COVID-19 Pandemic Simulation

Ahmed Medhat Mohamed

2018-01207

Zewail City for Science and Technology
June 26, 2021

Abstract

The persistent spread of the novel pandemic disease COVID-19 is increasing dramatically since the beginning of the second wave. The novel coronavirus has infected 75,471,414 people around the world and killed 1,672,034 people [1]. Moreover, the susceptible-exposed-infectious-recovered (SEIR) model is one of the classic compartmental models used in studying the epidemiology of infectious diseases [2].

SEIR model can be used for predicting the future outcomes of the pandemic like the number of infections and deaths based on the present statistics [2]. This general SEIR model can be modified to perform a simulation for the current COVID-19 pandemic by adding the appropriate parameters which are specific to the novel coronavirus [3]. Such a simulation can enable us to estimate the future outcomes and the date of the peaks of infections to enforce the required pre-cautions and medical resources in prior [3].

Therefore, the aim of this project is to build an SEIR model using the infectivity parameters (e.g. basic reproduction number R_o) of Sars-CoV2 virus to predict the outcomes of COVID-19 in Egypt by using Egypt's demographic data and statistics (e.g. population size, age ratios, death rate, etc..).

1. Introduction

Coronaviruses (CoVs) belong to an extended family of viruses that cause respiratory diseases in human that can range from common cold to severe illnesses like the severe acute respiratory syndrome (SARS) [5]. Bats are the main reservoirs of coronaviruses since there is a commensal symbiosis (an interaction between two species where one species gain benefit while the other one neither benefit nor are harmed) between bats and coronaviruses [5]. On the other hand, the relationship between coronaviruses and humans is a parasitic symbiosis (an interaction between two species where one species gains benefit while the other one is harmed) [5]. This infectious respiratory disease COVID-19 is caused by severe acute respiratory syndrome coronavirus 2 (Sars-CoV2) which belongs to the *coronaviridae* family and originated in Wuhan, China by the end of 2019 [5]. The origins of the virus are still debatable, but the most evidence supported scenario is that its origins are natural. The results of the genomic analysis of Sars-CoV2 showed that it is very closely related to a coronavirus that infects bats and pangolin which proposes the idea that bats are the original hosts and pangolins are the intermediate hosts that narrowed the jump from bats to humans, that is the most common evolutionary scenario in the past pandemics like swine flu H1N1, Spanish Flu and MERS-CoV [6].

The susceptible-exposed-infectious-recovered (SEIR) model is a deterministic model that is used very frequently in epidemiological studies [2]. SEIR belongs to the compartmental mathematical models which are models that divide the population of interest (N) into compartments, such as, susceptible (S), exposed (E), infected (I) and recovered (R) [2]. Given the fact that most infectious agents have a latent (incubation) period there is a compartment for the exposed proportion of the population which are carriers but show no symptoms [2]. Additionally, the susceptible compartment refers to the fraction of the population that is not immune to the disease [2]. The values of the previously mentioned compartments vary with time which can be represented by a system of ordinary differential equations (ODEs) that can be solved numerically to calculate these values at any time point; thus, enabling conducting a simulation for the pandemic through a time interval [2]. Moreover, SEIR model can be used for conducting a simulation for COVID-19 disease by using the appropriate parameters to predict the future outcomes in order to implement effective control strategies and policies [3][4]. This project aims at comparing different numerical approaches to solve the system of ODEs like Euler's method, and Heun's method against the analytical (true) solution.

2. Problem definition

The mathematical modelling of COVID-19 disease can be conducted by dividing the initial population number (N) into the following compartments that vary with time (t): susceptible, S(t), exposed, E(t), infected-infectious, I(t) and recovered, R(t) [3][4]. This SEIR model will include the vital dynamics which are the birth rate and natural death rate which will vary the original population number (N_o) and susceptible number (S). The differential equations that govern this mathematical model are:

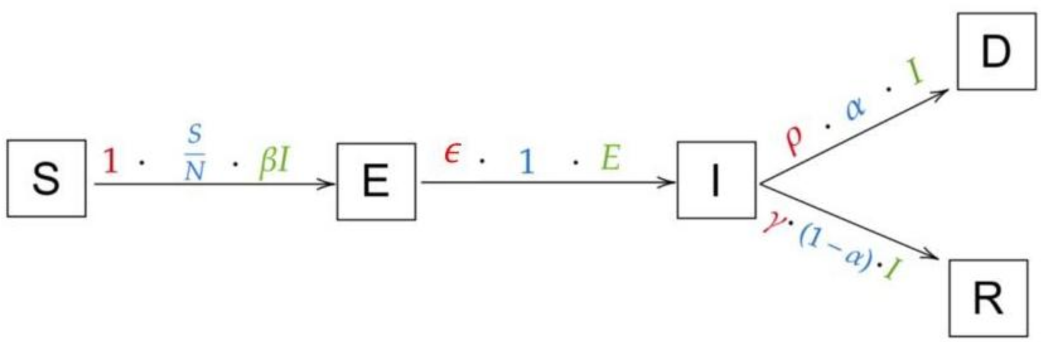


Figure 1. SEIR model parameters [4]

2.1. Without vital dynamics

$$\frac{dS}{dt} = -\beta S \frac{I}{N} \quad (2.1)$$

$$\frac{dE}{dt} = \beta S \frac{I}{N} - \epsilon E \quad (2.2)$$

$$\frac{dI}{dt} = \epsilon E - ((1-\alpha)\gamma + \alpha\rho)I \quad (2.3)$$

$$\frac{dR}{dt} = (1-\alpha)\gamma I \quad (2.4)$$

2.2. With vital dynamics

$$\frac{dS}{dt} = vN - \mu S - \beta S\frac{I}{N} \quad (2.5)$$

$$\frac{dE}{dt} = \beta S\frac{I}{N} - (\mu + \epsilon)E \quad (2.6)$$

$$\frac{dI}{dt} = \epsilon E - ((1-\alpha)\gamma + \mu + \alpha\rho)I \quad (2.7)$$

$$\frac{dR}{dt} = (1-\alpha)\gamma I - \mu R \quad (2.8)$$

Where v is the per-capita birth rate (birth per individual per unit time which is days in that case), μ is the per-capita natural death rate (death per individual per day), α is virus-induced death rate and (1-α) is the probability of death, γ is the recovery rate of infected individuals (the reciprocal of the infectious period IP), ρ is the death rate of infected individuals (also the reciprocal of the infectious period IP), β is the probability of infection, and ϵ is the rate of transmission from exposed to infected (the reciprocal of latent period LP) [2][3][4].

Since the basic reproductive number of SARS-CoV2 (R_o) equals β / γ, then β can be calculated by multiplying the recovery rate γ by R_o [2]. Also, the population number at time t is $N(t) = S(t) + E(t) + I(t) + R(t) \leq N_o$.

The death population can be calculated by the following differential equation:

$$\frac{dD}{dt} = \alpha\rho I(t) \quad (2.9)$$

Moreover, to calculate the death rate (α) during an ongoing pandemic we used the following formula [8]:

$$\alpha = \frac{Number\ of\ deaths}{Number\ of\ deaths + Number\ of\ recovered} \quad (2.10)$$

3. Methodology

3.1. Numerical solutions

3.1.1. Forward Euler Method:

The previous system of differential equations will be solved using the forward Euler finite-difference scheme which discretize the differential equations by representing the time domain [0, T] in the form of N+1 finite points (t_0, t_1, t_2 … t_N), where N is the total number of days. Followed by solving the differential equations at each time point [3][7]. Therefore, the value of t_n is calculated by the following equation where Δt is the spacing between the time points and n = 1, 2, 3, … N:

$$t_n = n\Delta t$$

So, each ODE in the model can be reduced into *N* equations as follows:

A. Without vital dynamics:

$$S^{n+1} = S^n + \Delta t\left(-\beta S^n \frac{I^n}{N^n}\right) \quad (3.1)$$

$$E^{n+1} = E^n + \Delta t\left(\beta S^n \frac{I^n}{N^n} - \epsilon E^n\right) \quad (3.2)$$

$$I^{n+1} = I^n + \Delta t(\epsilon E^n - ((1-\alpha)\gamma + \alpha\rho)I^n) \quad (3.3)$$

$$R^{n+1} = R^n + \Delta t((1-\alpha)\gamma I^n) \quad (3.4)$$

$$D^{n+1} = D^n + \Delta t(\alpha\rho I^n) \quad (3.5)$$

$$N^n = S^n + E^n + I^n + R^n \quad (3.6)$$

B. With vital dynamics:

$$S^{n+1} = S^n + \Delta t\left(vN - \mu S^n - \beta S^n \frac{I^n}{N^n}\right) \quad (3.7)$$

$$E^{n+1} = E^n + \Delta t\left(\beta S^n \frac{I^n}{N^n} - (\mu + \epsilon)E^n\right) \quad (3.8)$$

$$I^{n+1} = I^n + \Delta t(\epsilon E^n - ((1-\alpha)\gamma + \mu + \alpha\rho)I^n) \quad (3.9)$$

$$R^{n+1} = R^n + \Delta t((1-\alpha)\gamma I^n - \mu R^n) \quad (3.10)$$

$$D^{n+1} = D^n + \Delta t(\alpha\rho I^n) \quad (3.11)$$

$$N^n = S^n + E^n + I^n + R^n \quad (3.12)$$

Although Euler's method usually succeeds in capturing trends, there is a fundamental source of error caused by the assumption that the derivative at the beginning of the interval applies to the rest of the interval. This error can be reduced by taking a smaller step-size, but still the error accumulates considerably over large intervals [14].

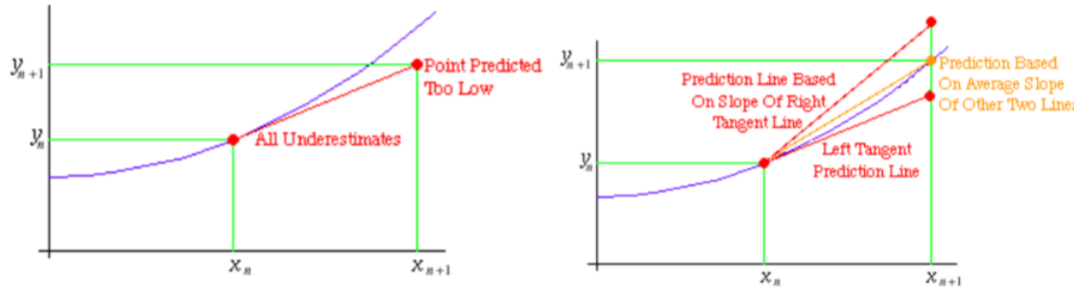
3.1.2. Modified Euler Method (Heun's method):

Heun's method attempts to correct the intrinsic error in Euler's method by adding a modification to estimating the slope of the interval. It calculates two derivatives for the interval, one at the initial point and another one at the end of the interval, then it takes the average of the two derivatives [14]. This method incorporates Euler method (predictor step) to calculate the slope at the beginning of the interval, then the slope is calculated again at the end of the interval, finally a corrector step which averages the two slopes is applied; thus, this makes Heun's method a predictor-corrector approach [14].

$$y_{n+1}^0 = y_n + \Delta t f(y_n, x_n) \text{ (\textit{Euler's Predictor at initial point})}$$

$$y'_{n+1} = f(y_{n+1}^0, x_{n+1}) \text{ (\textit{Slope at end point})}$$

$$y_{n+1} = y_n + \Delta t \frac{f(y_n, x_n) + f(y_{n+1}^0, x_{n+1})}{2} \text{ (\textit{Corrector})}$$



Euler's method

Modified Euler method

Figure 2. Graphical representation for Euler's method and Heun's method

3.1.3. Runge Kutta Fourth Method (RK4):

The Runge-Kutta method finds approximate value of y for a given x. Only first order ordinary differential equations can be solved by using the Runge Kutta 4th order method. Below is the formula used to compute next value yn+1 from previous value yn. The value of n is 0, 1, 2, 3, …. (x – x0)/h. Here h is step height and xn+1 = x0 + h, where:

$$K_1 = hf(x_n, y_n)$$
$$K_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$
$$K_3 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$$
$$K_4 = hf(x_n + h, y_n + k_3)$$
$$y_{n+1} = y_n + k_1/6 + k_2/3 + k_3/3 + k_4/6 + O(h^5)$$

3.2. Initial conditions

The initial condition data was collected from Egypt's section on Worldmeter and E_o was estimated by the addition of the new cases within 6 days which is the latent (incubation period) of COVID-19 [9][10][11].

Variable	Initial Value
N_o	103,342,110
S_o	103,202,639
E_o	8,156
I_o	18,958
R_o	112,826

D_o	7,687

Table 1. Initial values of the model

3.3. Parameters

The R_o of COVID-19 is estimated to be within the range 2-3, so we will use the average R_o = 2.5 [12]. The death rate was calculated using the CDC's formula to calculate case fatality rate during an ongoing pandemic (equation 2.10) and the reported statistics giving a death rate (α) of 6.38% [8][10]. Then the recovery and death rates are IP^{-1} (reciprocal of infection period) as mentioned previously; so, γ and $\rho = 14^{-1} = 0.0714$ [2]. Moreover, the probability of infection can be calculated by multiplying the recovery rate γ by R_o; thus, β = 0.0714*2.5 = 0.1785 [2]. The rate of transmission from exposed to infected is LP^{-1} (reciprocal of latent period); therefore, $\epsilon = 6^{-1}$ = 0.1667 [3][4]. The per capita birth rate (v) and the death rate (μ) for Egypt was collected from reported statistics [13].

Parameter	Value
R_O	2.5
$\boldsymbol{\alpha}$	0.0638
β	0.1785
$\boldsymbol{\gamma}$	0.0714
$\boldsymbol{\rho}$	0.0714
$\boldsymbol{\epsilon}$	0.1667
$\boldsymbol{v}$	0.0000784
μ	0.0000171

Table 2. Values of model parameters

3.4. Tools used

a) Python 3.7.0: Programming language Python was used to develop the SEIR model, because it is a dynamic, and interpreted language that enables smooth data preprocessing, analysis and visualization. Therefore, it is well fitted to our corresponding objective. The model was built using the Google Colaboratory interface.

b) Hardware and environment: Dell Inspiron 5000 series Laptop Intel core i7 running on Microsoft Office 10 pro. Education operating system was used for the script development.

c) Packages and modules: The implementation required importing the following python libraries: NumPy (for vector operations), Matplotlib (for visualizing the simulation), and SciPy (ODEInt module was used for the analytical solution).

4. Results and discussion

4.1. What is ODEInt and how does it work

The odeint library provides a number of algorithms to solve initial value problems of ODEs. The focus of the library are explicit methods, although some implicit routines are also available. Its main advantage over other ODE libraries is a strict separation of the numerical algorithms from the underlying arithmetical computations. Odeint provides a number of numerical schemes to find an approximate solution for such ODEs. Those schemes can be divided into two main groups: explicit and implicit methods. The main focus of odeint lies on explicit methods, where several algorithm families are implemented, like explicit Runge-Kutta methods, extrapolation methods, linear multistep methods and others. For implicit methods, only semi-implicit Runge-Kutta schemes are provided.

4.1.1. Simulation with ODEInt Function

We solved the two systems of differential equations 2.1 (does not involve the vital dynamics) and 2.2 (involves the vital dynamics) analytically using SciPy's module ODEInt over a time interval of 200 days and plotted the solutions using Matplotlib (Figure 3). There was no significant difference in the results of both systems, because the vital dynamics values where almost equal to zero (v = 0.0000784 and μ = 0.0000171); therefore, incorporating these variables into the equations (2.2) did not affect the simulation significantly. Furthermore, the maximum number of infections is expected to be 16,958,313 people on day 144. The maximum number of infections would be smaller and shifted at an earlier time point, if the effect of a lockdown was implemented in this model which will reduce R_o by time and consequently β.

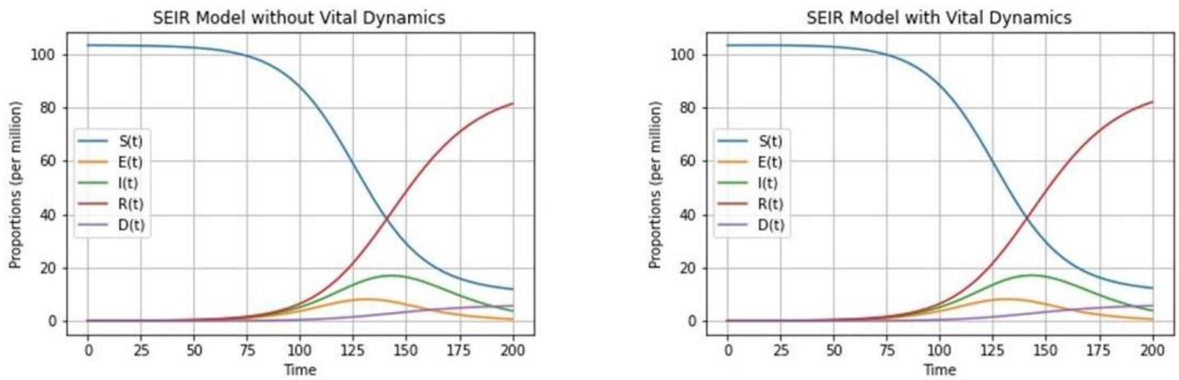


Figure 3. SEIR model with ODEInt

4.2. Numerical solutions

4.2.1. Euler's Method

The system of differential equations 2.1 was solved using forward Euler's method as mentioned in section 3.1.1. We implemented the equations in section 3.1.1.A and ignored 3.1.1.B since there was no significant difference when the vital dynamics variables were implemented as mentioned previously. A time interval of 200 days was chosen with a step size Δt of 1 and the results of the simulation were visualized using Matplotlib (Figure 4). Additionally, the maximum number of infections is expected to be 17,256,430 people (error = 1.76%) on day 146 (with 3 days difference from the analytical solution). Finally, a comparison between the numerical and analytical solutions of the different compartments (susceptible, exposed, infected, recovered and dead) was performed in figure 5.

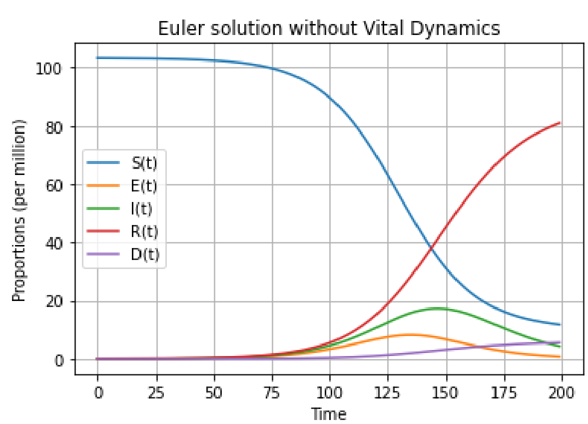


Figure 4. SEIR model Euler's numerical solution

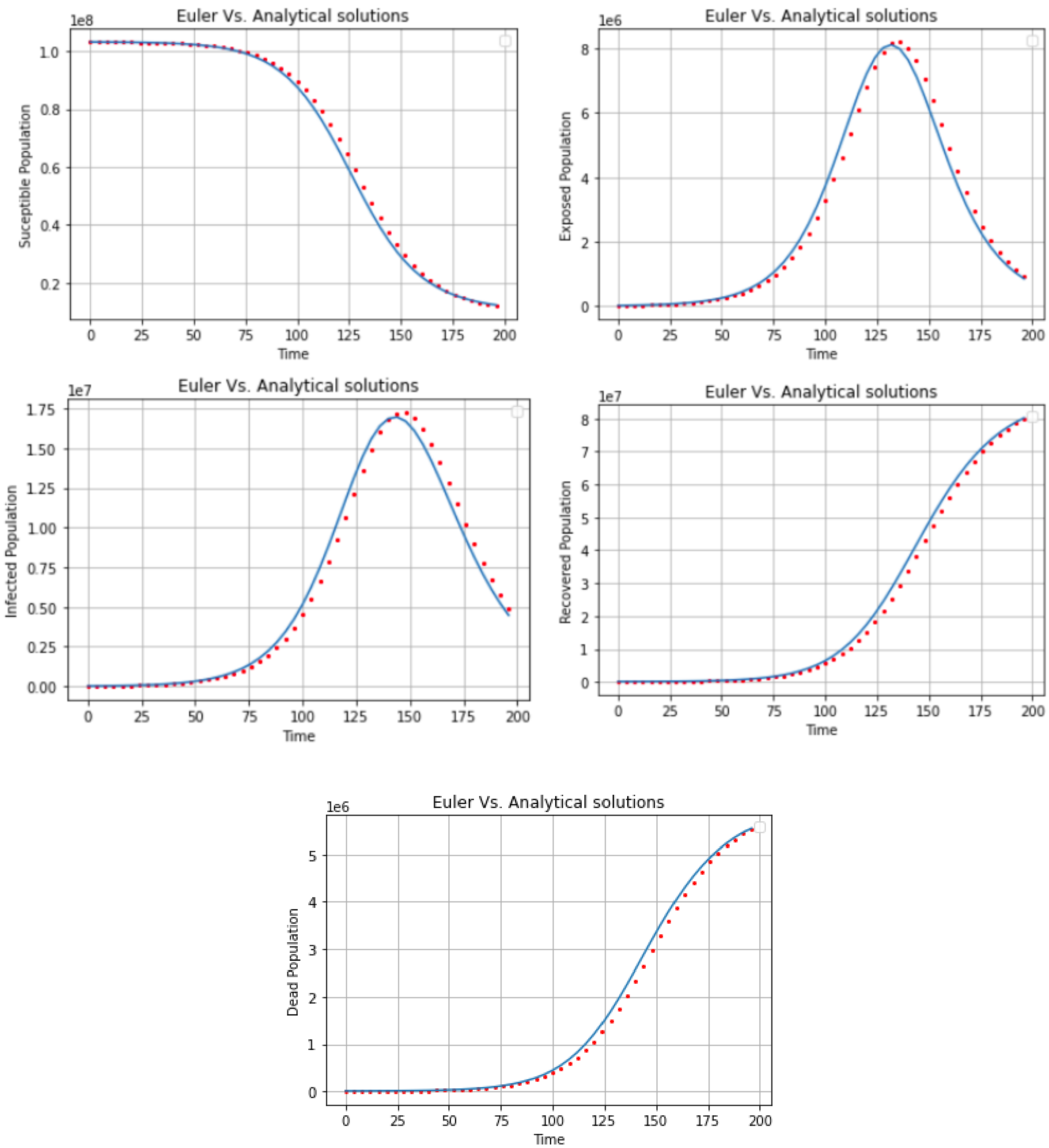


Figure 5. A comparison between Euler's and analytical solutions on different compartments

As seen in figure 5 there is a considerable amount of accumulated error across the 200 steps caused by the fundamental assumption of Euler's method that the slope approximation at the beginning of the interval can be applied to across the rest of the interval [14]. This method caught the trend successfully, but the curve is obviously shifted.

4.2.2. Modified Euler Method

The system of differential equations 2.1 was solved using forward Euler's method as mentioned in section 3.1.2. We implemented the predictor and corrector approaches that were mentioned previously on each equation of the SEIR system of ODEs in an iterative manner. A time interval of 200 days was chosen with a step size Δt of 1 and the results of the simulation were visualized using Matplotlib (Figure 5). Furthermore, the maximum number of infections is expected to be 16,653,724 people (error = 1.79%) on day 143 (1 day difference from the analytical solution). Finally, a comparison between this numerical method and analytical solutions of the different compartments (susceptible, exposed, infected, recovered and dead) was performed in figure 6.

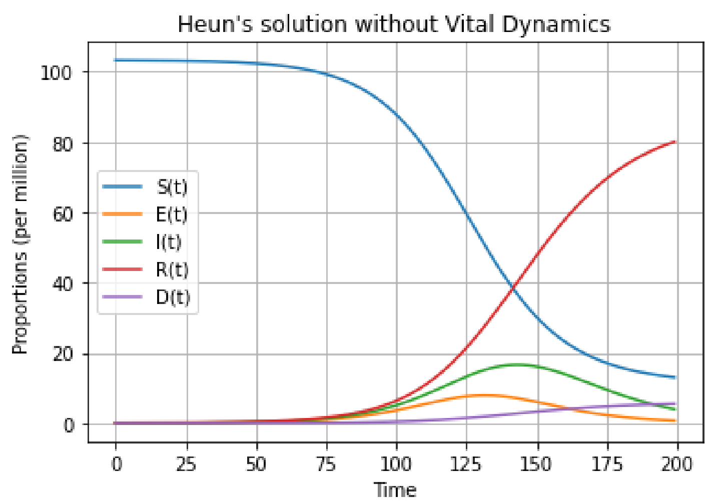


Figure 5. SEIR model Heun's numerical solution

According to figure 6, it is obvious that Heun's modified implementation of Euler's method corrected for the shift error that was represented in figure 4 to a great extent.

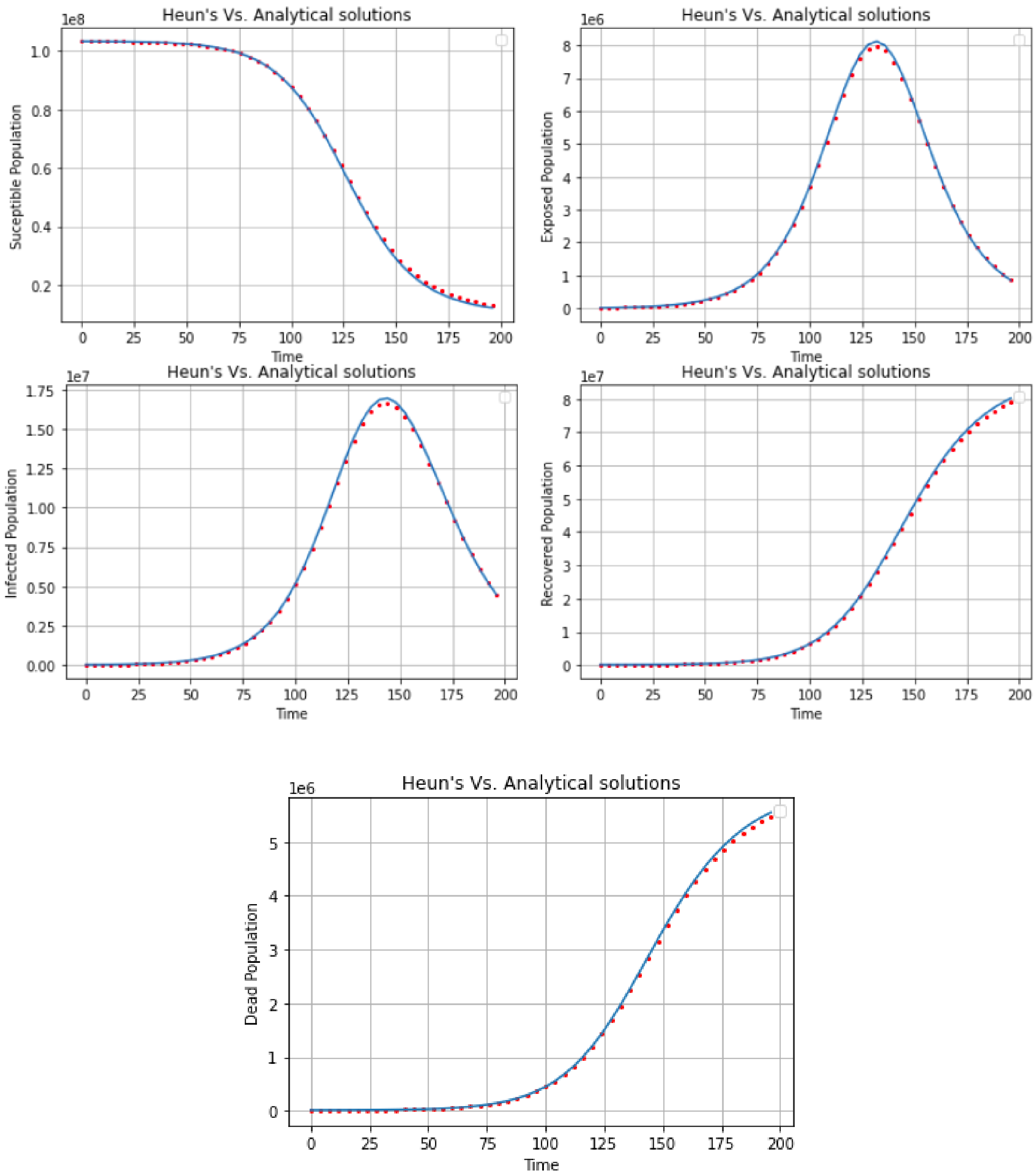
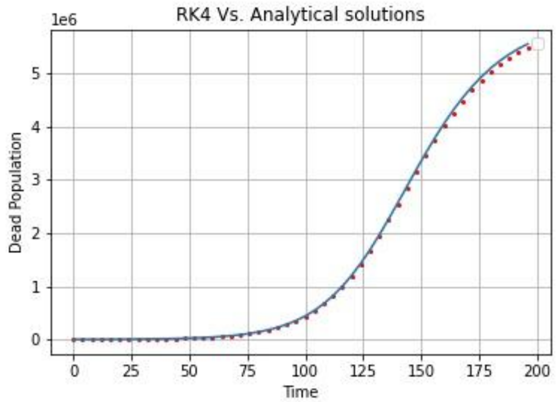
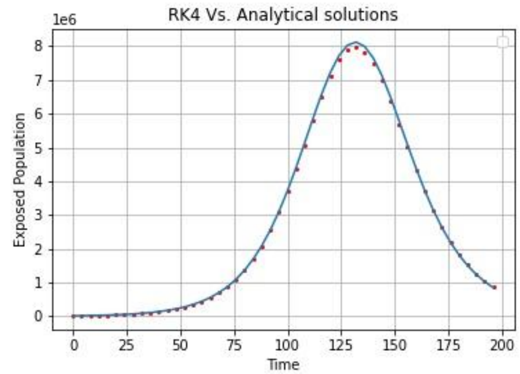
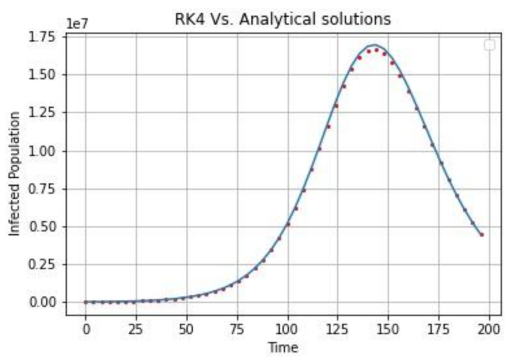


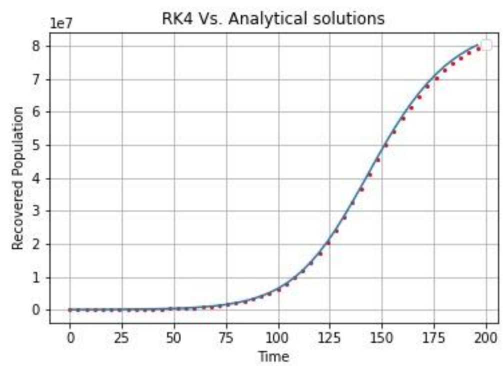
Figure 5. A comparison between Heun's and analytical solutions on different compartments

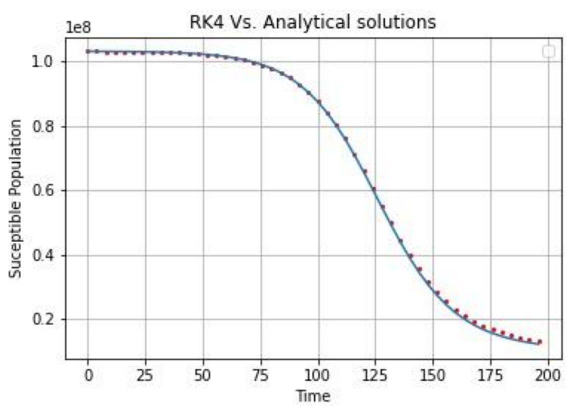
4.2.3. Runge Kutta Method











References

[1] "Coronavirus Cases:" *Worldometer*. [Online]. Available: https://www.worldometers.info/coronavirus/. [Accessed: 18-Dec-2020].

[2] "SEIR and SEIRS models¶," *SEIR and SEIRS models - HIV Model documentation*. [Online]. Available: https://docs.idmod.org/projects/emod-hiv/en/latest/model-seir.html. [Accessed: 18-Dec-2020].

[3] J. M. Carcione, J. E. Santos, C. Bagaini, and J. Ba, "A Simulation of a COVID-19 Epidemic Based on a Deterministic SEIR Model," *Frontiers*, 15-May-2020. [Online]. Available: https://doi.org/10.3389/fpubh.2020.00230. [Accessed: 18-Dec-2020].

[4] H. Froese, "Infectious Disease Modelling: Beyond the Basic SIR Model," *Medium*, 22-Apr-2020. [Online]. Available: https://towardsdatascience.com/infectious-disease-modelling-beyond-the-basic-sir-model-216369c584c4. [Accessed: 13-Jan-2021].

[5] "The species Severe acute respiratory syndrome-related coronavirus: classifying 2019-nCoV and naming it SARS-CoV-2," Nature News, 02-Mar-2020. [Online]. Available: https://www.nature.com/articles/s41564-020-0695-z. [Accessed: 18-Dec-2020].

[6] K. G. Andersen, A. Rambaut, W. I. Lipkin, E. C. Holmes, and R. F. Garry, "The proximal origin of SARS-CoV-2," Nature News, 17-Mar-2020. [Online]. Available: https://www.nature.com/articles/s41591-020-0820-9. [Accessed: 18-Dec-2020].

[7] "Finite difference methods for first-order ODEs¶," *Finite difference methods for first-order ODEs - Finite difference methods for first-order ODEs 1.0 documentation*. [Online]. Available: http://hplgit.github.io/INF5620/doc/notes/decay-sphinx/main_decay.html. [Accessed: 18-Dec-2020].

[8] "Estimating mortality from COVID-19," *World Health Organization*. [Online]. Available: https://www.who.int/news-room/commentaries/detail/estimating-mortality-from-covid-19. [Accessed: 02-Jan-2021].

[9] "Egypt Population (LIVE)," Worldometer. [Online]. Available: https://www.worldometers.info/world-population/egypt-population/. [Accessed: 02-Jan-2021].

[10] " Egypt," *Worldometer*. [Online]. Available: https://www.worldometers.info/coronavirus/country/egypt/. [Accessed: 02-Jan-2021].

[11] "Report of the WHO-China Joint Mission on Coronavirus", [online] Available: https://www.who.int/docs/default-source/coronaviruse/who-china-joint-mission-on-covid-19-final-report.pdf

[12] J. Hilton and M. J. Keeling, "Estimation of country-level basic reproductive ratios for novel Coronavirus (COVID-19) using synthetic contact matrices," 2020. [Online]. Available: https://www.medrxiv.org/content/10.1101/2020.02.26.20028167v1. [Accessed: 02-Jan-2021].

[13] Countrymeters.info, “Egypt Population,” *Countrymeters*. [Online]. Available: https://countrymeters.info/en/Egypt#:~:text=8%2C102%20live%20births%20average%20per,day%20(73.66%20in%20an%20hour). [Accessed: 12-Jan-2021].

[14] Numerical Methods--Heun's Method. [Online]. Available: http://calculuslab.deltacollege.edu/ODE/7-C-2/7-C-2-h.html. [Accessed: 26-Jun-2021].