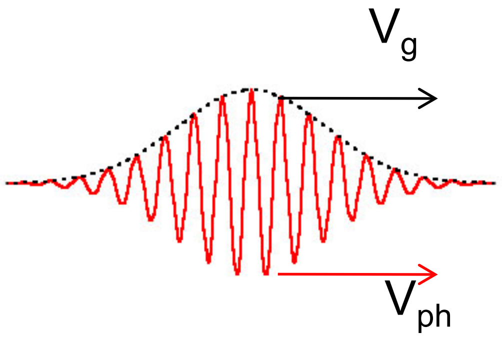
Fiber Optic Networks



$V_g \neq V_{ph}$

Lecture 3

Stephen E. Ralph

Wave Equation
Fiber Types

Spring 2026

Plane Waves (Review)

- The wave equation for linear media
  - Linear means that the material parameters, ε and μ, do not depend on the field E
  - We have also ignored a term proportional to $\nabla\varepsilon/\varepsilon$
  - The units are Volts/m^3

$$\nabla^2\vec{E} = \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2}$$

- The solution to the wave equation is (by separation of variables)

$$\psi(r,t) = \psi_o(r)\phi(t) = \psi_o e^{-ik\bullet r}e^{i\omega t} + cc$$

ψ_o is the amplitude

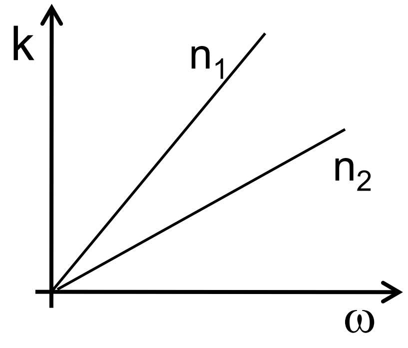
k is the wavevector or spatial frequency [1/cm]

ω is the angular frequency [1/s]

$$|k| = \omega\sqrt{\mu\varepsilon}$$

Plane Wave Dispersion

k = ω/(c/n) : Slope = n/c



$n_2 < n_1$ and n≠n(ω)

- There is a relationship between the spatial and angular frequencies and that relationship depends on the properties of the medium
  - Direction of **k** is the direction of propagation
  - Recognizing that that $\overline{c = 1}/\sqrt{\epsilon\mu}$ lets us write k = ω/(c/n) and k = 2π/(λ/n)
  - Where $n = \sqrt{\epsilon_r}$

Phase Velocity

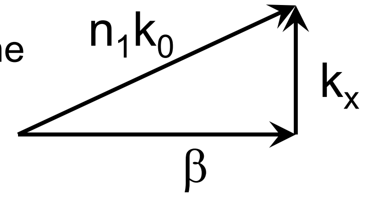
- We examine the phase velocity of a single monochromatic wave, polarized along y and propagating along +x. The field is given by:

$$E_y = E_0 \cos\left(\omega t - \beta x\right)$$

- Examination of constant phase of the argument of the cosine yields

$$\boxed{V_{ph} = \left(\frac{\beta}{\omega}\right)^{-1}}$$

NOTE: For guided waves we typically use the longitudinal part of the wave vector β to distinguish it from unguided plane waves where we typically use k



Dispersion
Phase Velocity = ω/β = c/n

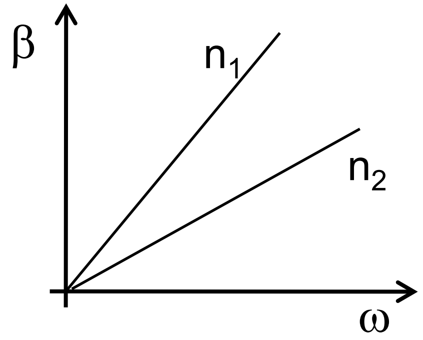
- For lossless dielectrics $\beta = \omega\sqrt{\mu\varepsilon}$ then

$$V_{ph} = \frac{1}{\sqrt{\mu\varepsilon}} = C\frac{1}{\sqrt{\mu_r\varepsilon_r}}$$

Typically (for non-magnetic materials) $\mu_r = 1$

$$V_{ph} = C\frac{1}{\sqrt{\varepsilon_r}} = \frac{C}{n}$$

$n \equiv$ index of refraction



$n_2 < n_1$ and we presume $n \neq n(\omega)$

Group Velocity

- Examine only two frequency components
  - Our two frequencies are:

$$\omega_0 + \Delta\omega \quad \text{freq 1}$$
$$\omega_0 - \Delta\omega \quad \text{freq 2}$$

Then β has two values

$$\beta_0 + \Delta\beta \quad \text{for 1}$$
$$\beta_0 - \Delta\beta \quad \text{for 2}$$

Frequency 1 after propagation:

$$E_y^{(1)} = E_0 \cos\left[\left(\omega_0 + \Delta\omega\right)t - \left(\beta_0 + \Delta\beta\right)x\right]$$

Frequency 2 after propagation:

$$E_y^{(2)} = E_0 \cos\left[\left(\omega_0 - \Delta\omega\right)t - \left(\beta_0 - \Delta\beta\right)x\right]$$

- Find the total field by superposition

$$E_y = E_y^{(1)} + E_y^{(2)}$$

Write as a product using

$$2\cos(x)\cos(y) = \cos(x+y) + \cos(x-y)$$

$$E_y = 2E_0 \cos\left(\omega_0 t - \beta_0 x\right)\cos(\Delta\omega_0 t - \Delta\beta x)$$

What we might expect from a single frequency

A very slow modulation since $\Delta\omega$ is small

Group Velocity

- The phase velocity of the underlying high frequency ω_0 travels at

$$v_p = \frac{\omega_0}{\beta_0}$$

- The group velocity is the speed at which the envelope or pulse travels
- In this case, the pulse envelope is described by $\cos(\Delta\omega t - \Delta\beta x)$
- Follow the same logic used to determine phase velocity in order to compute group velocity

$$\Delta\omega t - \Delta\beta x = constant$$

$$x(t) = \frac{\Delta\omega t}{\Delta\beta} + constant$$

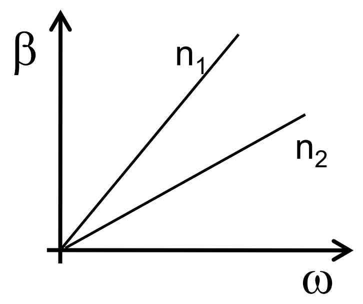
$$v_g = \frac{dx}{dt} = \frac{\Delta\omega}{\Delta\beta}$$

- Let $\Delta\omega \to 0$

$$v_g = \Delta\omega \to 0 \, \frac{\Delta\omega}{\Delta\beta} = \frac{d\omega}{d\beta}$$

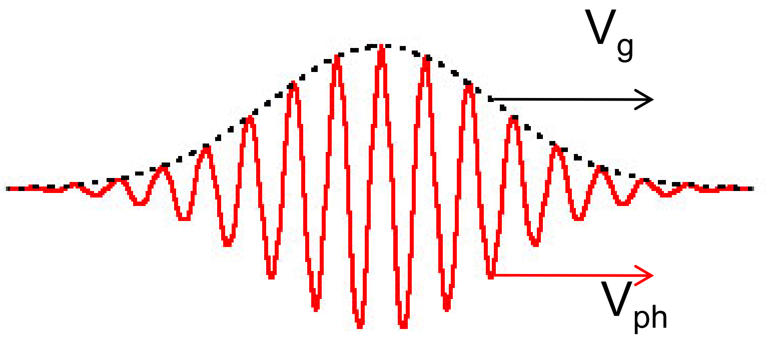
Phase and group velocity are generally not equal

Wave Dispersion
Group Velocity = $d\omega/d\beta$ = 1/slope



$n_2 < n_1$ and we presume $n \neq n(\omega)$

Group Velocity (Review)



$V_g \neq V_{ph}$

The group velocity is also frequency dependent
This results in group velocity dispersion or GVD

- We can define a group index N_g

$$V_g = \frac{d\omega}{d\beta} = \left[\frac{d\beta}{d\omega}\right]^{-1} = \left[\frac{d}{d\omega}\left(\frac{\omega n(\omega)}{c}\right)\right]^{-1} = \left[\frac{n}{c} + \frac{\omega}{c}\frac{dn}{d\omega}\right]^{-1}$$

$$= \frac{1}{\frac{n}{c} + \frac{\omega}{c}\frac{dn}{d\omega}}$$

$$\equiv \frac{c}{N_g} \text{ where } N_g \text{ is the group index}$$

$$\text{and } N_g \equiv n + \omega\frac{dn}{d\omega}$$

In using $\beta(\omega)$ we have not identified the mechanism of the frequency dependence. Therefore these results for phase and group velocity are valid for all mechanisms that impact $\beta(\omega)$

Plane Wave Propagation

- The wave equation for linear media

$$\nabla^2 \vec{E} = \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

- The solution to the wave equation by separation of variables

$$\psi(r,t) = \psi_0(t)\phi(t) = \psi_0 e^{j\vec{k}\cdot\vec{r}} e^{j\omega t} + cc$$

- The previous solution has assumed that the reduced wave equation applies

- The reduced wave equation applies in all linear cases
  - That is the dielectric constant behaves in a linear fashion and hence each of the distinct frequency components which make up the optical wave can be treated independently in the frequency domain

- Taking out time dependence

$$\left(\frac{\partial^2}{\partial z^2} + \frac{\omega^2}{c^2}\epsilon(\omega)\right)E(z,\omega) = 0$$

  - Solutions can have the form

$$E(\omega, z_2) = E(\omega, z_1)e^{\boxed{-j\beta(\omega)(z_2 - z_1)}}$$

"electromagnetic propagator"

  - Where

$$\beta^2(\omega) = \epsilon(\omega)\frac{\omega^2}{c^2}$$

$$\beta(\omega) = \beta_r(\omega) + j\alpha(\omega)/2$$

Plane Wave Propagation

$$\beta^2(\omega) = \epsilon(\omega)\frac{\omega^2}{c^2}$$

- This is a very powerful result: We only need to know $\epsilon(\omega)$ to find the phase of the various spectral components. This allows us to reconstruct the pulse at any position by inverse Fourier Transform

- For guided waves we require $\beta(\omega)$ or equivalently the effective index of refraction of the guided mode

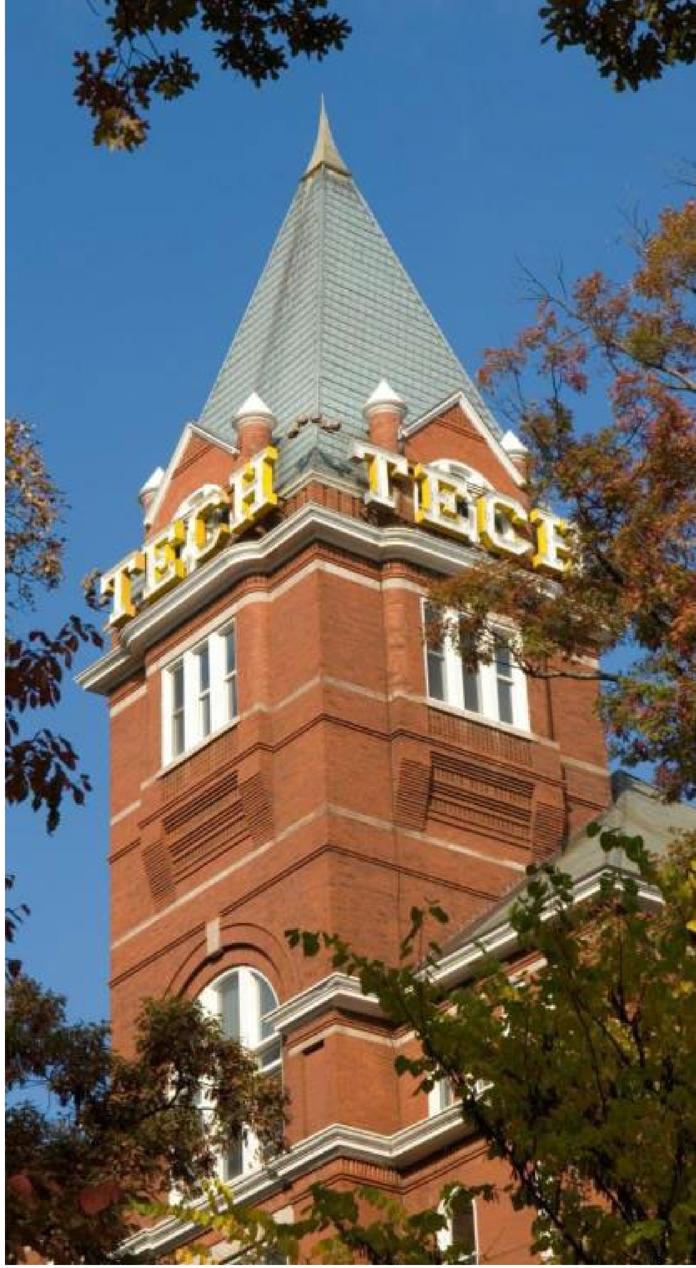
- The method of solution is now clear:

1) Determine $E(\omega, z_1)$ by FT of the initial known temporal shape (implicit is that you know the underlying phase as well)

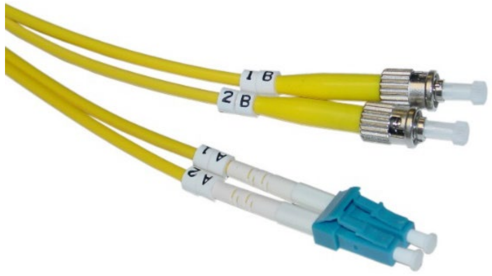
2) Determine the new (complex) spectrum by multiplying the input spectrum $E(\omega, z_1)$, by the complex propagator

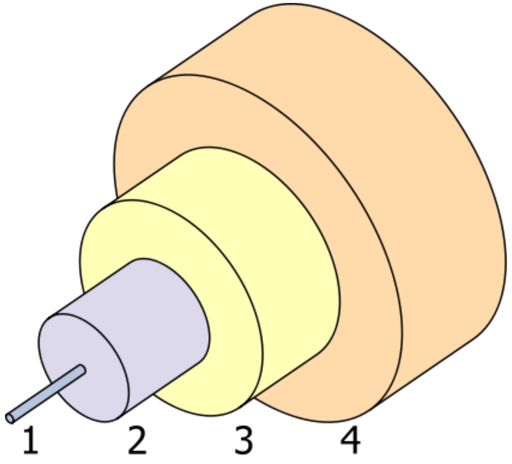
3) Determine the new output pulse shape by IFT of the new spectrum

Optical Fiber: Structure, Ray Propagation, Manufacture

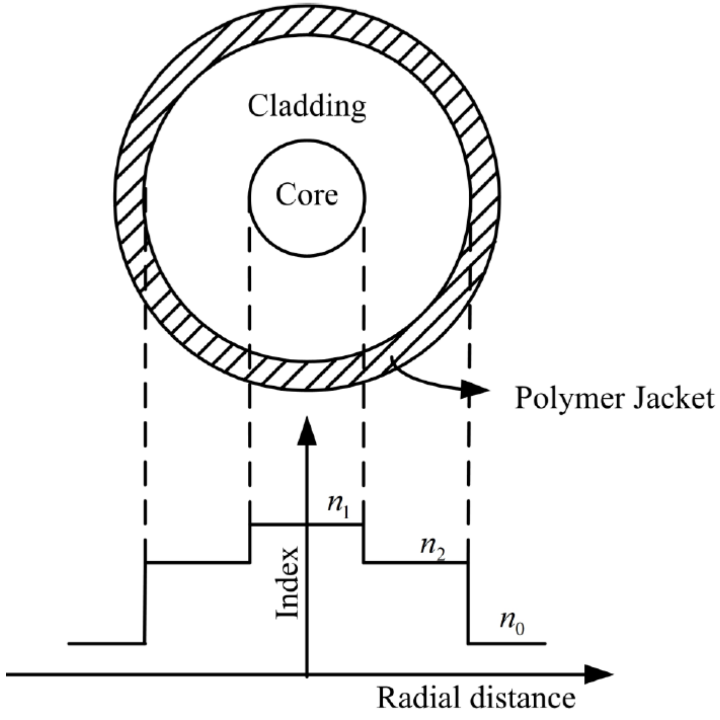


Optical Fiber

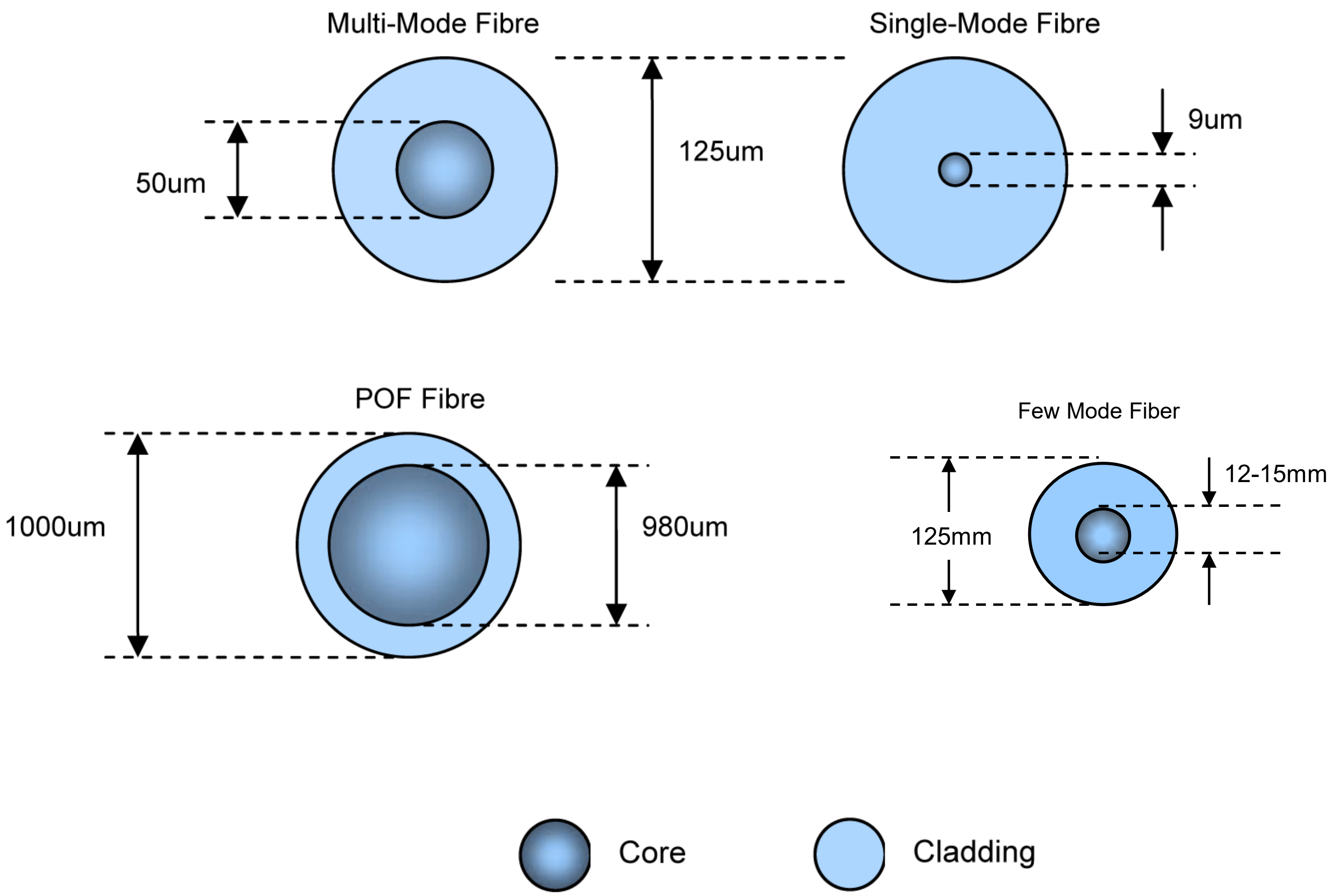




1. Core
2. Cladding
3. Buffer
4. Jacket

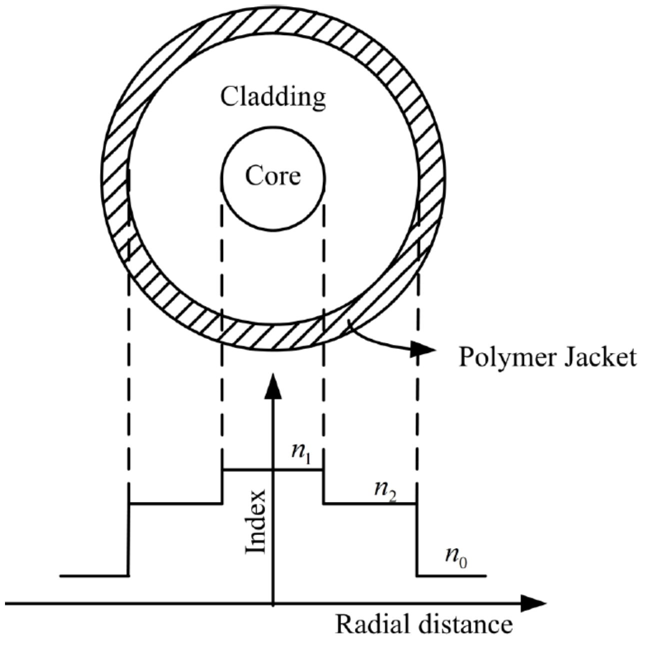


Types of Fiber: Core Size



Types of Fiber: Composition

- Glass or plastic or both or neither
  - Plastic core and cladding: PCP or POF
  - Glass core: plastic cladding PCS, glass cladding SCS
  - Air core/air cladding: Microstructured fiber
- Plastic core
  - Somewhat more flexible
  - lower cost due to easier "termination"
  - Higher attenuation than glass fiber
  - Higher mode coupling
- Glass core
  - Lowest attenuation; SCS *dominates telecom*
- Microstructured fiber
  - Array of holes in silica
  - Wide control of propagation characteristics
  - Current area of research



Types of Fiber: Core Size

- Fiber core size
  - Single mode fiber: SMF
    - Core size is somewhat larger than λ
  - Multimode fiber: MMF
    - Core size is much larger than λ
    - 100s to 1000s of modes
  - Few mode fiber: FMF ***NEW***
    - Core size is somewhat larger than λ
    - 2 to 12 modes

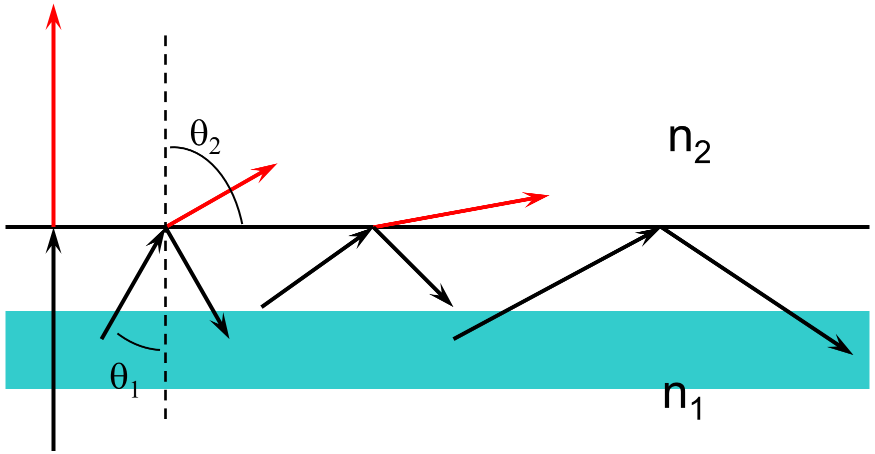
- Waveguide Modes
  - The boundary conditions on the optical wave require the wave to propagate only in certain allowed *transverse modes*
  - Smaller fiber cores allow fewer transverse modes
  - Number of modes is proportional to

$$(a/\lambda)^2$$

  - where *a* is the core radius, λ is the wavelength in vacuum
  - valid for large numbers of modes

Optical Confinement Mechanism: TIR

- Optical energy Confinement by *total internal reflection* (TIR)
  - The boundary condition, continuous tangential E field and constant phase establishes the angle of refraction when a wave is transmitted across a dielectric interface
  - Can also be derived from Fermat's principle of *least time (see Feynman QED)*
  - Independent of polarization
  - When the incident material index is larger than the transmitted index there is an angle at which the condition cannot be met with energy propagating away in the second material
  - **TIR means ALL energy is reflected, not just most, ALL: it is a lossless reflection**



Boundary condition:

$E_{tangental}$ is continuous across boundary

$[E_{normal}$ is discontinuous across boundary$]$

yields Snells Law of refraction:

$$n_1 Sin(\theta_1) = n_2 Sin(\theta_2)$$

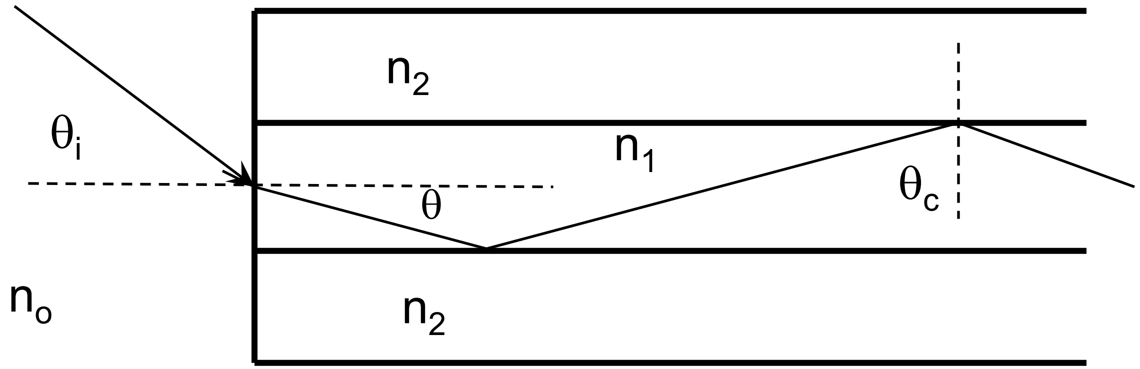
At a critical angle θ_2 is 90°

$$Sin(\theta_c) = \frac{n_2}{n_1} Sin(\theta_2)$$

for angles greater than θ_c all power is reflected back into the first medium

Numerical Aperture NA

- The NA is $n_o \sin(\theta)$ where θ is the maximum external angle (half angle) of an *accepted* ray into the fiber
  - Internal to the core the max angle is the critical angle
  - Telecom GRIN MMF has a NA of ~0.22



TIR considerations: $Sin(\theta_c) = \frac{n_2}{n_1}$ (2)

$$\cos\theta_c = \sqrt{1 - \sin^2\theta_c} \quad (3)$$

$$n_o \sin\theta_i = n_1 \sin\theta$$

$$= n_1 \cos\theta_c \quad \text{since } \theta = \frac{\pi}{2}$$

$$\sin\theta = \frac{n_1}{n_o}\cos\theta_c \quad (1)$$

(2) $\Rightarrow$ (3) $\Rightarrow$ (1) $\Rightarrow$ $n_o \sin\theta_i = \sqrt{n_1^2 - n_2^2}$

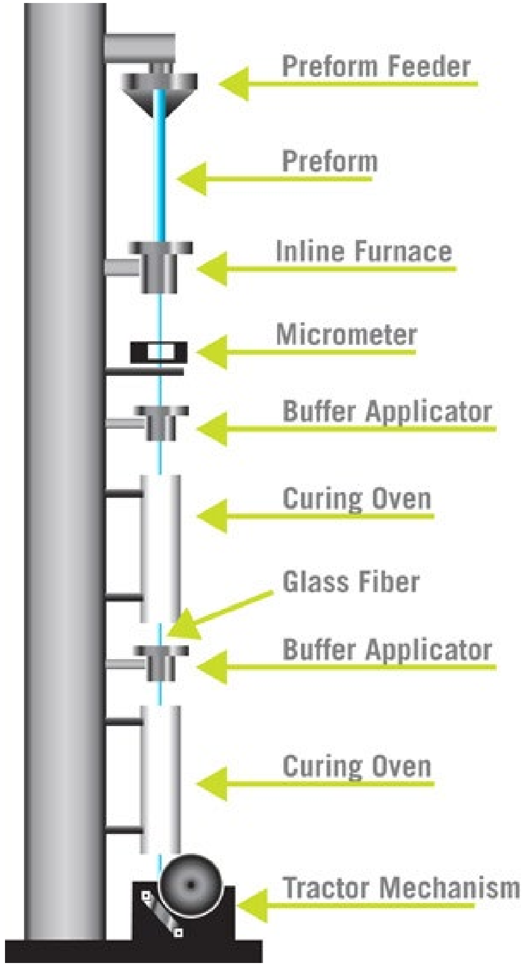
$$NA = \sqrt{n_1^2 - n_2^2}$$

The Numerical Aperture can also be written as

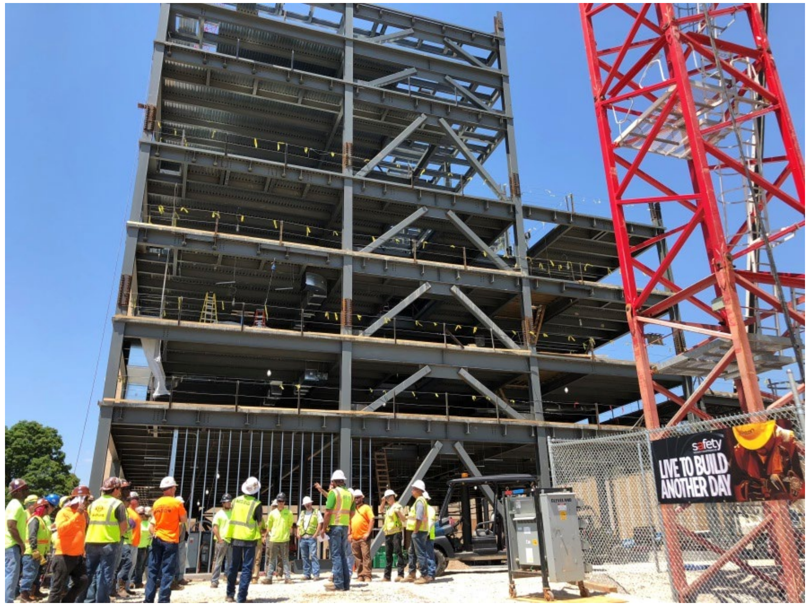
$$\mathrm{NA} \approx \mathrm{n}_1 (2\Delta)^{1/2} \text{ where } \Delta = (n_1 - n_2)/n_1$$

Glass Fiber Manufacturing

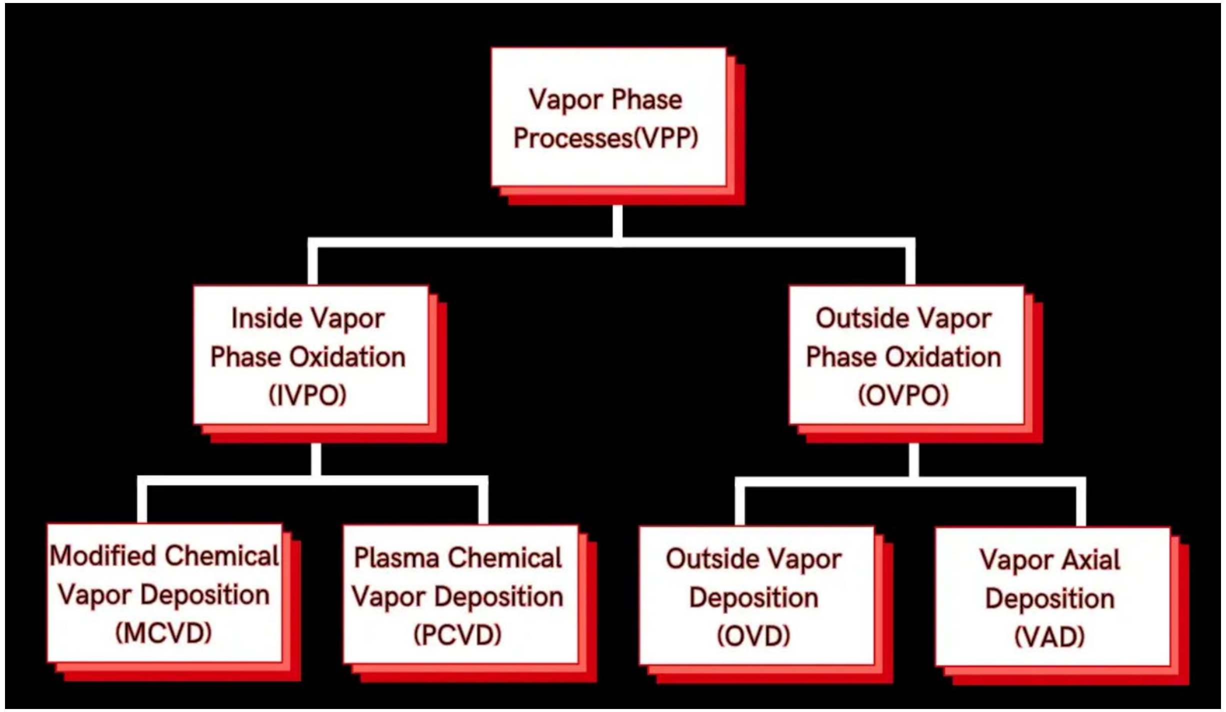
- Optical fiber is drawn from a preform
  - Preform is ~2 meter by ~2 cm in diameter (some are larger)
  - Preform refractive-index profile is a larger, scaled version of the required profile in the finished fiber
  - GeO_2 or P_2O_5 are used to increase the refractive index i.e. the core is "doped" the cladding is typically pure silica
  - The index of refraction is wavelength dependent
  - B_2O_3 and F can be used in the cladding to decrease the refractive index
  - Some fiber uses pure silica core to reduce loss



Ref: FIS Instruments



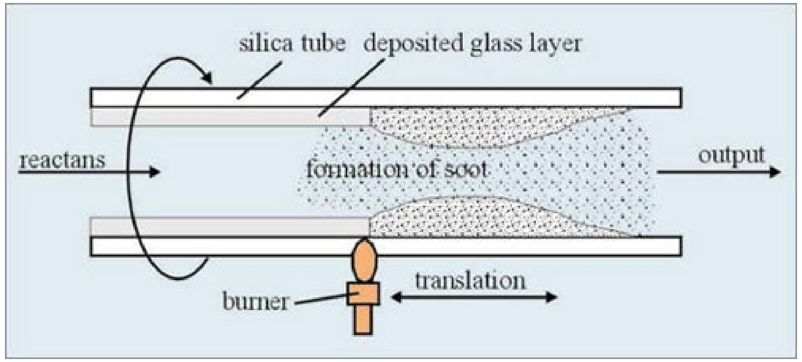
Preform Fabrication Methods



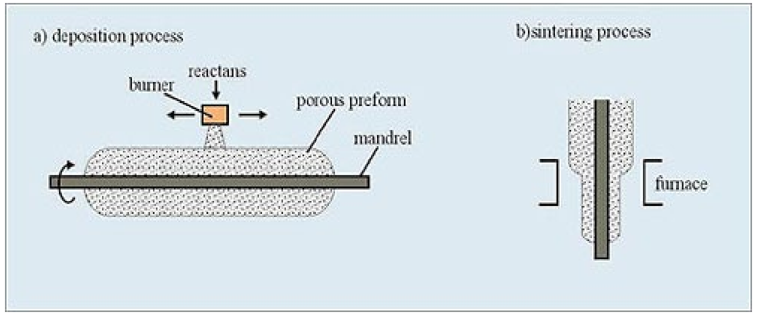
- MCVD offers flexibility for specialty fiber R&D with lower material efficiency
- PCVD enhances deposition rate and achieves radial dopant profiles
- OVD allows large preform dimensions with thick deposits
- VAD allows continuous preform growth for maximum productivity

Preform Fabrication

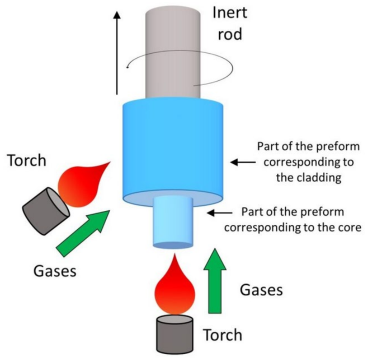
- Preform fabrication (1974 Bell Labs: improves traditional Chemical Vapor Deposition (CVD) methods)
  - Modified chemical vapor deposition (MCVD) ***OFS Inc.***
    - The cladding and core are deposited on the inside of a fused silica tube
    - The refractive-index profile is controlled by adding the dopants as deposition progresses
    - The tube is then heated and collapsed into a solid rod



  - Outside vapor deposition (OVD) ***Corning Inc.***
    - Layers are deposited on the outside of a rotating mandrel using flame hydrolysis
    - The mandrel's thermal expansion coefficient is larger than that of the preform, so the mandrel drops out after cooling
    - Without care this process is more susceptible to producing a central dip in the refractive index profile

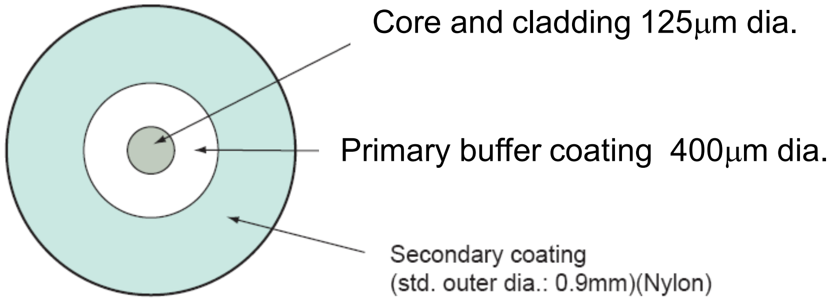


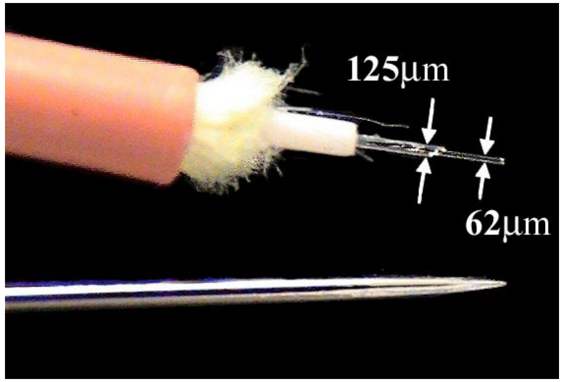
  - Vapor Axial Deposition (VAD) – Fabricating long preforms
    - Vapor Axial Deposition is widely used for large-scale production of preforms
    - Unlike OVD, the deposition occurs axially enabling continuous single-step preform growth
    - The VAD process enables the fabrication of large preforms suitable for drawing very long lengths of optical fiber, up to 250 km



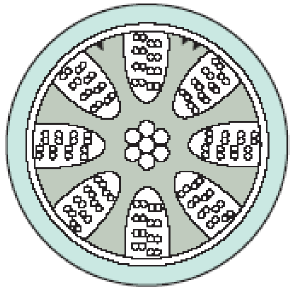
Fiber Manufacturing and Cabling

- Primary Coating
  - Deposited during draw process
    - Protection from abrasion
    - Surface flaws cause cracks to form
    - Protection from H_2O
    - Incursion of H_2O increases loss
    - Plastic
    - Strips “cladding modes”
    - Prevents inter fiber coupling

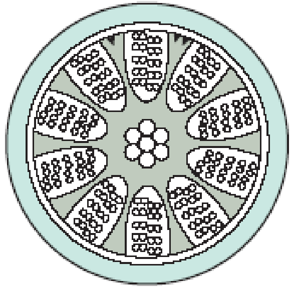




- Cable
  - Jacket contains one to many fibers
    - Mechanical strength added
    - Protection from H_2O
    - Protection from environmental chemicals



128 core type



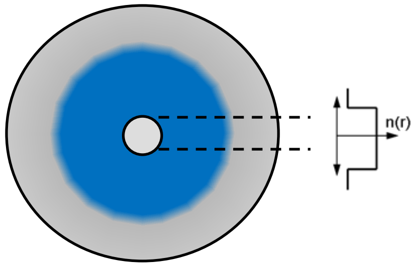
200 core type

Fiber Parameters

- Numerical aperture
  - Measure of the acceptance cone
  - More important for multimode fibers and incoherent sources
- Attenuation
  - ~0.20 dB/km at 1550nm
  - Best new commercially available fiber with pure silica core is ~0.18dB/km
  - Best reported loss is ~0.15 dB/km
  - ~3 dB/km at 850nm
- Birefringence
  - Index of refraction depends on E-field orientation
- Polarization-maintaining fiber
  - Sufficient birefringence purposely built in to maintain E field orientation
  - Needed to connect components, not used for transport fiber
- Dispersion
  - Intermodal dispersion; Applies to multi and few mode fibers
    - Typically the bandwidth limiting mechanism
  - Chromatic dispersion
    - Typically the bandwidth limiting mechanism for single-mode fibers
  - Polarization-mode dispersion (stochastic process)
    - Group Velocity depends on E field orientation
    - Leads to pulse spreading
    - Limiting mechanism for dispersion managed links
- Optical nonlinear coefficients n_2
  - Index of refraction is intensity dependent $n= n+I\, n_2$

Types of Single-Mode Fiber

- SSMF (standard SMF) or SMF-28 (Corning brand name)
  - Most widely deployed, introduced in 1986
  - Also known by the Standard which defines the specification: ITU-T **G.652** fibers
  - Also known as "non-dispersion shifted fiber" N-DSF
  - Zero-dispersion wavelength lies between 1300 nm and 1324 nm
  - Low water peak at 1400nm version termed ITU-T G.652.C
    - AKA SMF-28e or AllWave Fiber (OFS brand name)
  - Index profile is a simple step-index structure
  - Modal area is ~80μm^2
- SMF/DS (dispersion shifted: DSF) ITU-T G.653
  - For single channel operation at 1550 nm, now uncommon
  - Zero dispersion engineered to be in the region between 1500nm to 1600nm
  - More complex index profile
- SMF-LS TrueWave Fiber (OFS brand name) (nonzero dispersion shifted: NZ-DSF) ITU-T G.655
  - For WDM operation in the 1550 nm region
  - Small, but non zero dispersion near 1550nm
  - Insures adjacent channels have different group velocity minimizes nonlinear interactions between channels

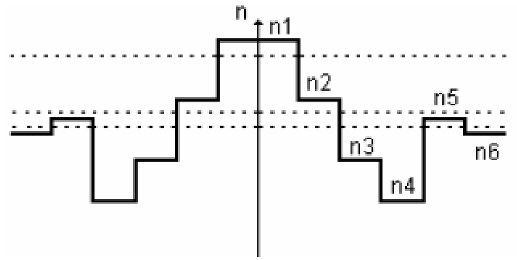


Step index profile

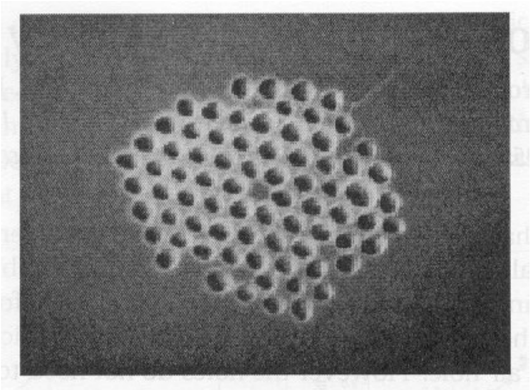
International Telecommunication Union (ITU-T), is a global standardization body for telecommunication systems and vendors

Types of Single-Mode Fiber

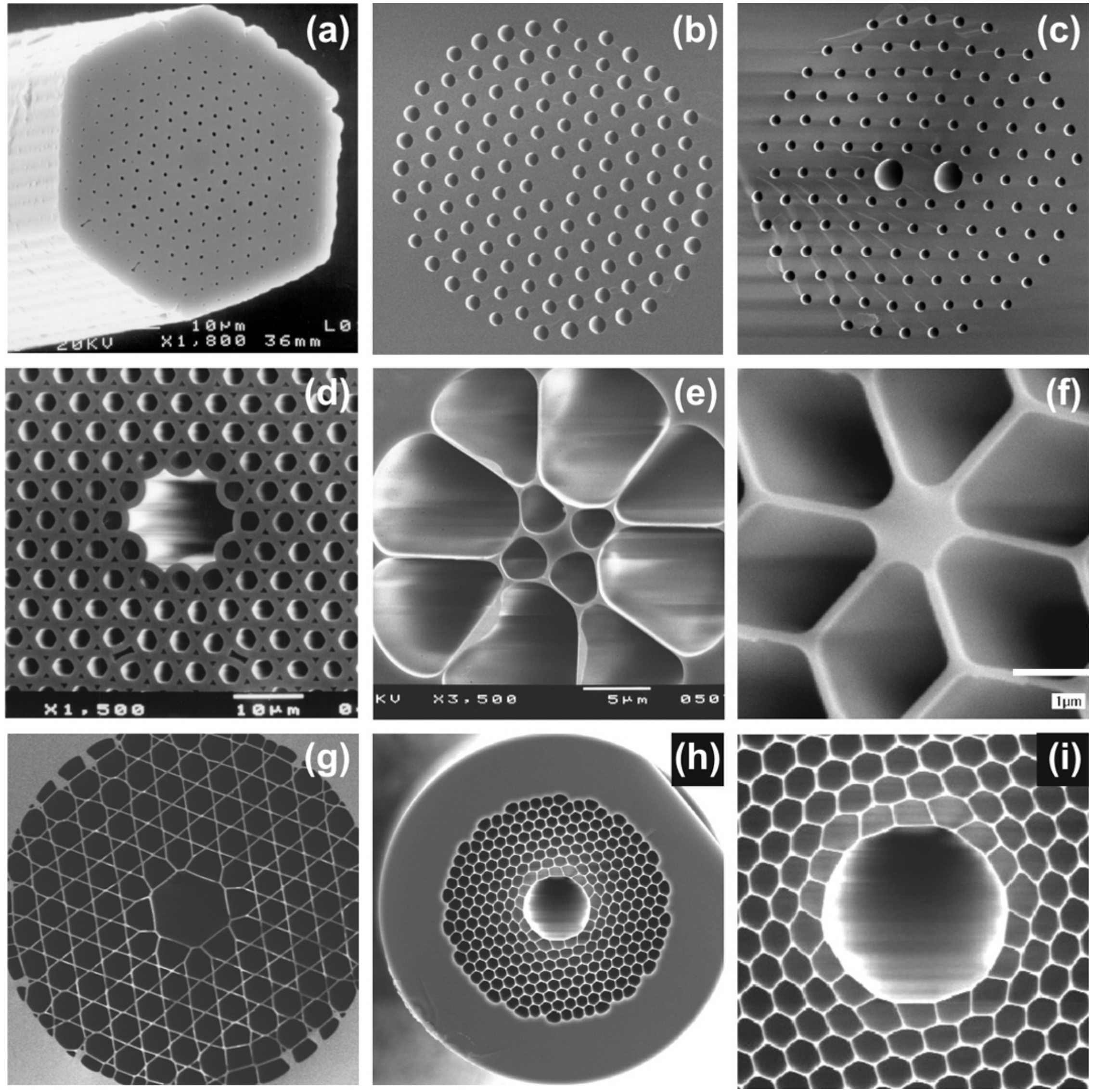
- LEAF (large effective area fiber) Corning brand name
  - Latest generation fiber developed in mid 90s
  - For better performance with high-capacity DWDM systems
- Ultra Large Area Fiber (ULAF OFS brand name)
  - New type of fiber with complex index profile
  - Allows larger mode size to reduce nonlinear effects but maintains single mode character
  - Challenging to control bending losses
  - Modal area ~120μm^2



- Microstructured Fiber
  - A NEW CLASS OF OPTICAL FIBER
    - An array of holes surrounding a solid glass core
    - The fiber is generally undoped
    - Losses are larger than conventional fiber
  - Confinement occurs by one of two methods
    - Effective Index: air spaces lower the effective index
    - Interference effects due to the periodicity of the array of holes(photonic crystal fiber PCF)
  - Allows for control of core size and dispersion



Photonic Crystal Fiber



a) First Working PCF
b) Single mode solid core
c) Polarization preserving solid core
d) First hollow core PCF
e) PCF made of SF-6
f) Highly nonlinear, core size 800nm
g) Hollow core with Kangome Lattice
h) Hollow core PCF
i) Detail of h)

Ref: Phillip St. John Russell in Optical Fiber Telecommunications IV, ed. by Kaminow, Li and Wilner