Correlation functions in the AdS/CFT correspondence

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1 Introduction

1.1 AdS/CFT

The AdS/CFT correspondence is a remarkable duality between gravity in anti-de Sitter space in $d+1$ dimensions and field theory in d dimensions on the conformal boundary of that space. It was introduced by Maldacena in 1997 [1] and further elucidated by Witten [2] and Gubser, Klebanov, and Polyakov [3] shortly thereafter. In this work we will motivate the correspondence and make use of it in computing two- and three-point correlation functions. In doing so, we will introduce Witten diagrams, an analog to Feynman diagrams for gravity in AdS. Although the AdS/CFT correspondence is often discussed in the context of string theory, here we will make no connection to string theory.

1.2 Preliminaries of AdS

We will briefly review some basic features of anti-de Sitter space and introduce some choices of coordinates. In Lorentzian signature, one can embed AdS_{d+1} in $\mathbb{R}^{d,2}$. Let X_{-1} and X_0 be the time directions and $\vec{\mathbf{X}} = X_1 \dots X_d$ the space directions. Then AdS_{d+1} is the universal cover of the hyperboloid defined by

$$-X_{-1}^2 - X_0^2 + \left|\vec{\mathbf{X}}\right|^2 = -\ell^2 \tag{1.1}$$

where ℓ is a constant. We can write the metric in terms of global coordinates (t, ρ):

$$\mathrm{d}s^2 = \ell^2(-\cosh^2\rho\,\mathrm{d}t^2 + \mathrm{d}\rho^2 + \sinh^2\rho\,\mathrm{d}\Omega_{d-1}{}^2). \tag{1.2}$$

Alternatively, we can choose Poincaré coordinates $(t, z_0, \vec{\mathbf{z}} = z_1 \dots z_{d-1})$ where the metric takes the form

$$\mathrm{d}s^2 = \frac{\ell^2}{z_0^2}(-\,\mathrm{d}t^2 + \mathrm{d}z_0{}^2 + \mathrm{d}\vec{\mathbf{z}}^2). \tag{1.3}$$

In these coordinates, $z_0 \in (0, \infty)$ and $z_0 = 0$ corresponds to the conformal boundary.

It is useful to Wick-rotate the t coordinate to obtain Euclidean AdS. In Euclidean signature, the metric in Poincaré coordinates is

$$\mathrm{d}s^2 = \frac{\ell^2}{x_0^2}(\mathrm{d}x_0{}^2 + \mathrm{d}\vec{\mathbf{x}}^2) \tag{1.4}$$

where $\vec{\mathbf{x}} = x_1 \dots x_d$. These are the coordinates used throughout this work. The conformal boundary is at $x_0 = 0$. Moreover, these coordinates clearly demonstrate that the boundary is conformally Euclidean. Likewise, the boundary of Lorentzian AdS is conformally equivalent to Minkowski spacetime.

2 Boundary behaviour of AdS

2.1 Geometry of the boundary

The metric is singular on the boundary of AdS. This is apparent from taking the $x_0 \to 0$ limit in Poincaré coordinates. However, suppose we perform a conformal transformation $g \to \omega^2 g$, where ω is a function with a zero of order one on the boundary. For example, we could take $\omega = x_0$. Then we are free to take the limit of the metric at the boundary, so there is a well-defined notion of distance on the boundary. However, we could conformally rescale ω and this procedure would still make sense, which means the boundary metric is only defined up to conformal transformations. Moreover, choosing $\omega = x_0$ and taking the $x_0 \to 0$ limit of equation 1.4, we see the boundary is conformally flat.

The symmetry group of Euclidean AdS_{d+1} is $SO(d+1,1)$ (for Lorentzian signature it is $SO(d,2)$). This is easiest to see by embedding AdS_{d+1} in $\mathbb{R}^{d+1,1}$. AdS_{d+1} corresponds to the hyperboloid

$$-X_0^2 + \left|\vec{\mathbf{X}}\right|^2 = -\ell^2 \tag{2.1}$$

where X_0 is the time direction and $\vec{\mathbf{X}} = X_1 \dots X_{d+1}$ are the space directions. The hyperboloid is invariant under rotations and boosts, but not translations, and so the symmetry group is $SO(d+1,1)$. Both the AdS metric and the conformal geometry on the boundary are preserved by $SO(d+1,1)$ transformations.

2.2 Fields on the boundary

Consider a massless, real, scalar field ϕ defined on AdS_{d+1}. The Lagrangian is $\mathcal{L} = \frac{1}{2}\sqrt{g}(\boldsymbol{\nabla}\phi)^2$ and the equation of motion is the Laplace equation $\nabla^2\phi = 0$. Now suppose we wish to extend ϕ to the boundary of AdS_{d+1}. In Poincaré coordinates (see Eqn. 1.4), this is realized by extending ϕ to $x_0 = 0$.

Remarkably, specifying ϕ on the boundary—that is, insisting $\phi(0,\vec{\mathbf{x}}) = \phi_0(\vec{\mathbf{x}})$—uniquely determines ϕ in the bulk. There is more than one way to demonstrate this. We will prove it using Green's functions, which will be useful in calculating correlation functions later on. We want to find a function G satisfying the Laplace equation whose value at $x_0 = 0$ is $G(0,\vec{\mathbf{x}}) = \delta^d(\vec{\mathbf{x}})$. To do so, first consider a different function $K = cx_0^d$, where c is a constant. For a function which only depends on x_0, the Laplacian is

$$\nabla^2 K(x_0) = \frac{1}{\sqrt{g}}\frac{\mathrm{d}}{\mathrm{d}x_0}(\sqrt{g}g^{00}\frac{\mathrm{d}K}{\mathrm{d}x_0}) = x_0^{d+1}\frac{\mathrm{d}}{\mathrm{d}x_0}(x_0^{-d+1}\frac{\mathrm{d}K}{\mathrm{d}x_0}). \tag{2.2}$$

We see that $K = cx_0^d$ solves the equation of motion $\nabla^2 K = 0$. K diverges in the $x_0 \to \infty$ limit, while we desire to find a G which diverges as $x_0 \to 0$. We can implement an $SO(d+1,1)$ transformation and our function will still solve the equation of motion. In particular, if we choose the transformation

$$x_i \to \frac{x_i}{x_0^2 + |\vec{\mathbf{x}}|^2} \tag{2.3}$$

then the point at $x_0 \to \infty$ is mapped to $x_0 \to 0$. The resulting function,

$$G(x_0,\vec{\mathbf{x}}) = c\frac{x_0^d}{(x_0^2 + |\vec{\mathbf{x}}|^2)^d} \tag{2.4}$$

has the desired divergence at $x_0 = 0$ and $\vec{\mathbf{x}} = \vec{\mathbf{0}}$, while it vanishes identically for $x_0 = 0$ and $\vec{\mathbf{x}} \neq \vec{\mathbf{0}}$. By choosing c appropriately, G becomes a properly normalized delta function. Note that G is, in essence, the familiar Poisson kernel. Also, G is often called the boundary-to-boundary propagator.

So the boundary value of ϕ, denoted by $\phi_0(\vec{\mathbf{x}})$, determines a bulk solution for ϕ:

$$\phi(x_0,\vec{\mathbf{x}}) = \int \mathrm{d}\vec{\mathbf{x}'}\, G(x_0,\vec{\mathbf{x}} - \vec{\mathbf{x}'})\phi_0(\vec{\mathbf{x}}) = \int \mathrm{d}\vec{\mathbf{x}'}\, \frac{cx_0^d\phi_0(\vec{\mathbf{x}'})}{(x_0^2 + \left|\vec{\mathbf{x}} - \vec{\mathbf{x}'}\right|^2)^d}. \tag{2.5}$$

It is easy to show that this solution is unique. Suppose there existed another solution ϕ', satisfying the same boundary condition but distinct from the ϕ defined by the previous equation. The Laplace equation is linear, so $\psi \equiv \phi' - \phi$ also solves the equation of motion; furthermore, ψ is 0 on the boundary. But since $\nabla^2\psi = 0$, we must have

$$0 = \int_M \sqrt{g}\psi\nabla^2\psi = -\int_M \sqrt{g}(\boldsymbol{\nabla}\psi)^2. \tag{2.6}$$

(The integral is taken over the entire bulk of AdS space, M.) The second equality uses integration by parts wherein the surface term vanishes due to the boundary condition. Therefore $\boldsymbol{\nabla}\psi = 0$, which, together with the boundary condition, implies $\psi = 0$, contradicting our initial assumption that ϕ and ϕ' were distinct. We conclude that the solution given by equation 2.5 is unique.

3 Partition functions and the GKPW dictionary

In this section we will precisely and mathematically state the AdS/CFT correspondence. Given a conformal field theory with a single field $\mathcal{O}$ defined on the boundary ∂M of AdS, we may define the generating functional:

$$Z_{CFT}[\phi_0] = \int \mathcal{D}\mathcal{O}\exp\left(-S[\mathcal{O}] - \int_{\partial M}\sqrt{h}\phi_0\mathcal{O}\right) = \left\langle \exp\left(-\int_{\partial M}\sqrt{h}\phi_0\mathcal{O}\right)\right\rangle. \tag{3.1}$$

In the above equation, h is the induced metric on the boundary, $S[\mathcal{O}]$ denotes the action of the field theory, and the ϕ_0 field provides a source term for $\mathcal{O}$. Correlation functions are given by functional derivatives of the generating functional with respect to ϕ_0. This generalizes easily to multiple fields; we introduce a source field $\phi_{0,i}$ for each field $\mathcal{O}_i$ and replace the integrand in the exponent with $\sum_i \phi_{0,i}\mathcal{O}_i$.

On the other hand, we have seen that ϕ_0 uniquely determines a bulk field ϕ satisfying the classical equation of motion. We may also define the gravitational partition function

$$Z_{grav}[\phi_0] = \int \mathcal{D}\phi\mathcal{D}g\exp(-S_E[\phi, g]) \tag{3.2}$$

where the path integral is taken over all possible ϕ and g satisfying the boundary condition, and S_E is the Euclidean action. In the classical approximation, $\log Z_{grav}[\phi_0] = -S_E[\phi]$, where ϕ is taken to be the on-shell solution given by equation 2.5.

The AdS/CFT correspondence asserts that

$$Z_{CFT} = Z_{grav}. \tag{3.3}$$

This formulation of the correspondence is called the GKPW dictionary, after Gubser, Klebanov, and Polyakov [3], and Witten [2].

4 The two-point function

Next we will explicitly compute the two-point correlation function in the AdS/CFT correspondence. The propagator between two points on the boundary of AdS space is given at tree level by the diagram in Figure 1. The boundary of AdS is represented by a circle, while the squiggly line signifies the propagating field. Position-space Feynman diagrams in AdS like this one are called Witten diagrams. We work in the limit where gravity is weakly coupled so that we can approximate the gravitational path integral using the on-shell action. For a free field, this propagator is exact, whereas in a more general theory loop diagrams may provide quantum corrections at higher orders in perturbation theory. In that case, the diagram we consider represents the leading-order term.

In light of the GKPW dictionary, one expects that the boundary-to-boundary AdS_{d+1} propagator is equal to the two-point correlation function in a d-dimensional CFT on the boundary. The form of the latter is severely restricted by conformal symmetry. In Euclidean coordinates $\vec{\mathbf{x}} = x_1 \ldots x_d$ the two-point function of

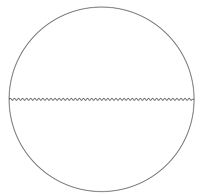

Figure 1: Witten diagram for two-point function.

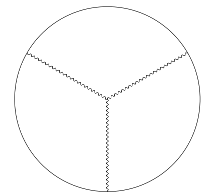

Figure 2: Tree-level Witten diagram for three-point function.

a field $\mathcal{O}$ obeys $\langle \mathcal{O}(\vec{\mathbf{x}})\mathcal{O}(\vec{\mathbf{x}} + \Delta\vec{\mathbf{x}})\rangle \sim |\Delta\vec{\mathbf{x}}|^{-2\Delta}$, where Δ is a constant, characteristic of $\mathcal{O}$, called the scaling dimension.

Using Stokes' theorem and integration by parts, the Euclidean action can be written as

$$S_E = \frac{1}{2}\int_M \mathrm{d}x_0\,\mathrm{d}\vec{\mathbf{x}}\,\sqrt{g}(\boldsymbol{\nabla}\phi)^2 = \frac{1}{2}\int_{\partial M}\mathrm{d}\vec{\mathbf{x}}\,\sqrt{h}\phi\vec{\mathbf{n}}\cdot\boldsymbol{\nabla}\phi - \frac{1}{2}\int_M \mathrm{d}x_0\,\mathrm{d}\vec{\mathbf{x}}\,\sqrt{g}\phi\nabla^2\phi \tag{4.1}$$

where M is AdS_{d+1}, $\vec{\mathbf{n}}$ is a unit vector normal to the conformal boundary, and h is the induced metric on the boundary. The second term vanishes on-shell. The boundary can be thought of as the hypersurface Σ_ϵ defined by $x_0 = \epsilon$, in the $\epsilon \to 0$ limit. Thus $\sqrt{h} = \epsilon^{-d}$, and $n^\mu = (\epsilon, \vec{\mathbf{0}})$. It follows that

$$S_E = \frac{1}{2}\lim_{\epsilon\to 0}\int_{\Sigma_\epsilon}\mathrm{d}\vec{\mathbf{x}}\,\epsilon^{-d+1}\phi\frac{\partial\phi}{\partial x_0}. \tag{4.2}$$

. Recalling our expression for ϕ in terms of the boundary value ϕ_0 (equation 2.5), we find

$$\left.\frac{\partial\phi}{\partial x_0}\right|_{x_0=\epsilon} = cd\epsilon^{d-1}\int \mathrm{d}\vec{\mathbf{x'}}\,\frac{\phi_0(\vec{\mathbf{x'}})}{(\epsilon^2 + \left|\vec{\mathbf{x}} - \vec{\mathbf{x'}}\right|^2)^d}\Big(1 - \frac{2\epsilon^2}{\epsilon^2 + \left|\vec{\mathbf{x}} - \vec{\mathbf{x'}}\right|^2}\Big). \tag{4.3}$$

The second term in the parentheses drops out when we take the $\epsilon \to 0$ limit. Furthermore, we have by definition $\lim_{\epsilon\to 0}\phi(\epsilon, \vec{\mathbf{x}}) = \phi_0(\vec{\mathbf{x}})$. We can therefore conclude

$$S_E = \frac{cd}{2}\int \mathrm{d}\vec{\mathbf{x}}\,\mathrm{d}\vec{\mathbf{x'}}\,\frac{\phi_0(\vec{\mathbf{x}})\phi_0(\vec{\mathbf{x'}})}{\left|\vec{\mathbf{x}} - \vec{\mathbf{x'}}\right|^{2d}}. \tag{4.4}$$

The two-point correlation function is

$$\frac{\delta}{\delta\phi_0(\vec{\mathbf{x_1}})}\frac{\delta}{\delta\phi_0(\vec{\mathbf{x_2}})}S_E\Big|_{\phi=0} \sim \frac{1}{|\vec{\mathbf{x_1}} - \vec{\mathbf{x_2}}|^{2d}} \tag{4.5}$$

which is of the form we anticipated. The scaling dimension is d.

5 The three-point function

The simplest field for which we can evaluate a non-trivial three-point function is massless ϕ^3 theory. The tree-level contribution is represented by the diagram in Figure 2. Since we cannot analytically solve the equation of motion, we will attempt to find a perturbative solution at leading order in the coupling. The Euclidean action is

$$S_E = \frac{1}{2}\int_M \mathrm{d}x_0\,\mathrm{d}\vec{\mathbf{x}}\,\sqrt{g}((\boldsymbol{\nabla}\phi)^2 + \frac{\eta}{3}\phi^3). \tag{5.1}$$

We again rewrite this using Stokes' theorem, integration by parts, and the equation of motion:

$$\begin{aligned} S_E &= \frac{1}{2}\int_{\partial M} \mathrm{d}\vec{\mathbf{x}}\,\sqrt{h}\phi\vec{\mathbf{n}}\cdot\boldsymbol{\nabla}\phi - \frac{1}{2}\int_M \mathrm{d}x_0\,\mathrm{d}\vec{\mathbf{x}}\,\sqrt{g}\phi\nabla^2\phi + \frac{1}{2}\int_M \mathrm{d}x_0\,\mathrm{d}\vec{\mathbf{x}}\,\sqrt{g}\frac{\eta}{3}\phi^3 \\ &= \frac{1}{2}\int_{\partial M} \mathrm{d}\vec{\mathbf{x}}\,\sqrt{h}\phi\vec{\mathbf{n}}\cdot\boldsymbol{\nabla}\phi - \frac{1}{2}\int_M \mathrm{d}x_0\,\mathrm{d}\vec{\mathbf{x}}\,\sqrt{g}\frac{\eta}{3!}\phi^3. \end{aligned} \tag{5.2}$$

Next we will solve the equation of motion, $\nabla^2\phi = \frac{1}{2}\eta\phi^2$, up to order η. Recall that $G(x_0, \vec{\mathbf{x}})$ solves the Laplace equation and behaves as a delta function on the boundary. Similarly, we can define a propagator $\tilde{G}(x_0, x_0', \vec{\mathbf{x}})$ satisfying

$$\nabla^2\tilde{G}(x_0, x_0', \vec{\mathbf{x}}) = \frac{1}{\sqrt{g(x_0)}}\delta(x_0 - x_0')\delta^d(\vec{\mathbf{x}}) = x_0^{d+1}\delta(x_0 - x_0')\delta^d(\vec{\mathbf{x}}). \tag{5.3}$$

$\tilde{G}$ is called the bulk-to-bulk propagator. One can write an explicit expression for $\tilde{G}$ using the hypergeometric function, but this will not be necessary. However, we will make use of the relation

$$\lim_{\epsilon\to 0}\tilde{G}(\epsilon, x_0, \vec{\mathbf{x}}) \sim \epsilon^d G(x_0, \vec{\mathbf{x}}); \tag{5.4}$$

that is, the bulk-to-bulk propagator approaches the bulk-to-boundary propagator near the boundary. This statement is proved in, for instance, [4].

At lowest order, ϕ is given by Equation 2.5. Subsituting this back into the equation of motion yields

$$\nabla^2\phi = \frac{\eta}{2}\int \mathrm{d}\vec{\mathbf{x}'}\int \mathrm{d}\vec{\mathbf{x}''}\,G(x_0, \vec{\mathbf{x}} - \vec{\mathbf{x}'})\phi_0(\vec{\mathbf{x}'})G(x_0, \vec{\mathbf{x}} - \vec{\mathbf{x}''})\phi_0(\vec{\mathbf{x}''}). \tag{5.5}$$

We can solve this to first order in η using $\tilde{G}$:

$$\phi(x_0, \vec{\mathbf{x}}) = \phi^{(0)}(x_0, \vec{\mathbf{x}}) + \frac{\eta}{2}\int \mathrm{d}x_0'\,\mathrm{d}\vec{\mathbf{x}'}\,\sqrt{g}\tilde{G}(x_0, x_0', \vec{\mathbf{x}} - \vec{\mathbf{x}'})[\phi^{(0)}(x_0', \vec{\mathbf{x}'})]^2 + O(\eta^2); \tag{5.6}$$

where

$$\phi^{(0)}(x_0, \vec{\mathbf{x}}) = \int \mathrm{d}\vec{\mathbf{x}'}\,G(x_0, \vec{\mathbf{x}} - \vec{\mathbf{x}'})\phi_0(\vec{\mathbf{x}'}) \tag{5.7}$$

is the zeroth-order solution.

Finally we must evaluate the on-shell action, which contains a boundary term and a bulk term. We will first evalute the $O(\eta)$ contribution of the boundary term, given by

$$S_E^{boundary} = \frac{1}{2}\int_{\partial M} \mathrm{d}\vec{\mathbf{x}}\,\sqrt{h}\phi\vec{\mathbf{n}}\cdot\boldsymbol{\nabla}\phi = \frac{1}{2}\lim_{\epsilon\to 0}\int_{\Sigma_\epsilon} \mathrm{d}\vec{\mathbf{x}}\,\epsilon^{-d+1}\phi\frac{\partial\phi}{\partial x_0}. \tag{5.8}$$

We now compute

$$\begin{aligned} \left.\frac{\partial\phi}{\partial x_0}\right|_{x_0=\epsilon} &= O(\eta^0) + \frac{\eta}{2}\int \mathrm{d}x_0'\,\mathrm{d}\vec{\mathbf{x}'}\,\sqrt{g}\,\left.\frac{\partial\tilde{G}(x_0, x_0', \vec{\mathbf{x}} - \vec{\mathbf{x}'})}{\partial x_0}\right|_{x_0=\epsilon}[\phi^{(0)}(x_0', \vec{\mathbf{x}'})]^2 \\ &\xrightarrow{\epsilon\to 0}\sim \frac{\eta d}{2}\epsilon^{d-1}\int \mathrm{d}x_0'\,\mathrm{d}\vec{\mathbf{x}'}\,\sqrt{g}G(x_0', \vec{\mathbf{x}} - \vec{\mathbf{x}'})[\phi^{(0)}(x_0', \vec{\mathbf{x}'})]^2 \end{aligned} \tag{5.9}$$

dropping the $O(\eta^0)$ term in the second line. Recall that, as always, $\lim_{\epsilon\to 0}\phi(\epsilon, \vec{\mathbf{x}}) = \phi_0(\vec{\mathbf{x}})$. The contribution to the action is therefore proportional to

$$\eta\int \mathrm{d}x_0'\,\mathrm{d}\vec{\mathbf{x}}\,\mathrm{d}\vec{\mathbf{x}'}\,\sqrt{g}G(x_0', \vec{\mathbf{x}} - \vec{\mathbf{x}'})\phi_0(\vec{\mathbf{x}})[\phi^{(0)}(x_0', \vec{\mathbf{x}'})]^2 = \eta\int \mathrm{d}x_0'\,\mathrm{d}\vec{\mathbf{x}'}\,\sqrt{g}[\phi(x_0', \vec{\mathbf{x}'})]^3. \tag{5.10}$$

Conveniently, the $O(\eta)$ piece of the bulk term in the action has the same form:

$$S_E^{bulk} = -\frac{\eta}{3!}\int \mathrm{d}x_0\,\mathrm{d}\vec{\mathbf{x}}\,\sqrt{g}\phi^3 = -\frac{\eta}{3!}\int \mathrm{d}x_0\,\mathrm{d}\vec{\mathbf{x}}\,\sqrt{g}[\phi^{(0)}]^3 + O(\eta^2). \tag{5.11}$$

It follows that the order η piece of the on-shell action is given by

$$S_E \sim \eta\int \mathrm{d}x_0\,\mathrm{d}\vec{\mathbf{x}}\,\sqrt{g}[\phi^{(0)}(x_0,\vec{\mathbf{x}})]^3 \sim \eta\int \mathrm{d}x_0\,\mathrm{d}\vec{\mathbf{x}}\,\sqrt{g}\Big[\int \mathrm{d}\vec{\mathbf{x}'}\,\frac{x_0^d\phi_0(\vec{\mathbf{x}'})}{(x_0^2+\left|\vec{\mathbf{x}}-\vec{\mathbf{x}'}\right|^2)^d}\Big]^3. \tag{5.12}$$

This implies the three-point correlation function takes the form

$$\langle\phi_0(\vec{\mathbf{x_1}})\phi_0(\vec{\mathbf{x_2}})\phi_0(\vec{\mathbf{x_3}})\rangle = \frac{C}{|\vec{\mathbf{x_1}}-\vec{\mathbf{x_2}}|^d|\vec{\mathbf{x_2}}-\vec{\mathbf{x_3}}|^d|\vec{\mathbf{x_3}}-\vec{\mathbf{x_1}}|^d} \tag{5.13}$$

where C is a constant of order η. This is indeed the correct form for a three-point function of a CFT operator with scaling dimension d, in accordance with the GKPW dictionary. Higher order terms in the perturbation series manifest as corrections to C, maintaining the overall form for the three-point function dictated by conformal invariance.

6 Conclusions

6.1 Beyond the massless scalar

In calculating correlation functions we considered only a single, massless, real scalar field. In this section we will state some results for more general correlation functions without proof. Derivations of these results may be found in the References; they proceed largely in the same manner as the simple examples of the previous two sections.

The mass m of a bulk field determines the scaling dimension Δ of the dual CFT operator. In particular, we have

$$\Delta = \frac{1}{2}(d+\sqrt{d^2+4m^2}) \tag{6.1}$$

which agrees with our result $\Delta = d$ for the massless case. One could also consider the two-point correlation function between two different bulk scalars ϕ_1 and ϕ_2, which is dual to the two-point function for two different CFT operators $\mathcal{O}_1$ and $\mathcal{O}_2$. We then find that the correlation function vanishes unless the dual operators have the same scaling dimension, $\Delta_1 = \Delta_2 \equiv \Delta$, in which case the correlation function is proportional to $\left|\vec{\mathbf{\Delta x}}\right|^{-2\Delta}$. This is exactly what one expects in a conformal field theory.

Just as scalar fields in the bulk source CFT operators on the boundary, spin-one gauge fields source spin-one currents on the boundary. The GKPW dictionary takes essentially the same form for gauge fields:

$$Z_{grav}[A_{0,i}] = \left\langle \exp\left(-\int_{\partial_M}\sqrt{h}\sum_i A_i\cdot J_i\right)\right\rangle \tag{6.2}$$

where A_i are the gauge fields with boundary value $A_{0,i}$ and J_i are the currents. Furthermore, the spin-2 bulk graviton is dual to the energy-momentum tensor. Remarkably, the masslessness of gauge fields and the graviton, which is required by gauge invariance, ensures that the dual currents and energy-momentum tensor are conserved.

6.2 Outlook

In this work we have explicitly computed two- and three-point correlation functions in the AdS/CFT correspondence, introducing Witten diagrams as a calculational tool. This barely scratches the surface of

AdS/CFT; enormous progress has been made in the field over the past twenty years. Moreover, we have made no mention of the importance of the correspondence in string theory, nor its applications to QCD and in condensed matter physics, nor its generalizations such as Kerr/CFT. The interested reader is referred to the numerous reviews in the literature.

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