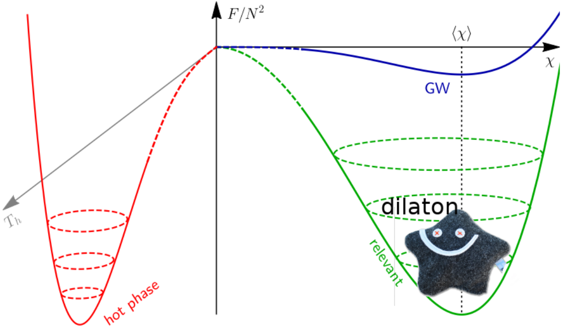
Relevant stabilization

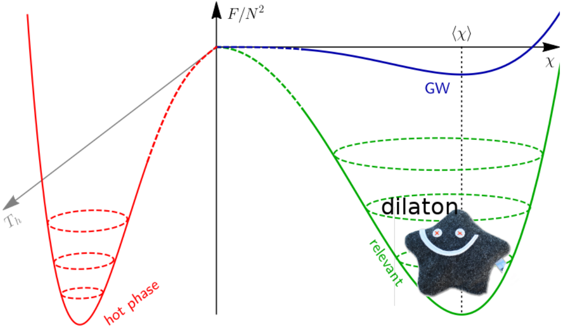


Ameen Ismail

UMD EPT seminar

13 March 2023

Relevant stabilization



(on conformal symmetry breaking + phase transitions)

arXiv:2301.10247

with C. Csáki, M. Geller, and Z. Heller-Algazi

Conformal theories

4D conformal group generators: $P_\mu, J_{\mu\nu}$, D , K_μ

4D Poincaré

Generates dilatations
(scale transformations)

Conformal invariance $\Rightarrow$ **scale invariance**

Q: so how can this be applicable to the real world?

Answer

$\boxed{\textbf{Spontaneously broken conformal symmetry!}}$

Consequences:

- Goldstone boson: dilaton χ
- Large UV/IR hierarchy from scale invariance?

Answer

Spontaneously broken conformal symmetry!

Consequences:

- Goldstone boson: dilaton χ
- Large UV/IR hierarchy from scale invariance?

Applications of conformal sectors

The Minimal Composite Higgs Model

Kaustubh Agashe [a], Roberto Contino [a], Alex Pomarol [b]

Higgs as a Holographic Pseudo-Goldstone Boson

Roberto Contino[a], Yasunori Nomura[b] and Alex Pomarol[c]

KK Parity in Warped Extra Dimension

Kaustubh Agashe [a,b], Adam Falkowski [c,d], Ian Low [e,f,g], Géraldine Servant [c,]

A Large Mass Hierarchy from a Small Extra Dimension

Lisa Randall

Raman Sundrum

Crunching Dilaton, Hidden Naturalness

Csaba Csáki,[1] Raffaele Tito D'Agnolo,[2] Michael Geller,[3] and Am

[1]*Department of Physics, LEPP, Cornell University, Ithaca, NY 14*

ut de Physique Théorique, Université Paris Saclay, CEA, F-91191 Gif-

[3]*School of Physics and Astronomy, Tel-Aviv University, Tel-Aviv 69*

Unparticle Physics

Howard Georgi,*

Continuum Dark Matter

Csaba Csáki,[a] Sungwoo Hong,[a,b,c] Gowri Kurup,[a,d] Seung J. Lee,[e] Maxim Perelstein,[a] and Wei Xue[f]

Applications of conformal sectors

Compositeness to stabilize a large hierarchy

5D dual via AdS/CFT: warped extra dimensions, RS, holographic CH, etc.

(+ many other applications!)

The dilaton, χ

Goldstone boson of broken scale invariance

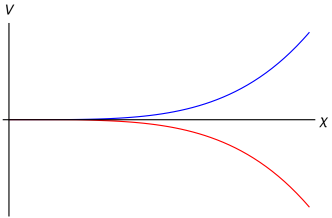
Vev $\langle\chi\rangle$ is the symmetry breaking scale

Understand PT via low-energy effective action

Dictated by symmetry: only **scale-invariant quartic term** permitted

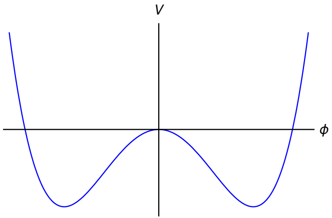
The need for stabilization

Dilaton:
$V(\chi) = \lambda\chi^4$



Runaway vev! $\langle\chi\rangle \to 0$ or ∞

Global U(1):
$V(\phi) = -\mu^2|\phi|^2 + \lambda|\phi|^4$



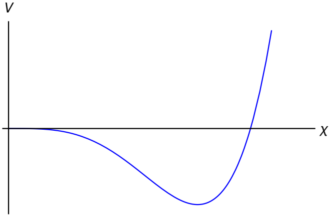
Stable vev

Goldberger–Wise mechanism

Small explicit breaking: nearly marginal operator (dimension $4+\epsilon$) gets a vev

$$V(\chi) = -\lambda\chi^4 + \lambda_{\mathrm{GW}}\chi^{4+\epsilon}/\mu_{\mathrm{UV}}^{\epsilon}$$

Stable minimum, large UV/IR hierarchy:
$\langle\chi\rangle \simeq \mu_{\mathrm{UV}}(\lambda_{GW}/\lambda)^{1/\epsilon}$



Light dilaton, $m^2/\langle\chi\rangle^2 \sim \epsilon$

Early-universe behaviour

At high T: symmetry restored, theory is a hot, deconfined CFT

GW mechanism has ramifications for transition to broken (cold, confined) phase:

- supercooled and strongly first-order
- PT does not complete until well below weak scale (or sometimes not at all!)
- strong stochastic gravitational wave signals

Relevant stabilization preview

Relevant operator (dimension $d < 4$) with small, technically natural coefficient γ gets a vev

Again a stable minimum, large hierarchy: $\langle\chi\rangle \simeq \mu_{\mathrm{UV}}\gamma^{1/(4-d)}$

Admits holographic 5D interpretation — more on this later

Main effects:

- heavier dilaton
- prompt PT $\Rightarrow$ no supercooling, weaker GW signals
- harder to calculate (but we'll make do)

Relevant stabilization

- The mechanism
- The phase transition

The 4D picture

Deform CFT with relevant operator: $\delta L = g_d \mathcal{O}$, with dimension $d < 4$

g_d very small at UV scale $\mu_{\rm UV}$ (take $\mathcal{O}$ to be odd under a Z_2 for technical naturalness)

Coupling grows in the IR and triggers spontaneous breaking:

$$g_d(\mu) = g_d(\mu_{\rm UV})(\mu_{\rm UV}/\mu)^{4-d}$$

Spurion analysis

Restore scale invariance and discrete Z_2:

assign g_d scaling dimension $4 - d$, odd under Z_2

Then: g_d^2 has scaling dimension $8 - 2d$

So $g_d^2 \chi^{2d-4}$ has dimension 4 and can show up the in the dilaton potential

Effective potential

Dilaton potential

$$V_{\text{eff}}(\chi) = \lambda\chi^4 - \lambda_{2\nu}\mu_{\text{UV}}^{4-2\nu}\chi^{2\nu}$$

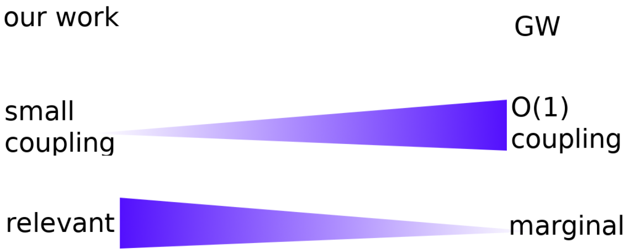
where $\nu = d - 2$

$\lambda_{2\nu} \propto \gamma^2$ where $\gamma = g_d(\mu_{\text{UV}})\mu_{\text{UV}}^{d-4} \ll 1$ parametrizes explicit breaking

Stable minimum, large hierarchy: $\langle\chi\rangle \sim \mu_{\text{UV}}\gamma^{1/(2-\nu)} \ll \mu_{\text{UV}}$

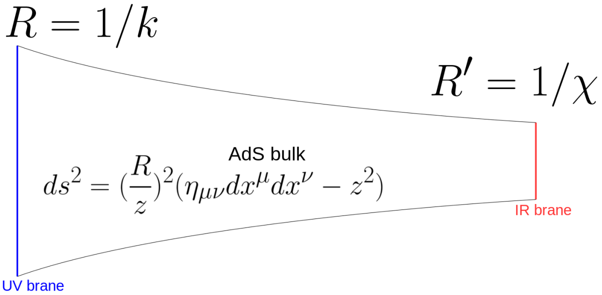
Comparison with Goldberger–Wise

In limit $\gamma \lesssim 1, \nu = 2 - \epsilon$ we approach GW: $\langle \chi \rangle \sim \mu_{\rm UV} \gamma^{1/\epsilon}$



The view from 5D

Warped extra dimension with bulk scalar Φ (c.f. Goldberger–Wise)



(as usual, bulk CC $-24M_5^3k^2$, brane tensions $\pm 24M_5^3k$)

AdS/CFT dictionary

UV cutoff $\mu_{\mathrm{UV}} \to k$, inverse AdS curvature / UV brane

Dilaton $\chi \to 1/R'$, radion fluctuations of IR brane

RG flow from μ_{UV} to $\mu_{\mathrm{IR}} \to$ motion from R to R'

Number of colours $N^2/(16\pi^2) = M_5^3/k^3$

Scalar action

$$S_\Phi = \int d^5x\sqrt{g}\left(\frac{1}{2}g^{MN}\partial_M\Phi\partial_N\Phi - \frac{1}{2}m^2\Phi^2\right) - \int d^4x\sqrt{g_{\rm ind}}V_{\rm UV} - \int d^4x\sqrt{g_{\rm ind}}V_{\rm IR}$$

Bulk mass controls localization of scalar profile

$$V_{\rm IR} = \frac{1}{2}m_{\rm IR}\Phi^2$$
$$V_{\rm UV} = \frac{1}{2}m_{\rm UV}\Phi^2 + \gamma k^{5/2}\Phi$$

γ is technically natural; the tadpole softly breaks a Z_2

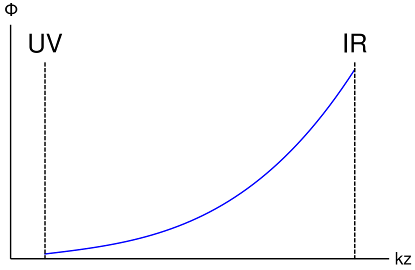
Scalar profile

Solve 5D EOM for Φ: $\Phi = \Phi_0(kz)^{2-\nu}(1+\Phi_1(kz)^{2\nu})$

where $\nu = \sqrt{4 + m^2/k^2}$ (assume $\nu < 2$ so profile grows in IR)

$\Phi_{0,1}$ controlled by BCs:

- $\Phi_0 \propto \gamma k^{3/2}$
  (small UV brane vev)
- $\Phi_1 \propto (\chi/k)^{2/\nu}$



Effective potential, again

Integrate out bulk matter, substitute in solution to 5D EOM (kills bulk contribution): Bellazzini *et al.* 1305.1319 Csáki *et al.*, 2205.15324

$$
\begin{aligned}
V(\chi) &\supset -\int dz \mathcal{L}_\Phi \\
&= -\Phi'(R)\Phi(R) + V_{\mathrm{UV}} + (R/R')^4 \left((R'/R)\Phi'(R')\Phi(R') + V_{\mathrm{IR}}\right) \\
&\sim \gamma^2 k^{4-2\nu} \chi^{2\nu}
\end{aligned}
$$

(ignoring a constant and some $\mathcal{O}(1)$ factors that depend on $m_{\mathrm{IR}}, m_{\mathrm{UV}}$)

Matches our 4D expectation

Effective potential, again

Full potential:

4D CC

$$V_{\text{eff}}(\chi) = \frac{24M_5^3}{k^3}\left(\boxed{\lambda\chi^4} - \boxed{\lambda_{2\nu}k^{4-2\nu}\chi^{2\nu}} + V_1\right)$$

Quartic from brane tension mistune

Relevant term, $\lambda_{2\nu} \propto -k^3/M_5^3\gamma^2$

Stable minimum at
$\langle\chi\rangle = k(\lambda_{2\nu}\nu/2\lambda)^{1/(4-2\nu)} \sim k\gamma^{1/(2-\nu)} \ll k$

Effective potential, again

Tune 4D CC $V(\langle\chi\rangle) = 0$, use holographic relation $N^2/16\pi^2 = M_5^3/k^3$, rewrite potential

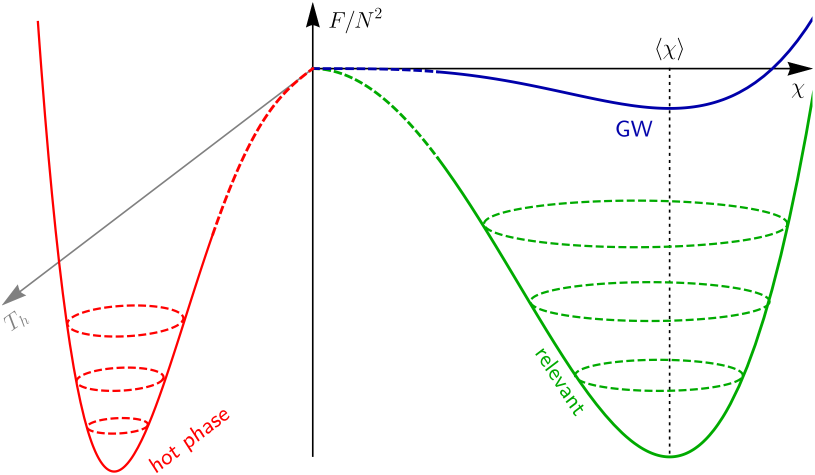
Dilaton effective action

$$S = \int d^4x \frac{3N^2}{4\pi^2}(\partial\chi)^2 - V(\chi),$$

$$V(\chi) = \frac{3N^2\lambda}{2\pi^2}\langle\chi\rangle^4 \left[(\chi/\langle\chi\rangle)^4 - 1 - \frac{(\chi/\langle\chi\rangle)^{2\nu} - 1}{\nu/2} \right]$$

Mass $m_\chi^2 = 8\lambda(2-\nu)\langle\chi\rangle^2$ — unsuppressed

Potential diagram



Summary so far

Relevant operator with small, technically natural coefficient γ gets a vev

5D interpretation:

- bulk scalar with small, Z_2-breaking tadpole on UV brane and appropriate bulk mass term
- gets small VEV on UV brane which grows in the IR and triggers breaking

Stable minimum, large hierarchy, and unsuppressed dilaton mass

Relevant stabilization

- The mechanism
- The phase transition

Phase transition basics

As universe cools: unbroken, deconfined phase $\rightarrow$ broken, confined phase

At critical temp. $T_c \propto \langle \chi \rangle$, free energies of phases equal

PT actually happens at nucleation temp. $T_n < T_c$

Need bubble nucleation rate $\Gamma \sim T^4 e^{-S_b} > H^4$

$\rightarrow$ governed by bounce action S_b

How small of a bounce action?

Remember: PT proceeds when $\Gamma \sim T^4 e^{-S_b} > H^4$

Hubble: $H^2 = F_{\rm deconf}(T)/(3M_{\rm Pl}^2) \approx \pi^2 N^2 T_c^4/(24 M_{\rm Pl}^2)$

$\Rightarrow$ need $S_b \lesssim 4 \log(M_{\rm Pl}/T_c)$

Upshot: TeV-scale $\langle\chi\rangle$, T_c requires $S_b \lesssim 140$

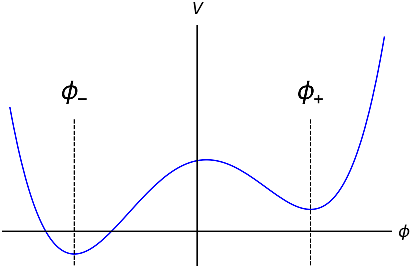
Agashe *et al.* 1910.06238
von Harling, Servant 1711.11554

Intuition for the bounce

"Bounce" $=$ solution to Euclidean EOM

$$\ddot{\phi} + \nabla^2\phi = V'(\phi)$$

w/ $\phi(\tau \to \pm\infty) = \phi_-$,
$\phi(|\vec{x}| \to \infty) = \phi_+$,
$\dot{\phi}(\tau = 0) = 0$

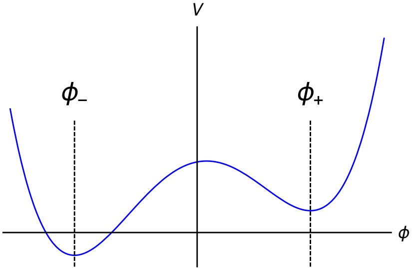


The bounce interpolates between vacua

Intuition for the bounce

"Bounce" $=$ solution to Euclidean EOM

$$S_b = \int d\tau \int d^3\vec{x}$$
$$\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\nabla\phi)^2 + V(\phi)$$



Lowest-action sol'n usually has $O(3)$ or $O(4)$ symmetry

Challenges in the bounce action

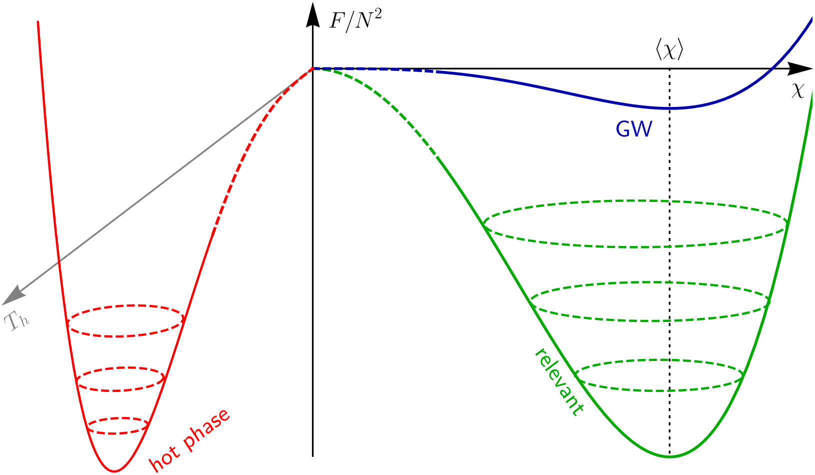
"Bounce" $=$ solution to Euclidean EOM interpolating b/w vacua

i.e. we need solution to dilaton EOM interpolating from $\chi = \langle\chi\rangle$ to $\chi = 0$

Calculability issues from treating bounce action in dilaton EFT:

- breaks down near origin, $T > M_{\mathrm{KK}} \sim \chi$
- cannot calculate part of bounce in deconfined phase

Calculability issues



Thin-wall limit

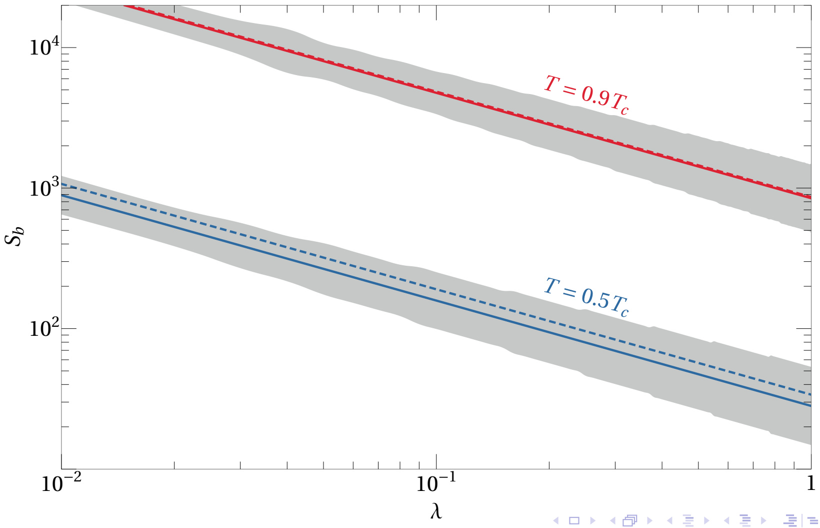
For $\delta = 1 - T/T_c$, we find:

$$S_b = \frac{N^2}{3^{1/4}\lambda^{3/4}\delta^2}F(\nu)^3,$$
$$F(\nu) = \left(\frac{\nu}{2-\nu}\right)^{3/4}\int_0^1 dx\sqrt{\frac{1-x^{2\nu}}{\nu/2} - (1-x^4)}$$

Remarks:

- Thin-wall approximation corresponds to $\delta \ll 1$
- **No** enhancement by $1/\epsilon$
- $N^2/\lambda^{3/4}$ scaling holds outside thin-wall limit

Comparison with numerics ($N = 5$, $\nu = 1.2$)



Comments on numerics

Quantify sensitivity to noncalculable regime
$0 < \chi < T(\chi/M_{\mathrm{KK}}) \approx T/2.1$:

- rescale $V \to (1+\epsilon)V$ in that region
- compute variation in bounce action, $dS_b/d\epsilon$
- take $S_b^{-1} dS_b/d\epsilon$ as relative error

Implications for the PT

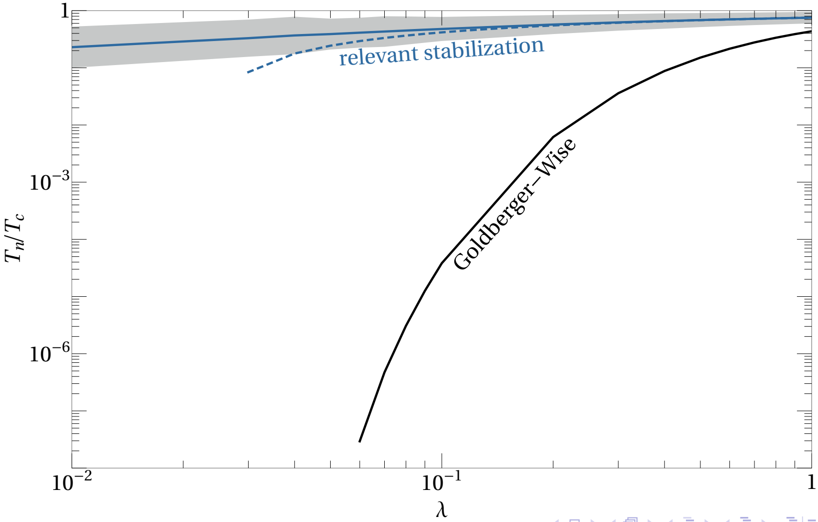
Deeper dilaton potential leads to smaller bounce action

Smaller S_b leads to prompt PT, no substantial supercooling

Prompt PT:

- avoids issues of eternal inflation
- weakens gravitational wave signals

Nucleation temp. (vs. GW with $\epsilon = -1/20$)



Gravitational waves from PTs

Stochastic GW signals from bubble collisions

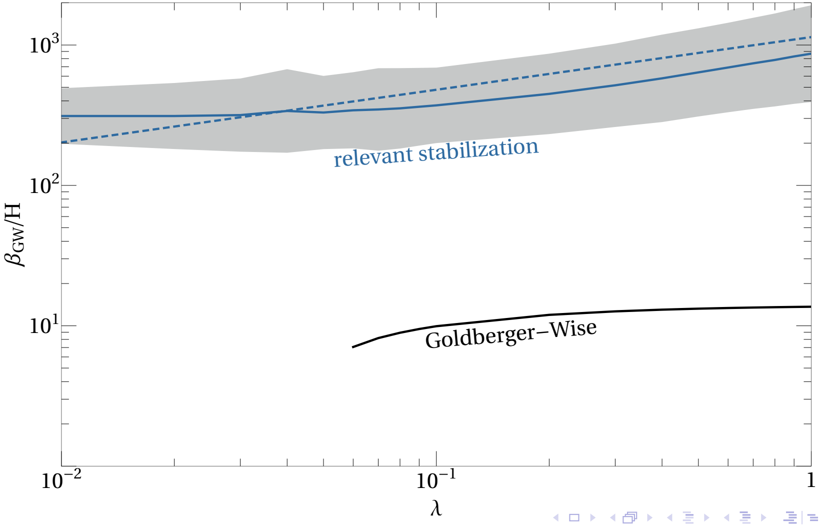
Key quantity:

$$\boxed{\text{(Inverse) } \textbf{duration of PT } \frac{\beta_{\rm GW}}{H} \approx T \frac{dS_b}{dT}\bigg|_{T=T_n}}$$

Controls signal strength $\propto (\beta_{\rm GW}/H)^{-2}$, peak freq. $\propto \beta_{\rm GW}/H$

Expect $\beta_{\rm GW}$ is larger in our mechanism

PT duration vs. Goldberger–Wise



Gravitational wave spectra

Recall: signal $\propto (\beta_{\rm GW}/H)^{-2}$, $f_p \propto \beta_{\rm GW}/H$

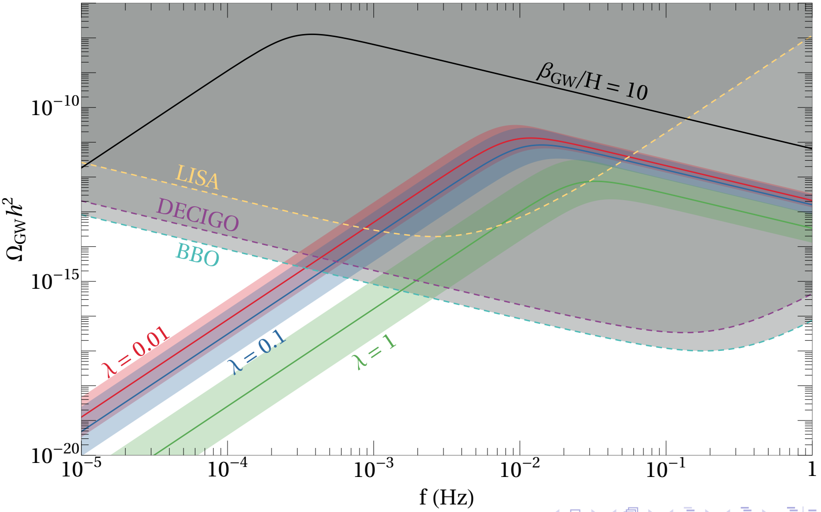
$$\Omega_{\rm GW} h^2 = 1.3 \times 10^{-6} \left(\frac{H}{\beta_{\rm GW}}\right)^2 \left(\frac{100}{g_*}\right)^{1/3} \frac{3.8(f/f_p)^{2.8}}{1+2.8(f/f_p)^{3.8}},$$

$$f_p = 3.8 \times 10^{-5} \text{ Hz} \frac{\beta_{\rm GW}}{H} \frac{T}{1 \text{ TeV}} \left(\frac{g_*}{100}\right)^{1/6}$$

Caprini *et al.* 1512.06239
Caprini *et al.* 1910.13125

So signals should be **weaker** and shifted towards **higher** frequency

Experimental reaches



Takeaways

Relevant stabilization offers an alternative to Goldberger–Wise

Differences in pheno and early-universe behaviour:

- heavier dilaton, leading to lower bounce action
- prompt PT, no supercooling nor eternal inflation

Gravitational waves weaker, but probably still observable at next-gen detectors

Outlook

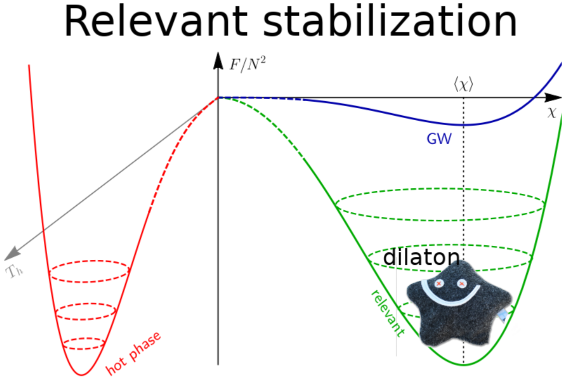
Inherent limitations of the dilaton effective theory: cannot describe hot phase, breaks down at small $\langle\chi\rangle$

Black brane–RS bounce configuration in 5D?

Similar work has proven fruitful see Aharony *et al.* hep-th/0507219

$\rightarrow$ no reason to believe we can't find the full bounce action

Thank you!



more info:

`arxiv.org/abs/2301.10247`

`ai279@cornell.edu`

`ameenismail.github.io`

Boundary conditions

$$V_{\mathrm{UV}} = \frac{1}{2} m_{\mathrm{UV}} \Phi^2 + \gamma k^{5/2} \Phi$$
$$V_{\mathrm{IR}} = \frac{1}{2} m_{\mathrm{IR}} \Phi^2$$

BCs:

$$2\Phi'(R) = m_{\mathrm{UV}} \Phi(R)$$
$$-2(R'/R)\Phi'(R') = m_{\mathrm{IR}} \Phi(R') + \gamma k^{5/2}$$

Boundary conditions

Solve 5D EOM: $\Phi = \Phi_0(kz)^{2-\nu}(1+\Phi_1(kz)^{2\nu})$

Then

$$\Phi_0 = -\frac{\gamma k^{3/2}}{\tau_{\rm UV} + \Phi_1(\tau_{\rm UV} - 4\nu)} \simeq -\frac{\gamma k^{3/2}}{\tau_{\rm UV}}$$
$$\Phi_1 = -\frac{\tau_{\rm IR}(R/R')^{2\nu}}{\tau_{\rm IR} + 4\nu}$$

with

$$\tau_{\rm UV,IR} = m_{\rm UV,IR}/k \mp (4-2\nu)$$

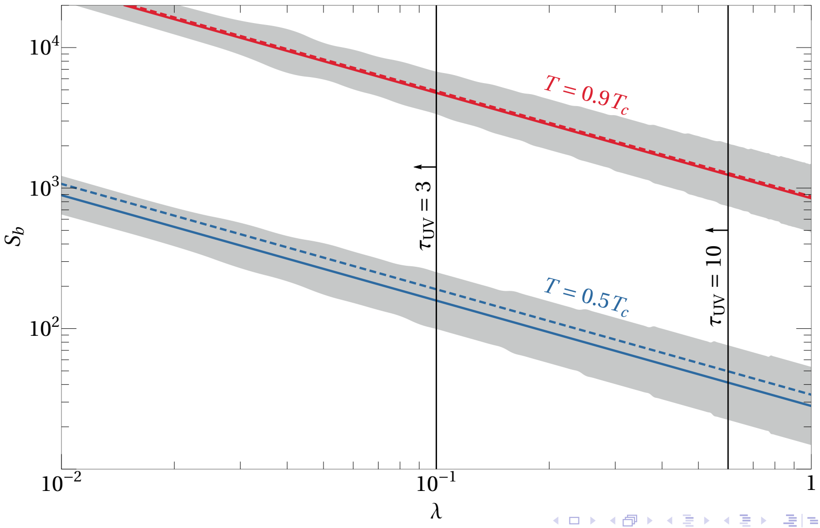
Consistency conditions

Need dilaton EFT to be valid:

- dilaton lighter than first KK mode (scalar or graviton), $m_\chi \lesssim M_{\rm KK}$
- small backreaction on metric, $V_{\rm IR}(\Phi) + 24\lambda M_5^3 k < \Lambda_{\rm IR}$

Yield upper bound $\lambda \lesssim \mathcal{O}(0.1\text{–}1)$, depending on $\tau_{\rm UV} = m_{\rm UV}/k - (4 - 2\nu)$

Comparison with numerics ($N = 5$, $\nu = 1.2$)



Critical temperature calculation

As universe cools: unbroken, deconfined phase $\to$ broken, confined phase

Free energies

- $F_{\mathrm{conf}}(\chi) \approx V(\chi)$
- $F_{\mathrm{deconf}}(T) = -\pi^2 N^2 T^4/8 + 3N^2\lambda(2-\nu)\langle\chi\rangle^4/(2\pi^2\nu)$

Constant determined by common limit $T, \chi \to 0$
Critical temperature: $T_c = (\langle\chi\rangle/\pi)\,(12\lambda(2-\nu)/\nu)^{1/4}$

Bounce action in thin-wall limit

$O(3)$-symmetric bounce action $S_b = (16\pi/3)S_1^3/(\Delta V^2 T)$

Wall tension $S_1 = \sqrt{3N^2/2\pi^2}\int_0^{\langle\chi\rangle} d\chi\sqrt{2V(\chi)} \propto N^2\lambda^{1/2}\langle\chi\rangle^3$

$\Delta V \propto N^2(T_c^4 - T^4) \propto N^2\lambda\langle\chi\rangle^4\delta$, where $\delta \equiv 1 - T/T_c$

$T = T_c(1-\delta) \propto \lambda^{1/4}\langle\chi\rangle(1-\delta)$

Thus we expect $S_b \sim N^2/(\lambda^{3/4}\delta^2)$

PT duration in thin-wall limit

Recall $S_b = \dfrac{N^2}{3^{1/4}\lambda^{3/4}\delta^2} F(\nu)^3 \propto \delta^{-2}$, with $\delta = 1 - T/T_c$

Then $\beta_{\mathrm{GW}}/H = T(dS_b/dT)|_{T=T_n} = 2S_b/\delta$

(to be evaluated at T_n, when $S_b \approx 140$)

Expect this to be **larger** than in Goldberger–Wise

Bubble wall collision domination

Ratio of energy released in PT to radiation bath:

$$\alpha = \frac{15N^2}{4g_*}\left(\frac{T_c^4}{T_n^4} - 1\right)$$

Bubble collisions dominate when $\alpha \gg \alpha_\infty$:

$$\alpha_\infty = \frac{30}{24\pi^2 g_* T_n^2}\sum c_i m_i^2$$

(sum over particles i which acquire a mass m_i during PT)

Bubble wall collision domination

Assume techni-quarks confine into mesons with mass $\sim \langle \chi \rangle$

Demanding $\alpha > \alpha_\infty$ requires $\lesssim 200$ mesonic dofs