

Fiber Optic Communications

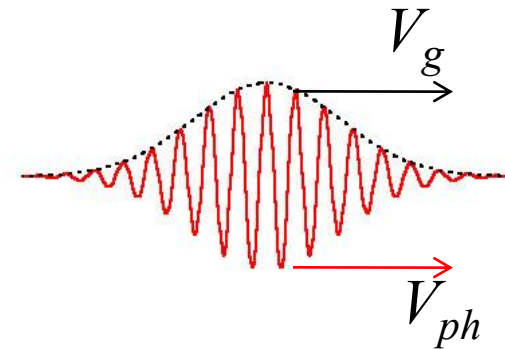


Lecture 4

Stephen E. Ralph

Fiber Modes
Multimode Fiber

Spring 2026



Homework Due Wednesday 4 Feb 2026

Review basic E&M, Chapter 1 Kumar and Deen

Plane waves, relationships between wavelength, velocity, k-vector

Phase and group velocity, physical meaning and mathematical definition

Read Chapter 2 and 3 Kumar and Deen

Read Journal articles

- Fiber-optic transmission and networking: the previous 20 and the next 20 years,
 - P. J. Winzer, D. T. Neilson, and A. R. Chraplyvy
- Future Optical Networks
 - Michael J. O'Mahony, Christina Politi, Dimitrios Klonidis, Reza Nejabati, and Dimitra Simeonidou, JLT V24, 2006

1) Kumar and Deen **Problems 1.9**,

2) Kumar and Deen **1.15**

3) Pulse Propagation

Compute and plot the temporal shape and spectral content of a single optical pulse. Assume the carrier is essentially monochromatic at center wavelength 1552nm (plot both amplitude and phase, use a frequency axis centered on the carrier and denote frequency offset from carrier in units of GHz). The pulse should have the following characteristics:

- a) Single raised cosine pulse with pulse width corresponding to 32 Gbps, and excess bandwidth factor of 0.1. The FWHM should be = symbol slot T .
- b) Roughly estimate the temporal spread of the pulse after 10km and 100km in standard fiber. (Suggestion: estimate Δf to be $=1/T$).
- c) The pulse in part a) after propagating through 10km and 100km of standard fiber. Neglect loss ($\alpha=0$) and nonlinear (intensity dependent) effects. Plot the temporal shape. Indicate both real time units (ps) and a bit slot unit. Choose a time scale appropriate for the anticipated temporal spread i.e. at least 10ns for the 1000km case. You also need sufficient temporal resolution, at least 20 points in the original pulse shape or ~ 1 ps.

Also plot the phase in the frequency domain. Show only the quadratic phase behavior (i.e. remove any linear phase behavior). Be sure to unwrap any phase discontinuities.

4) A Gaussian pulse (100ps FWHM) is launched into 80km of SSMF fiber with initial power of +15 dBm. The central wavelength of the pulse is 1550nm. State clearly any assumptions.

- a) If you measure the power at the end of the fiber, what value are you likely to observe? (Answer both in mW and dBm)
- b) What is the loss parameter α ?
- c) What is the pulse width (FWHM) at the end of the fiber?
- d) At the input to the fiber what is the spectral content ($\Delta\omega$ FWHM) if the pulse initially unchirped?
- e) At the output of the fiber the pulse is chirped. Is it positively or negatively chirped? Does this mean the “blues” are ahead of or behind the “reds”? What is the spectral content ($\Delta\omega$ FWHM)?
- f) What is the value of the chirp parameter at the output of the fiber? Write an expression for the pulse shape which includes the chirp parameter.

Waves Summary

Wave Equation(1D case)

$$\frac{d^2 E_x}{dz^2} = \mu\epsilon \frac{d^2 E_x}{dt^2}$$

$$\nabla^2 E + n^2(\omega)k_o^2 E = 0$$

3D with $E = E e^{i\omega t}$

Solution

$$E_x(z, t) = E_{xo} \cos(\omega t - kz)$$

where $\omega = 2\pi f$

$$k = \frac{2\pi}{\lambda}$$

Examination of constant phase of the argument of the cosine solution yields the phase velocity

$$V_p = \frac{\omega}{k}$$

Examination of the cosine solution in the wave equation shows

$$\frac{\omega}{k} = \frac{1}{\sqrt{\mu\epsilon}}$$

Thus

$$V_p = \frac{\omega}{k} = \frac{1}{\sqrt{\mu\epsilon}} = \frac{C}{\sqrt{\mu_r(\omega)\epsilon_r(\omega)}}$$

Known as the dispersion relation, i.e. how k (space) relates to ω (time)

Phase velocity in vacuum

$$V_p = \frac{\omega}{k_o} = \frac{1}{\sqrt{\mu_o\epsilon_o}} = 2.99 \times 10^8 \text{ m/s} = C$$

$$\frac{\omega}{k_o} = \frac{2\pi f}{2\pi/\lambda_o} = f\lambda_o = C$$

Electromagnetic Modes

MODES: *Stable electric field distributions in waveguides, optical resonators or in free space that satisfy Maxwell's equations and boundary conditions*

WAVEGUIDE MODES: *Solution of the wave equation that satisfies the boundary conditions and has a spatial distribution does not change with propagation*

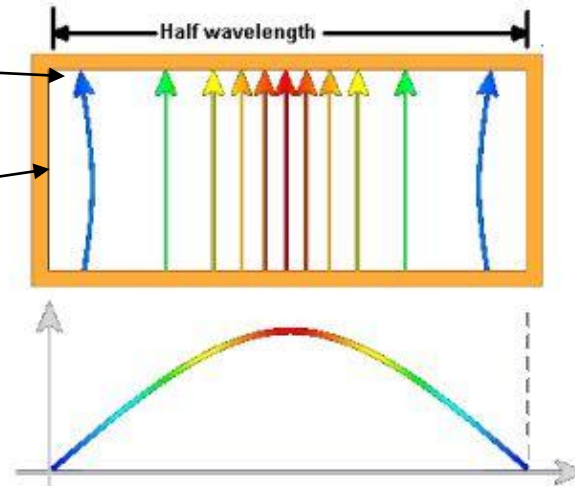
■ Confined structures: Waveguide Modes

- For waveguides, the stable condition for a mode requires that the shape of the complex amplitude profile in the transverse dimensions remains exactly constant
- For metal waveguides the E-field must be zero at the metal boundary

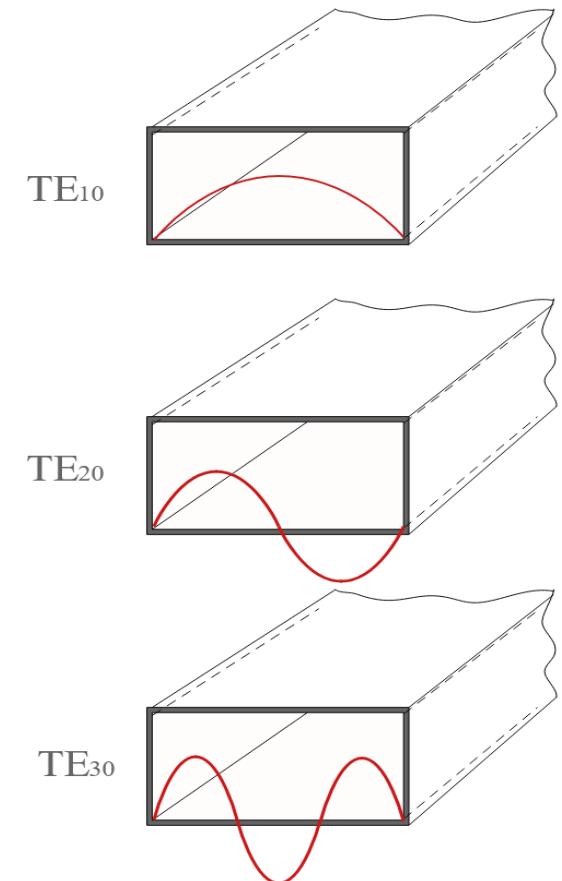
TE mode E field in a waveguide

Normal E-field starts and terminates with charge

Tangential E-field is zero at conductor

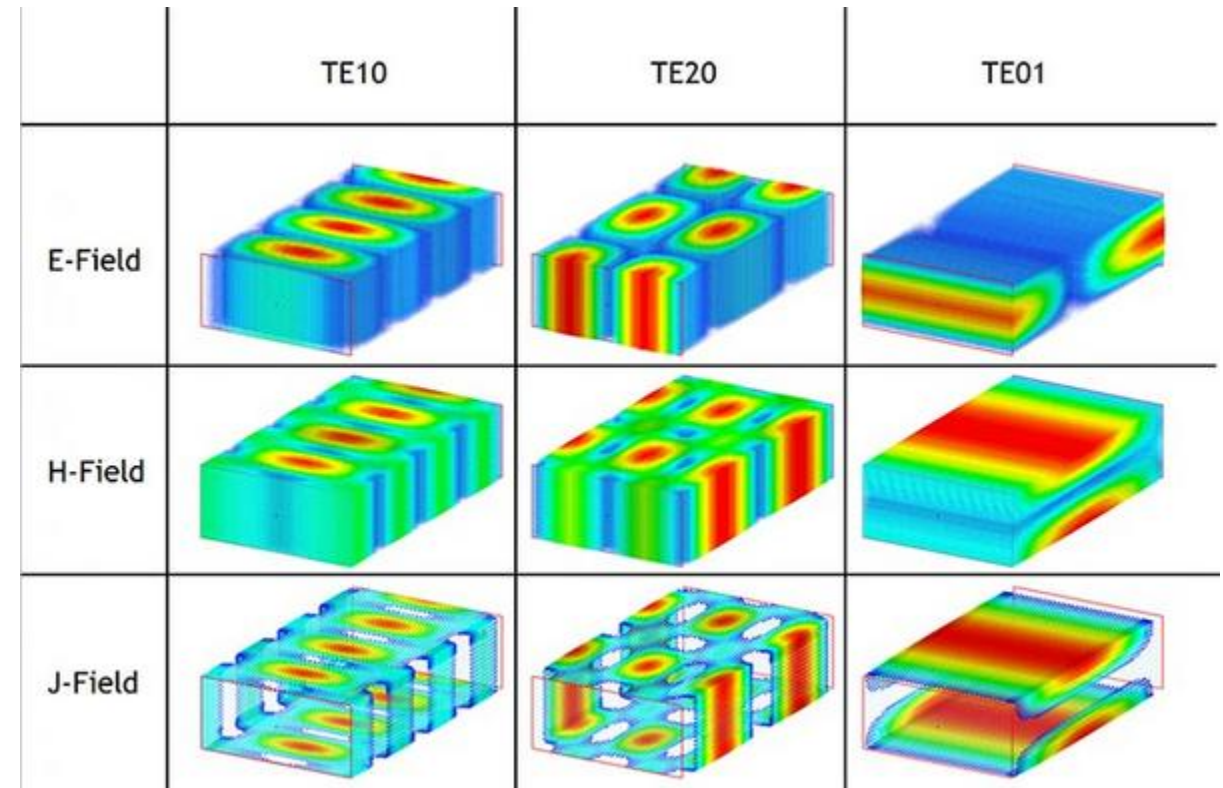


Metal Waveguide



Mode Classification

- Transverse electromagnetic (TEM) modes
 - Neither electric nor magnetic field in the direction of propagation
- Transverse electric (TE) modes
 - No electric field in the direction of propagation. These are sometimes called H modes because there is only a magnetic field along the direction of propagation (H is the conventional symbol for magnetic field)
- Transverse magnetic (TM) modes
 - No magnetic field in the direction of propagation. These are sometimes called E modes because there is only an electric field along the direction of propagation
- Hybrid modes
 - Non-zero electric and magnetic fields in the direction of propagation



Rectangular transverse mode patterns TEM(mn)

Electromagnetic Modes

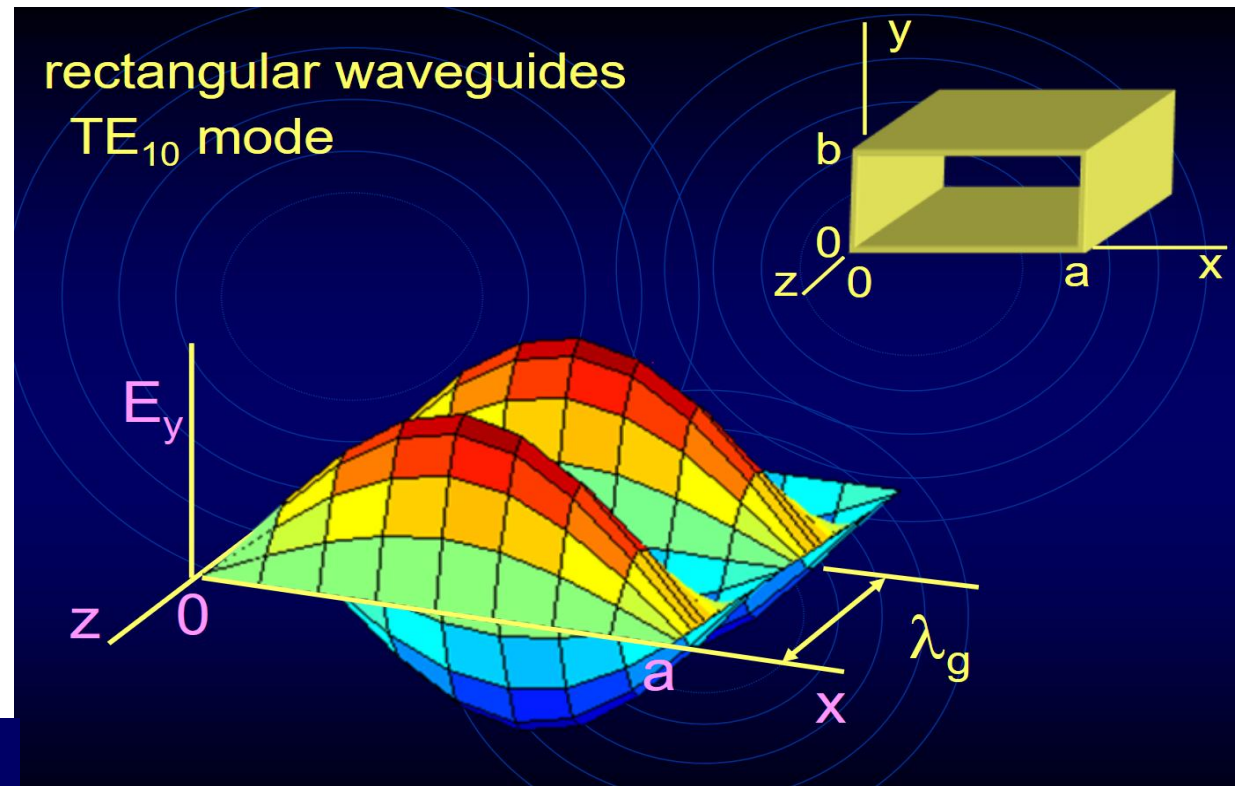
MODES: *Stable electric field distributions in waveguides, optical resonators or in free space that satisfy Maxwell's equations and boundary conditions*

WAVEGUIDE MODES: *Solution of the wave equation that satisfies the boundary conditions and has a spatial distribution does not change with propagation*

■ Confined structures: Waveguide Modes

- For waveguides, the stable condition for a mode requires that the shape of the complex amplitude profile in the transverse dimensions remains exactly constant
- For metal waveguides the E-field must be zero at the metal boundary

Metal Waveguide



Electromagnetic Modes

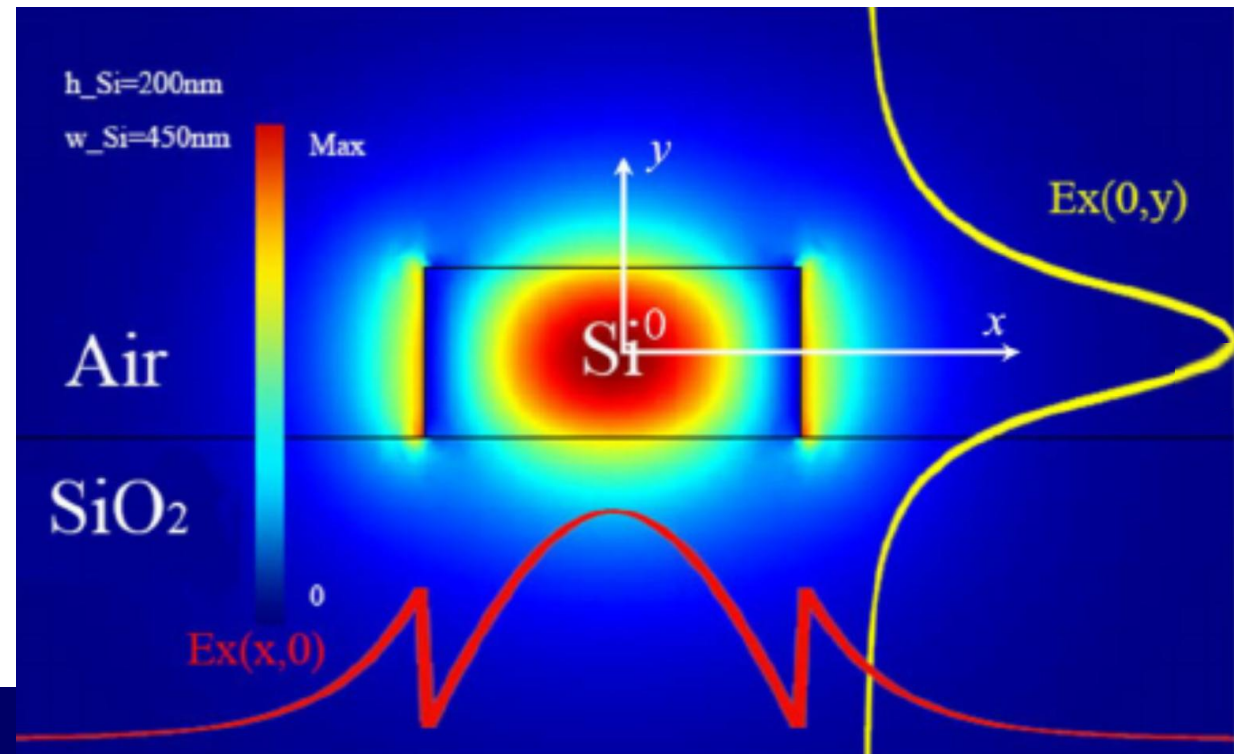
MODES: *Stable electric field distributions in waveguides, optical resonators or in free space that satisfy Maxwell's equations and boundary conditions*

WAVEGUIDE MODES: *Solution of the wave equation that satisfies the boundary conditions and has a spatial distribution does not change with propagation*

■ Confined structures: Waveguide Modes

- For dielectric waveguides, like glass optical fiber, the field and energy exists in the cladding
- For cylindrical dielectric waveguides the modes are described by Bessel functions
- The lowest order mode is approximately Gaussian
- A waveguide has only a finite number of *guided* propagation modes *for a specified wavelength*, the intensity distributions have a finite extent around the waveguide core

Dielectric Waveguide



Electromagnetic Modes

MODES: Stable electric field distributions in waveguides, optical resonators or in free space that satisfy Maxwell's equations and boundary conditions

■ Confined structures: Waveguide Modes

- For cylindrical dielectric waveguides the modes are described by Bessel functions, however the lowest order mode is nearly Gaussian
- A waveguide supports only a finite number of *guided* propagation modes for a specified wavelength and waveguide size

Dielectric Waveguide

Note: Free-space also has stable modes including plane Waves and Gaussian beams

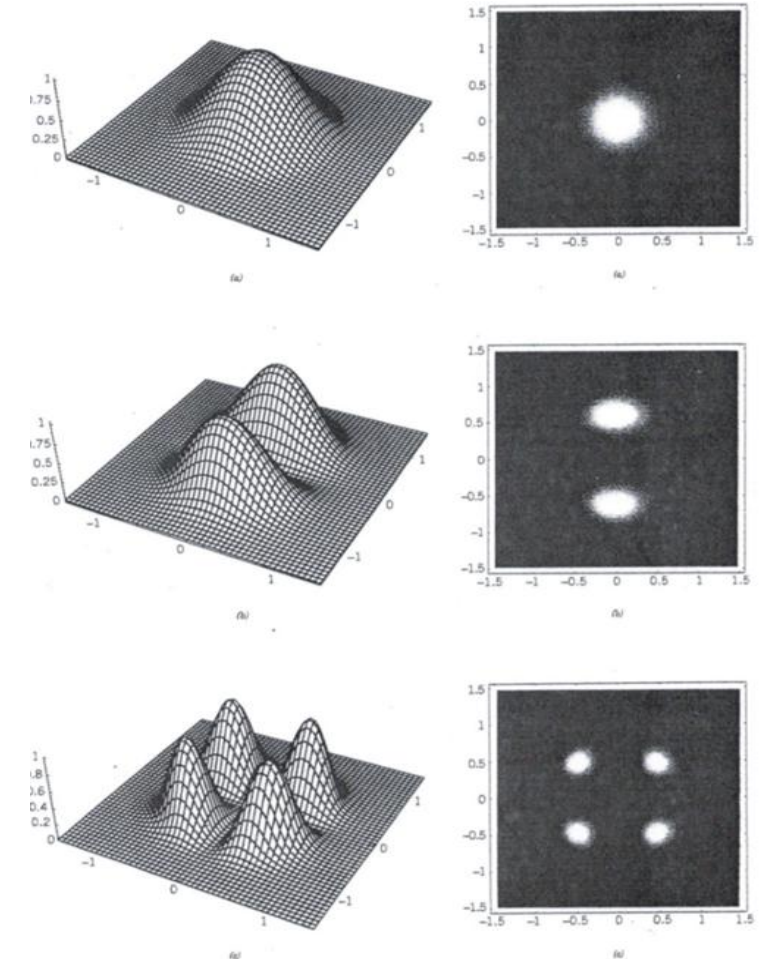
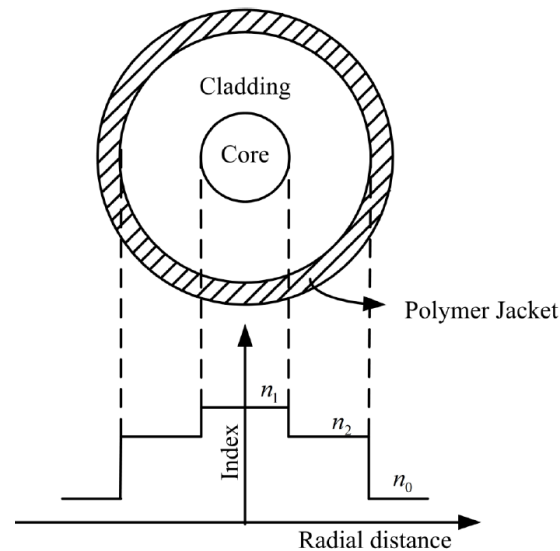
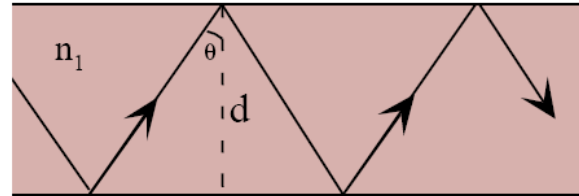


Figure 3.9. Intensity plots for the six LP modes, with $a = 1$. (a) LP_{01} : $u = 2$. (b) LP_{11} : $u = 3$. (c) LP_{21} : $u = 4.5$. (d) LP_{02} : $u = 4.5$. (e) LP_{31} : $u = 5.6$. (f) LP_{12} : $u = 6.3$.

Transverse Resonance Condition

- The transverse resonance condition is an analytical technique used to determine the propagation constants and mode conditions in waveguides
- The effects of the boundary conditions may be viewed as a requirement on the roundtrip phase across the waveguide
- A guided wave must form a self-consistent standing wave pattern in the transverse direction (perpendicular to propagation)
- In the ray picture the wave accumulates phase as it travels across the core
- It also acquires additional phase shifts upon reflection at the cladding boundaries
- TRC: For a mode to exist, the total round-trip phase shift *in the transverse direction* must equal an integer multiple of 2π
 - This restricts the allowed values of k_x and hence only discrete values of β are allowed
 - The TRC is also a dispersion relation
 - Note ϕ are sometimes written as -2Φ



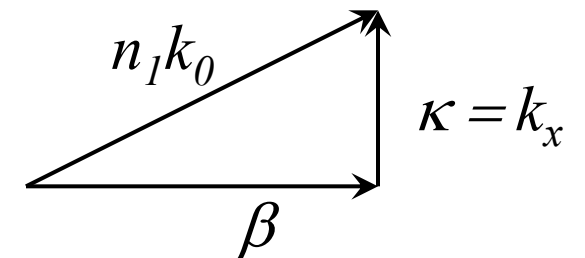
$$2d \left[\frac{2\pi n_1}{\lambda} \cos(\theta) \right] + \phi_{12}(\lambda) + \phi_{13}(\lambda) = 2\pi m$$

$$2d \left[k_{x, \text{film}} \right] + \phi_{12}(\lambda) + \phi_{13}(\lambda) = 2\pi m$$

$$2d \left[\frac{2\pi n_1}{\lambda} \cos(\theta) \right] \quad \text{Phase shift for passing through core}$$

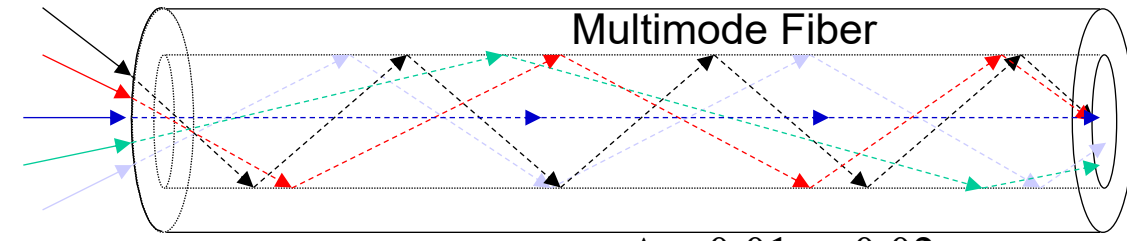
$$\phi_{12}(\lambda) \quad \text{Phase shift due to TIR at core-cladding interface}$$

$$\phi_{13}(\lambda) \quad \text{Phase shift due to TIR at core-cladding interface}$$



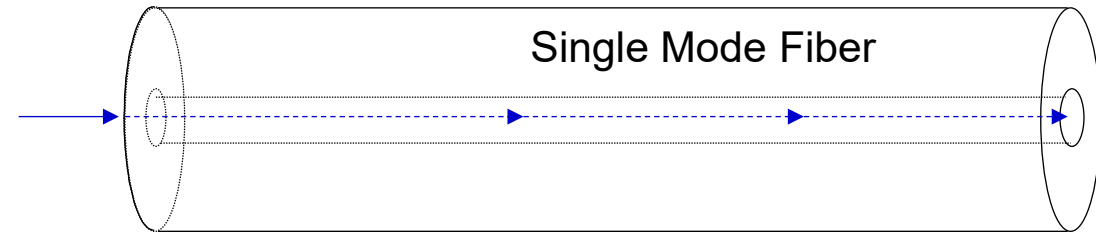
Types of Fiber: Core Size

Ray Picture



$\Delta \approx 0.01$ to 0.02

Core $n_1 = \sim 1.480$ - 1.500
Cladding $n_2 = \sim 1.444$
 $\Delta \approx 0.01$ to 0.03

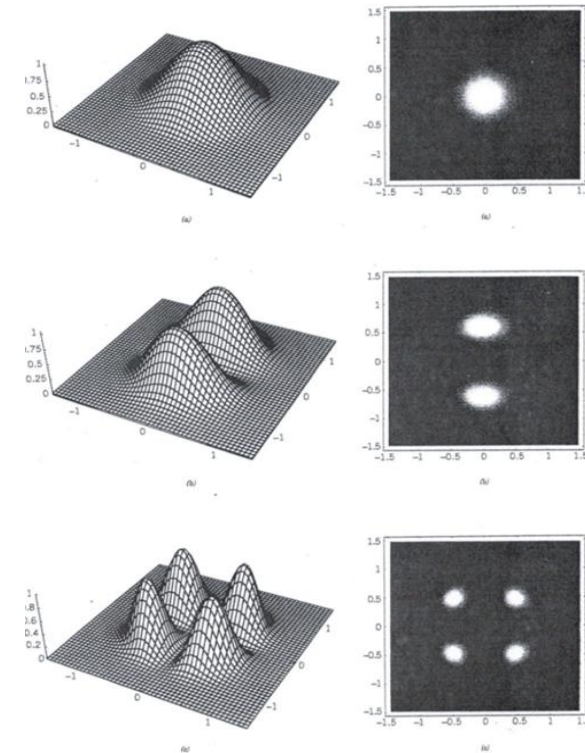


Core $n_1 = \sim 1.450$
Cladding $n_2 = \sim 1.444$
 $\Delta \approx 0.0036$ to 0.004

Index contrast

$$\Delta = \frac{n_1^2 - n_2^2}{2n_1^2} \approx \frac{n_1 - n_2}{n_1}$$

Intensity Profiles



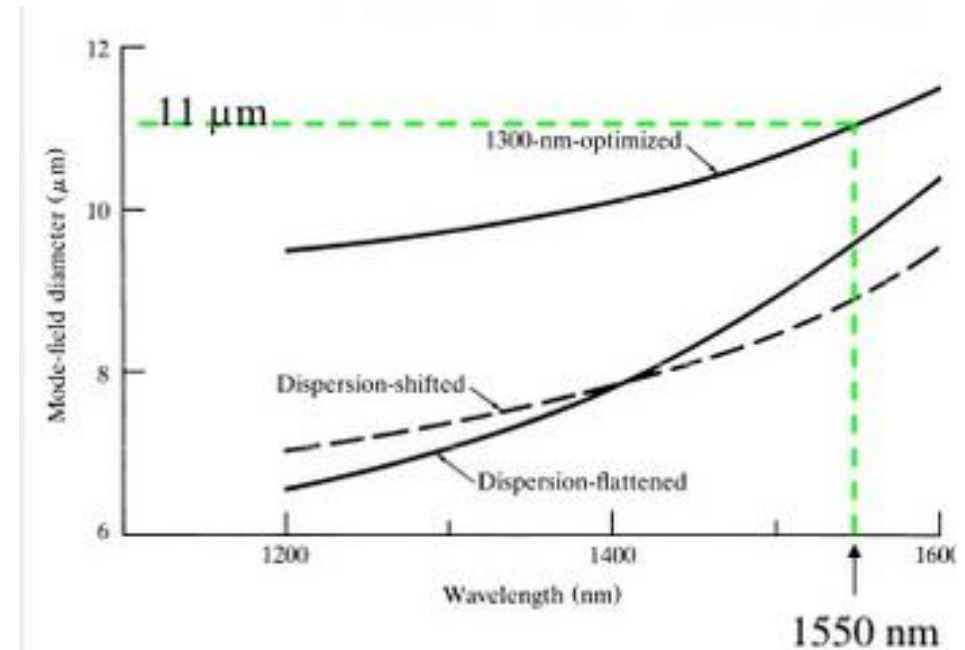
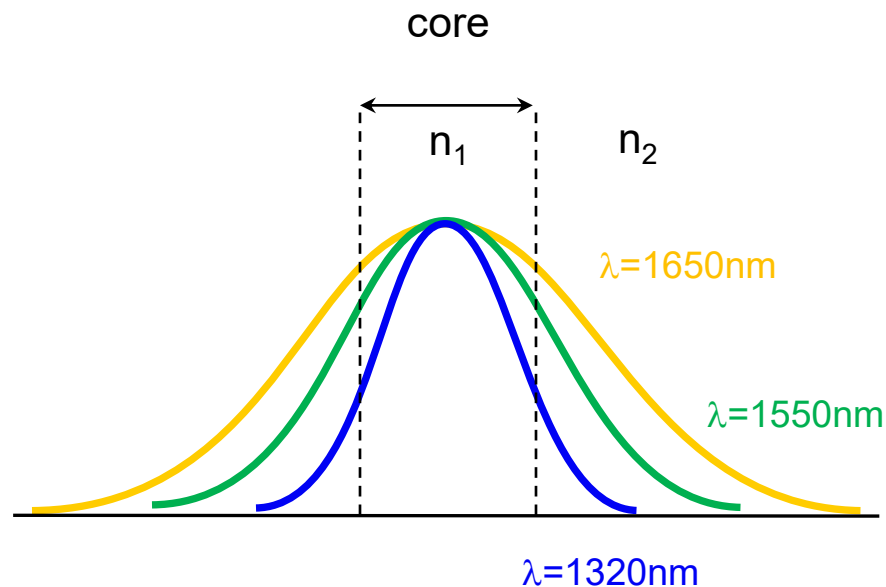
Lowest order Mode

Figure 3.9. Intensity plots for the six LP modes, with $a = 1$. (a) LP_{01} : $u = 2$. (b) LP_{11} : $u = 3$. (c) LP_{21} : $u = 4.5$. (d) LP_{02} : $u = 4.5$. (e) LP_{31} : $u = 5.6$. (f) LP_{12} : $u = 6.3$.

- Waveguides support modes that satisfy the EM boundary conditions between the core and cladding
- Very small cores only support one transverse mode (even the smallest core supports at least one mode)
- Larger cores support many transverse optical modes that exhibit different (more complex) intensity profiles

Mode Size vs. Wavelength

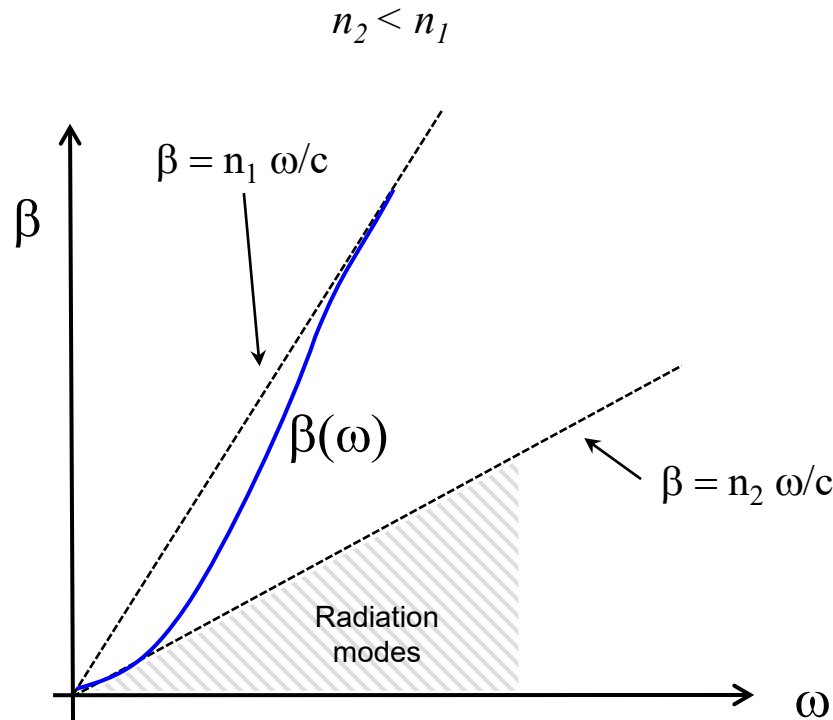
For a given mode, the distribution of field (and power) changes with wavelength



- For low frequencies (large λ) the mode is spatially large and extends significantly into the cladding thus the effective index is very close to that of the cladding n_2
- For high frequencies (short λ) the mode is spatially confined mostly to the core thus the effective index is very close to that of the core n_1

Dispersion β vs ω

$$V_{ph} = \left(\frac{\omega}{\beta} \right) \Rightarrow \beta = \frac{\omega}{V_{ph}} = \frac{\omega}{c/n} = n \frac{\omega}{c}$$



Knowledge of $\beta(\omega)$ is required to understand signal propagation

n_1 and n_2 are functions of ω
In the figure we neglect this dependence

- For low frequencies (large λ) the mode is spatially large and extends significantly into the cladding thus the effective index is very close to that of the cladding n_2
- For high frequencies (short λ) the mode is spatially confined to the core thus the effective index is very close to that of the core n_1



Fiber Modes

Fiber Modes and Waveguide Dispersion (summary)

- For a given mode, the group velocity depends on the wavelength (even if the material dispersion is negligible)
 - The precise mode field distribution changes slightly when the wavelength changes
 - The relative fraction of the electric field in the core versus cladding changes
 - Thus the *effective index* changes and the group velocity must change
- We need to determine the mode field profile and $\beta(\omega)$ in fibers
 - The solution is analytic for step index fibers but is typically solved numerically
 - We start with the wave equation (*Helmholtz* equation) in cylindrical coordinates
 - The 3D wave equation $\nabla^2 E + n^2(\omega)k_0^2 E = 0$

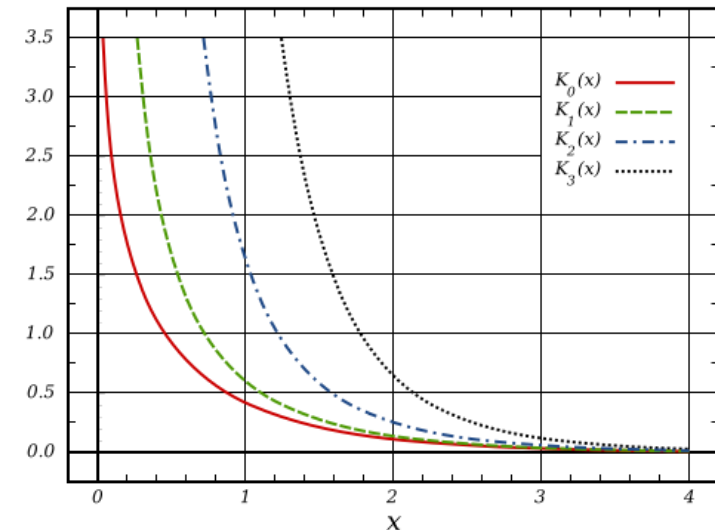
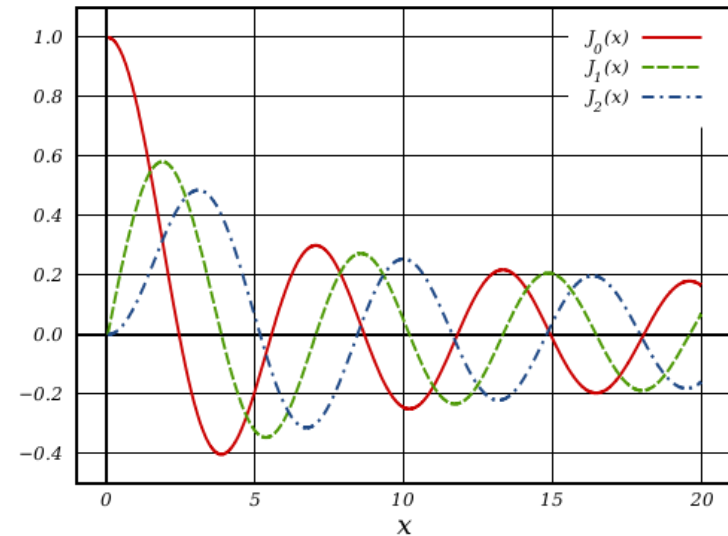
—becomes:
$$\frac{\partial^2 E_z}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial E_z}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2 E_z}{\partial \phi^2} + \frac{\partial^2 E_z}{\partial z^2} + n^2 k_0^2 E_z = 0, \quad n = \begin{cases} n_1 & \text{for } \rho \leq a, \\ n_2 & \text{for } \rho > a, \end{cases} \quad k_0 = 2\pi/\lambda_0$$

Equivalent to Keiser Eq 2.36

- We have written just the axial component E_z , there are of course six variables E_z, E_ρ, E_ϕ and H_z, H_ρ, H_ϕ , but only two are independent
- The frequency dependence is not explicitly written

Optical Fiber Modes

- The optical fiber has a circular waveguide
- Solve Maxwell's equations with cylindrical symmetry
- Solutions vary with radius ρ and angle ϕ
 - Fields in the core are non-decaying
 - J, Y Bessel functions of the first and second kind
 - Fields in the cladding are decaying
 - A boundary condition is $E \Rightarrow 0$ as $\rho \Rightarrow \infty$
 - K modified Bessel functions of second kind
 - In a rectangular system the fields decay exponentially
- There are two mode number to specify each distinct mode
 - n is the radial mode number or mode index
 - m is the angular mode number or azimuthal mode index



Finding the Solution

- Separation of variables suggests

$$E_z(\rho, \phi, z) = F(\rho)\Phi(\phi)Z(z) \quad \text{Equivalent to Keiser Eq 2.38}$$

- Resulting in 3 separate equations (ODEs), one for each variable

$$\frac{d^2 Z}{dz^2} + \beta^2 Z = 0$$

β is a separation constant

$$Z(z) = A_1 e^{i\beta z} + A_2 e^{-i\beta z}$$

Z dependence is identical to plane wave

$$\frac{d^2 \Phi}{d\phi^2} + m^2 \Phi = 0$$

m^2 is a separation constant

$$\Phi(\phi) = B_1 e^{im\phi} + B_2 e^{-im\phi}$$

ϕ dependence is periodic in ϕ

$$\frac{d^2 F}{d\rho^2} + \frac{1}{\rho} \frac{dF}{d\rho} + \left(\underbrace{n^2 k_0^2 - \beta^2}_{\text{Keiser's } q^2} - \frac{m^2}{\rho^2} \right) F = 0$$

Keiser's q^2

The solutions to this are Bessel functions (note: n here is index of refraction)

- Equation for ρ dependence has Bessel function solutions

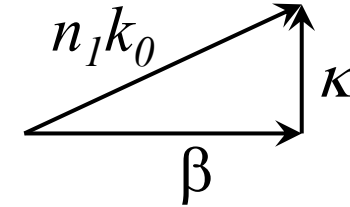
$$F(\rho) = \begin{cases} AJ_m(\kappa\rho) + A'Y_m(\kappa\rho) & \text{for } \rho \leq a, \\ CK_m(\gamma\rho) + C'I_m(\gamma\rho) & \text{for } \rho > a, \end{cases}$$

- J , Y , K and I , are types of Bessel functions and A , A' , C , and C' are constants

Solution

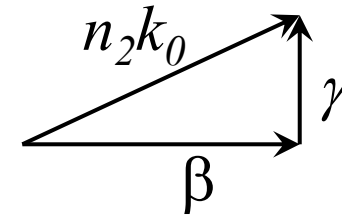
- In the core the propagation constant β is related to the free space wavevector:

$$\kappa^2 = n_1^2 k_0^2 - \beta^2$$



- In the cladding the evanescent decay constant

$$\gamma^2 = \beta^2 - n_2^2 k_0^2$$



- Boundary conditions (field must be finite and go to zero as $\rho \rightarrow \infty$)

$$E_z = \begin{cases} A J_m(\kappa \rho) \exp(im\phi) \exp(i\beta z) & \text{for } \rho \leq a \\ C K_m(\gamma \rho) \exp(im\phi) \exp(i\beta z) & \text{for } \rho > a \end{cases}$$

- Similar relationships exist for H_z
- Using Maxwell's Equations, expressions can be determined for, E_ρ , E_ϕ and, H_r , H_ϕ in terms of E_z and H_z

Eigenvalue Equation for β

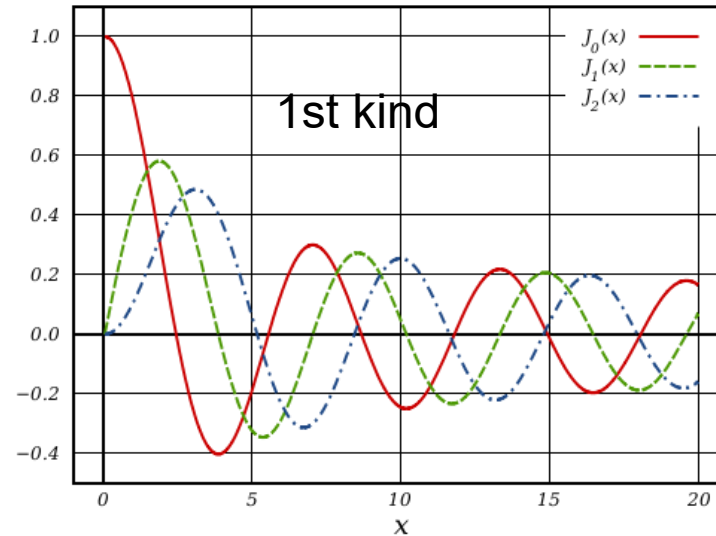
- Using the boundary conditions for continuous tangential E and H across the core-cladding interface gives four equations for continuity of E_z , E_ϕ , H_z , and H_ϕ
- These four equations have solution when the determinant of the corresponding matrix is zero, yielding:

$$\left(\frac{J'_m(\kappa a)}{\kappa J_m(\kappa a)} + \frac{K'_m(\gamma a)}{\gamma K_m(\gamma a)} \right) \left(\frac{J'_m(\kappa a)}{\kappa J_m(\kappa a)} + \frac{n_2^2}{n_1^2} \frac{K'_m(\gamma a)}{\gamma K_m(\gamma a)} \right) = \left(\frac{m\beta}{n_1 a k_0} \right)^2 \left(\frac{1}{\kappa^2} + \frac{1}{\gamma^2} \right)^2 \quad \text{Keiser 2.54}$$

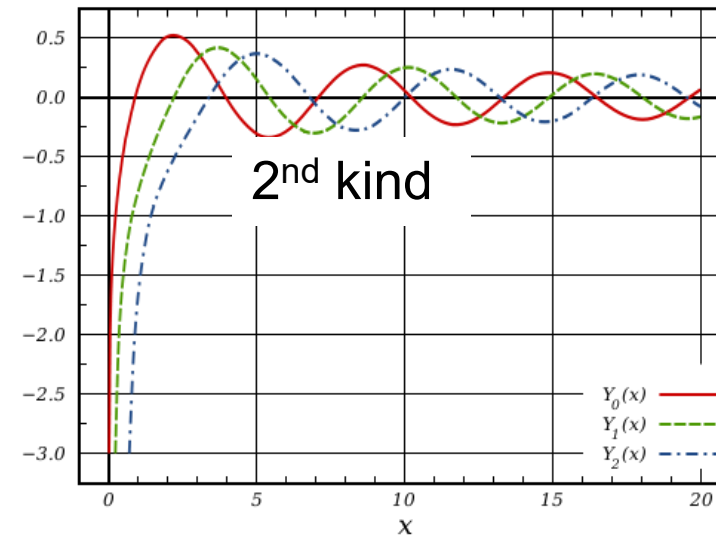
- This provides a method to determine β for any given set of k_o , a , n_1 and n_2
- There are multiple solutions for each azimuthal mode index m , $m = 0, 1, 2, 3 \dots$
- The solutions are identified in terms of m and n and β_{mn}
- Where n is the radial mode index, $n = 1, 2, 3 \dots$
- In general E_z and H_z are both non zero except for $m = 0$
- We therefore call these hybrid modes designated HE_{mn} and EH_{mn} depending on whether H_z is $> E_z$ or not
- For $m=0$ we do have TE_{0n} and TM_{0n}

Bessel Functions

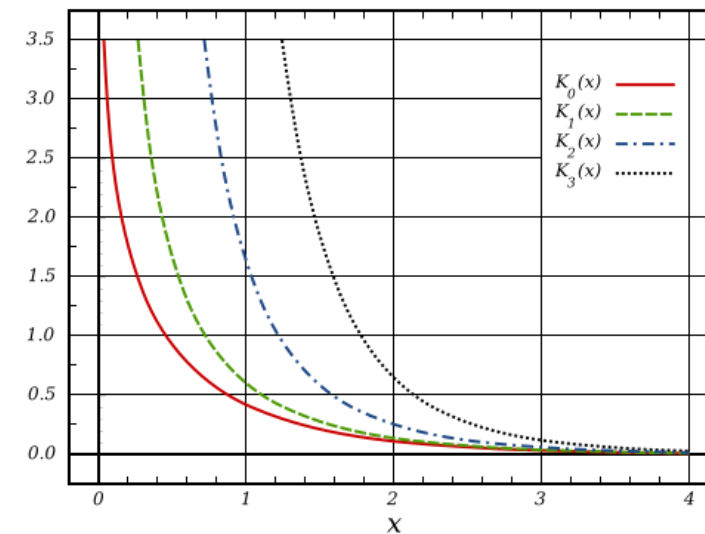
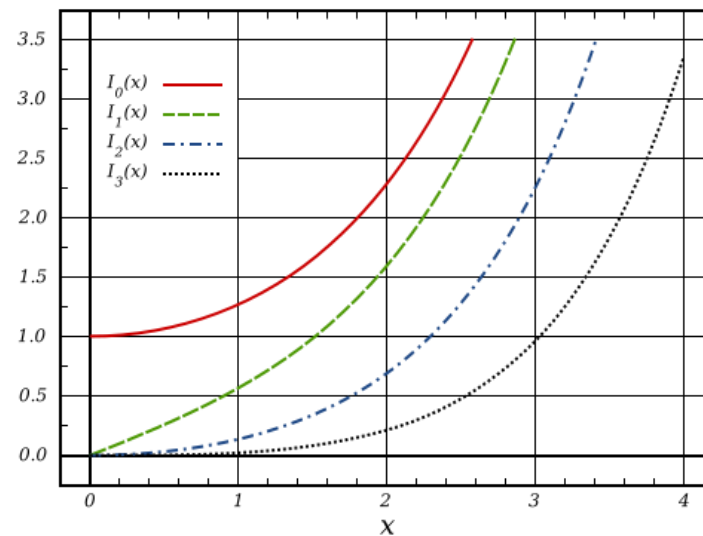
Bessel functions of the first kind: J_α



Bessel functions of the second kind: Y_α

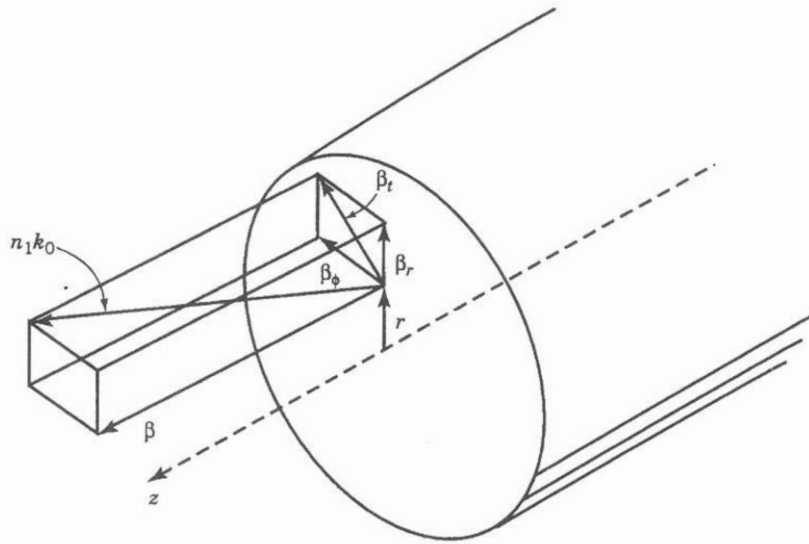


Modified Bessel functions: I_α, K_α
special case with purely imaginary argument



Ray Picture

- A *meridional* ray has no ϕ component –it passes through the z axis
- Ray propagation with a component in the ϕ direction, is called a *skew* ray
- Skew rays exhibit a spiral-like path down the core, never crossing the z axis

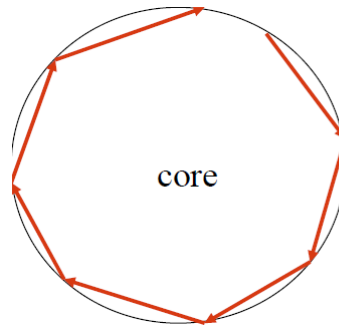


$$(n_1 k_0)^2 = \beta_r^2 + \beta_\phi^2 + \beta^2 = \beta_t^2 + \beta^2$$

- **TE** ($E_z = 0$) and **TM** ($H_z = 0$) modes are possible
 - These modes correspond to meridional rays and pass through the fiber axis
- We refer to these modes as **TE_{lm}** and **TM_{lm}** modes
 - Two indices to describe the two degrees of freedom

Hybrid Modes

- **Hybrid modes:** both E_z and H_z are nonzero
 - These are skew rays (helical path without passing through the fiber axis)
- The modes are written as \mathbf{HE}_{lm} and \mathbf{EH}_{lm} depending on whether the components of \mathbf{H} or \mathbf{E} make the larger contribution to the transverse field



- The full set of circular optical fiber modes therefore comprises: **TE**, **TM** (meridional rays), **HE** and **EH** (skew rays) **modes**

Effective Index and V Number

- Each mode is uniquely characterized by a specific $\beta(\omega)$
 - We can therefore define a mode index of refraction (mode index) \bar{n}
 - The mode propagates with the effective index constrained by the core and cladding indices: $n_1 > \bar{n} > n_2$
 - The mode reaches cutoff when $\gamma=0$ or $\bar{n} = n_2$
- We also define a normalized frequency V

$$V = k_0 a (n_1^2 - n_2^2)^{\frac{1}{2}} \approx \left(\frac{2\pi}{\lambda} \right) a n_1 \sqrt{2\Delta}$$

$$V = \frac{2\pi}{\lambda} a NA$$

- And normalized propagation constant b :

$$b = \frac{(\beta / k_0)^2 - n_2^2}{n_1^2 - n_2^2} \approx \frac{\beta / k_0 - n_2}{n_1 - n_2} = \frac{\bar{n} - n_2}{n_1 - n_2}$$

- The behavior of β vs. V depicts the fiber mode structure
 - When TE_{01} and TM_{01} modes reach cutoff then the fiber is single mode
 - This is equivalent to $J_0(V)=0$
 - or $V < 2.405$

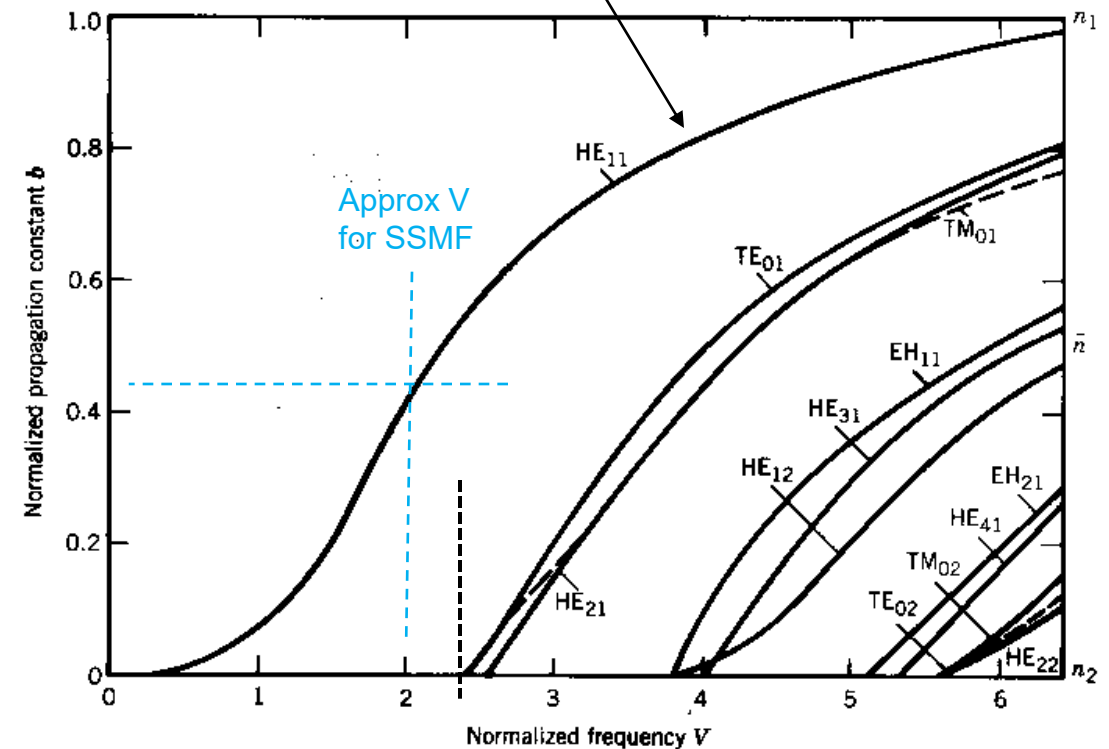
$$\beta_1 = \frac{1}{V_g} = \frac{d\beta}{d\omega} = \frac{N_g}{c}$$

$$N_g \equiv \bar{n} + \omega \frac{d\bar{n}}{d\omega}$$

$$\Delta = (n_1 - n_2) / n_1$$

$$NA = \sqrt{n_1^2 - n_2^2}$$

HE_{11} is a hybrid mode and it has the lowest cutoff (at $V=0$), i.e. does not cutoff



Waveguide Dispersion

- Waveguide dispersion is generally the smallest contributor to the total dispersion
 - It can dominate near the dispersion minimum i.e. $\lambda=1300\text{nm}$ for single mode waveguide
 - It must be included since it typically shifts the dispersion dominated by material dispersion
- Waveguide dispersion: wave vector varies with wavelength
- In order to determine waveguide dispersion we must return to the eigenvalue equation which defines the allowed modes
 - The EV equation was written in terms of the transverse wavevector κ
 - The transverse wavevector depends on:

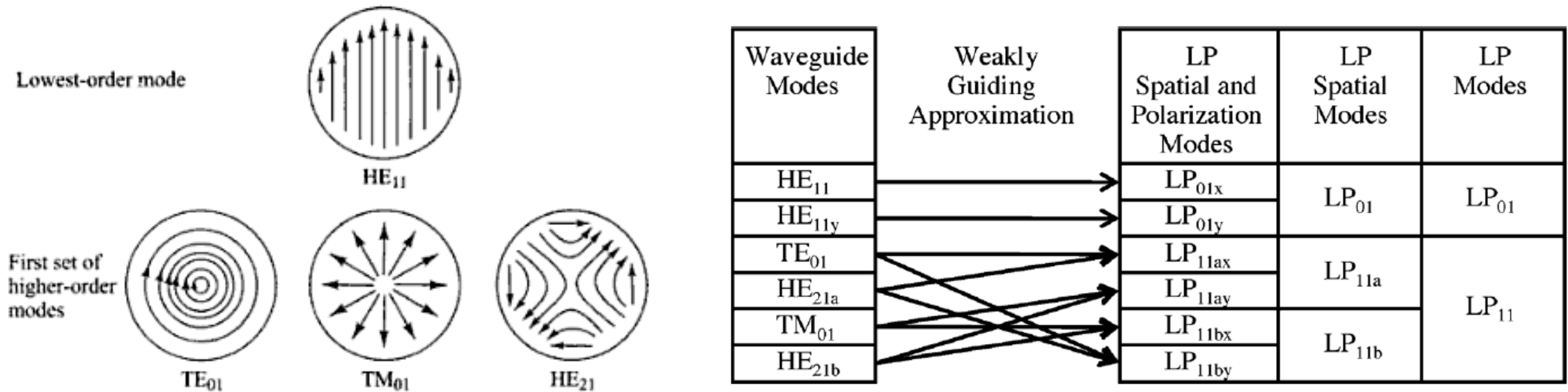
$$\kappa = \sqrt{k_o^2 n_1^2 - \beta^2}$$

- When the wavelength changes, k_o changes and hence the propagation constant β must change
- κ is fixed for a particular mode, it is a solution to the EV equation
- The mode shape stays essentially the same when the wavelength changes [we are assuming that the wavelength changes are small, consistent with that needed to support data streams]

Weakly Guided modes

- For telecom ***Special case of cylindrical modes***
- Typical of all common optical fibers index contrast is low fibers $\Delta \ll 1$
 - Weakly guided modes $n_1 \sim n_2$, determinant equation simplifies, modes become degenerate
 - E_z and H_z are both nearly zero
 - Designated LP modes for Linearly Polarized
- The propagation is *preferentially along the fiber axis* and the *field is predominantly transverse*
- Modes are *approximated by two linearly polarized components*
 - both E_z and H_z are *approximately zero*)

Fiber LP modes



- Relation between the LP modes and the “real” exact waveguide modes HE_{11x}, HE_{11y}, TE₀₁, TM₀₁, HE_{21a}, and HE_{21b} of the six-mode FMF

LP modes: Normalized Propagation Constant

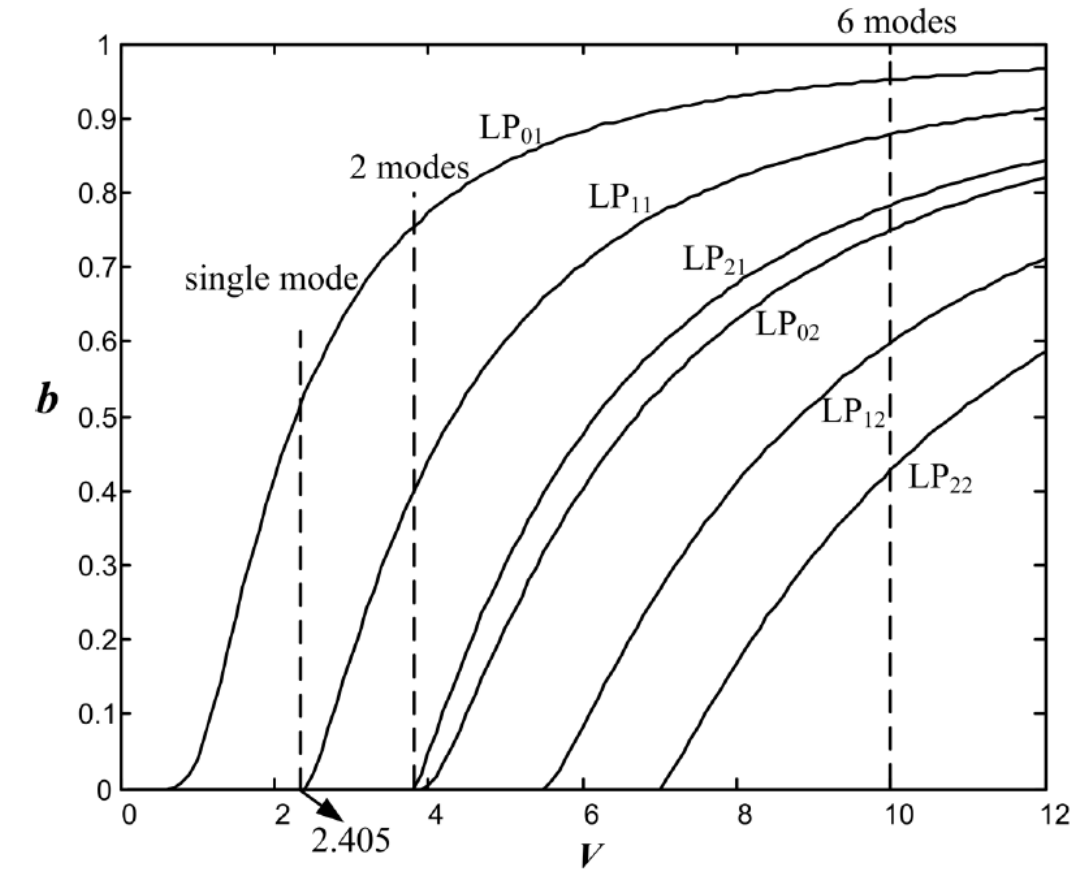
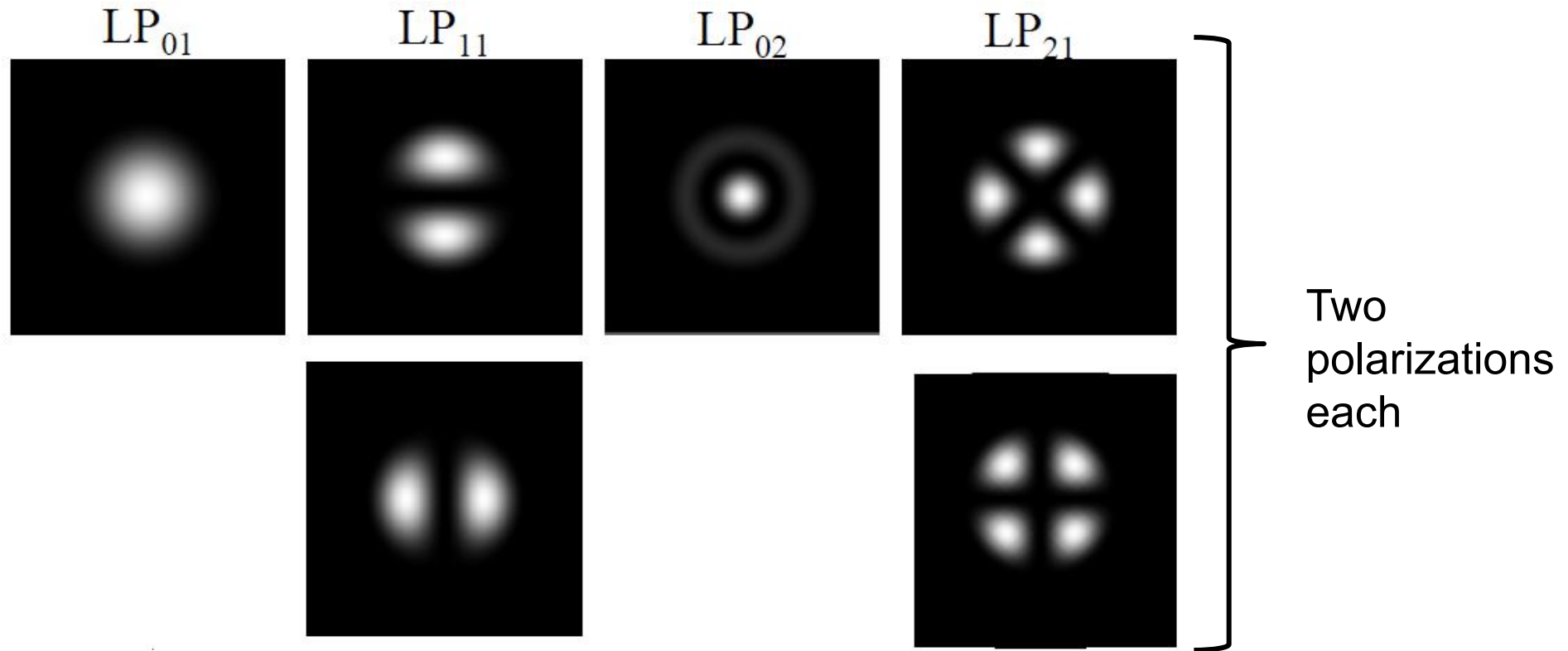


Fig 2.16 Kumar and Deen

Intensity profiles for the lowest LP modes



Gaussian Approximation for the LP₀₁ mode

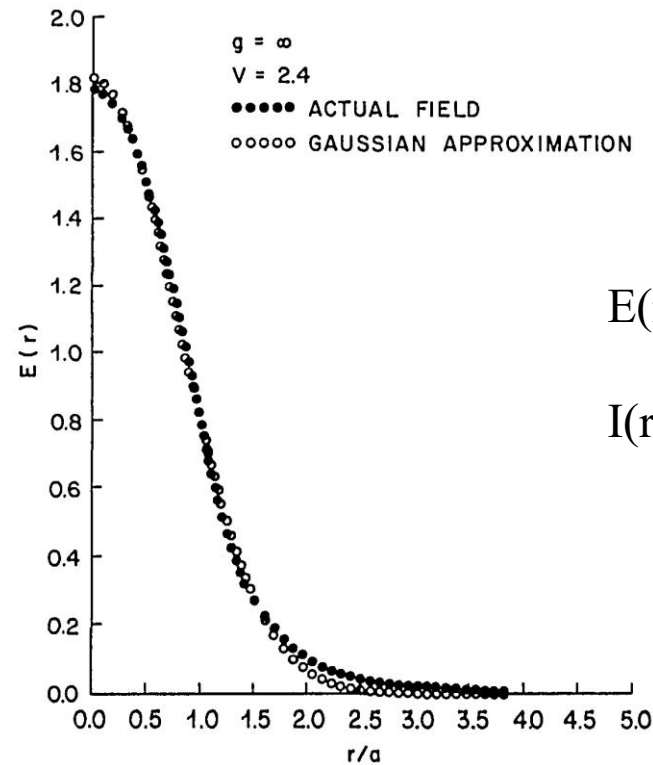


FIG. 8

Citation

D. Marcuse, "Gaussian approximation of the fundamental modes of graded-index fibers," J. Opt. Soc. Am. **68**, 103-109 (1978);
<https://www.osapublishing.org/josa/abstract.cfm?uri=josa-68-1-103>

$$E(r) = E(0) \exp(-r^2 / w_0^2)$$

$$I(r) = I(0) \exp(-2r^2/w_0^2)$$

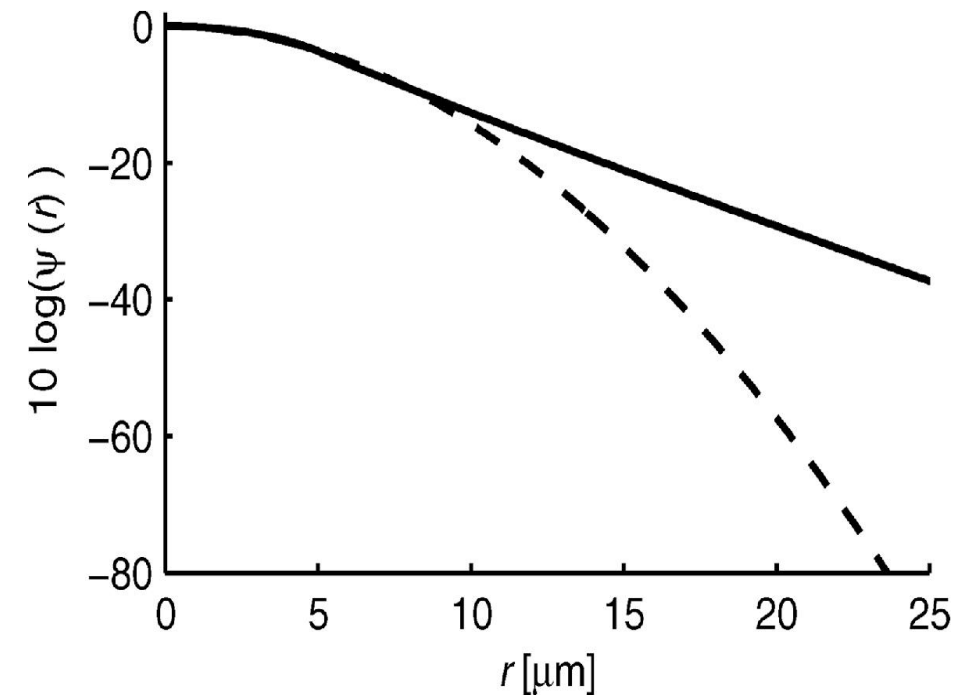


Fig. 2

Citation

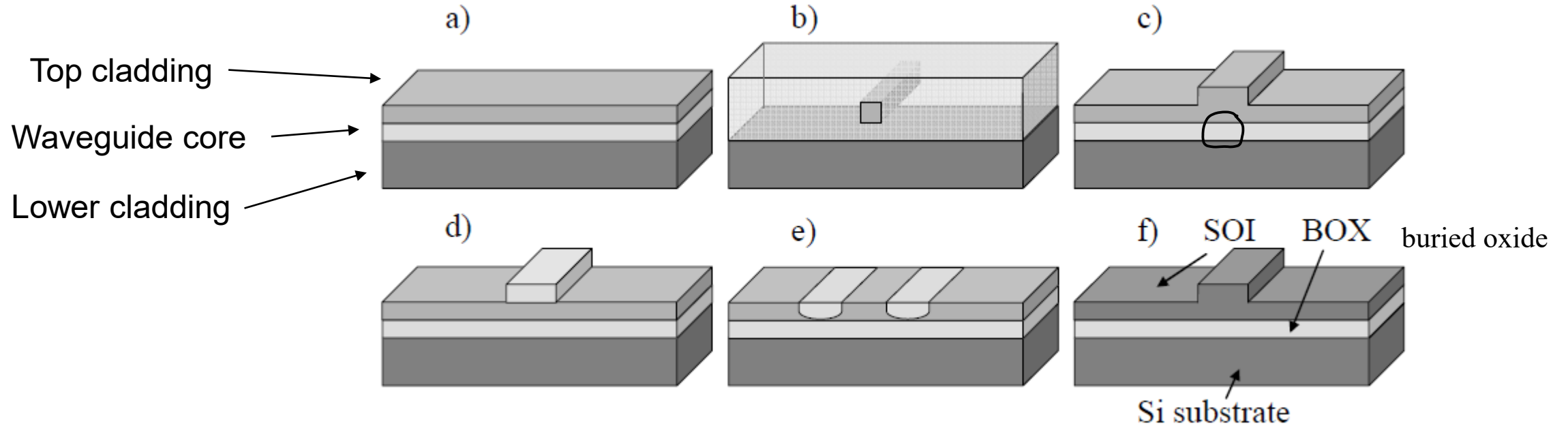
Vincent Trudel, Yves St-Amant, "One-dimensional single-mode fiber-optic displacement sensors for submillimeter measurements," Appl. Opt. **48**, 4851-4857 (2009);
<https://www.osapublishing.org/ao/abstract.cfm?uri=ao-48-26-4851>

- The LP₀₁ mode intensity varies with radius as $J_0^2(ur/a)$ inside the core and as $K_0^2(wr/a)$ in the cladding
 - The resultant intensity profile closely fits a Gaussian function width w_0 , known as the mode-field radius defined as the radial distance from the core center to the $1/e^2$ point of the intensity profile



More on Guided Modes

Planar Guided Wave Structures



a) Slab waveguide

b) strip waveguide

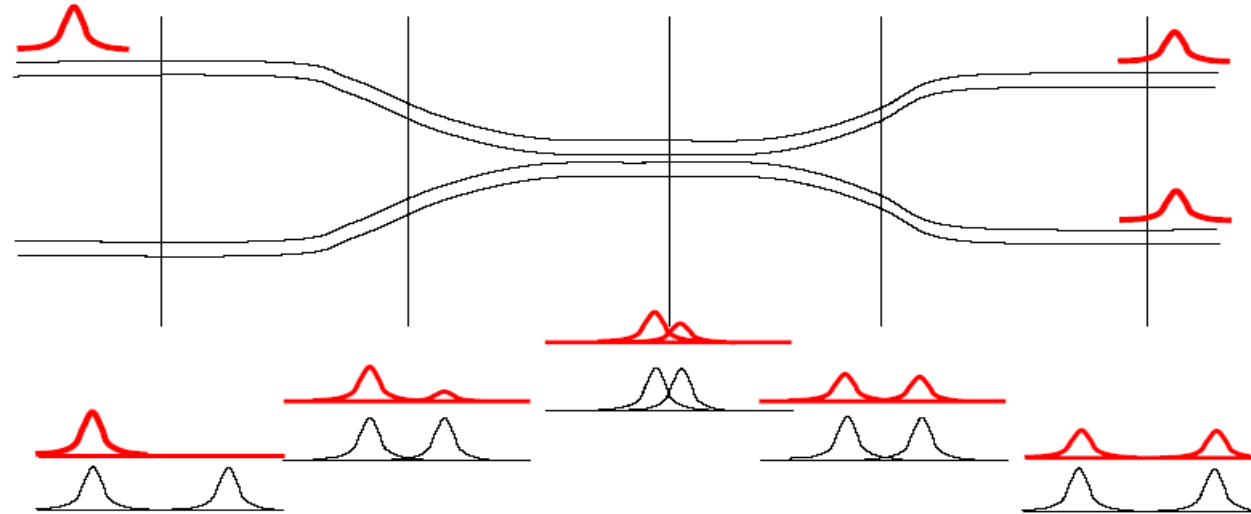
c) rib waveguide

d) strip-loaded waveguide

e) diffused waveguide

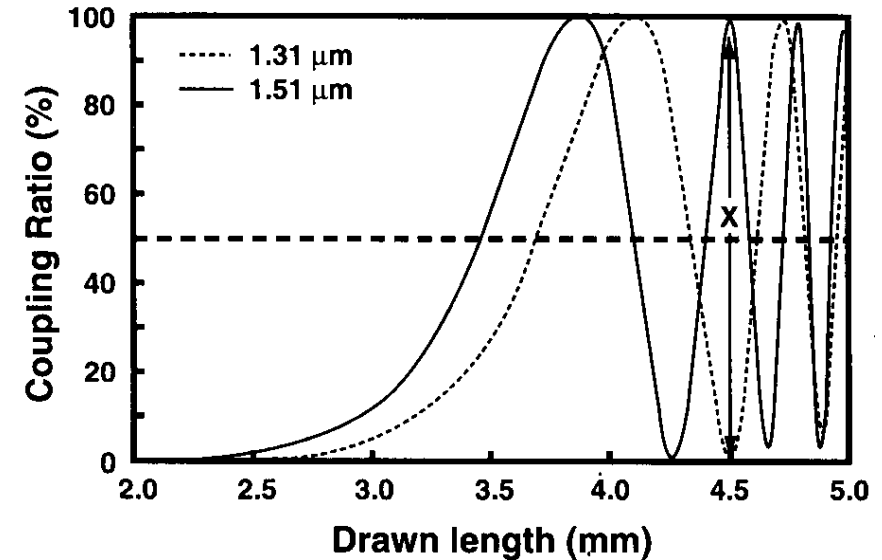
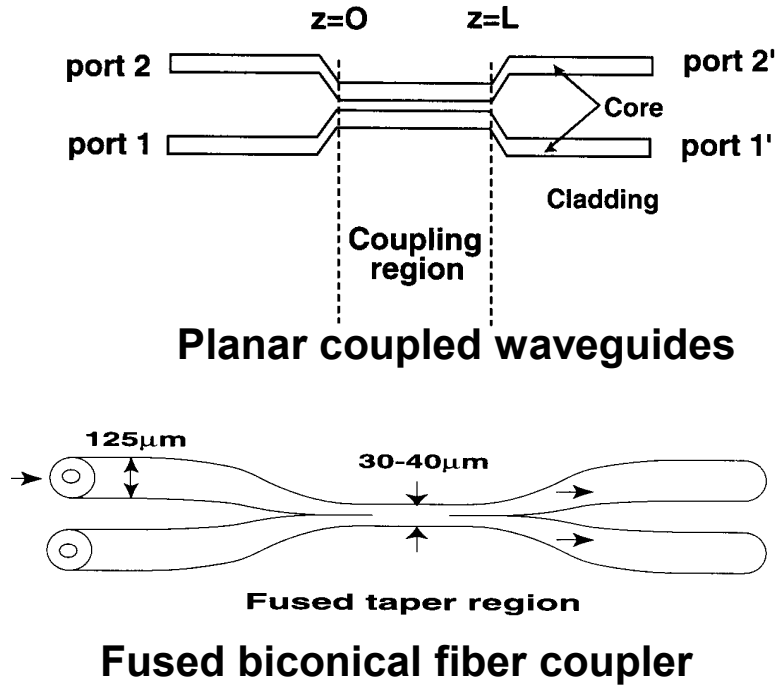
f) rib-type SOI waveguide

Coupling Between Adjacent Waveguides



- Two adjacent optical modes exchange power
 - Phase matching
 - Spatial overlap
- Directional coupler: no power is coupled into backward waves
- 3dB coupler: input power is equally split between two output guides

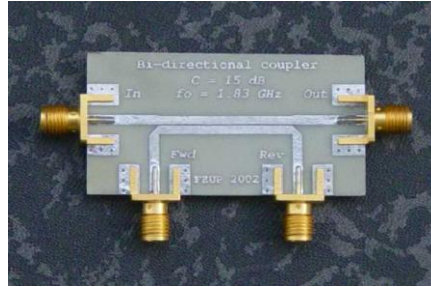
Optical Couplers



Length determines coupling ratio

- 2x2 couplers
 - Waveguides are tapered, V number decreases, expanding evanescent field
 - Controlling parameters
 - Size of reduced waveguide, coupling region length, difference in waveguide size
- Coupled mode theory allows a description of the wave transverse overlap
- Relative phase of each input mode is critical to performance

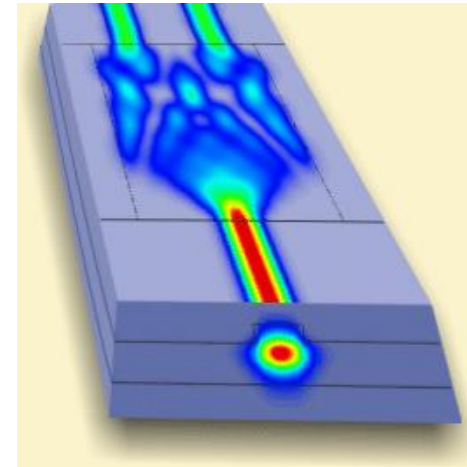
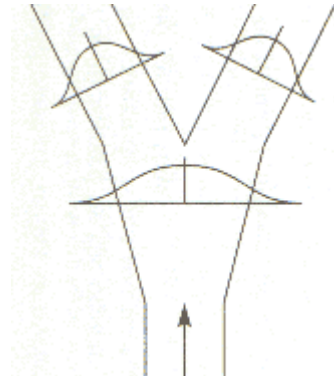
Couplers



RF: microstrip

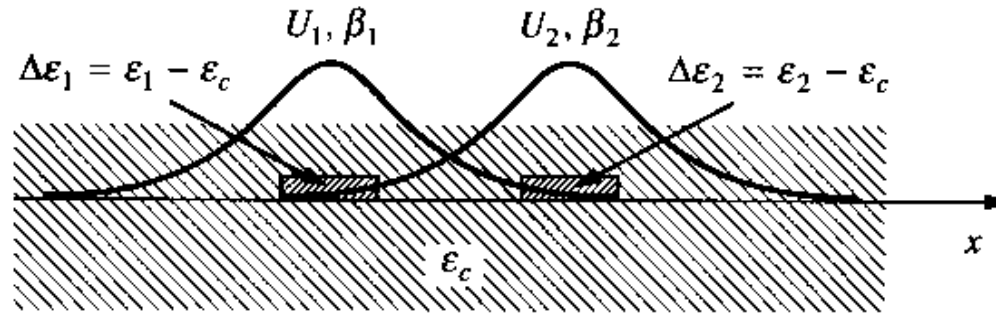


RF: metal waveguide



Optical: index guided

Co-directional coupling



$$P = (\epsilon_2 - \epsilon)E_2 = \epsilon_0(n_2^2 - n^2)E_2$$

- For coupling between the modes we need to add a dielectric perturbation to the wave equations
 - Without the dielectric perturbation the situation is just two waves passing each other in a dielectric
 - $\Delta\epsilon_2$ acts as a perturbation for E_1
 - $\Delta\epsilon_1$ acts as a perturbation for E_2
- The polarization perturbation is of the form

$$k_{12} = \frac{k_0^2}{2\beta_1} \frac{\int (\epsilon_1 - \epsilon_c) E_1 \cdot E_2 dA}{\int |E_1|^2 dA}$$

Coupled mode theory; Two identical waveguides

- Using the wave equation, we find added source terms which originate from both waveguides

- $E(x,y,z)=A(z)E(x,y)$
- $A(z)$: **complex field** amplitude including a phase term $e^{-j\beta z}$
- $E(x,y)$: Field distribution in one waveguide

- Couple Mode Equations

- β is the same for both waveguides (synchronous case)

- Initial conditions

- Solutions

driven waveguide lags
 90° behind phase of
driving waveguide

$$\frac{dA_0(z)}{dz} = -j\beta A_0(z) - j\kappa A_1(z)$$

$$\frac{dA_1(z)}{dz} = -j\beta A_1(z) - j\kappa A_0(z)$$

$$A_0(0) = 1$$

$$A_1(0) = 0$$

$$A_0(z) = \cos(\kappa z) \exp(-j\beta z)$$

$$A_1(z) = -j \sin(\kappa z) \exp(-j\beta z)$$

$$\beta = \beta_r - j \frac{\alpha}{2}$$

α : Loss coefficient

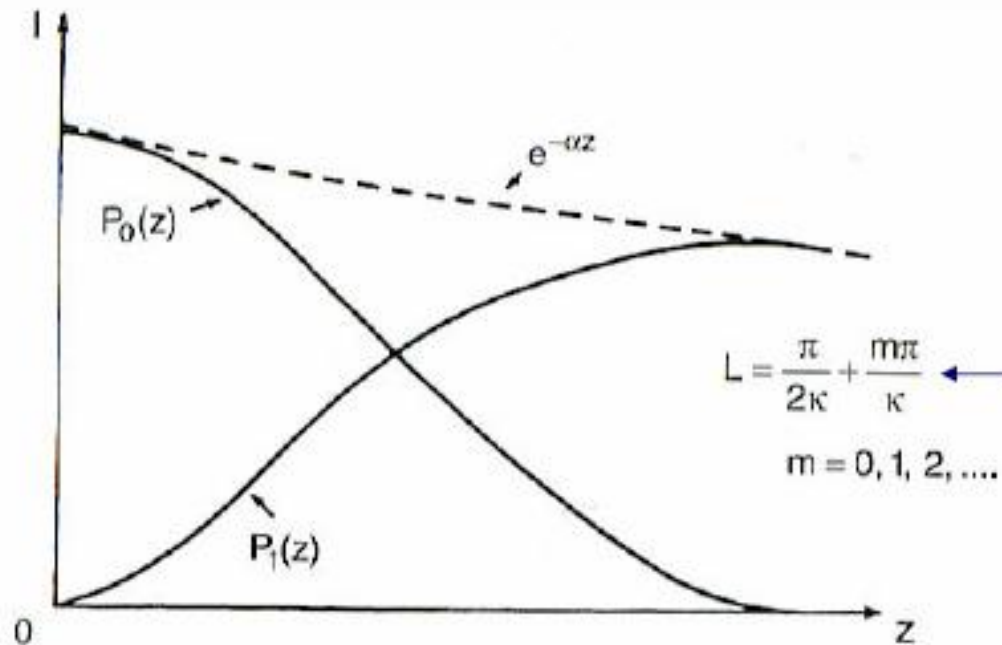
New source
terms

κ : Coupling
coefficient

Power Transfer: Synchronous case

- Power Flow
 - Periodic

$$P_0(z) = |A_0(z)|^2 = \cos^2(\kappa z) \exp(-\alpha z)$$
$$P_1(z) = |A_1(z)|^2 = \sin^2(\kappa z) \exp(-\alpha z)$$



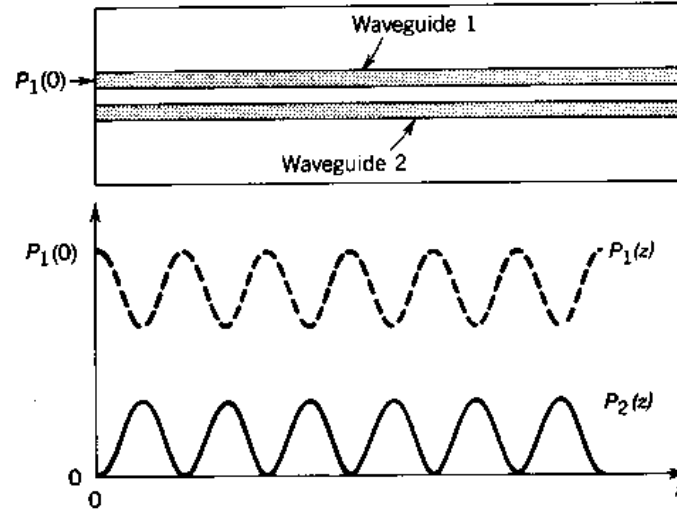
When $\kappa z = \pi/2$ all power is transferred

Length needed to transfer all power from one waveguide to the other

Calculated power distribution along the length of the waveguide

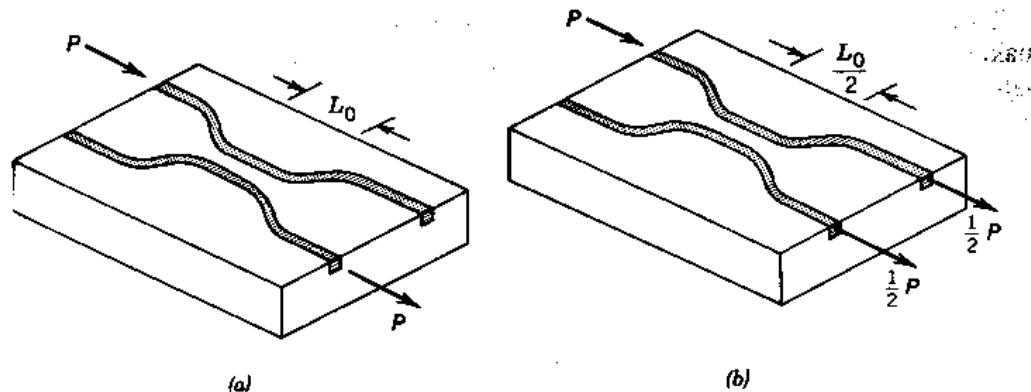
Periodic Exchange of Power

- In the absence of loss the power exchange continues indefinitely
- Coupling between two modes is not symmetric, $\kappa_{\nu\mu} \neq \kappa_{\mu\nu}^*$, unless $\beta_\nu = \beta_\mu$
- For synchronous waveguides the coupling is complete
- For asynchronous waveguides complete power transfer does not occur
- For both cases the power transfer is periodic



Asynchronous case

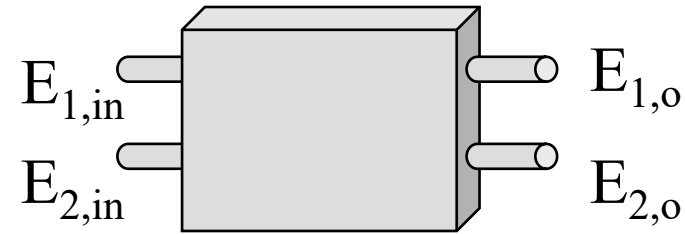
Optical Couplers



- Couplers length depends on the need
 - a) complete switching of power from one guide to the other
 - b) splitting the power half and half i.e. a 3dB coupler
 - Can also create other couplers 10%, 5% etc
 - Dynamic coupler made by changing $\Delta\beta$ using electro-optic effect
- NB: there is a relative **phase change** between the two outputs and the relative input phase matters
 - This impacts how you cascade these devices

Optical Signal Control: Scattering Matrix Representation

- Optical Hybrids

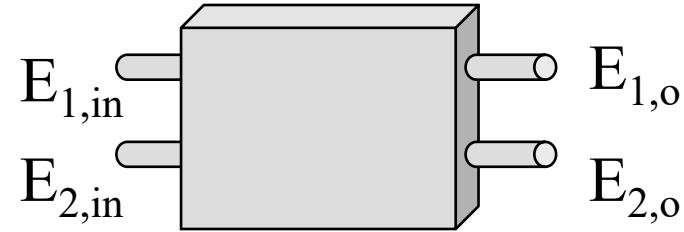


$$\begin{bmatrix} E_{o1} \\ E_{o2} \end{bmatrix} = e^{-j\beta l} \begin{bmatrix} \cos(kl) & j \sin(kl) \\ j \sin(kl) & \cos(kl) \end{bmatrix} \begin{bmatrix} E_{i1} \\ E_{i2} \end{bmatrix}$$

- Where κ is the coupling coefficient
- For $\kappa l = \pi/4$ the device is a 3dB coupler
 - Each term then has magnitude $1/\sqrt{2}$

Optical Signal Control: Scattering Matrix Representation

■ Optical Hybrids



$$E_o = SE_i$$

Where E are column vectors and S is the transfer matrix

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22}e^{i(\phi_{22})} \end{bmatrix}$$

by use of conservation of power and assuming lossless device

$$S = \begin{bmatrix} \sqrt{1-k} & \sqrt{k} \\ \sqrt{k} & -\sqrt{1-k} \end{bmatrix}$$

Where k is the coupling coefficient. For $k = 1/2$ we have a 3dB coupler or π -hybrid since the lossless assumption required $\phi_{22} = \pi$ the π hybrid is an essential element of a balanced receiver used in a coherent link