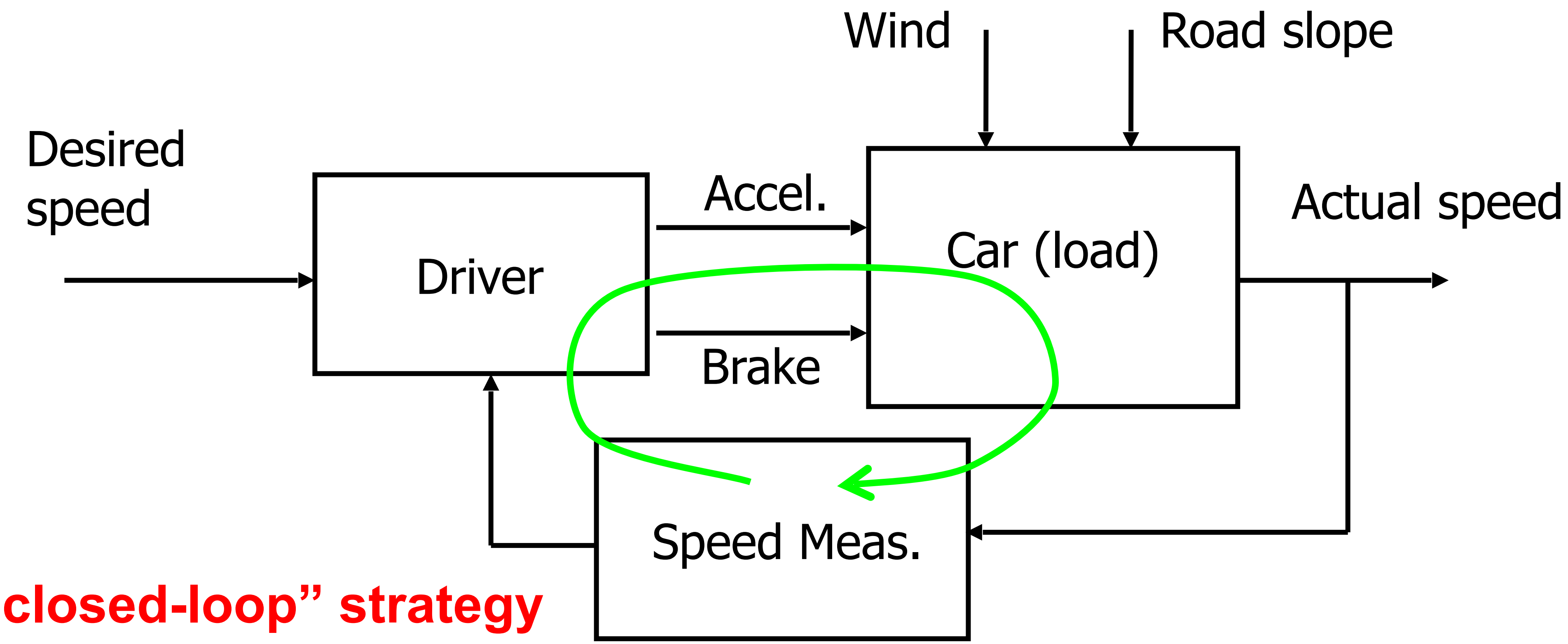
The time-behaviour of the speed for given acceleration and braking actions depends on:

- Initial speed
- Vehicle parameters
- External actions/factors
- ...

Typically uncertain

Speed measurement enables to mitigate the uncertainty

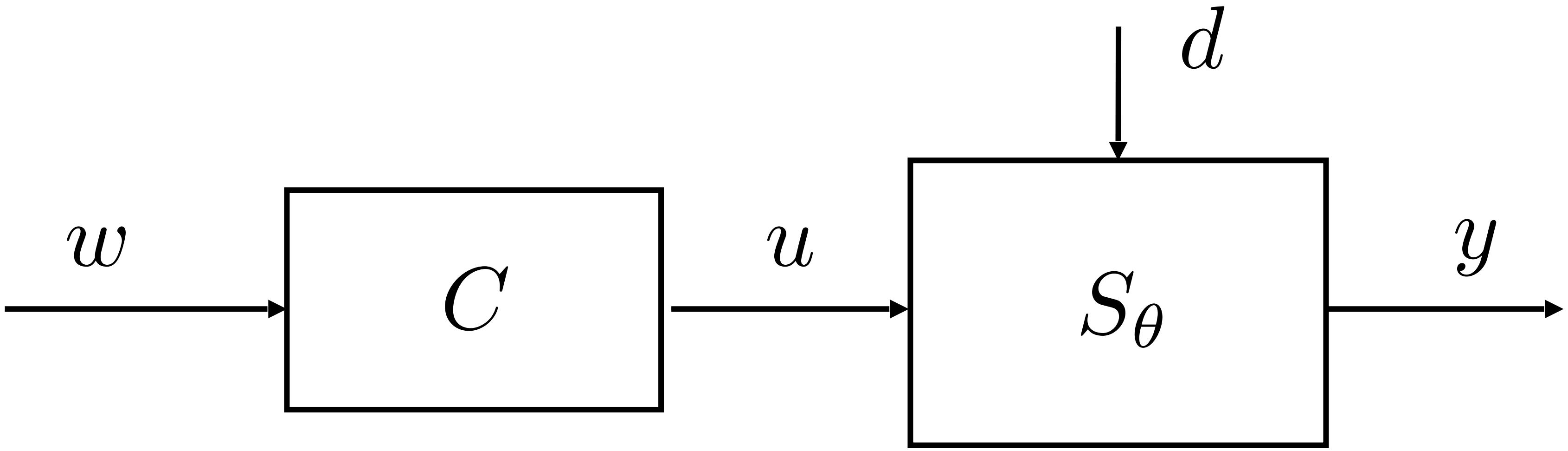
Example: Speed Control



“closed-loop” strategy

Effective in the presence of uncertainty

The Control System



- S_θ : system to be controlled
- C : controller
- θ : system's parameters
- y : controlled variable (output)
- u : control variable (accessible to the controller)
- d : disturbance
- w : reference variable (set-point)

The Controller Objective

To act on u so that $y \simeq w$ even in the presence of uncertainty

Typically:

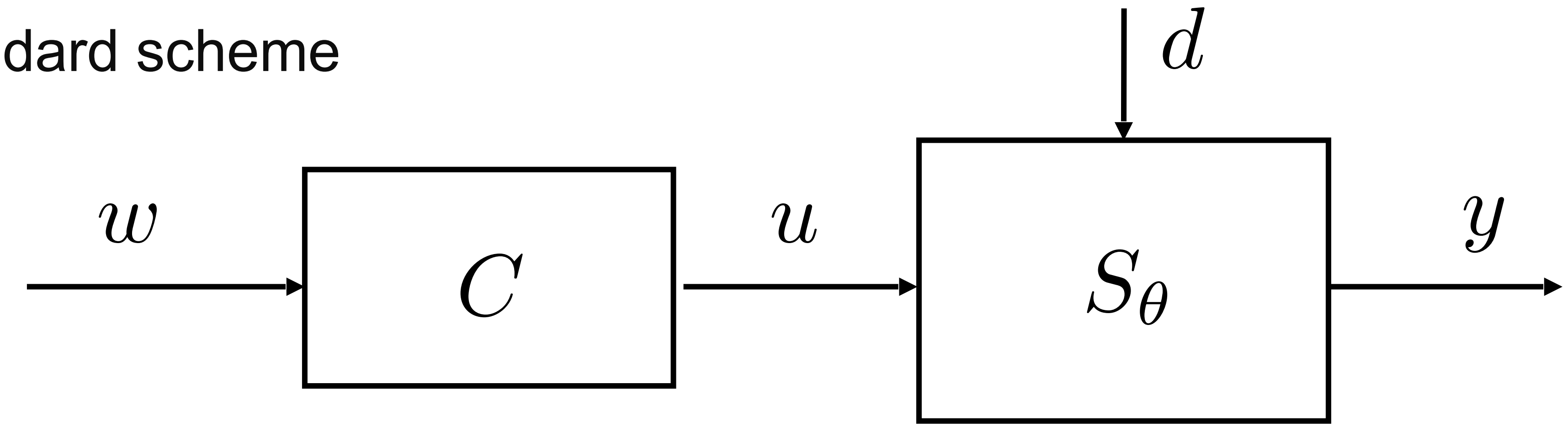
$$d = \bar{d} + \Delta d, \qquad |\Delta d| < \bar{D}$$

$$\theta = \bar{\theta} + \Delta\theta$$

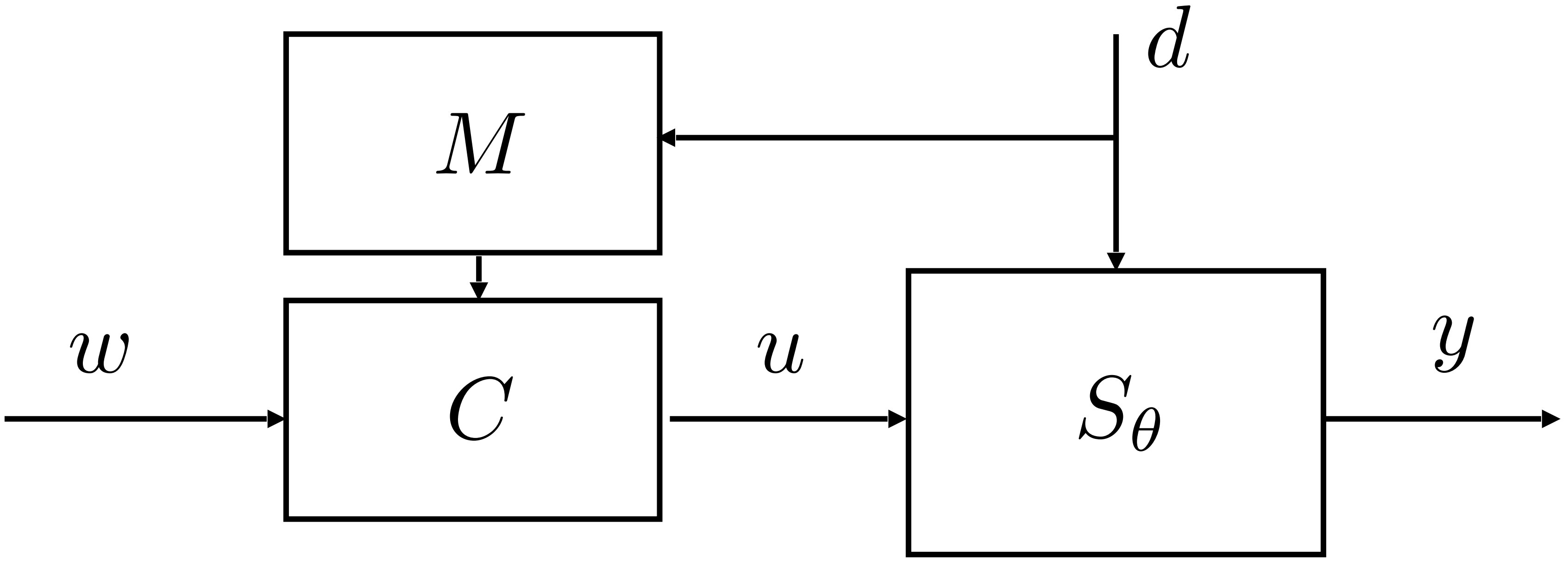
where $\bar{d}, \bar{\theta}$ are **known nominal values** of d, θ

Open-Loop Control Strategies

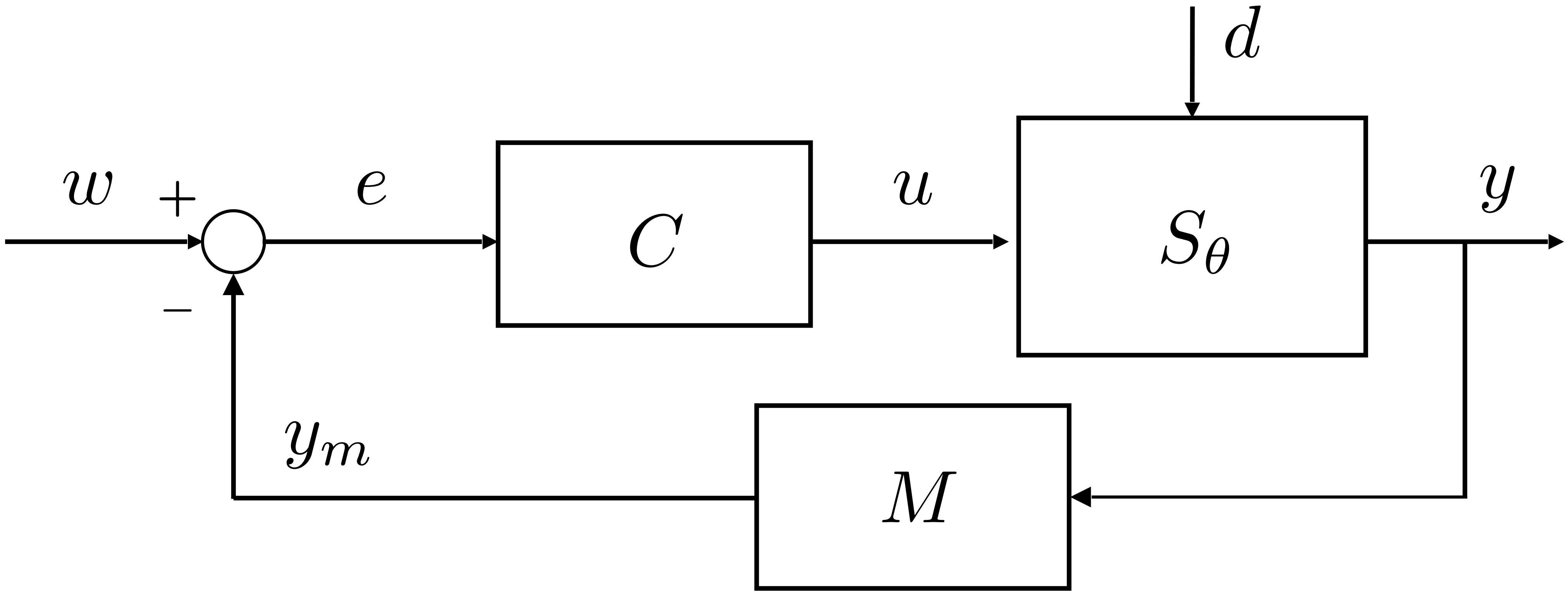
a) Standard scheme



b) Scheme with **feedforward** disturbance rejection



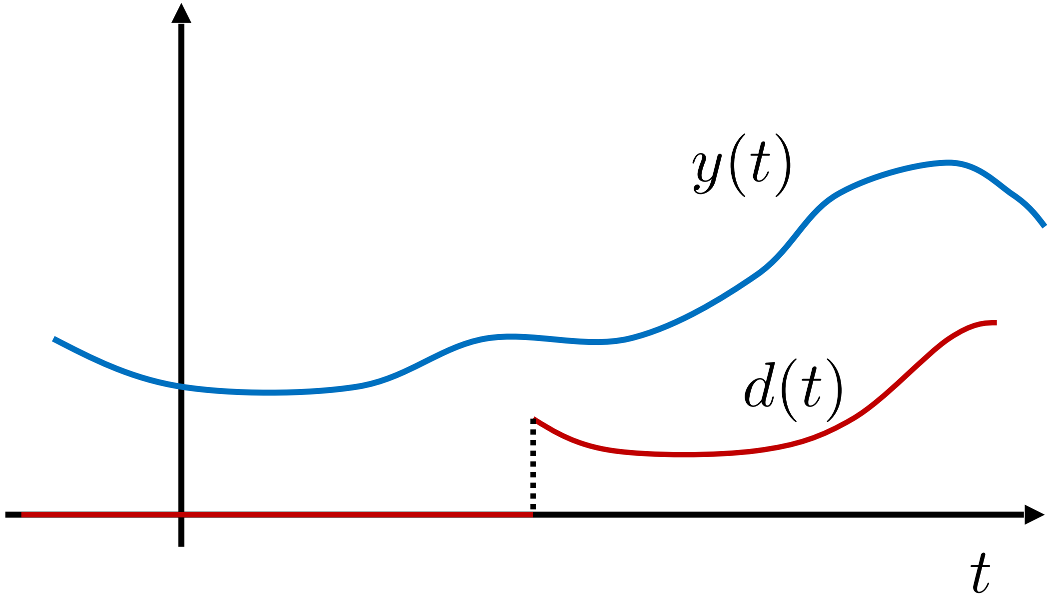
Standard scheme



Assumptions on Continuous-time Variables

$$t \in \mathbb{R}$$
$$y(t) \in \mathbb{R}$$
$$d(t) \in \mathbb{R}$$
$$\vdots$$

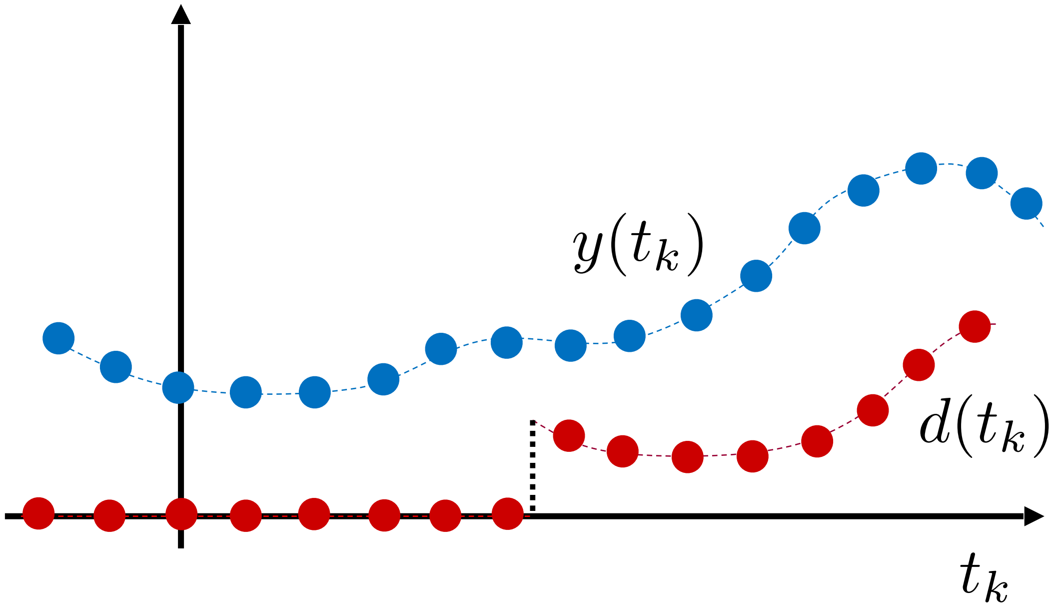
Continuous-time variables



Assumptions on Discrete-time/Sampled-time Variables

Discrete-time variables

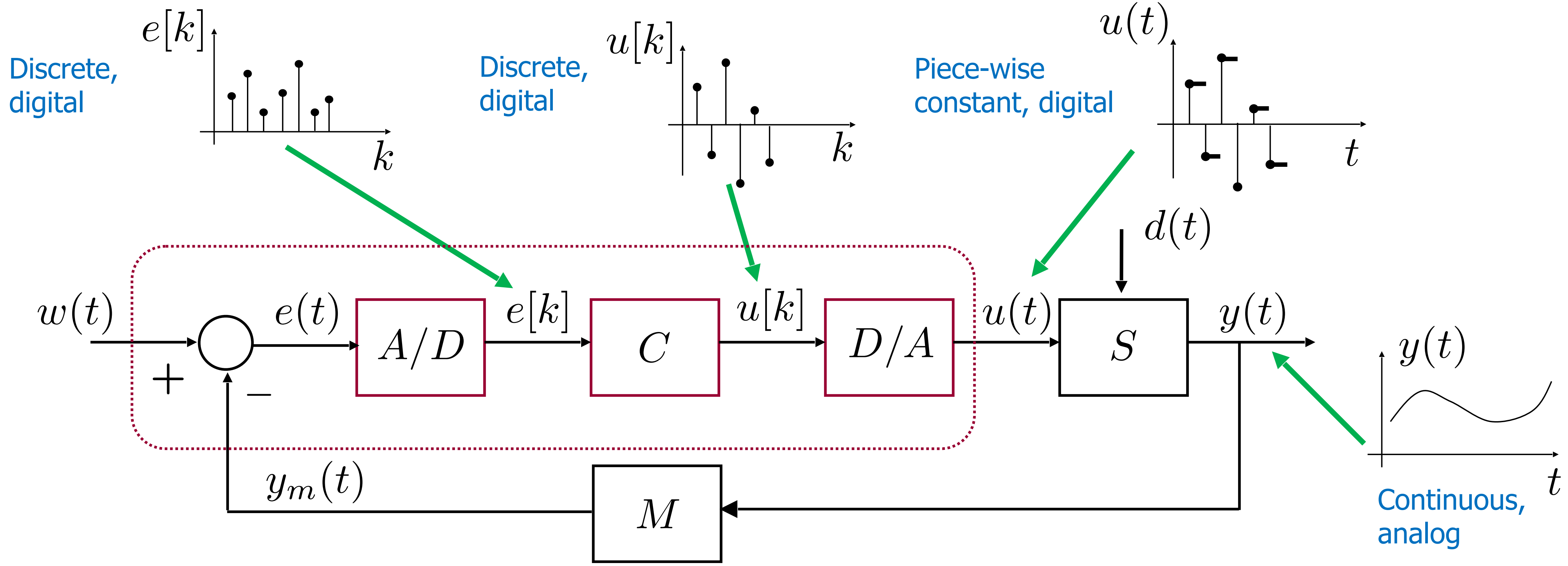
$$t_k,\ k \in \mathbb{Z}$$
$$y(t_k) \in \mathbb{R}$$
$$d(t_k) \in \mathbb{R}$$
$$\vdots$$



In case of regular sampling: $y(t_k)\ :\ t_k = kT_s,\ k \in \mathbb{Z}$

$$y[k], d[k], \ldots\ k \in \mathbb{Z}$$

Digital Control: Typical Structure



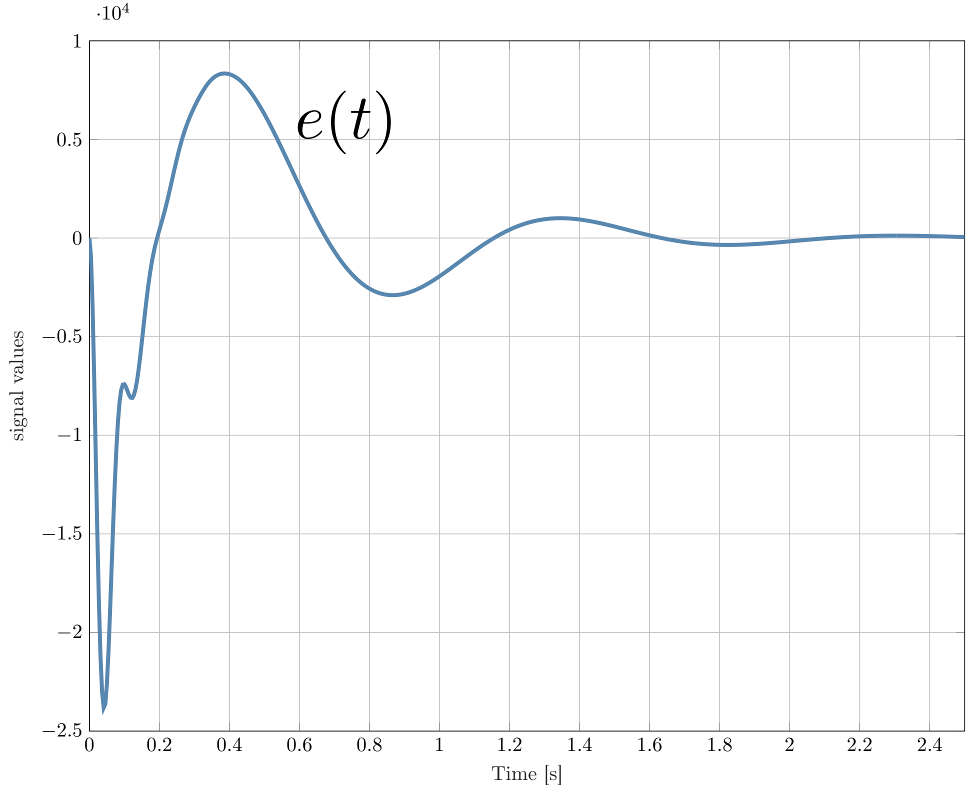
From a mathematical point of view, digital control systems are **hybrid systems** since continuous-time and discrete-time variables are present at the same time

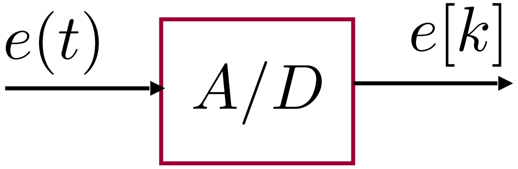
- **Analog** variables: their values change with continuity
- **Digital** variables: their values are quantised

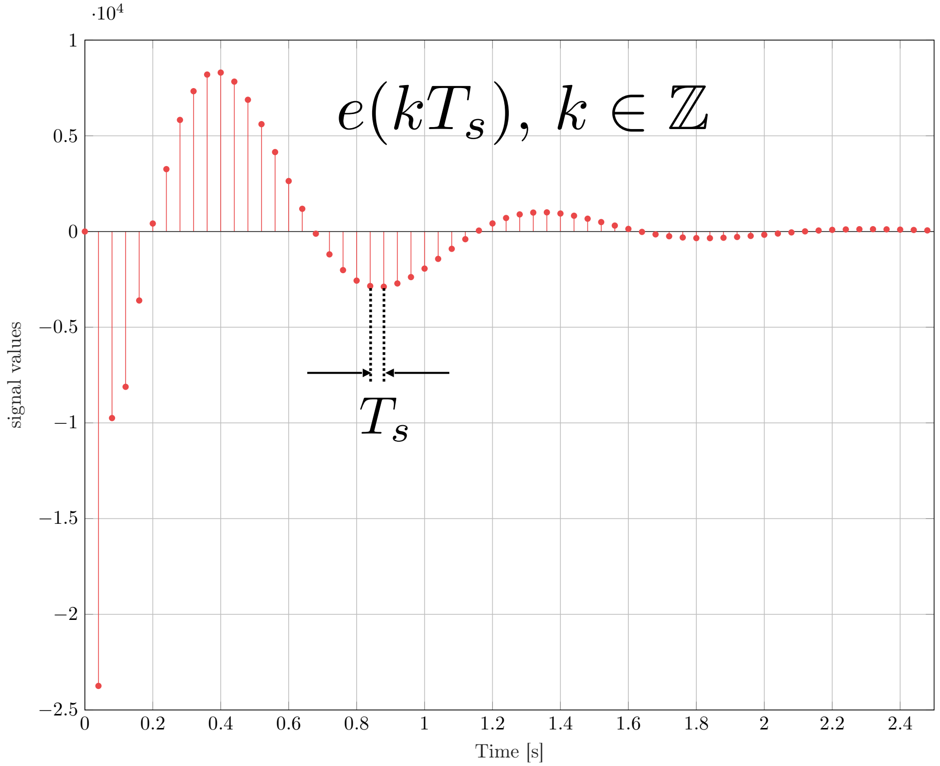
Analog Controller vs. Digital Controller

- The analog controller receives analog inputs in continuous-time and yields analog outputs in continuous-time
- The digital controller is implemented on a digital computing device (micro-controller, DSP card, PLC, etc.)
- Digital computing devices can only elaborate digital sampled-time variables, and suitable conversion devices are needed at the interface with the controlled system: Analog/Digital Converters (**A/D**) Digital/Analog Converters (**D/A**)
- A/D and D/A converters need to be synchronised via a clock signal with period T_s (**sampling time**)
- The control unit gets the input variables from the A/D converter and yields the output variable to the D/A converter **only on the clock time-instants**
- Such variables are denoted as **discrete-time** variables
- When there is no risk of ambiguity, we drop the sampling-time notation and only keep the discrete-time index: $t_k = kT_s \quad \Longrightarrow \quad k$

A/D Conversion







Ideally, the A/D conversion implements the **sampling mechanism**

$$\{e(t) : \; t = kT_s, \; k \in \mathbb{Z}\} \quad \Longrightarrow \quad e[k]$$

with **sampling frequency**

$$f_s = \frac{1}{T_s}$$

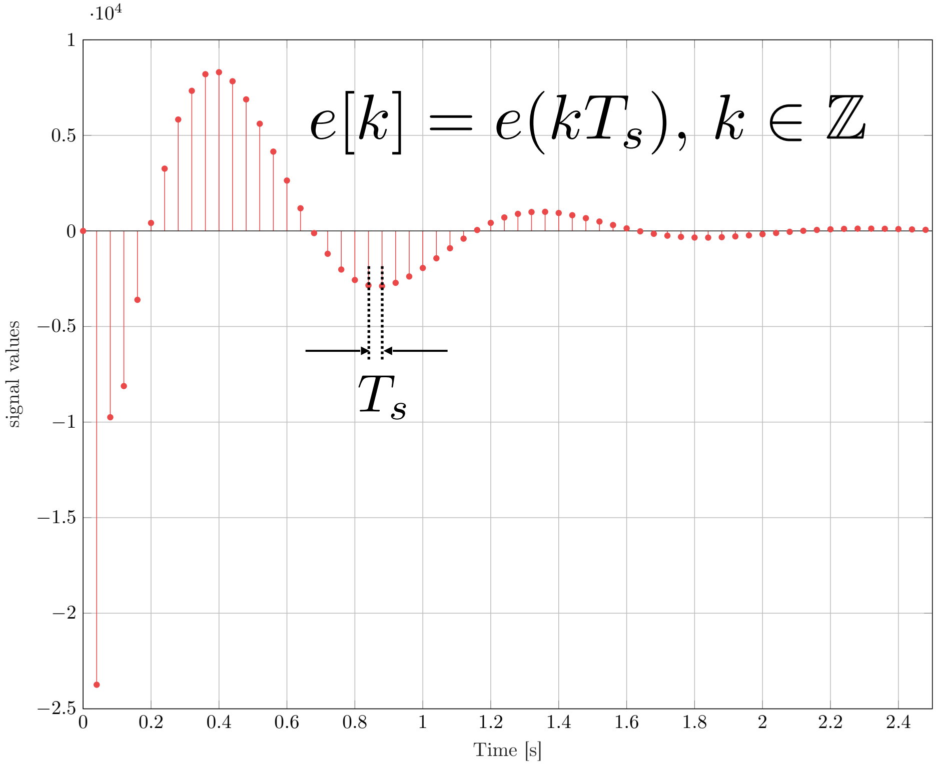
A/D Conversion (contd.)

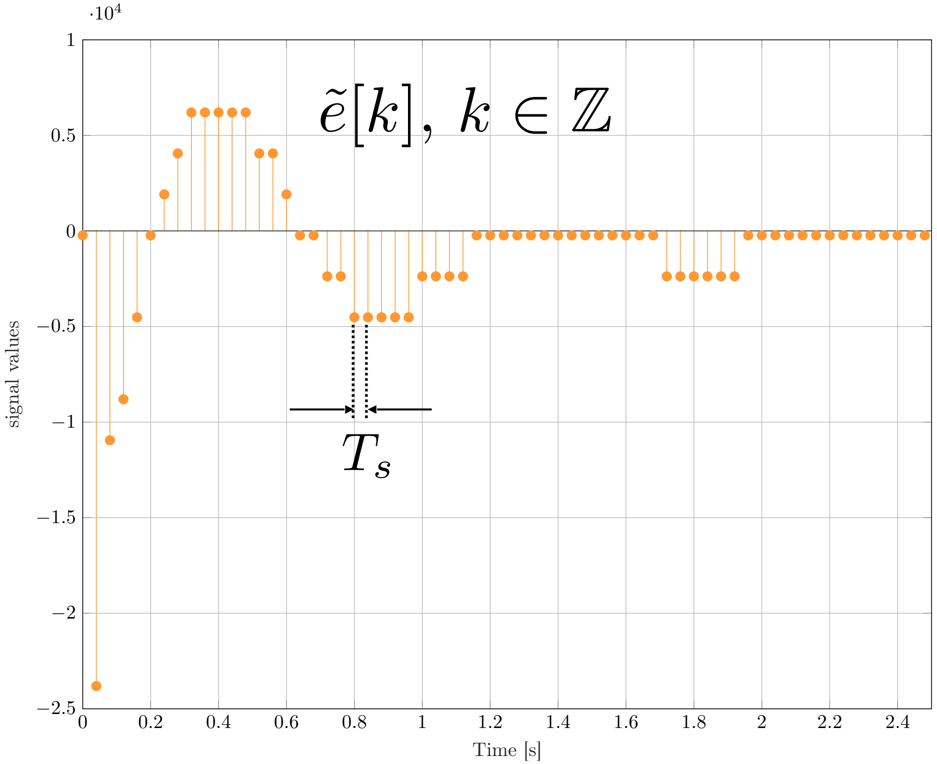
The A/D conversion implies:

- loss of information (continuous-time/discrete-time)
- quantisation noise and distortion (analog to digital)

$$e(t) \;\Rightarrow\; e[k],\ k \in \mathbb{Z} \;\Rightarrow\; \tilde{e}[k] \neq e[k],\ k \in \mathbb{Z}$$

0 1 10 10 . . .



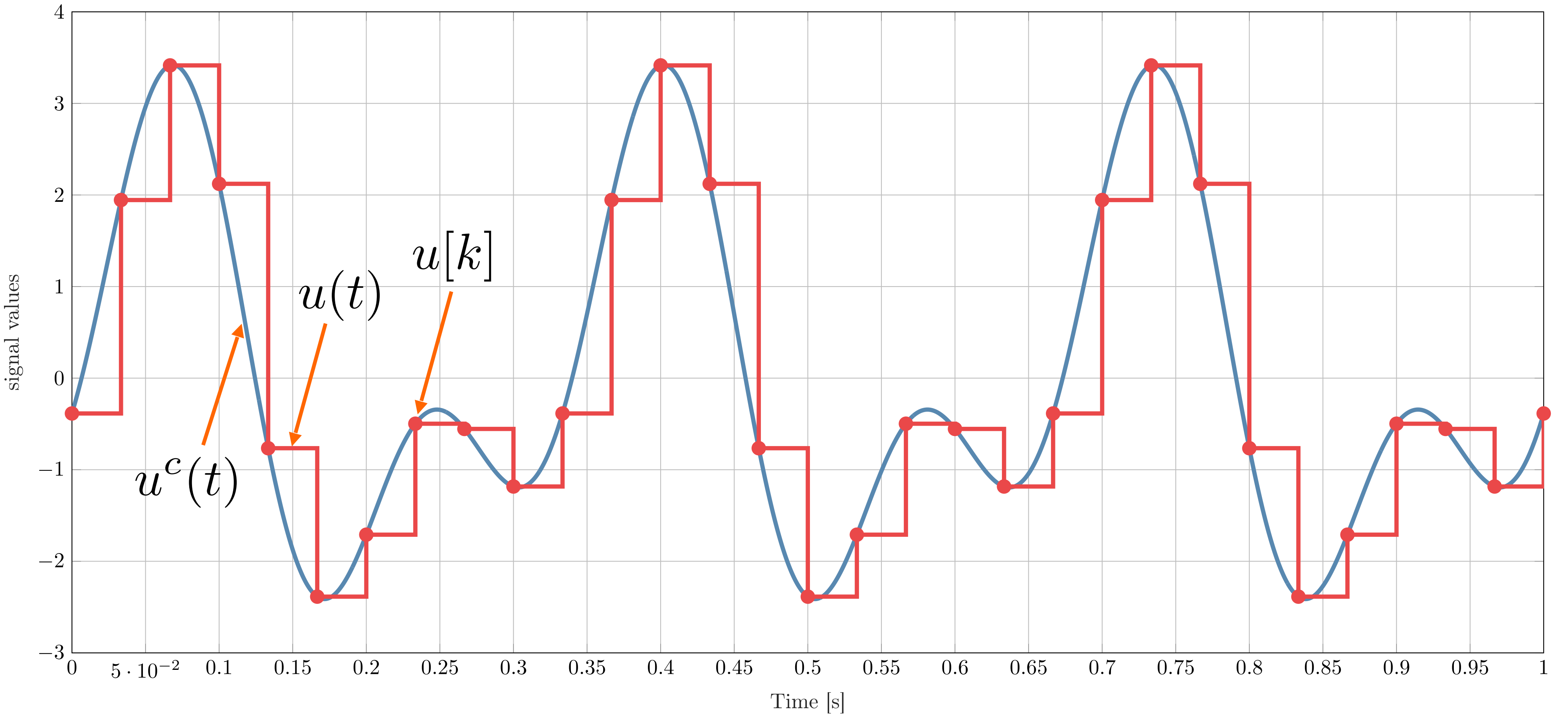


[In this course, we suppose $\tilde{e}[k] = e[k],\ k \in \mathbb{Z}$]

D/A Conversion

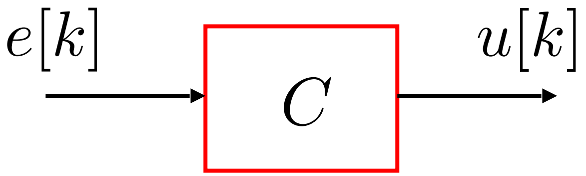
Zero-order hold (ZOH): $u(t) = u[k]\,,\; kT_s \leq t < (k+1)T_s,\; k \in \mathbb{Z}$

The D/A conversion using a ZOH is a stair-wise **delayed** approximation of the unknown underlying continuous time function $u^c(t)$



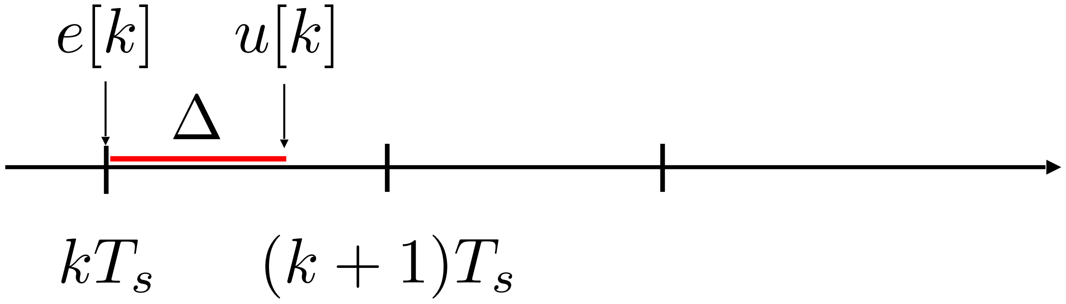
Digital Discrete-Time Controller

The controller is a discrete-time system, that is: a **computational algorithm**!



$$u[k] = f(u[k-1], u[k-2], \ldots, e[k], e[k-1], \ldots)$$

Temporisation:



The controller computation time should satisfy: $\Delta < T_s$

Requirements for an Analog Continuous-Time Control System

The basic requirement is:

$$\boxed{y(t) \simeq w(t)}$$

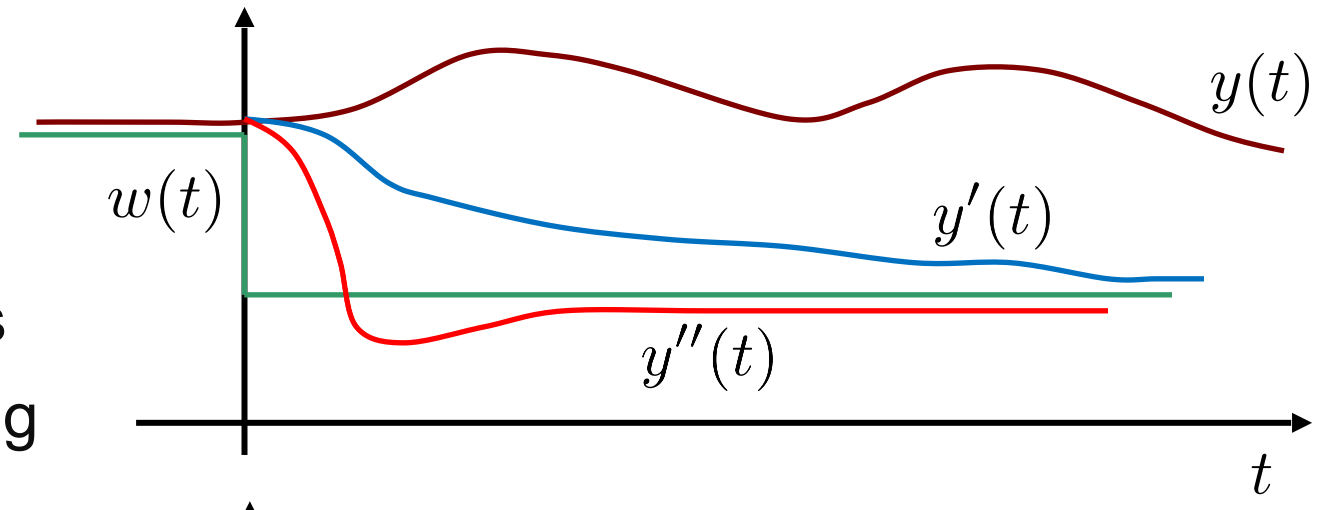
Introducing the error variable $e(t) = w(t) - y(t)$, the above requirement becomes:

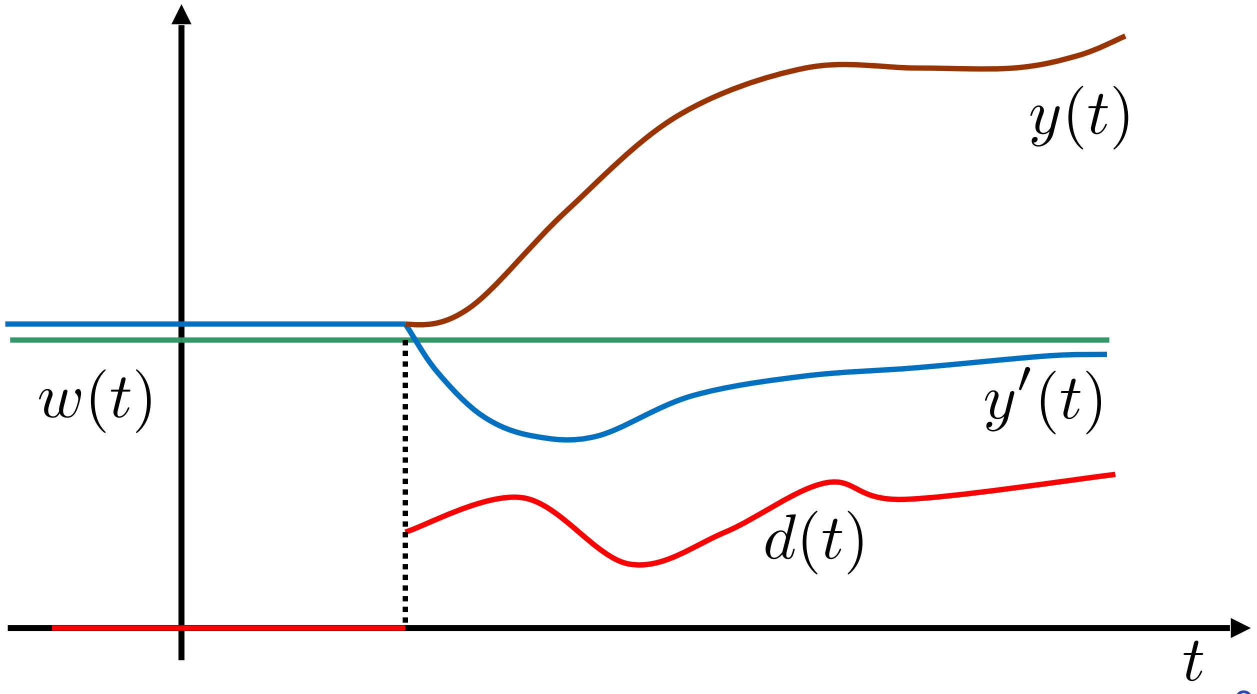
$$\boxed{e(t) \simeq 0 \text{ in all situations "of interest"}}$$

Without loss of generality, we focus on the continuous-time case. The discrete-time case is perfectly analogous

General Requirements of Control Systems

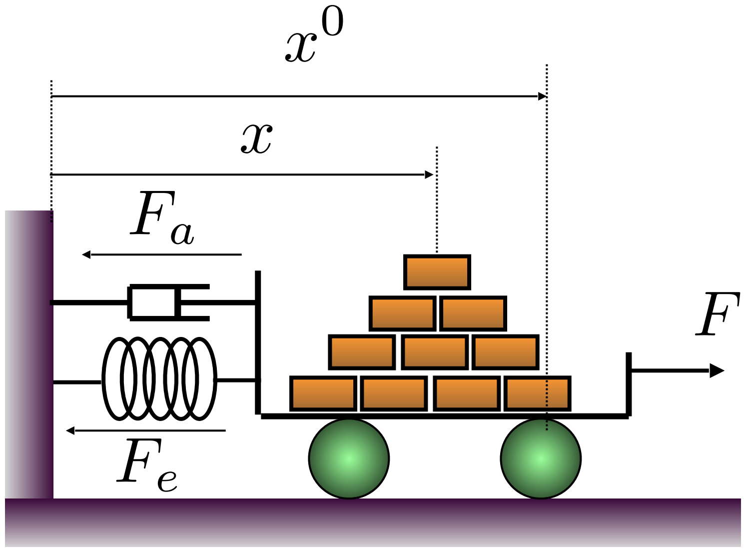
- (A) Static precision
  - ✦ In equilibrium conditions
- (B) Dynamic precision
  - ✦ Speed of response
  - ✦ Damping of possible oscillations
  - ✦ Capability of tracking fast-varying reference variables $w(t)$
- (C) Insensitivity to disturbances
  - ✦ Capability of rejecting a disturbance $d(t)$
- (D) Robustness
  - ✦ (A), (B), (C) in the presence of uncertain system's parameters θ





Example 2: Position Control

Mechanical system

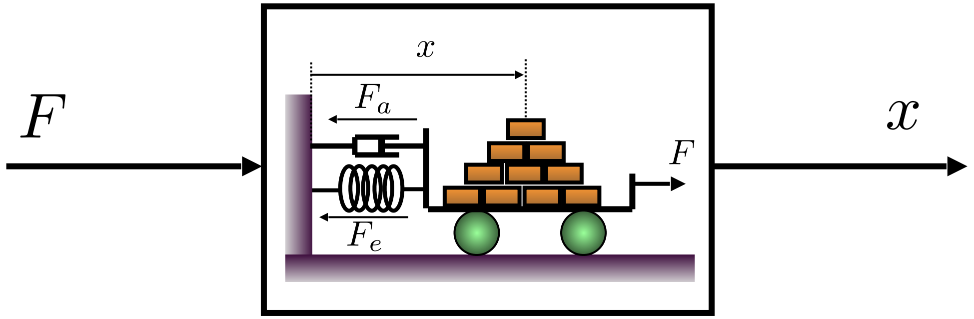


- Control input: external force F
- Controlled output: position of the cart x
- Reference output: desired position $w = x^0$
- Elastic spring force: $F_e = kx$
- Damper viscous friction force: $F_a = h\dot{x}$

Static Modelling

$$F = kx$$

Equilibrium of the forces
in **static** conditions

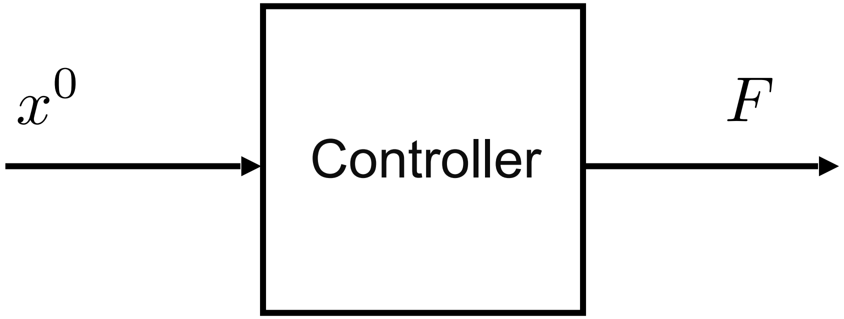


$$x = \frac{1}{k}F$$

output

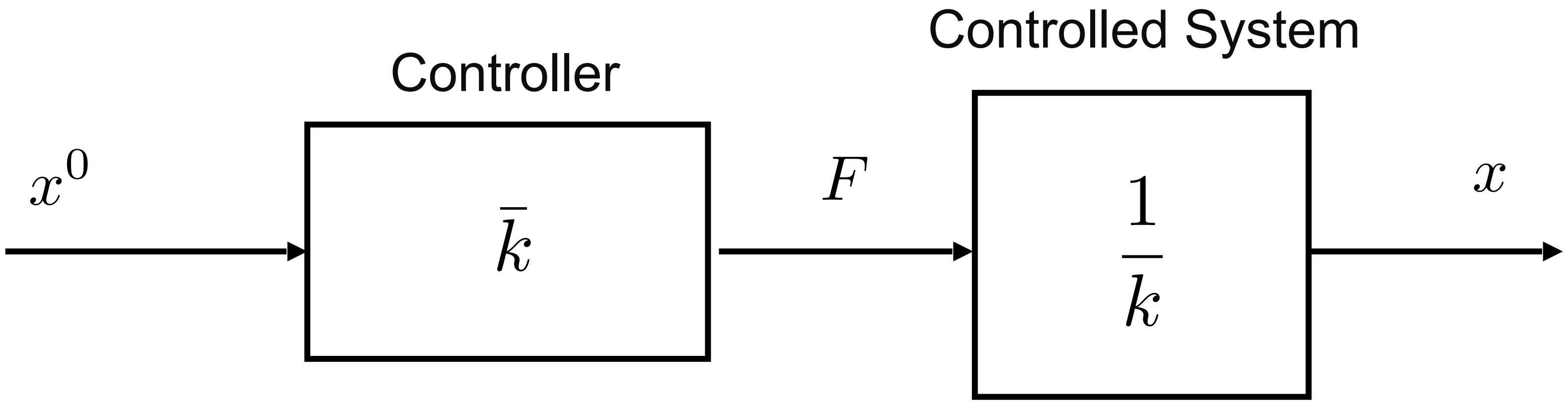
input

Open-Loop Control Strategy



$$F = \bar{k} x^o$$

$\bar{k}$ nominal value of the spring constant



$$x = \frac{\bar{k}}{k} x^o \quad \Longrightarrow \quad e = x^o - x = x^o \left(1 - \frac{\bar{k}}{k}\right)$$

Open-Loop Control Strategy (contd.)

- In nominal conditions $k = \bar{k}$ $\Longrightarrow$ $e = 0$

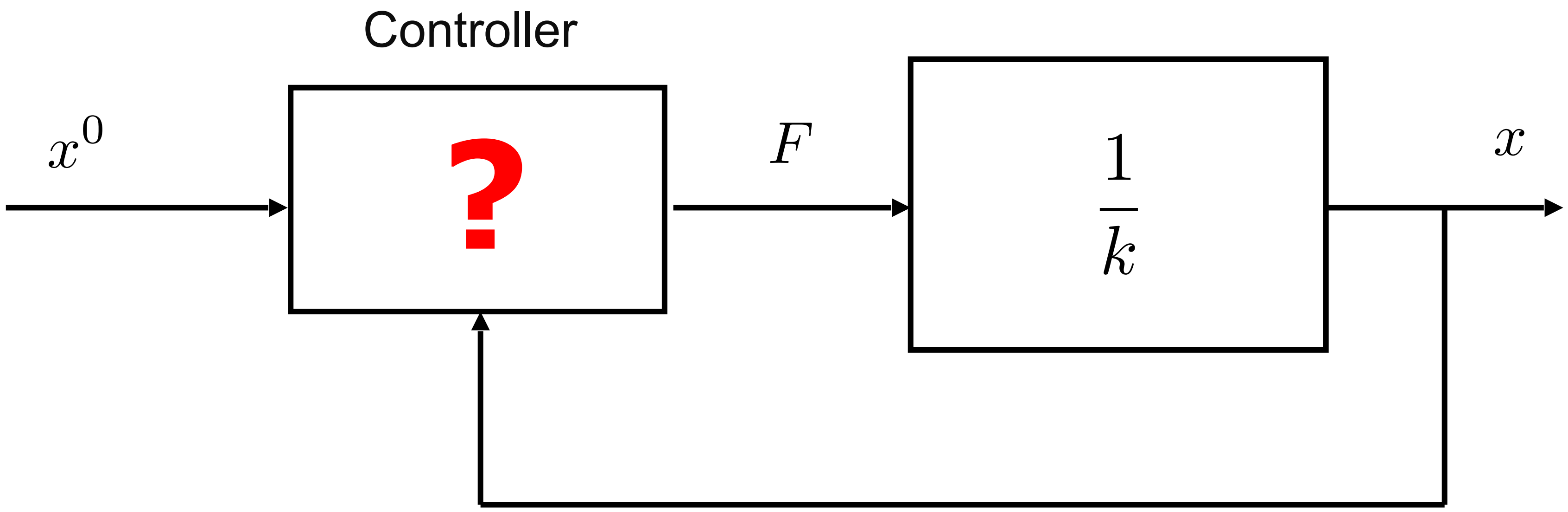
- In uncertain conditions $k \neq \bar{k}$ $\Longrightarrow$ $e = x^o \dfrac{\Delta k}{k} \neq 0$

$$\Delta k = k - \bar{k} \neq 0$$

uncertainty

There is no way to compensate for the uncertainty by an open-loop control strategy

Closed-Loop Control Strategy



Let us opt for a **proportional control** strategy:

$$F = \alpha \underbrace{(x^o - x)}_{e}, \;\; \alpha > 0$$

Closed-Loop Control Strategy (contd.)

Suppose $x^o \neq 0$:

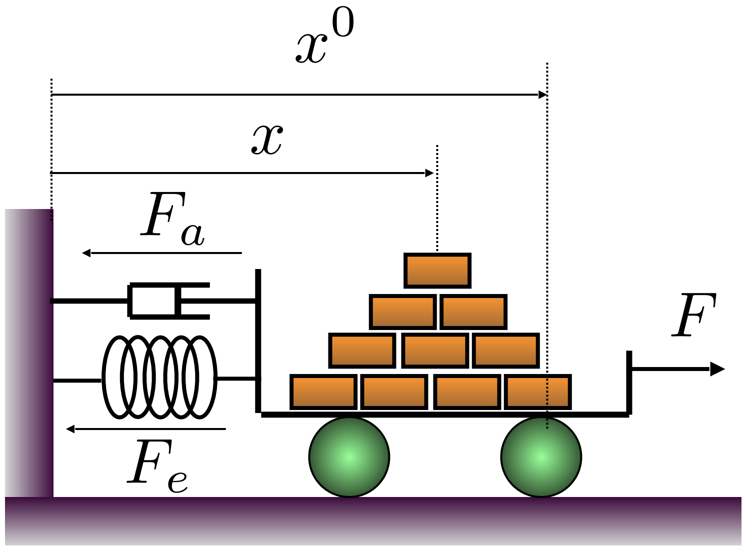
$$x = \frac{1}{k}F = \frac{1}{k}\alpha\left(x^o - x\right) \Rightarrow x\left(1 + \frac{\alpha}{k}\right) = \frac{\alpha}{k}x^o$$

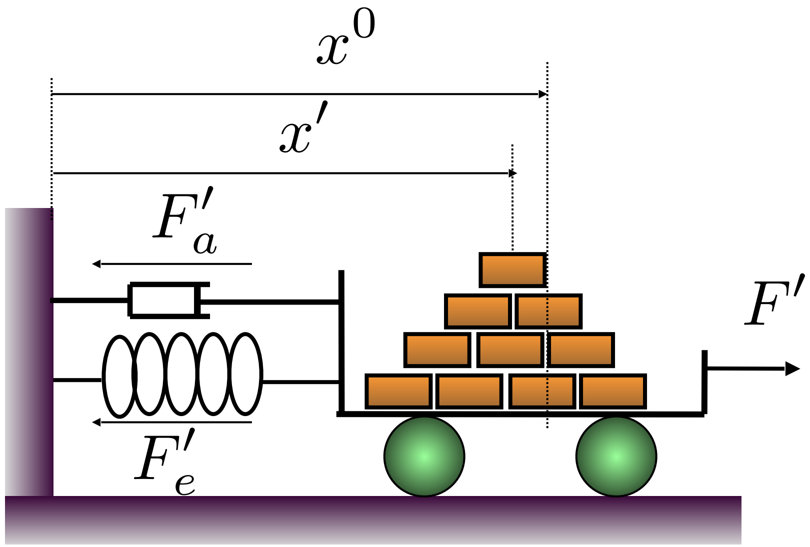
$$\Rightarrow x = \frac{\alpha/k}{1+\alpha/k}x^o \Rightarrow \boxed{e = x^o - x = \frac{1}{1+\alpha/k}x^o}$$

- In nominal conditions $k = \bar{k}$ $\Rightarrow$ $e \neq 0$

- In uncertain conditions $k \neq \bar{k}$ $\Rightarrow$ $e \simeq 0 \quad \text{if} \quad \alpha \gg k_{\max}$

Dynamic Modelling





$$F = \alpha \, \underbrace{(x^o - x)}_{e} \, , \;\; \alpha > 0$$

The effects of mass, spring, and damper are not negligible, and dynamic behaviours such as oscillations may occur

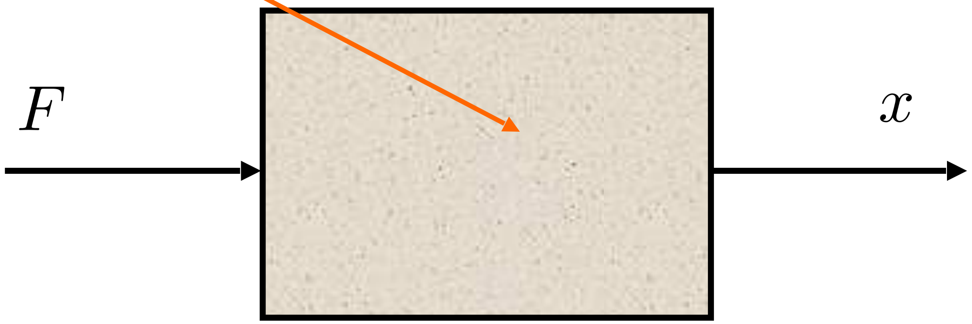
Models capturing the dynamic modes of behaviour are clearly necessary

Dynamic Modelling (contd.)

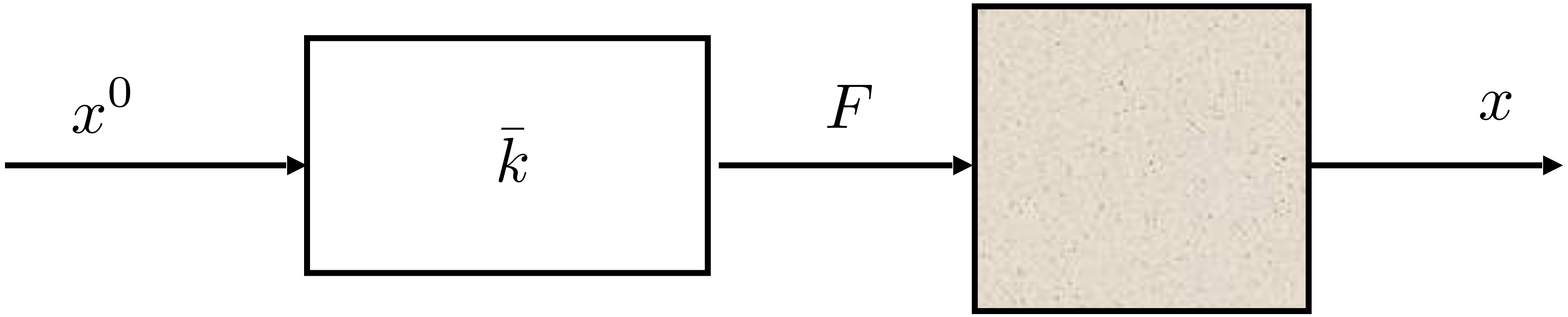
$$\text{sum of all external forces} = M\ddot{x}$$

$$F - kx - h\dot{x} = M\ddot{x}$$

$$M\ddot{x} + h\dot{x} + kx = F$$



Open-Loop Control Strategy



$$\underbrace{M\ddot{x} + h\dot{x}}_{\text{dynamic terms}} + \underbrace{kx = \underbrace{\bar{k}}_{\text{constant}} x^o}_{\text{condition for static equilibrium}}$$

$$\underbrace{\begin{matrix} x(0) \\ \dot{x}(0) \end{matrix}}_{\text{initial conditions}}$$

Open-Loop Control Strategy (contd.)

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Livescripts in MS Teams: see Part 1:
CART_OPENLOOP_CONTROL

From differential equations theory:

$$M\ddot{x} + h\dot{x} + kx = \bar{k}x^o$$

constant term due to open-loop control

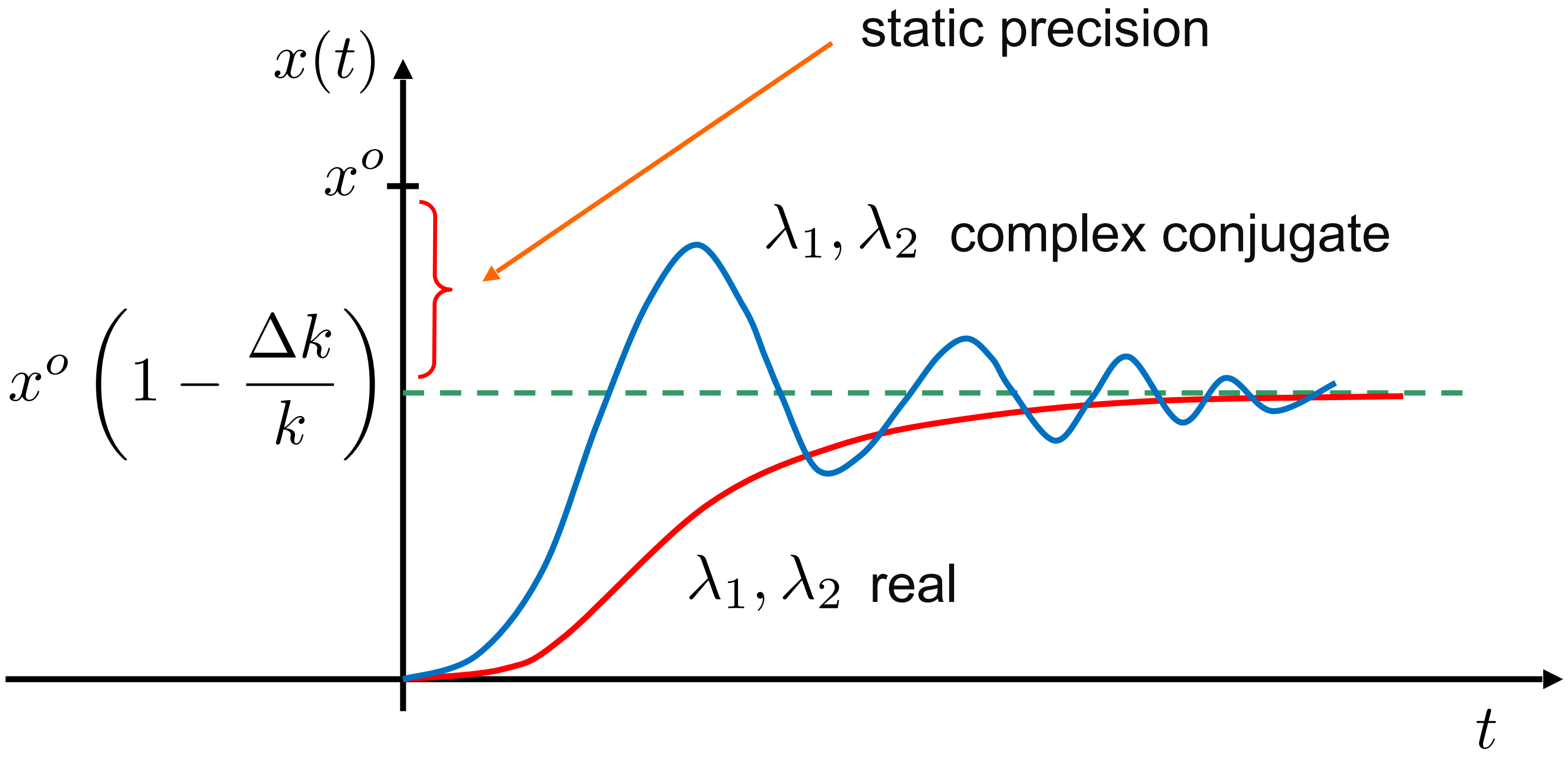
$$\underbrace{M\lambda^2 + h\lambda + k = 0}_{\text{polynomial algebraic equation}} \Rightarrow \underbrace{\lambda_1, \lambda_2}_{\text{roots}}$$

- If λ_1, λ_2 are real $\Rightarrow$ $x(t) = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t} + c_3$

- If λ_1, λ_2 are complex conjugate, that is $\lambda_1 = \sigma + j\omega,\ \lambda_2 = \sigma - j\omega$

$$x(t) = c_4 e^{\sigma t} \cos(\omega t + c_5) + c_6$$

Open-Loop Control Strategy (contd.)



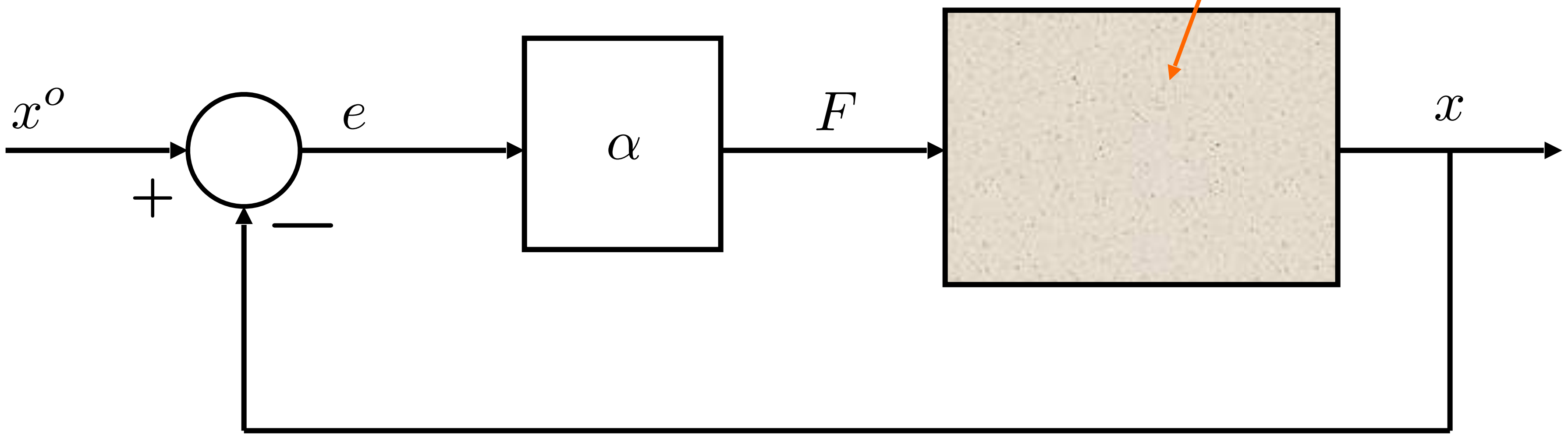
By the open-loop control strategy the dynamic behaviour cannot be modified since it only depends on the system's parameters M, k, h and not on the controller. Hence <u>dynamic precision cannot be modified by the controller</u>

Closed-Loop Control Strategy

Let us use again a **proportional closed-loop control** strategy:

$$F = \alpha \underbrace{(x^o - x)}_{e}\,, \;\; \alpha > 0$$

$$M\ddot{x} + h\dot{x} + kx = F$$



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Livescripts in MS Teams: see Part 1: CART_CLOSEDLOOP_P_CONTROLLER

Plugging in the proportional control scheme we get:

$$M\ddot{x} + h\dot{x} + kx = \alpha(x^o - x)$$

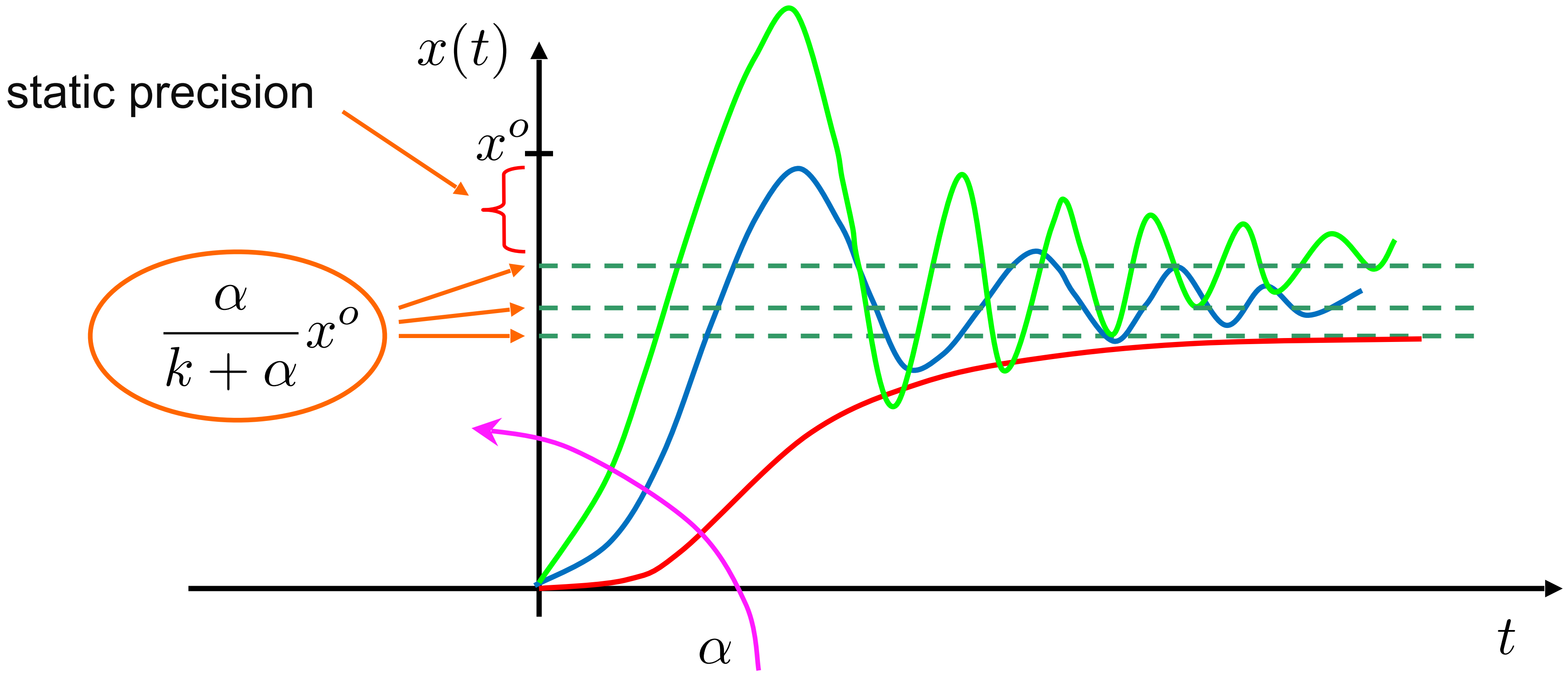
$$M\ddot{x} + h\dot{x} + (k+\alpha)x = \alpha x^o$$

the constant term of the algebraic equation is **influenced by the controller gain** α

$$\underbrace{M\lambda^2 + h\lambda + (k+\alpha) = 0}_{\text{polynomial algebraic equation}} \Rightarrow \underbrace{\lambda_1, \lambda_2}_{\text{roots}}$$

The roots λ_1, λ_2 can be modified by choosing α !!

Closed-Loop Control Strategy (contd.)



- **The dynamic precision also depends on the choice of the control gain**
- **Static and dynamic requirements are in contrast with each other: better static precision implies worse dynamic precision**

A Different Closed-Loop Control Strategy

Let us now opt for a **proportional/derivative** control scheme:

Livescripts in MS Teams: see Part 1: CART_CLOSEDLOOP_PD_CONTROLLER

$$F = \alpha\left(x^o - x\right) + \beta \frac{d}{dt}\left(x^o - x\right), \ \ \alpha, \beta > 0$$

$$M\ddot{x} + h\dot{x} + kx = \alpha(x^o - x) - \beta\dot{x}$$

$$M\ddot{x} + (h+\beta)\dot{x} + (k+\alpha)x = \alpha x^o$$

the first-order term of the algebraic equation is influenced by the **derivative controller gain** β

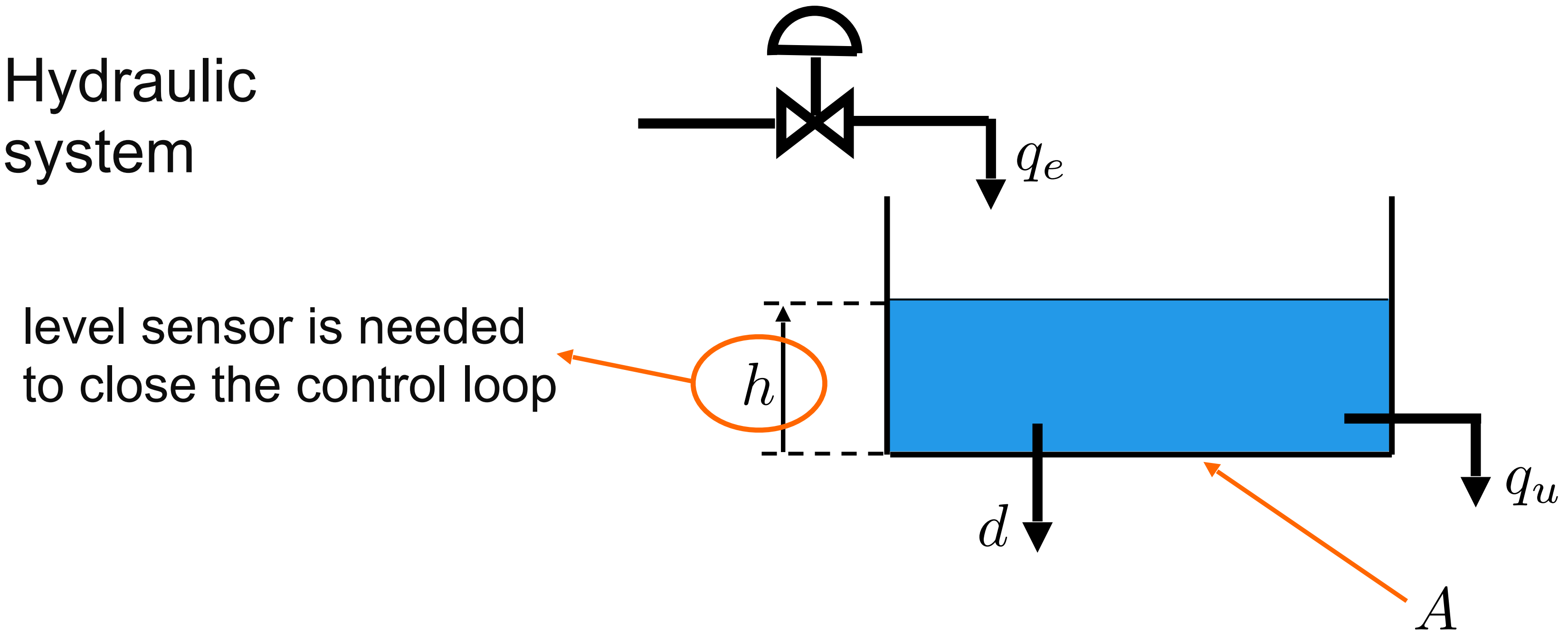
$$\underbrace{M\lambda^2 + (h+\beta)\lambda + (k+\alpha) = 0}_{\text{polynomial algebraic equation}} \Rightarrow \underbrace{\lambda_1, \lambda_2}_{\text{roots}}$$

the constant term of the algebraic equation is influenced by the **proportional controller gain** α

The roots λ_1, λ_2 can be modified by choosing α, β !!

Example 3: Tank Level Closed-Loop Control

Hydraulic system



- Control input: input flow-rate q_e
- Controlled output: level of liquid in the tank h
- Reference output: constant desired level in the tank $w = h^o$
- Out-flow rate: $q_u = kh$
- Disturbance: d

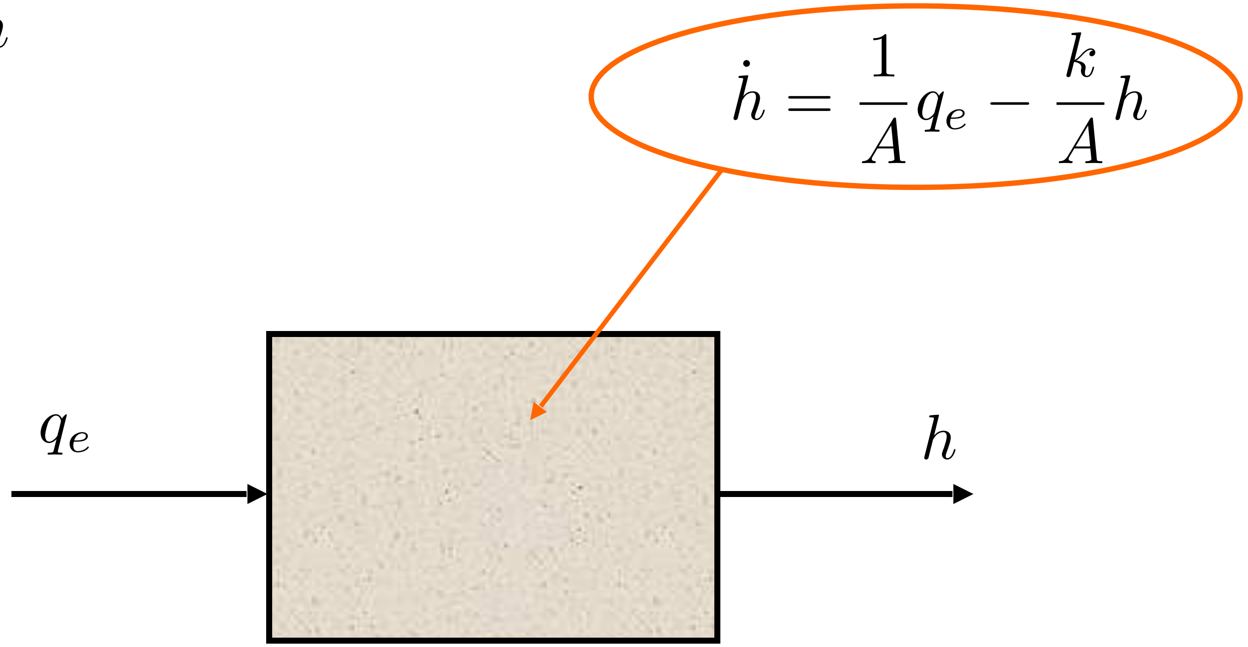
By the usual assumption of infinite-height of the tank and supposing (for the moment) that $d = 0$:

$$A\dot{h} = q_e - kh$$

conservation equation:

volume time-variation = input flow-rate - out flow-rate

$$\dot{h} = \frac{1}{A}q_e - \frac{k}{A}h$$

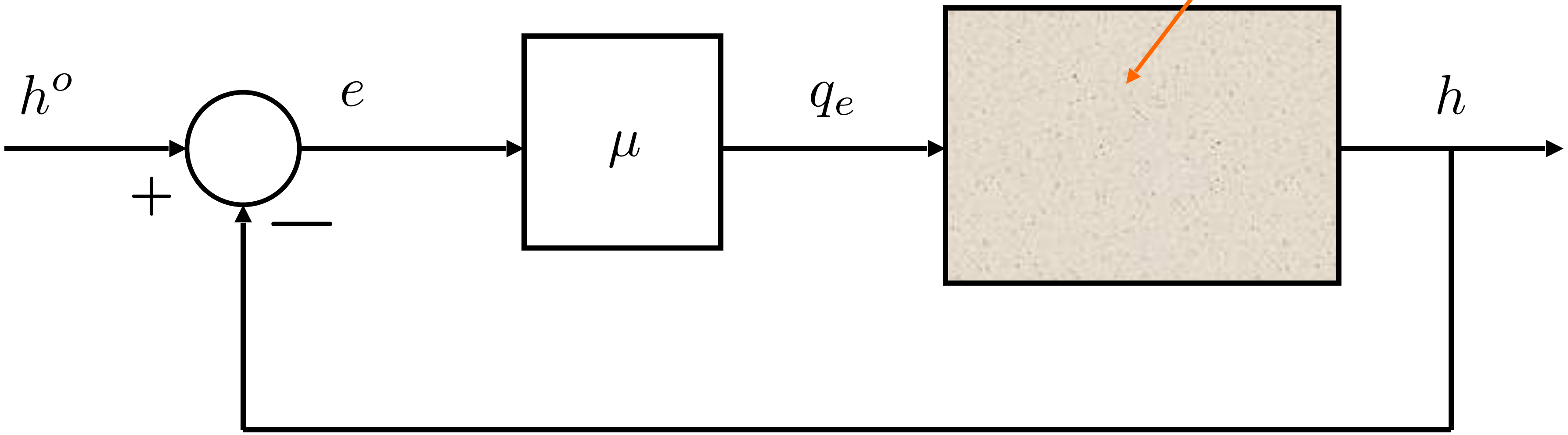


Closed-Loop Control Strategy

Let us opt for a **proportional control** strategy:

$$q_e = \mu \underbrace{(h^o - h)}_{e}, \ \ \mu > 0$$

$$\dot{h} = \frac{1}{A} q_e - \frac{k}{A} h$$



Closed-Loop Control Strategy (contd.)

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Livescripts in MS Teams: see Part 1: TANK_CLOSEDLOOP_P_CONTROLLER

Plugging-in the proportional control scheme we get:

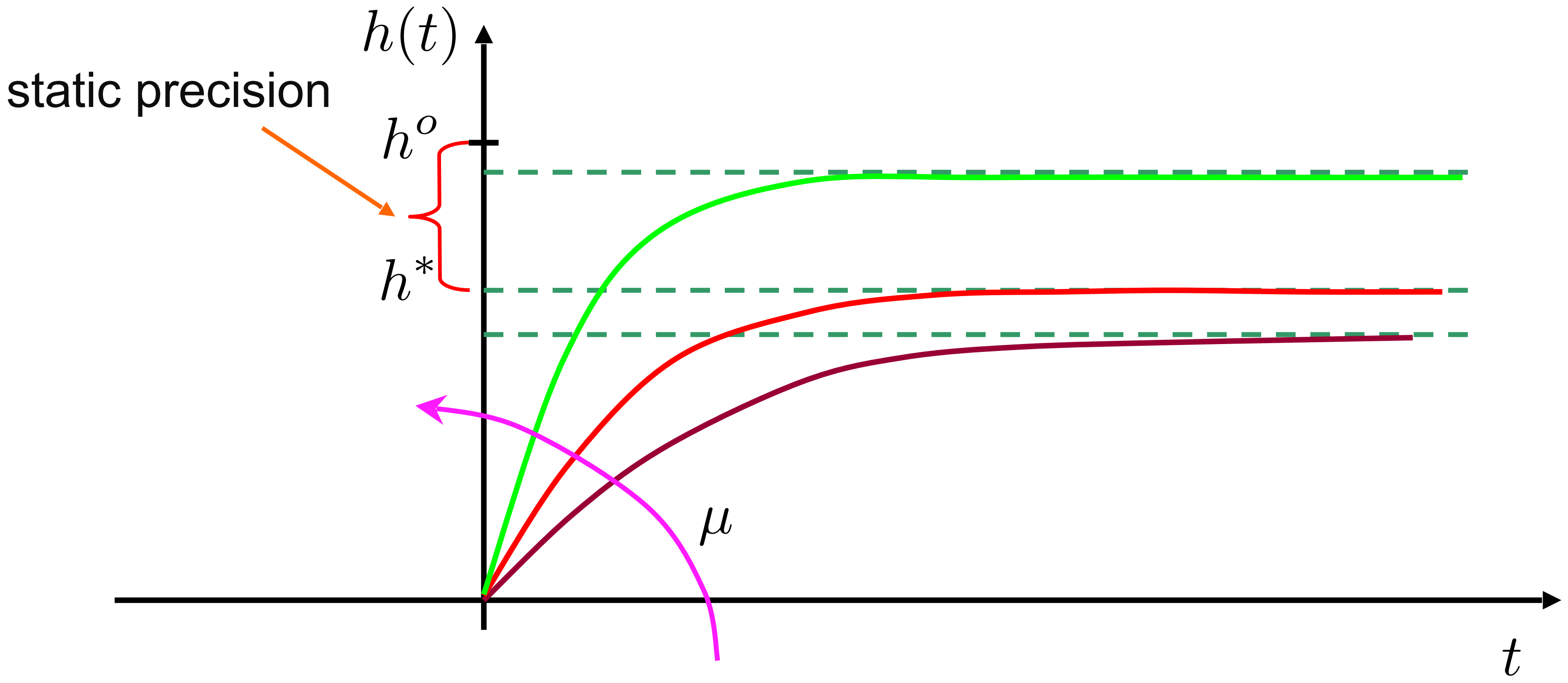
$$\dot{h} = \frac{1}{A}\mu\left(h^o - h\right) - \frac{k}{A}h$$

$$\dot{h} = \underbrace{-\frac{1}{A}\left(k+\mu\right)}_{\sigma > 0} h + \underbrace{\frac{\mu}{A}h^o}_{\text{constant}}$$

Assuming $h(0) = 0$ we get:

$$h(t) = \underbrace{\frac{\mu}{\mu + k}h^o}_{h^* = \lim_{t\to\infty} h(t)}\left(1 - e^{-\sigma t}\right), t \geq 0$$

Closed-Loop Control Strategy (contd.)



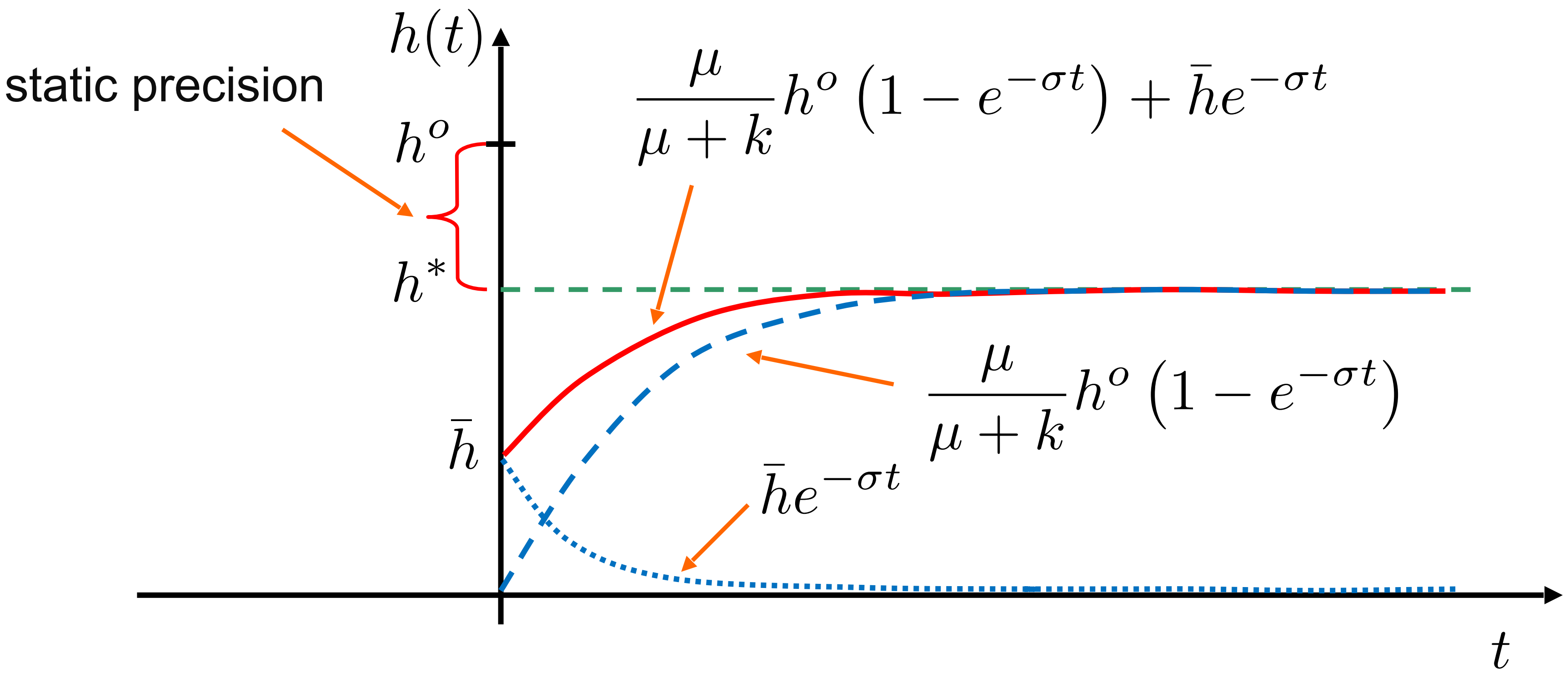
- **By increasing the control gain μ both the static and the dynamic precision improve**
- **In this hydraulic example, static and dynamic requirements are not in contrast with each other**

Non-Zero Initial Conditions

Assuming $h(0) = \bar{h} > 0$ we get:

$$h(t) = \underbrace{\frac{\mu}{\mu + k} h^o \left(1 - e^{-\sigma t}\right)}_{\text{effect of the control input}} + \overbrace{\bar{h} e^{-\sigma t}}^{\text{effect of the non-zero initial condition}}, t \geq 0$$

Non-Zero Initial Conditions (contd.)

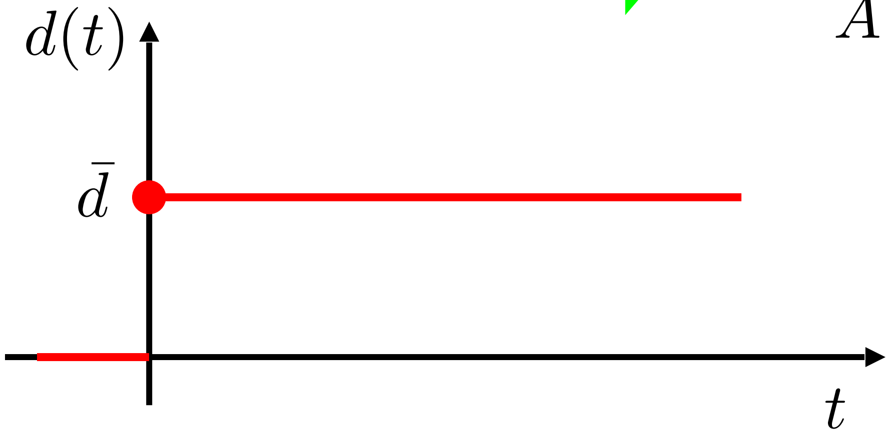


Dynamic Modelling with a Disturbance

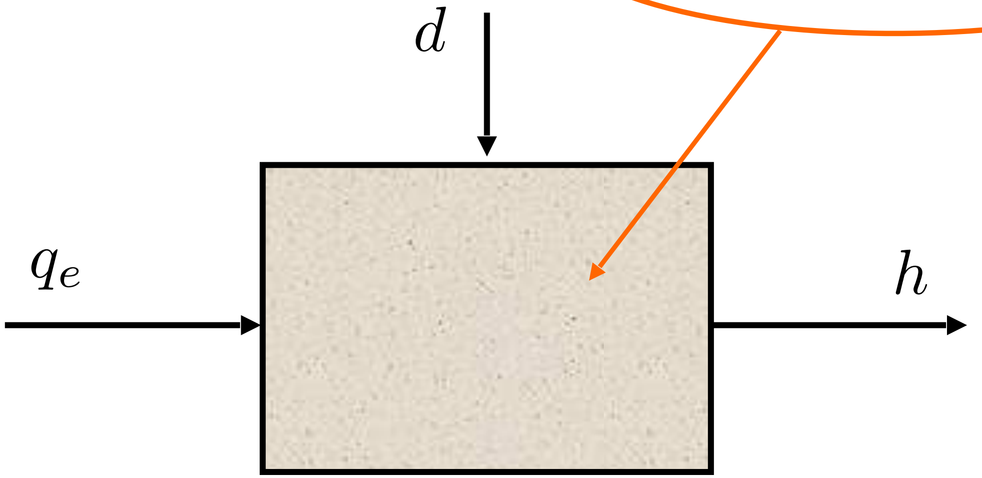
By the usual assumption of infinite-height of the tank and supposing that a step-like disturbance $d(t) = \bar{d},\ t \geq 0$ (leakage) is acting on the tank:

$$A\dot{h} = q_e - kh - \bar{d}$$

$$\dot{h} = \frac{1}{A}q_e - \frac{k}{A}h - \frac{1}{A}\bar{d}$$



$$\dot{h} = \frac{1}{A}q_e - \frac{k}{A}h - \frac{1}{A}\bar{d}$$



Closed-Loop Control Strategy

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Plugging in the proportional control scheme we get:

$$q_e = \mu \left(h^o - h\right), \;\; \mu > 0$$

$$\dot{h} = \frac{1}{A}\mu \left(h^o - h\right) - \frac{k}{A}h - \frac{1}{A}\bar{d}$$

$$\dot{h} = \underbrace{-\frac{1}{A}\left(k + \mu\right)}_{\sigma > 0} h + \underbrace{\frac{\mu}{A}h^o - \frac{1}{A}\bar{d}}_{\text{constant}}$$

Closed-Loop Control Strategy (contd.)

For simplicity suppose: $h(0) = h^* = \dfrac{\mu}{\mu + k} h^o$

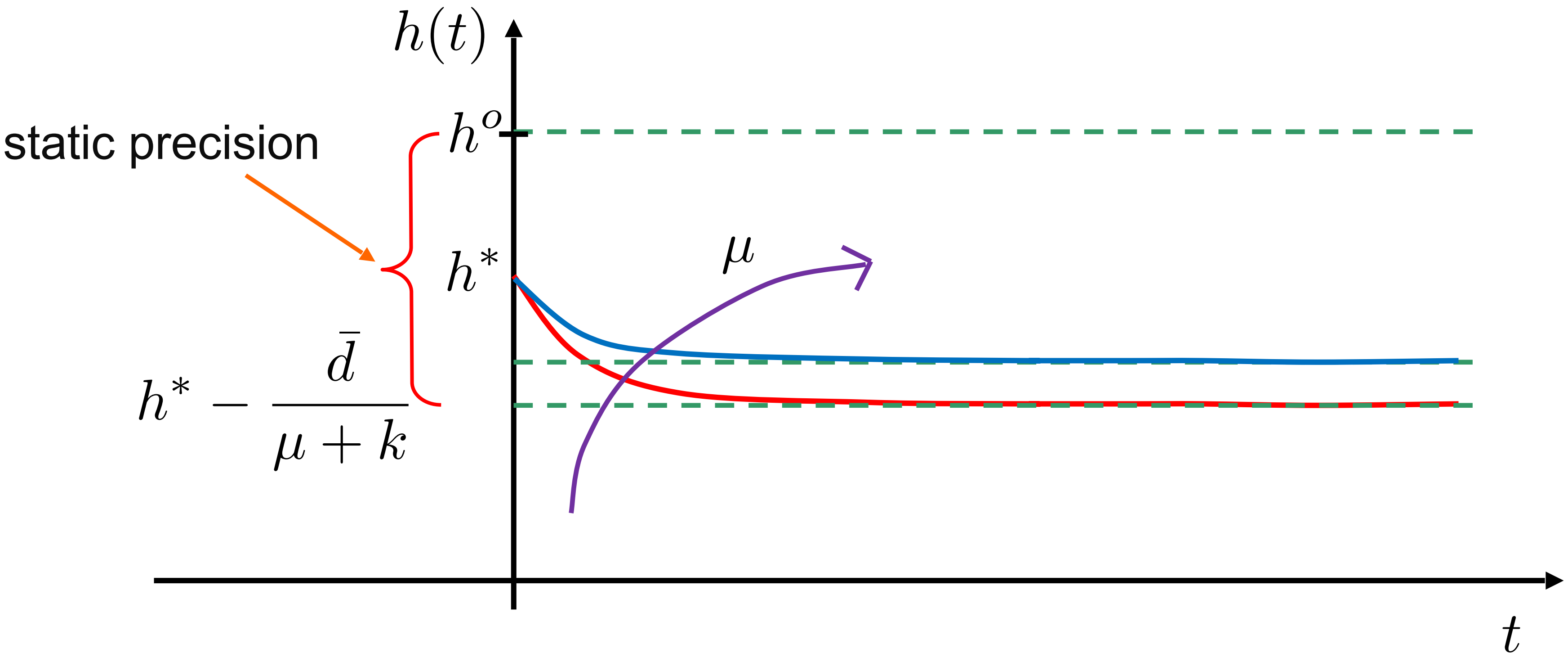
$$h(t) = \frac{\mu}{\mu+k} h^o \left(1 - e^{-\sigma t}\right) + h(0) e^{-\sigma t} - \frac{\bar{d}}{\mu+k}\left(1 - e^{-\sigma t}\right)$$

$$= h^* - \cancel{h^* e^{-\sigma t}} + \cancel{h^* e^{-\sigma t}} - \frac{\bar{d}}{\mu+k}\left(1 - e^{-\sigma t}\right)$$

$$h(t) = h^* - \frac{\bar{d}}{\mu+k}\left(1 - e^{-\sigma t}\right), t \geq 0$$

$$\lim_{t \to \infty} h(t) = h^* - \frac{\bar{d}}{\mu+k}$$

Closed-Loop Control Strategy (contd.)



Increasing the proportional gain μ improves the disturbance rejection performance

How to Improve the Static Precision?

A) Adding open-loop compensation actions:

$$q_e = \mu\left(h^o - h\right) + \overbrace{kh^o + \bar{d}}^{\text{open-loop action}}$$

$$\dot{h} = \frac{1}{A}\mu\left(h^o - h\right) + \frac{k}{A}h^o + \cancel{\frac{1}{A}\bar{d}} - \frac{k}{A}h - \cancel{\frac{1}{A}\bar{d}}$$

$$h(t) = h^o\left(1 - e^{-\sigma t}\right), t \geq 0$$

$$\lim_{t \to \infty} h(t) = h^o$$

<u>But</u>:

to implement the open-loop compensation actions, the knowledge of $k, \bar{d}$ is required

How to Improve the Static Precision?

B) Modifying the closed-loop controller by adding an **integral action** thus obtaining a **proportional/integral (PI) controller**:

Livescripts in MS Teams: see Part 1: TANK_CLOSEDLOOP_PI_CONTROLLER

$$q_e = \mu\,[h^o - h(t)] + \varphi \int_0^t [h^o - h(\tau)]d\tau \quad \mu, \varphi > 0$$

Justification:

$$q_e = \mu e(t) + \varphi \int_0^t e(\tau)d\tau \Rightarrow \dot{q}_e = \mu \dot{e}(t) + \varphi e(t)$$

Equilibrium:

$$\begin{cases} e(t) = \text{cost} = \bar{e} \\ q_e(t) = \text{cost} \end{cases} \Rightarrow 0 = \mu \cdot 0 + \varphi \cdot \bar{e}$$

$$\boxed{\bar{e} = 0}$$

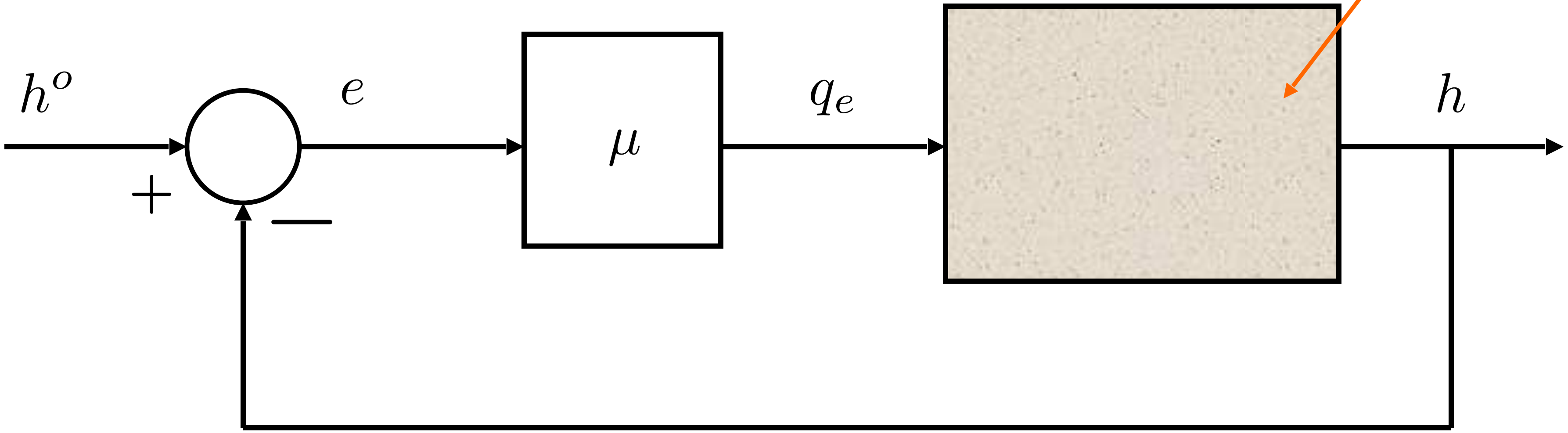
Example 3: Discrete-Time Implementation

Consider again the **proportional control** strategy:

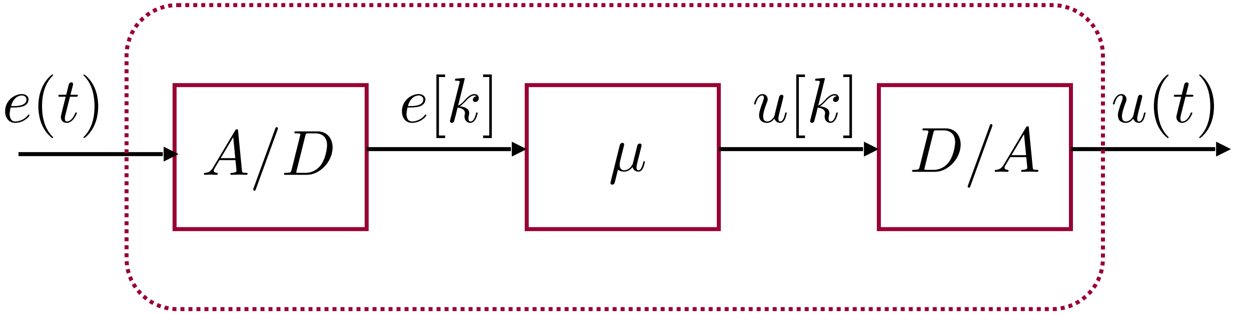
$$q_e = \mu \underbrace{(h^o - h)}_{e}\,, \;\; \mu > 0$$

and suppose for simplicity: $h(0) = 0, d = 0, A = 1, k = 0$

$$\dot{h} = q_e$$



Consider the discrete-time implementation of the controller:



$$e[k] = e(kT)\,, \quad u[k] = \mu\, e[k]\,, \quad u(t) = u[k],\ kT_s \leq t < (k+1)T_s$$

- $k = 0$
  $e[0] = 1$
  $u[0] = \mu\, e[0] = \mu$
  $u(t) = \mu,\ 0 \leq t < T_s$
  $h(t) = \mu\, t,\ 0 \leq t < T_s$

Example 3: Discrete-Time Implementation (contd.)

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- $k = 1$
  $e[1] = e(T_s) = w(T_s) - h(T_s) = 1 - \mu T_s$
  $u[1] = \mu\, e[1] = \mu(1 - \mu T_s)$
  $u(t) = \mu(1 - \mu T_s),\ T_s \le t < 2T_s$
  $h(t) = \mu\, T_s + \mu(1 - \mu T_s)(t - T_s),\ T \le t < 2T$

$$h(2T_s) = \mu\, T_s + \mu(1 - \mu T_s)T_s = 1 - (1 - \mu T_s)^2$$

- $k = 2$
  $e[2] = e(2T_s) = w(2T_s) - h(2T_s) = (1 - \mu T_s)^2$
  $\vdots$
  and so on
  $\vdots$

Example 3: Discrete-Time Implementation (contd.)

Livescripts in MS Teams: see Part 1: TANK_SAMPLED_P_CONTROLLER

Generalising, one gets:

$$
\begin{aligned}
e[k] &= (1-\mu T_s)^k\,, \quad k \geq 0 \\
u[k] &= \mu(1-\mu T_s)^k\,, \quad k \geq 0 \\
y[k] &= 1-(1-\mu T_s)^k\,, \quad k \geq 0
\end{aligned}
$$

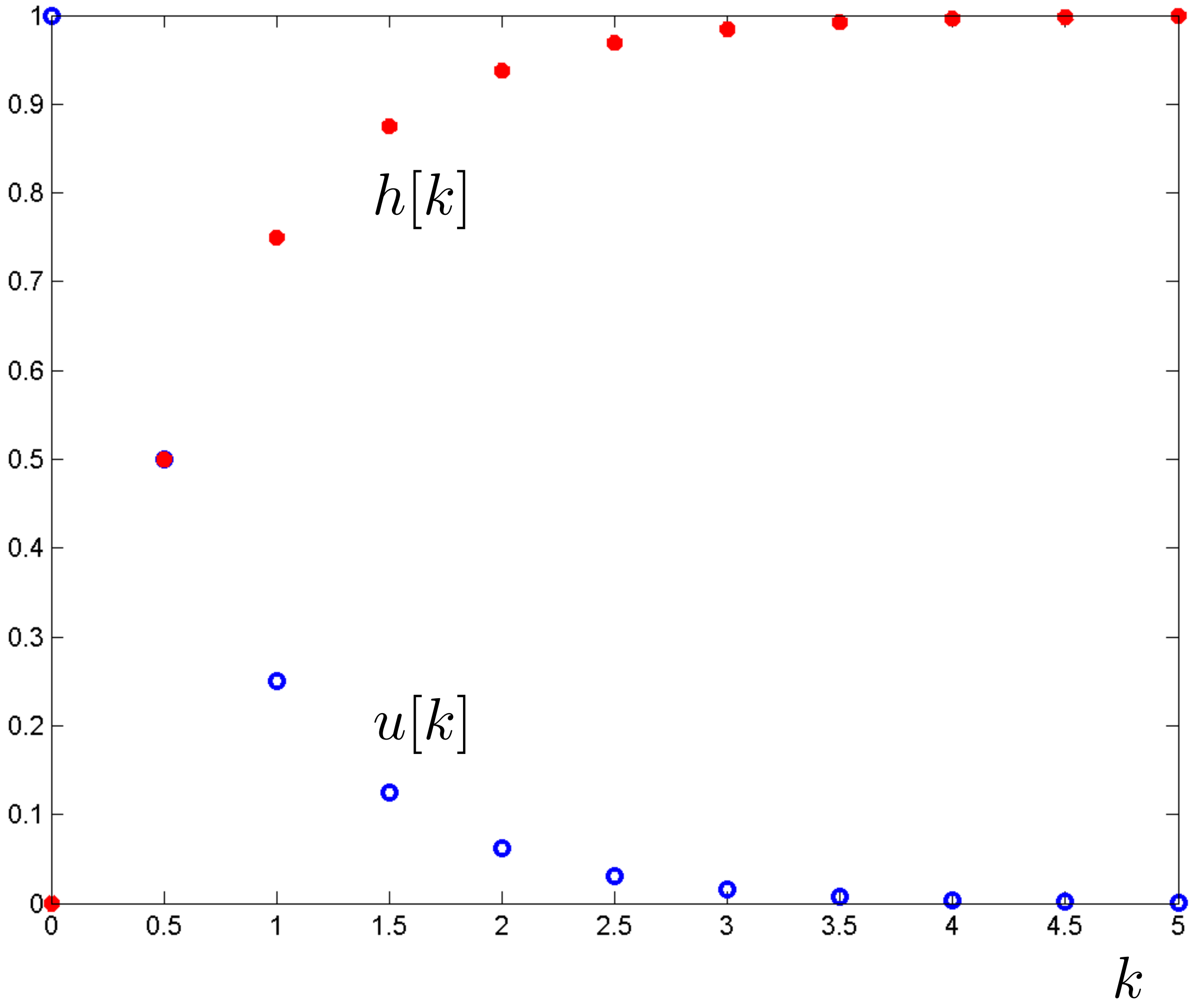
Notice that:

$$0 < \mu T_s < 2 \quad \Rightarrow \quad \lim_{k\to\infty} y[k] = 1$$

$$\mu T_s > 2 \quad \Rightarrow \quad \lim_{k\to\infty} y[k] = \infty$$

Example 3: Discrete-Time Implementation (contd.)

$$\mu = 1,\ T_s = 0.5$$



Example 3: Discrete-Time Implementation (contd.)

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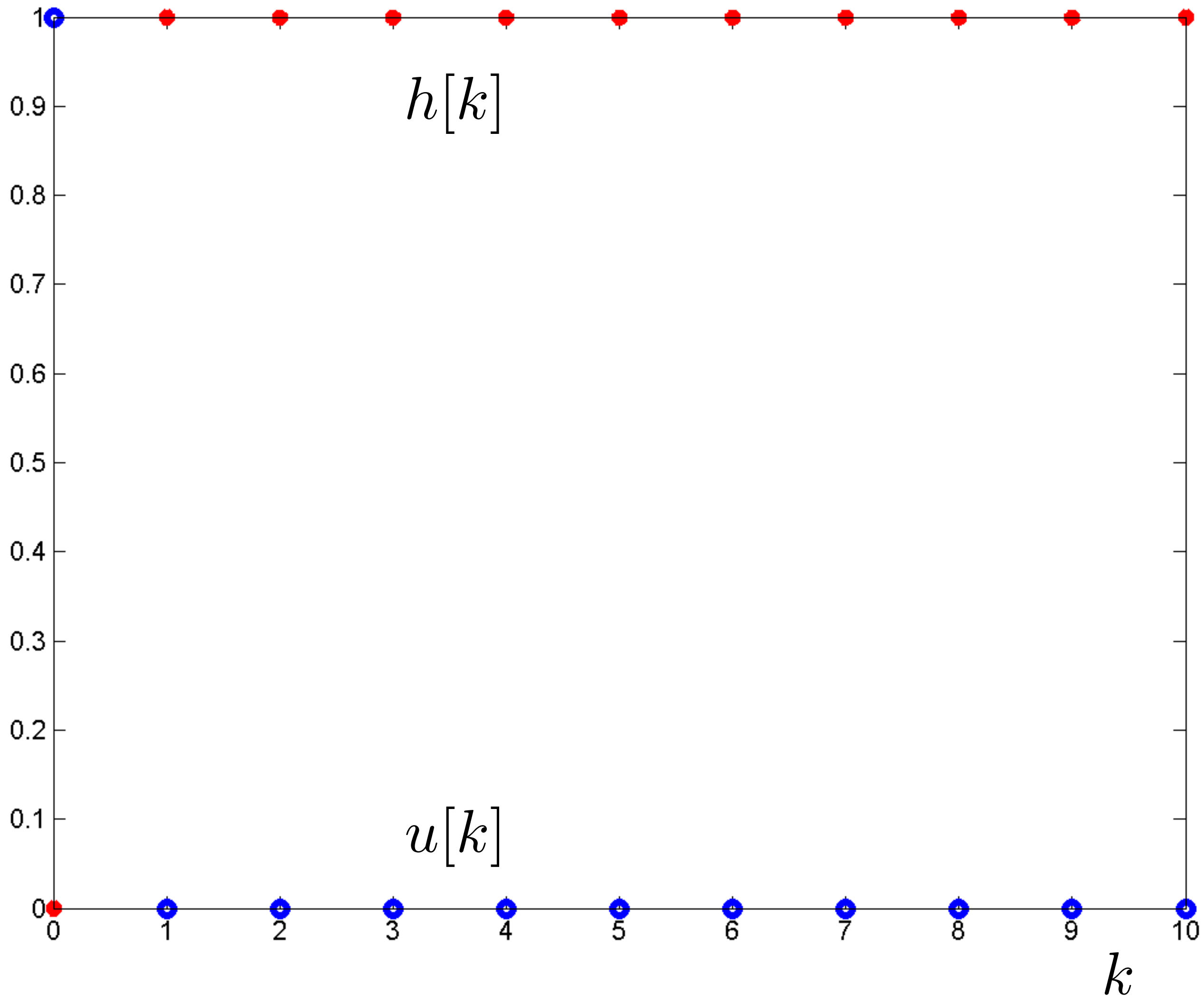
$$\mu = 1,\ T_s = 1$$



Example 3: Discrete-Time Implementation (contd.)

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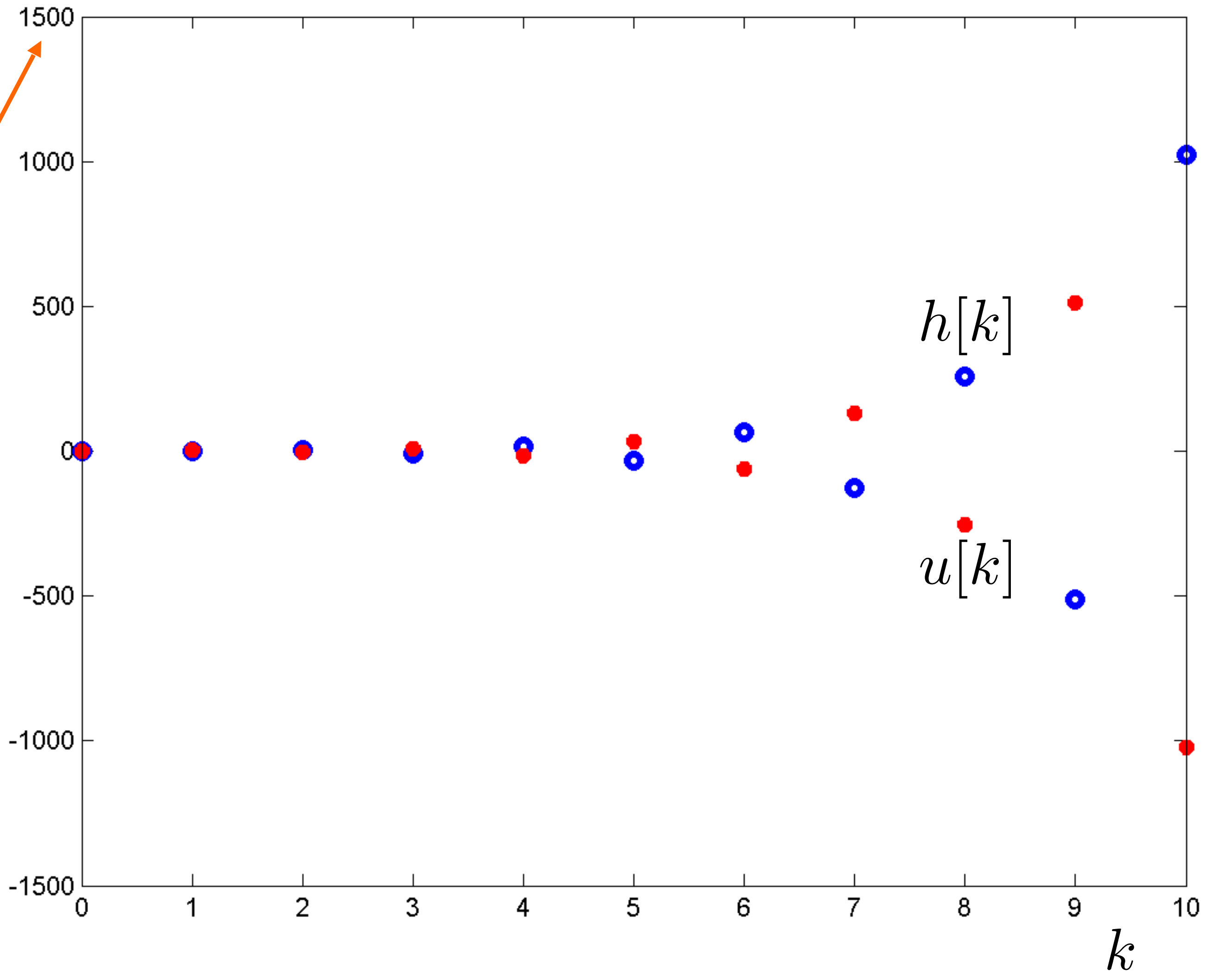
$$\mu = 3,\ T_s = 1$$

notice the scale



Remarks

- When the controller is implemented on a digital computation unit, the closed-loop performance also depend on the **choice of the sampling time**
- The analysis has to be carried out using **different mathematical tools**
- The digital implementation always causes **performance degradation**