



**PROJECT REPORT**

**ON**

# **MATHEMATICAL MODELLING OF INDUCTION MOTOR FOR VECTOR CONTROL**

**Submitted in Partial Fulfillment of the**

**Requirements for the Degree of**

**Bachelor of Engineering**

**In**

**ELECTRICAL ENGINEERING**

**By**

**Nimbalkar Gayatri Chandrakant(B120352548 )**

**Pawar Suraj Bharat (B120352553 )**

**Tamboli Ameer Shaukat (B120352567 )**

**Under the Guidance of**

**Prof. P. D. Upadhye**

**Department of Electrical Engineering,**

**VIDYA PRATISHTHAN'S KAMALNAYAN BAJAJ INSTITUTE OF  
ENGINEERING AND TECHNOLOGY, BARAMATI**

**2016-2017**

**Affiliated to**



**Savitribai Phule Pune University**



**Department of Electrical Engineering,  
Vidya Pratishthan's Kamalnayan Bajaj Institute of Engineering and Technology,  
Baramati**

## **CERTIFICATE**

This is to Certify that the Project Report Entitled  
**“Mathematical Modelling of Induction Motor For Vector Control”**

Submitted By

**Nimbalkar Gayatri Chandrakant(B120352548 )**

**Pawar Suraj Bharat(B120352553 )**

**Tamboli Ameer Shaukat (B120352567 )**

is a bonafide work carried out satisfactorily by them under supervision and guidance and it is submitted towards the partial fulfillment of the requirements of Savitribai Phule Pune University, Pune for the award of degree, Bachelor of Engineering (Electrical) during the academic year 2016-2017.

Place: Baramati

Date:

Guide  
( Prof. P. D. Upadhye)

Head of the Department

Principal  
Vidya Pratishthan's Kamalnayan Bajaj Institute Of Engineering and  
Technology, Baramati

## ACKNOWLEDGEMENT

The satisfaction and euphoria that accompany the successful completion of any task would be incomplete without the mention of the people who made it possible. Success is the epitome of hard work, perseverance and most of all those guidance and encouragement crowned our efforts and success. We owe a debt of thanks to our guide Mr. P. D. Upadhye, who stood as a backbone to our project, having worked meticulously all through with special vigilance and zeal. This contribution to the project is unbounded and mere words are not enough to express our deepest sense of gratitude. We thank Prof. N. B. Wagh, Head of the Department for his constant encouragement throughout the year. His advice and co-operation in the completion of project is really unforgettable. We are really indebted to him. We would avail this opportunity to thank our beloved principal Dr. M. G. Devamane, who has always been a source of inspiration and strength to us. We are also grateful to all teachers of our department who helped directly and indirectly and lent us useful suggestions in our project work.

Nimbalkar Gayatri Chandrakant

Pawar Suraj Bharat

Tamboli Ameer Shaukat

# ABSTRACT

Induction motors are widely used in many industrial applications due to their mechanical robustness and low cost. In industrialized countries, about 70% of all generated electric energy is used by electrical motors. In addition, more than 60% of all the electric energy converted into mechanical energy is consumed by pump and fan drives with induction motors. However, a drawback of induction motors is that a precise torque control cannot be easily achieved. So more and more emphasis is given to find out means of precise speed and torque control of induction motors.

The demonstration of an induction motor can be controlled like a separately excited dc motor, brought a renaissance in the high performance control of ac drives. In particular, the vector control, which guaranties high dynamic and static performance, has become very popular.

The principle of vector control of induction motor is based on the control of both the magnitude and the phase of each phase current and voltage. The vector control of induction motor is a digital implementation which demonstrates the capability of performing direct torque control and of achieving higher power conversion efficiency.

To understand vector control of 3 phase of induction motor it is necessary to study mathematical model of induction motor. To derive the mathematical model of a 3-phase induction motor, the theory of reference frames has been effectively used as an efficient approach. Dynamic models (mathematical models) are employed to better understand the behaviour of induction motor in both transient and steady state. The dynamic modelling sets all the mechanical equations for the inertia, torque and speed versus time. It also models all the differential voltage, currents and flux linkages between the stationary stator as well as the moving rotor. This project helps to understand the step by step Simulink implementation of induction motor using dq0 axis reference frame. For this purpose the relevant equations are stated at the beginning, and then a generalized model of a three-phase induction motor is developed and implemented in an easy to follow way. The obtained simulated results provides clear evidence that the reference frame theory is indeed an attractive algorithm to demonstrate the steady-state behavior of the induction motors.

# Contents

<b>Acknowledgment</b>	<b>iii</b>
<b>Abstract</b>	<b>iv</b>
<b>List of Figures</b>	<b>viii</b>
<b>List of Abbreviation</b>	<b>ix</b>
<b>1 INTRODUCTION</b>	<b>1</b>
1.1 Introduction . . . . .	1
1.2 Necessity . . . . .	2
1.3 Objective . . . . .	3
1.4 Organization . . . . .	3
<b>2 LITERATURE SURVEY</b>	<b>4</b>
<b>3 3-PHASE INDUCTION MOTOR</b>	<b>7</b>
3.1 Introduction . . . . .	7
3.2 Types of 3-phase IM:- . . . . .	8
3.2.1 Squirrel Cage Induction Motor . . . . .	8
3.2.2 Slip-Ring Induction Motor . . . . .	9
3.3 Principle of Operation . . . . .	10
<b>4 VECTOR CONTROL OF 3-PHASE INDUCTION MOTOR</b>	<b>12</b>
4.1 Introduction . . . . .	12
4.2 Space Vector Definition and Projection . . . . .	12
4.3 Clarke Transformation . . . . .	13
4.4 Park Transformation . . . . .	14
<b>5 IMPLEMENTATION OF MATHEMATICAL EQUATIONS USING MAT-LAB</b>	<b>15</b>
5.1 Introduction . . . . .	15
5.2 Induction Motor Model . . . . .	16

5.3	Matlab/Simulink Implementation . . . . .	18
<b>6</b>	<b>MATLAB/SIMULINK RESULTS</b>	<b>24</b>
6.1	Result for 3 HP IM . . . . .	24
6.2	Responses for different inputs . . . . .	27
6.2.1	Result for Unit step input . . . . .	27
6.2.2	Result for square wave input . . . . .	28
6.2.3	Result for ramp plus step input . . . . .	29
6.3	Comparative analysis with Ideal Model . . . . .	31
	<b>Conclusion</b>	<b>33</b>
	<b>Future Scope</b>	<b>34</b>
	<b>References</b>	<b>36</b>

## List of Figures

1	Construction of Induction Motor . . . . .	7
2	Section of Stator and Rotor . . . . .	8
3	Squirrel Cage Induction Motor . . . . .	9
4	Slip Ring Induction Motor . . . . .	9
5	Production of Torque . . . . .	11
6	Stator Space Vector Current and Its Component . . . . .	12
7	Clarke Transformation . . . . .	13
8	Park Transformation . . . . .	14
9	The dq Equivalent Circuit of Induction Motor . . . . .	15
10	Matlab Simulation /Simulink Model of 3-phase Induction Motor . . . . .	18
11	Internal Structure of dq Model . . . . .	19
12	Internal Structure of Flux Calculation . . . . .	20
13	Internal Structure of Calculating Flux $\Psi_{mq}$ , $\Psi_{md}$ and Currents $i_{qs}$ , $i_{dr}$ , $i_{ds}$ , $i_{qr}$ . . . . .	21
14	The Implementation of Torque Equation $T_e$ . . . . .	21
15	The Implementation of Angular Speed Equation $\omega_r$ . . . . .	21
16	Internal Structure of the Blocks of fig. 12 . . . . .	22
17	Internal Structure of the Blocks of fig. 12 . . . . .	22
18	Shows Calculation of Flux Linkages . . . . .	23
19	Implementation of dq Current Equations . . . . .	23
20	Stator a-phase Waveform With Respect to Time . . . . .	24
21	Stator b-phase Waveform With Respect to Time . . . . .	24
22	Stator c-phase Waveform With Respect to Time . . . . .	25
23	Rotor a-phase Waveform With Respect to Time . . . . .	25
24	Rotor b-phase Waveform With Respect to Time . . . . .	25
25	Rotor c-phase Waveform With Respect to Time . . . . .	26
26	Torque Waveform With Respect to Time . . . . .	26
27	Speed Waveform With Respect to Time . . . . .	26
28	Unit Step Input . . . . .	27
29	Stator Current Waveforms . . . . .	27
30	Rotor Current Waveforms . . . . .	27

31	Torque and Speed Waveforms . . . . .	28
32	Square Wave Input . . . . .	28
33	Stator Current Waveforms . . . . .	28
34	Rotor Current Waveforms . . . . .	29
35	Torque and Speed Waveforms . . . . .	29
36	Ramp Plus Step Input . . . . .	29
37	Stator Current Waveforms . . . . .	30
38	Rotor Current Waveforms . . . . .	30
39	Torque and Speed Waveforms . . . . .	30
40	Ideal model of induction motor . . . . .	31
41	Waveforms of current,speed and torque . . . . .	32



## List of Abbreviations

Abbreviation	Meaning
1. IM	Induction Motor
2. FOC	Field Oriented Control
3. PWM	Pulse Width Modulation
4. DC	Direct Current
5. AC	Alternating Current
6. DSP	Digital Signal Processing
7. dq0	Direct Quadrature Axis

# **1 INTRODUCTION**

## **1.1 Introduction**

Electrical Energy already constitutes more than 30 % of all energy usage on Earth and this is set to rise in the coming years. Its massive popularity has been caused by its efficiency of use, ease of transportation, ease of generation, and environment-friendliness. Part of the total electrical energy production is used to produce heat, light, in electrolysis, arc-furnaces, domestic heating etc. Another large part of the electrical energy production is used to convert mechanical energy via different kinds of electric drives viz., DC motors, synchronous motors and induction motors. Induction motors are often termed the workhorse of the industry. This is because it is one of the most widely used motor in the world. It is used in transportation and industries, and also in household appliances and laboratories. The major reasons behind the popularity of the induction motors are:

- 1] Induction motors are cheap compared to DC and synchronous motors. In this age of competition, this is a prime requirement for any machine. Due to its economy of procurement, installation and use, the induction motor is usually the first choice for an operation.
- 2] Squirrel-Cage induction motors are very rugged in construction. Their robustness enables them to be used in all kinds of environments and for long durations of time.
- 3] Induction motors have high efficiency of energy conversion. Also they are very reliable.
- 4] Owing to their simplicity of construction, induction motors have very low maintenance cost.
- 5] Induction motors have very high starting torque. This property is useful in applications where the load is applied before starting the motor.

Different applications require different optimum speeds. Speed control is a necessity in induction Motors because of the following factors:

- 1] Different processes require the motor to run at different speeds.
- 2] It provides torque control and acceleration control.
- 3] It ensures smooth operation.
- 4] It compensates fluctuating process parameters.

All these factors present a strong case for the implementation of vector control of 3- phase induction motor.

During the last few years the field of controlled electrical drives has undergone rapid expansion mainly due to the advantages of semiconductors in both power and signal electronics and culminating in micro-electronic microprocessors and DSPs. These technological improvements have enabled the development of really effective induction motor drive control with ever lower power dissipation hardware and ever more accurate control structures. The electrical drive controls become more accurate in the sense that not only the DC current and voltage controlled but also the three phase currents and voltages are managed by so-called vector controls. This Project describes the most efficient form of vector control method. Vector control method is based on three major points: the machine current and voltage space vectors, the transformation of a three phase speed and time dependent system into a two co-ordinate time invariant system.

To understand vector control of 3 phase of induction motor it is necessary to study mathematical model of induction motor and to derive the mathematical model of a 3-phase induction motor, the theory of reference frames has been effectively used as an efficient approach. Dynamic models (mathematical models) are employed to better understand the behaviour of induction motor in both transient and steady state. The dynamic modelling sets all the mechanical equations for the inertia, torque and speed versus time. It also models all the differential voltage, currents and flux linkages between the stationary stator as well as the moving rotor.

## **1.2 Necessity**

In 21<sup>st</sup> century electric drives continue to be widely used in industry. In industry 90 % of industrial drives are 3-phase induction motor. 3-phase induction motors have number of advantages and are widely used in industry. Because of these advantages induction motor finds application in engineering where maintenance free operation is required, in air craft drives where high power density is required and in the textile industry where dust and explosion-proof construction is required but one of the main problem linked with 3-phase induction motors drive is that speed control of induction motors which is very difficult. Hence for fine speed control applications DC motors are used in place of induction motors but the problems associated with DC drive are high initial cost, increased operation and

maintenance cost due to presence of commutator and brush arrangement and they should not be operated in inflammable environment as sparking occurs at brushes. To overcome all above mentioned problems associated with AC and DC drive and to improve dynamic performances of the induction motor at all speed there is need of vector control.

### **1.3 Objective**

- 1] The main objective of this project is to study the mathematical model of 3-phase induction motor and its implementation in MATLAB.
- 2] To study Clarke and Park transformation and simulate it using MATLAB.
- 3] Compare mathematical model of an induction motor with ideal model using MATLAB.

### **1.4 Organization**

In order to ease the reading of this report, and to create an overall image of the project a short description of the main chapters is given here.

Section 1 briefs about the need of vector control of induction motor and objectives of the project.

Section 2 gives an exhaustive literature survey about mathematical model of induction motor.

Section 3 gives brief information about induction motor like induction motor construction, types of induction motor and working principle.

Section 4 is about vector speed control method and Clarke, Park transformations.

Section 5 is about implementation of mathematical model of induction motor using MATLAB.

Section 6 gives results for the induction motor tested in simulated model.

section 7 is of comparative analysis.

## **2 LITERATURE SURVEY**

The step in this project was an extensive study of up-to-date relevant literature and background material. For the purposes of this project mainly available reports on mathematical modelling for vector control method were read and consulted. Out of vast literature survey that has been referred, few important ones are noted below which are worths in the project report.

**1] “Mathematical Modelling of an 3-Phase Induction Motor”, Using MATLAB/Simulink  
Mr. Punit L. Ratnani, Dr. A. G. Thosar.**

In this paper, few assumptions are made while deriving mathematical model of a 3-phase induction Motor. They are listed below.

1. Uniform air gap.
2. Squirrel cage type construction.
3. Balanced stator and rotor windings, with sinusoidally distributed winding.
4. Saturation and parameter change are neglected.

This paper gives information about step by step Matlab/Simulink implementation of induction motor using dq0 axis transformation of stator and rotor variables. Also this paper gives steady state model and equivalent circuit of the induction motor which is useful to study the performance of the machine in steady state.

From this paper we can conclude that the Matlab/Simulink is a reliable and sophisticated way to analyse and predict the behaviour of induction motors using the theory of reference frames.

**2]“ Induction Motor Modelling for Vector Control Purposes”, Helsinki University of Technology, Laboratory of Electromechanics , Report, Espoo 2000, 144 p.**

This paper helps to summarize the existing models and to develop new models, in order to obtain a unified approach on modelling of the induction machines for vector control purposes. Starting from vector control principles, the work suggests the d-q axes unified approach for all types of the induction motors. However, the space vector analysis is presented as a strong tool in modelling of the symmetrical induction motor. When an electrical motor is viewed as a mathematical system, with inputs and outputs, it can be analysed and described in multiple ways, considering different reference frames and state-space variables. All the mathematical possible models are illustrated in this paper. The suggestions

for what model is suitable for what application, are defined as well and the stability of these various models is analysed.

3]“**Comparative analysis of scalar and vector control of Induction motor through Modeling and Simulation**”, published by **Pabitra Kumar Behera<sup>1</sup>, Manoj Kumar Behera<sup>2</sup>, Amit Kumar Sahoo<sup>3</sup>**.

From this paper, an implementation of speed control of an induction motor (IM) using vector control method has been developed and simulated. The comparative study of vector control of IM and conventional v/f control of IM is done.

from this paper we are able to conclude that scalar control is cheap and well implementable method because of its advantages and simplicity, many applications in the industry operate with this control technique. On the other hand it is not satisfactory for the control of drives with fast dynamic behavior, since it gives slow response to transients. It is a low performance control, but it is a stable control technique. The field oriented control method operates with fast responses. So it satisfies the requirements of dynamic drives method to handle transients.

4]“**Dynamic Simulation of a Three-Phase Induction Motor Using Matlab Simulink**”, **Adel Aktaibi and Daw Ghanim**, graduate student members, IEEE, **M. A. Rahman**, life fellow, IEEE, Faculty of Engineering and Applied Science, Memorial University of Newfoundland St. Johns, NL, Canada, A1B 3X5.

In this paper, the theory of reference frames has been effectively used as an efficient approach to analyze the performance of the induction motor. This paper presents a step by step Simulink implementation of an Induction motor using dq0 axis transformations of the stator and rotor variables in the arbitrary reference frame. For this purpose, the relevant equations are stated and then a generalized model of a three-phase induction motor is developed and implemented. Simulated results obtained from this paper provides clear evidence that the reference frame theory is indeed an attractive algorithm to demonstrate the steady-state behaviour of the induction machines.

5]“**Modern power electronics and AC Drives**”, by **Bimal K. Bose**.

From this book we have studied the procedure of Clarke and Park transformation by changing frame of reference. Also we have studied the equivalent circuit implementation of electrical circuit in terms of resistances, inductance ,mutual inductance which is useful to

understand performance of IM. This book provides performance equations reference frames (stationary and dynamic) with respect to stator winding also it gives equivalent matrix formation of circuits, which is useful for the development of mathematical model.

6] **P. C. Krause, “Analysis of Electric Machinery”, McGraw-Hill Book Company, 1986.**

Detailed structure of mathematical model performance of induction motor is explained by this book. From this book, we have studied the basics of machines like construction, working principle, reference frame theory for the development of dq0 axis. This book also provides dynamic performances of motor during load changing conditions. It also helps to do simulation of Park and Clarke equations.

### 3 3-PHASE INDUCTION MOTOR

#### 3.1 Introduction

The 3-phase induction motor is a rotating electric machine designed to operate from a 3-phase source of alternating voltage. For variable speed drives, the source is normally an inverter that uses power switches to produce approximately sinusoidal voltages and currents of controllable magnitude and frequency. A cross-section of a two-pole induction motor is shown in Fig. 1. slots in the inner periphery of the stator accommodate 3-phase winding. The turns in each winding are distributed so that a current in a stator winding produces an approximately sinusoidally-distributed flux density around the periphery of the air gap. When three currents that are sinusoidal varying in time, but displaced in phase by  $120^\circ$  from each other, flow through the three symmetrically-placed windings, a radially-directed air gap flux density is produced that is also sinusoidally distributed around the gap and rotates at an angular velocity equal to the angular frequency of the stator currents.

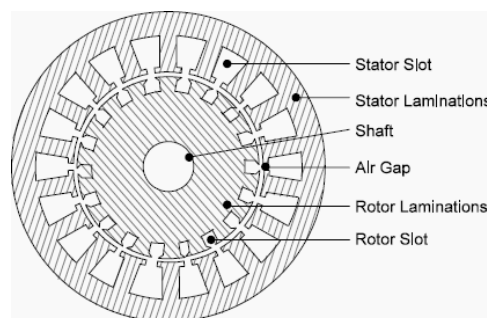


Figure 1: Construction of Induction Motor

3-phase IM is a rotating machine, unlike the transformer which is a static machine. Both the machines operate on ac supply. This machine mainly works as a motor, but it can also be run as a generator, which is not much used. Like all rotating machines, it consists of two parts - stator and rotor as shown in fig.1

In the stator (Fig.2), the winding used is a balanced three-phase one, which means that the number of turns in each phase, connected in star/delta, is equal. The windings of the three phases are placed  $120^\circ$  (electrical) apart, the mechanical angle between the adjacent phases being  $[(2 \times 120)/p]$ , where p is no. of poles. For a 4-pole ( $p = 4$ ) stator, the mechanical angle between the winding of the adjacent phases, is  $[(2 \times 120)/4] = 60^\circ$ . The



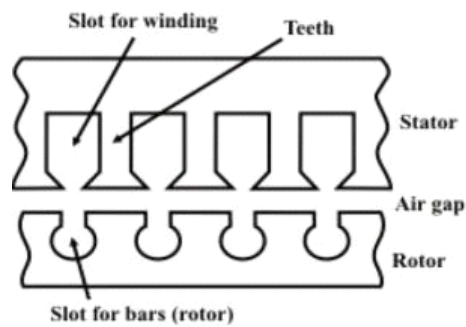


Figure 2: Section of Stator and Rotor

conductors, mostly multi-turn, are placed in the slots, which may be closed, or semi-closed, to keep the leakage inductance low. The start and return parts of the winding are placed nearly  $180^\circ$ , or  $(180^\circ - P)$  apart. The angle of short chording (P) is nearly equal to, or close to that value. The short chording results in reducing the amount of copper used for the winding, as the length of the conductor needed for overhang part is reduced. There are also other advantages. The section of the stampings used for both stator and rotor, is shown in Fig.1 The core is needed below the teeth to reduce the reluctance of the magnetic path, which carries the flux in the motor (machine). The stator is kept normally inside a support.

### **3.2 Types of 3-phase IM:-**

Depending on the construction of rotor, 3-phase IM is classified in two types:

- 1] Squirrel Cage Induction Motor
- 2] Slip-Ring Induction Motor

#### **3.2.1 Squirrel Cage Induction Motor**

The squirrel cage rotor (Fig.3) is mainly used, as it is cheap, rugged and needs little or no maintenance. It consists of copper bars placed in the slots of the rotor, short circuited at the two ends by end rings, brazed with the bars. This type of rotor is equivalent to a wound (slip-ring) one, with the advantage that this may be used for the stator with different no. of poles. The currents in the bars of a cage rotor, inserted inside the stator, follow the pattern of currents in the stator winding, when the motor (IM) develops torque, such that no. of poles in the rotor is same as that in the stator. If the stator winding of IM is changed, with

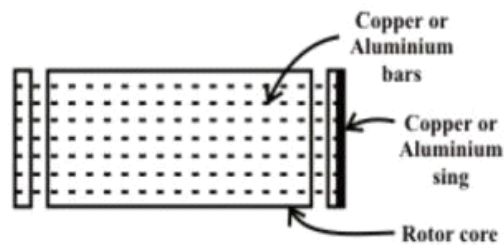


Figure 3: Squirrel Cage Induction Motor

no. of poles for the new one being different from the earlier one, the cage rotor used need not be changed, thus, can be same, as the current pattern in the rotor bars changes. But the no. of poles in the rotor due to the above currents in the bars is same as no. of poles in the new stator winding. The only problem here is that the equivalent resistance of the rotor is constant. Hence, at the design stage, the value is so chosen, so as to obtain a certain value of the starting torque, and also the slip at full load torque is kept within limits as needed.

### 3.2.2 Slip-Ring Induction Motor

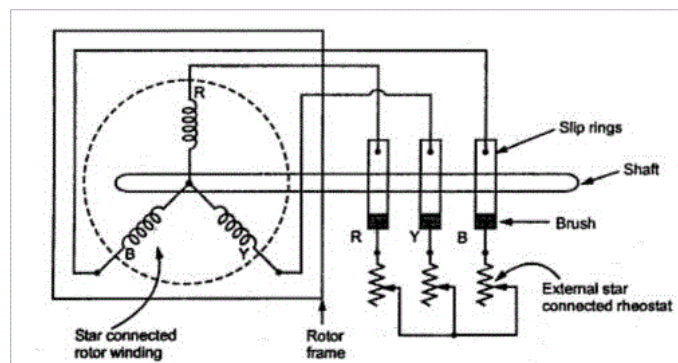


Figure 4: Slip Ring Induction Motor

Slip-Ring induction motor i.e., a wound rotor (slip ring) used has a balanced three-phase winding (Fig.4), being same as the stator winding, but no. of turns used depends on the voltage in the rotor. The three ends of the winding are brought at the three slip-rings, at which points external resistance can be inserted to increase the starting torque requirement. Other three ends are shorted inside. The motor with additional starting resistance is costlier, as this type of rotor is itself costlier than the cage rotor of same power rating, and additional cost of the starting resistance is incurred to increase the starting torque as required. But

the slip at full load torque is lower than that of a cage rotor with identical rating, when no additional resistance is used, with direct short-circuiting at the three slip-ring terminals. In both types of rotor, below the teeth, in which bars of a cage rotor, or the conductors of the rotor winding, are placed, lies the iron core, which carries the flux as is the case of the core in the stator. The shaft of the rotor passes below the rotor core. For large diameter of the rotor, a spider is used between the rotor core and the shaft. For a wound (slip-ring) rotor, the rotor winding must be designed for same no. of poles as used for the stator winding. If the no. of poles in the rotor winding is different from no. of poles in the stator winding, no torque will be developed in the motor. It may be noted that this was not the case with cage rotor, as explained earlier. The wound rotor (slip ring) shown in Fig.4 is shown as star-connected, whereas the rotor windings can also be connected in delta, which can be converted into its equivalent star configuration. This shows that the rotor need not always be connected in star as shown. The number of rotor turns changes, as the delta-connected rotor is converted into star-connected equivalent.

### **3.3 Principle of Operation**

The balanced three-phase winding of the stator is supplied with a balanced three- phase voltage. The current in the stator winding produces a rotating magnetic field, the magnitude of which remains constant. The axis of the magnetic field rotates at a synchronous speed  $n_s = [(2 \times f)/p]$ , a function of the supply frequency (f), and number of poles (p) in the stator winding. The magnetic flux lines in the air gap cut both stator and rotor conductors at the same speed. The emf in both stator and rotor conductors are induced at the same frequency, i.e. line or supply frequency, with number of poles for both stator and rotor windings (assuming wound one) being same. The stator conductors are always stationary, with the frequency in the stator winding being same as line frequency. As the rotor winding is short-circuited at the slip-rings, current flows in the rotor windings. The electromagnetic torque in the motor is in the same direction as that of the rotating magnetic field, due to the interaction between the rotating flux produced in the air gap by the current in the stator winding, and the current in the rotor winding. This is as per Lenz's law, as the developed torque is in such direction that it will oppose the cause, which results in the current flowing in the rotor winding. This is irrespective of the rotor type used cage or wound , with the

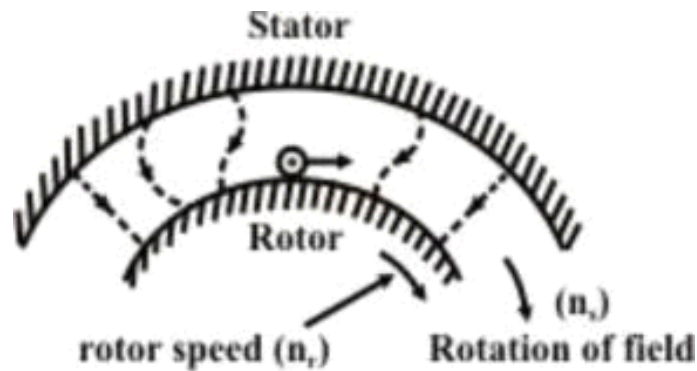


Figure 5: Production of Torque

cage rotor, with the bars short-circuited by two end-rings, is considered equivalent to a winding on the current in the rotor bars interacts with the air-gap flux to develop the torque, irrespective of the number of poles for which the winding in the stator is designed. Thus, the cage rotor may be termed as universal one. The induced emf and the current in the rotor are due to the relative velocity between the rotor conductors and the rotating flux in the air-gap, which is maximum, when the rotor is stationary. As the rotor starts rotating in the same direction, as that of the rotating magnetic field due to production of the torque as stated earlier, the relative velocity decreases, along with lower values of induced emf and current in the rotor. If the rotor speed is equal that of the rotating magnetic field, which is termed as synchronous speed, and also in the same direction, the relative velocity is zero, which causes both the induced emf and current in the rotor to be reduced to zero. Under this condition, torque will not be produced. So, for production of positive (motoring) torque, the rotor speed must always be lower than the synchronous speed. The rotor speed is never equal to the synchronous speed in an IM. The rotor speed is determined by the mechanical load on the shaft and the total rotor losses, mainly comprising of copper loss.

## 4 VECTOR CONTROL OF 3-PHASE INDUCTION MOTOR

### 4.1 Introduction

The vector control method is also known as Field Orientated Control (FOC) method. This method consists of controlling the stator currents represented by a vector. This control is based on projections which transform a three phase time and speed dependent system into a two co-ordinate (d and q co-ordinates) time invariant system. These projections lead to a structure similar to that of a DC machine control. Vector controlled machines need two constants as input references the torque component (aligned with the q co-ordinate) and the flux component (aligned with d co-ordinate). As vector Control is simply based on projections the control structure handles instantaneous electrical quantities. This makes the control accurate in every working operation (steady state and transient) and independent of the limited bandwidth mathematical model.

### 4.2 Space Vector Definition and Projection

The three-phase voltages, currents and fluxes of AC-motors can be analyzed in terms of complex space vectors. With regard to the currents, the space vector can be defined as follows. Assuming that  $i_a, i_b, i_c$  are the instantaneous currents in the stator phases, then the complex stator current vector is defined by

$$i_s = i_a + \alpha i_b + \alpha^2 i_c$$

The following diagram shows the stator current complex space vector:

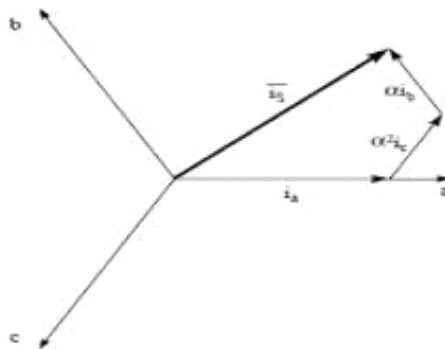


Figure 6: Stator Space Vector Current and Its Component

Where (a,b,c) are the three phase system axes. This current space vector depicts the three phase sinusoidal system. It still needs to be transformed into a two time invariant co ordinate system. This transformation can be split into two steps:

- 1] (a, b, c) to  $(\alpha, \beta)$  (the Clarke transformation) which outputs a two co-ordinate time variant system.
- 2] (a, b) to (d,q) (the Park transformation) which outputs a two co-ordinate time invariant system.

### 4.3 Clarke Transformation

The space vector can be transform in to another reference frame with only two orthogonal axis called  $(\alpha, \beta)$ . Assuming that the axis a and the axis  $\alpha$  are in the same direction we have the following vector diagram.

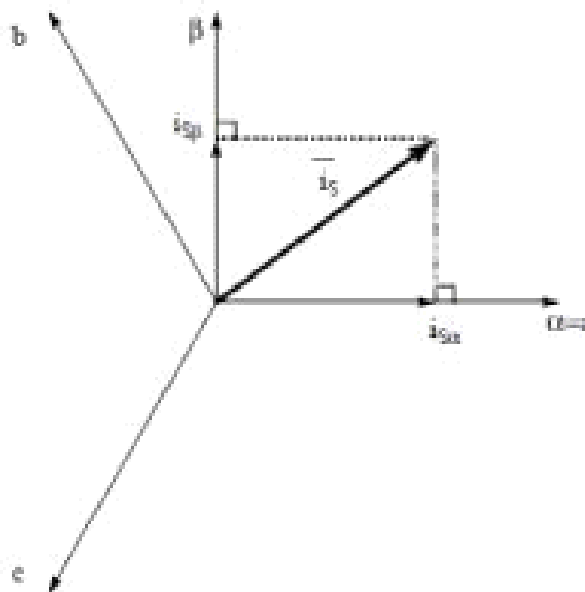


Figure 7: Clarke Transformation

The projection that modifies the three phase system into the  $(\alpha, \beta)$  two dimension orthogonal system is presented below.

$$i_{s\alpha} = i_a$$

$$i_{s\beta} = \frac{1}{\sqrt{3}}i_a + \frac{2}{\sqrt{3}}i_b$$

We obtain a two co-ordinate system  $(i_{s\alpha}, i_{s\beta})$  that still depends on time and speed.

#### 4.4 Park Transformation

This is the most important transformation in the vector control. In fact, this projection modifies a two phase orthogonal system  $(\alpha, \beta)$  into the d,q rotating reference frame. If we consider the d axis aligned with the rotor flux, the next diagram shows, for the current vector, the relationship from the two reference frame Where  $\theta$  is the rotor flux position. The

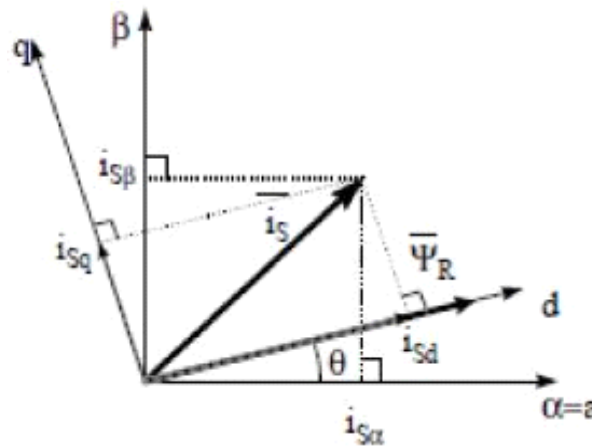


Figure 8: Park Transformation

flux and torque components of the current vector are determined by the following equations:

$$i_{sd} = i_{s\alpha} \cos \theta + i_{s\beta} \sin \theta$$

$$i_{sq} = -i_{s\alpha} \sin \theta + i_{s\beta} \cos \theta$$

These components depend on the current vector  $(\alpha, \beta)$  components and on the rotor flux position. If we know the right rotor flux position then, by this projection, the d,q component becomes a constant.

We can obtain a two co-ordinate system  $(i_{sd}, i_{sq})$  with the following characteristics:

- 1] Two co-ordinate time invariant system.
- 2] With  $i_{sd}$  &  $i_{sq}$  the direct torque control is possible and easy.

## 5 IMPLEMENTATION OF MATHEMATICAL EQUATIONS USING MATLAB

### 5.1 Introduction

The voltage & torque equations that describe the dynamic behavior of an induction motor are time-varying. It is successfully used to solve such differential equations and it may involve some complexity. A change of variables can be used to reduce the complexity of these equations by eliminating all time-varying inductances, due to electric circuits in relative motion, from the voltage equations of the motor. By this approach, a polyphase winding can be reduced to a set of two phase windings (q-d) with their magnetic axes formed in quadrature. In other words, the stator & rotor variables (voltages, currents and flux linkages) of an induction motor are transferred to a reference frame, which may rotate at any angular velocity or remain stationary. Such a frame of reference is commonly known in the generalized machines analysis as arbitrary reference frame.

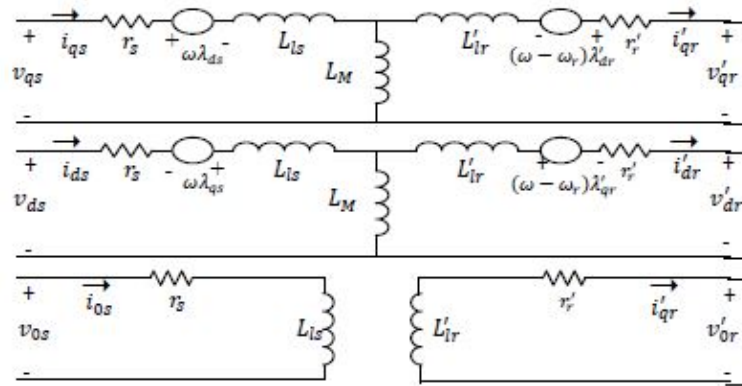


Figure 9: The dq Equivalent Circuit of Induction Motor

The dynamic analysis of the symmetrical induction motor in the arbitrary reference frame has been intensively used as a standard simulation approach from which any particular mode of operation may then be developed. Matlab/Simulink has an advantage over other machine simulators in modelling the induction machine using dq0 axis transformation. It can be a powerful technique in implementing the machine equations as they are transferred to a particular reference frame. Thus, every single equation among the model equations can



be easily implemented in one block so that all the machine variables can be made available for control and verification purposes. In this chapter, Matlab/Simulink is used to simulate the dynamic performance of an induction motor model whose stator and rotor variables are referred to an arbitrary reference frame. The provided machine model is simulated in a way that makes it easy for the reader to follow and understand the implementation process since it gives full details about Simulink structure of each of the model equations.

## 5.2 Induction Motor Model

Driving the model equations can be generated from the dq0 equivalent circuit of the induction motor shown in figure 9. The flux linkages equations associated with this circuit can be found as follows:

$$\frac{d\Psi_{qr}}{dt} = \omega_b[V_{qr} - \frac{(\omega_e - \omega_r)}{\omega_b}\Psi_{dr} + \frac{R_r}{X_{1r}}(\Psi_{mq} - \Psi_{qr})] \quad (1)$$

$$\frac{d\Psi_{ds}}{dt} = \omega_b[V_{ds} + \frac{\omega_e}{\omega_b}\Psi_{qs} + \frac{R_s}{X_s}(\Psi_{md} - \Psi_{ds})] \quad (2)$$

$$\frac{d\Psi_{qs}}{dt} = \omega_b[V_{qs} - \frac{\omega_e}{\omega_b}\Psi_{ds} + \frac{R_s}{X_{1s}}(\Psi_{mq} - \Psi_{qs})] \quad (3)$$

$$\frac{d\Psi_{dr}}{dt} = \omega_b[V_{dr} + \frac{(\omega_e - \omega_r)}{\omega_b}\Psi_{qr} + \frac{R_r}{X_{1r}}(\Psi_{md} - \Psi_{dr})] \quad (4)$$

where

$$\Psi_{md} = X_{m1}[\frac{\Psi_{ds}}{X_{1s}} + \frac{\Psi_{dr}}{X_{1r}}] \quad (5)$$

$$\Psi_{mq} = X_{m1}[\frac{\Psi_{qs}}{X_{1s}} + \frac{\Psi_{qr}}{X_{1r}}] \quad (6)$$

$$X_{m1} = \frac{1}{[\frac{1}{X_m} + \frac{1}{X_{1s}} + \frac{1}{X_{1r}}]} \quad (7)$$

Then substituting the values of the flux linkages to find the currents;

$$i_{qr} = \frac{1}{X_{1r}}(\Psi_{qr} - \Psi_{mq}) \quad (8)$$

$$i_{ds} = \frac{1}{X_{1s}}(\Psi_{ds} - \Psi_{md}) \quad (9)$$

$$i_{qs} = \frac{1}{X_{1s}}(\Psi_{qs} - \Psi_{mq}) \quad (10)$$

$$i_{dr} = \frac{1}{X_{1r}}(\Psi_{dr} - \Psi_{md}) \quad (11)$$

Based on the above equations, the torque and rotor speed can be determined as follows:

$$T_e = \left(\frac{3}{2}\right)\left(\frac{P}{2}\right)\left(\frac{1}{\omega_b}\right)(\Psi_{ds}i_{qs} - \Psi_{qs}i_{ds}) \quad (12)$$

$$\omega_r = \int \frac{P}{2J}(T_e - T_L) \quad (13)$$

Where P: number of poles; J: moment of inertia ( $kg/m^2$ ). For squirrel cage induction motor, the rotor voltages  $V_{qr}$  and  $V_{dr}$  in the flux equations are set to zero since the rotor cage bars are shorted. After deriving the torque and speed equations in term of d-q flux linkages and currents of the stator, the d-q axis transformation should now be applied to the machine input (stator) voltages.

The three-phase stator voltages of an induction motor under balanced conditions can be expressed as:

$$V_a = \sqrt{2}V_{rms} \sin(\omega t) \quad (14)$$

$$V_b = \sqrt{2}V_{rms} \sin\left(\omega t - \frac{2\Pi}{3}\right) \quad (15)$$

$$V_c = \sqrt{2}V_{rms} \sin\left(\omega t + \frac{2\Pi}{3}\right) \quad (16)$$

These three-phase voltages are transferred to a synchronously rotating reference frame in only two phases (d-q axis transformation). This can be done using the following two equations.

$$\begin{bmatrix} V_\alpha \\ V_\beta \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 & \frac{1}{2} & \frac{-1}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{-\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \quad (17)$$

Then, the direct and quadrature axes voltages are

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} V_\alpha \\ V_\beta \end{bmatrix} \quad (18)$$

The instantaneous values of the stator and rotor currents in three-phase system are ultimately calculated using the following transformation:

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} \quad (19)$$

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 & 0 \\ \frac{-\sqrt{1}}{2} & \frac{-\sqrt{3}}{2} \\ \frac{-\sqrt{1}}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} \quad (20)$$

### 5.3 Matlab/Simulink Implementation

In this section, the three phase induction motor model is simulated by using the Matlab/Simulink. Figure 10 depicts the complete Simulink scheme of the described induction motor model.

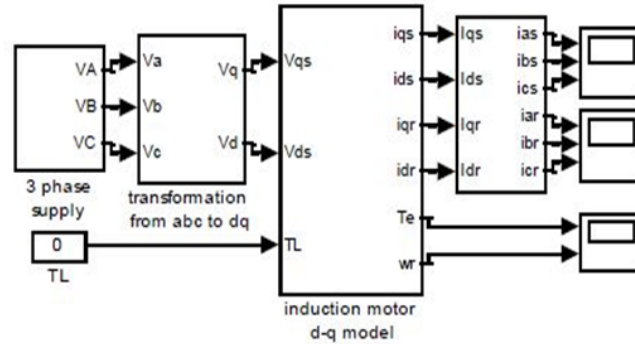


Figure 10: Matlab Simulation /Simulink Model of 3-phase Induction Motor

In this model the simulation starts with generating a three-phase stator voltages according to the equations (14, 15, 16), and then transforming these balanced voltages to two phase voltages referred to the synchronously rotating frame using Clarke and Park transformation as in equations (17, 18). After that the d-q flux linkage and current equations were implemented as to be demonstrated below.

Figure 11 illustrates the internal structure of the induction motor dq model by which the flux linkages, currents, torque and the rotor angular speed are calculated.

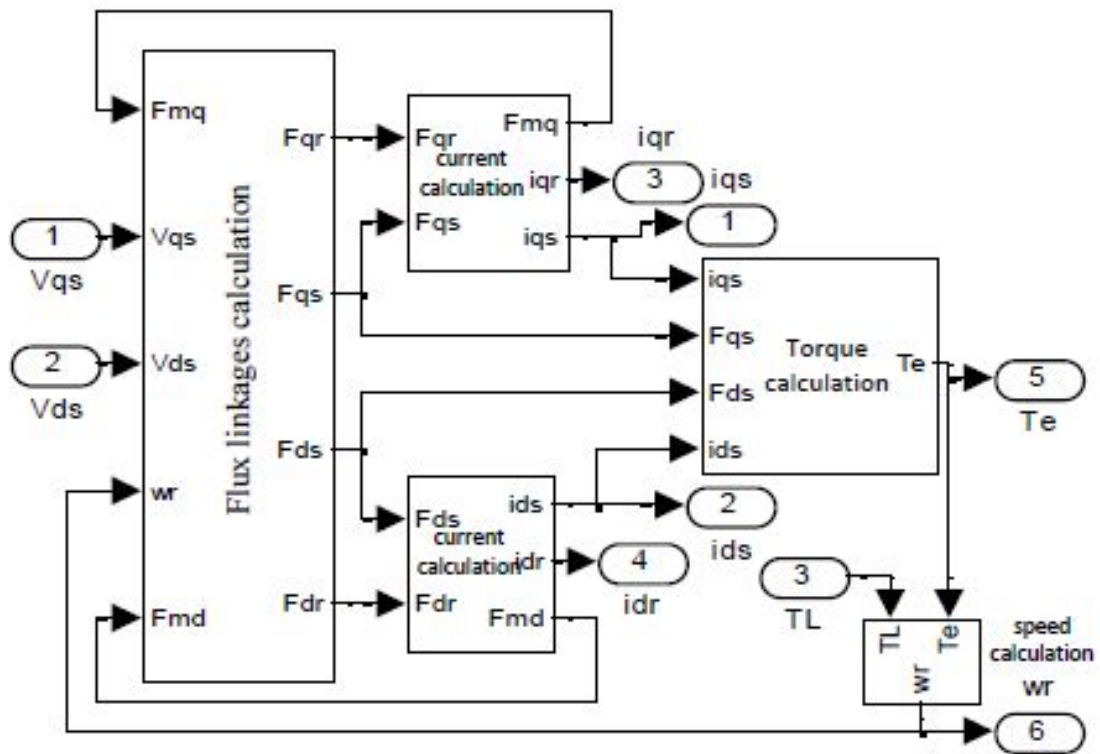


Figure 11: Internal Structure of dq Model

The Matlab/Simulink model to find the flux linkage ( $\Psi_{qs}, \Psi_{ds}, \Psi_{qr}, \Psi_{dr}$ ), as stated in equations (1)-(4) is shown in figure 12

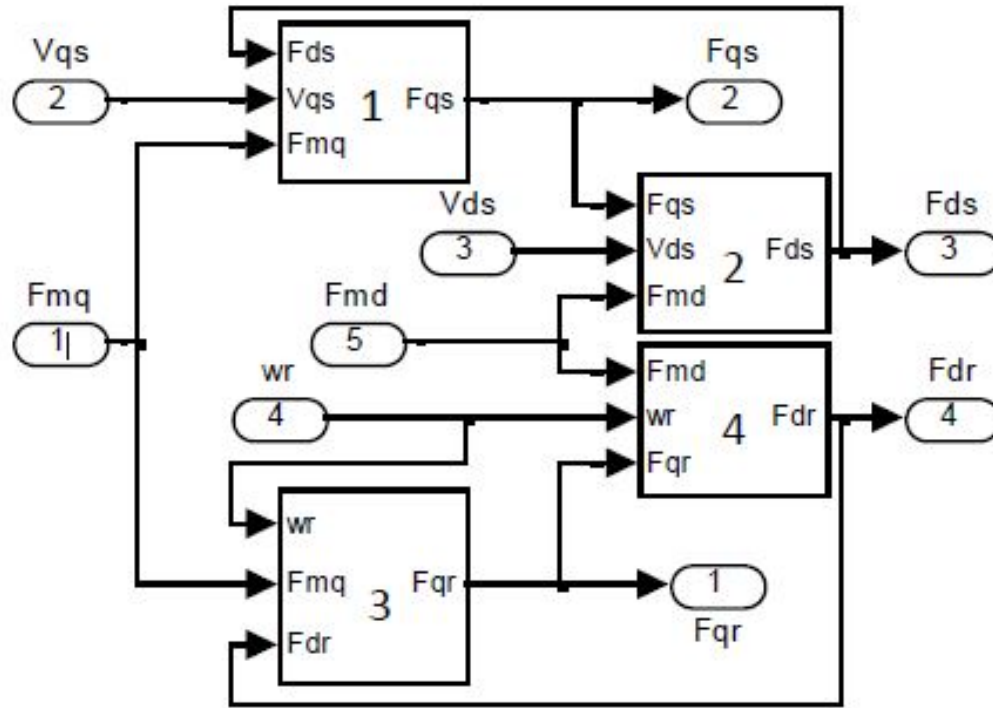


Figure 12: Internal Structure of Flux Calculation

Figure 13 shows the Simulink blocks used to calculate the currents  $i_{qs}, i_{dr}, i_{ds}, i_{qr}$  according to the equations (8),(11), also  $\Psi_{mq}, \Psi_{md}$ , in equations (5),(6).

Figures 14 and 15 show the implementation of torque  $T_e$  and angular speed  $\omega_r$  as expressed in equations (12), (13) respectively.

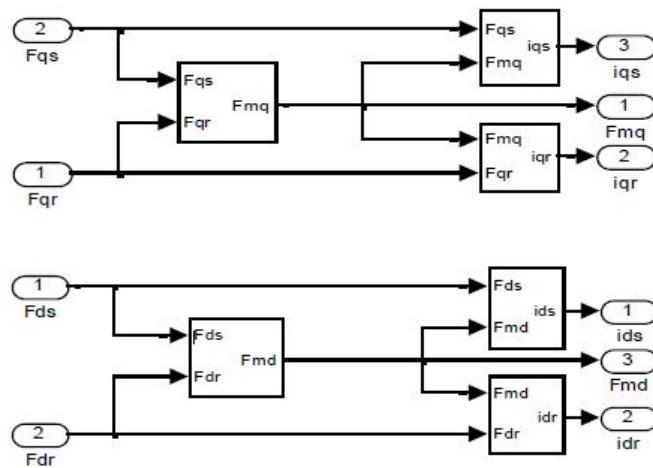


Figure 13: Internal Structure of Calculating Flux  $\Psi_{mq}$ ,  $\Psi_{md}$  and Currents  $i_{qs}$ ,  $i_{dr}$ ,  $i_{ds}$ ,  $i_{qr}$

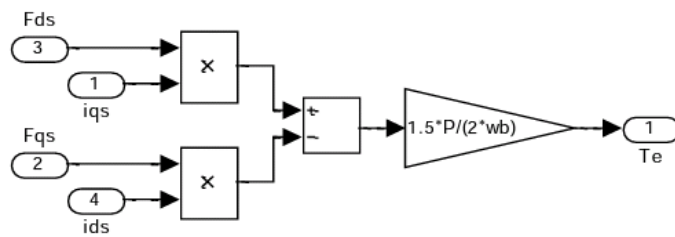


Figure 14: The Implementation of Torque Equation  $T_e$

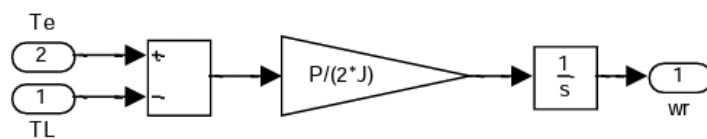


Figure 15: The Implementation of Angular Speed Equation  $\omega_r$

Figure.16 and 17 shows the internal structure of the blocks (1- 4) in figure 12 in which the equations (1)-(4) are implemented in Matlab/Simulink format.

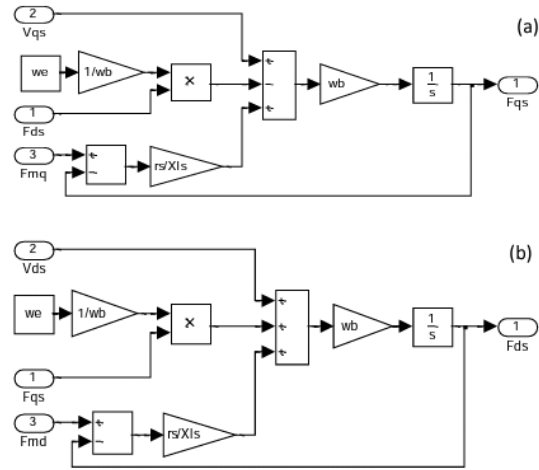


Figure 16: Internal Structure of the Blocks of fig. 12

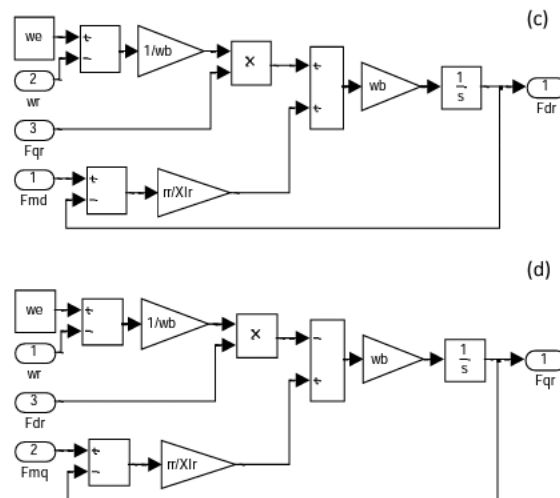


Figure 17: Internal Structure of the Blocks of fig. 12

Figure.18 presents the implementation of the flux linkages ,  $\Psi_{md}$  found in figure.13. Also, figure.19 depicts how the currents  $i_{qs}$ ,  $i_{dr}$ ,  $i_{ds}$ ,  $i_{qr}$  are constructed.

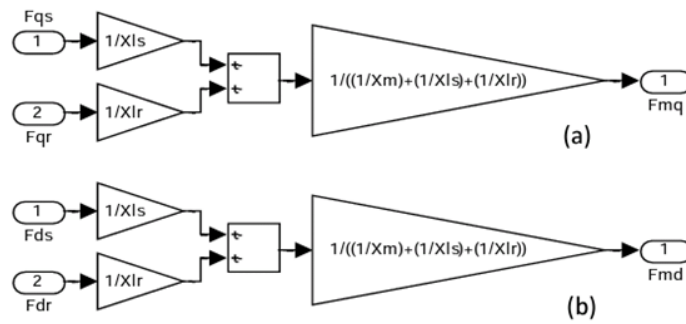


Figure 18: Shows Calculation of Flux Linkages

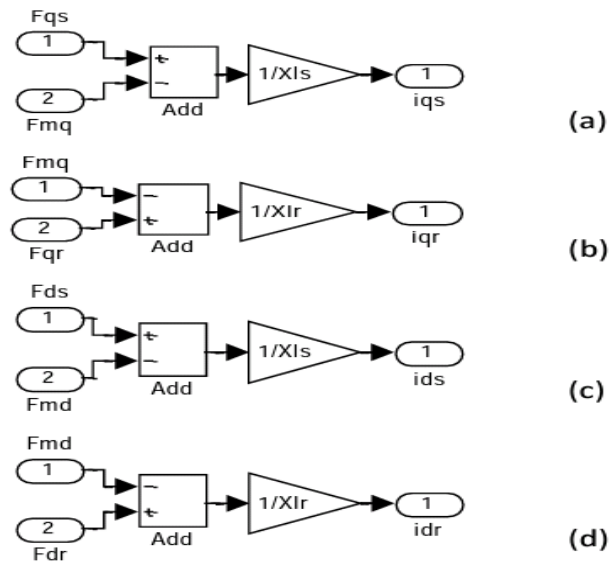


Figure 19: Implementation of dq Current Equations



## 6 MATLAB/SIMULINK RESULTS

### 6.1 Result for 3 HP IM

Induction motor of 3 HP was tested in this simulated model. The results of the simulation are given for the induction motor with the following specifications:

HP = 3,	$V_L = 220$ ,	$f = 50$
$R_s = 0.435$	$X_{ls} = 0.754$	$P = 4$
$R_r = 0.816$	$X_{lr} = 0.754$	$J = 0.089$
$X_m = 26.13$	$\text{rpm} = 1710$	$W_e = 157$
$\theta = 0$	$W_r = 157$	$W_b = 157$

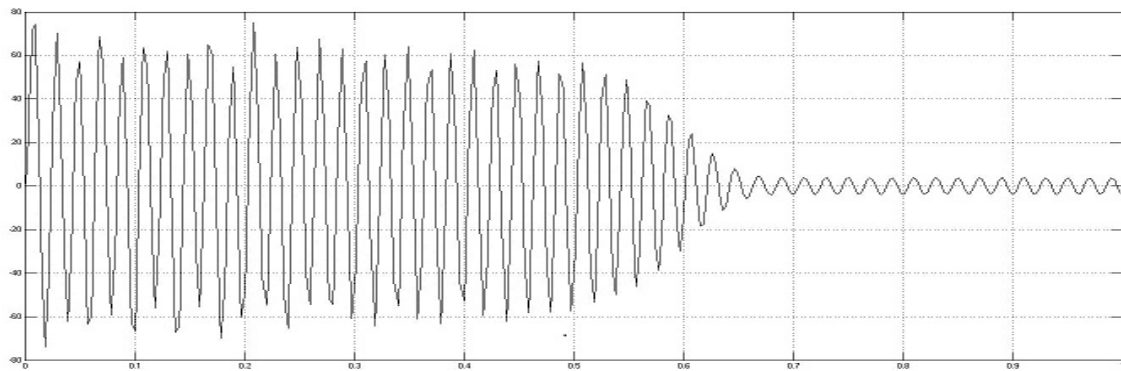


Figure 20: Stator a-phase Waveform With Respect to Time

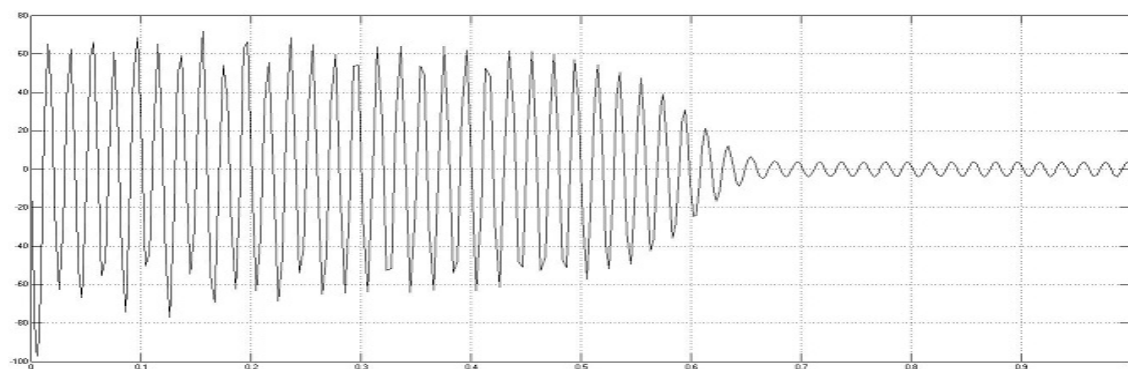


Figure 21: Stator b-phase Waveform With Respect to Time

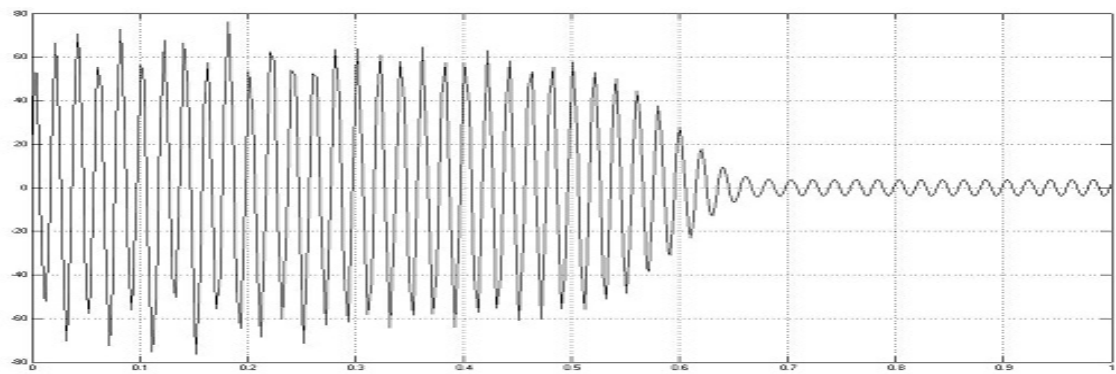


Figure 22: Stator c-phase Waveform With Respect to Time

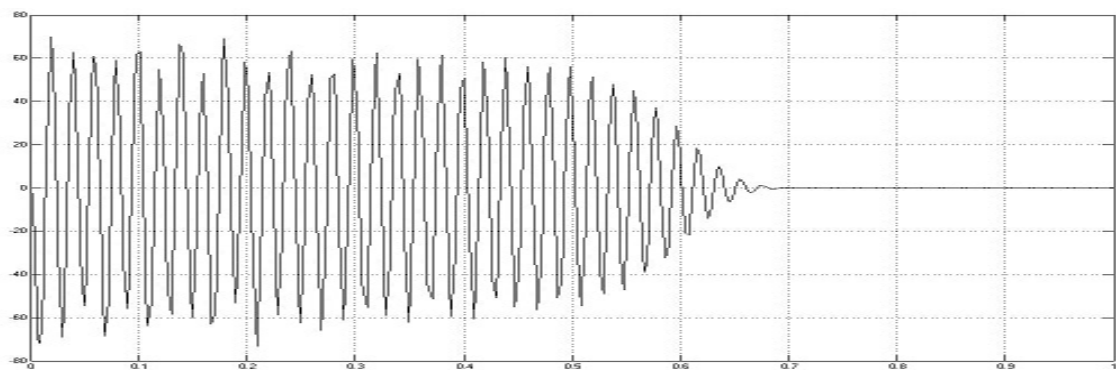


Figure 23: Rotor a-phase Waveform With Respect to Time

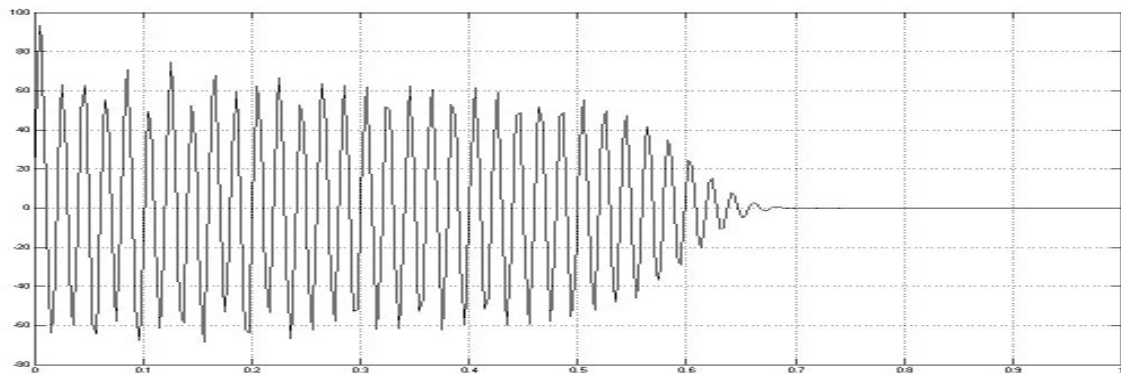


Figure 24: Rotor b-phase Waveform With Respect to Time

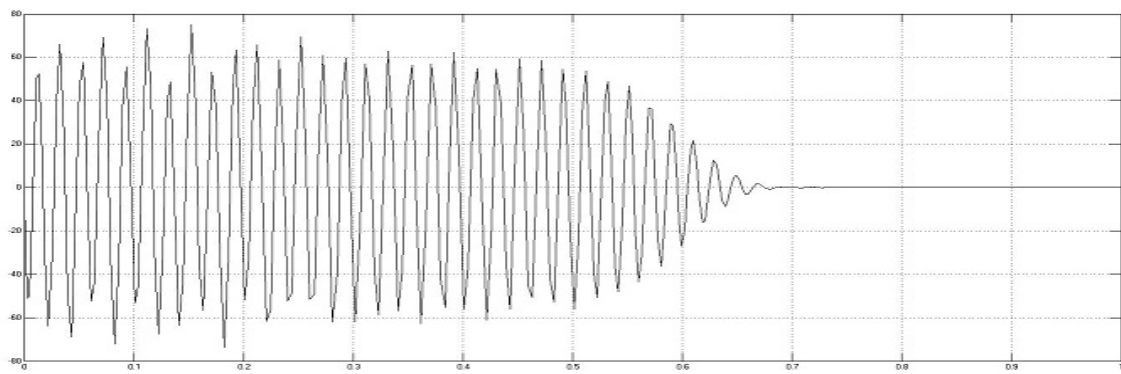


Figure 25: Rotor c-phase Waveform With Respect to Time

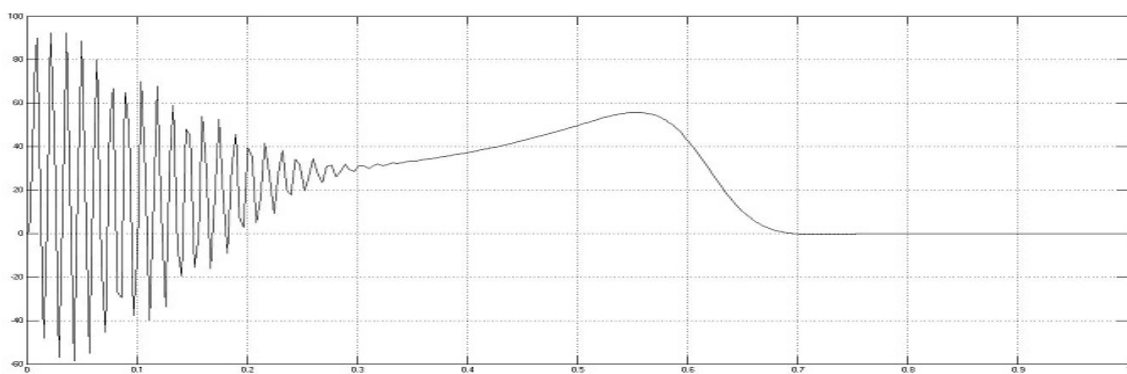


Figure 26: Torque Waveform With Respect to Time

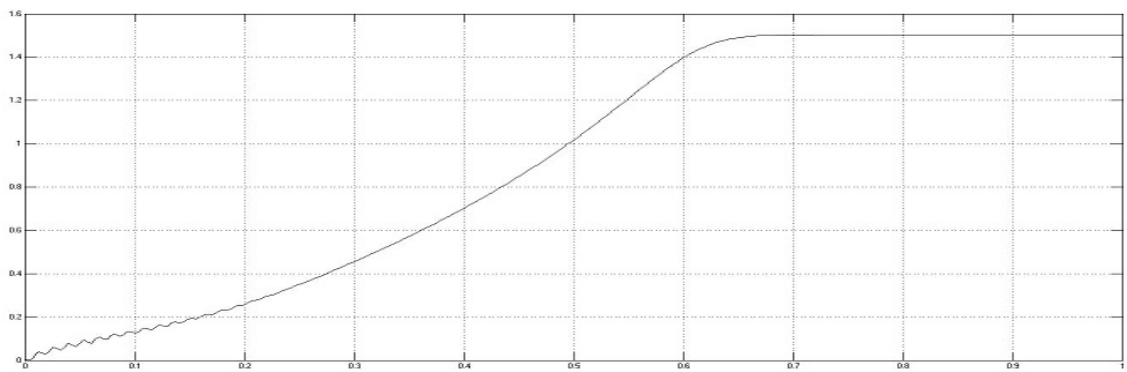


Figure 27: Speed Waveform With Respect to Time

## 6.2 Responses for different inputs

### 6.2.1 Result for Unit step input

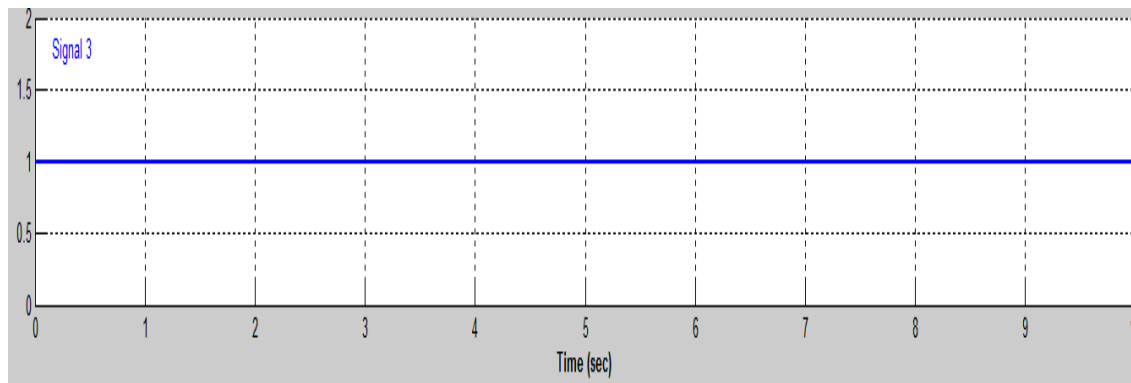


Figure 28: Unit Step Input

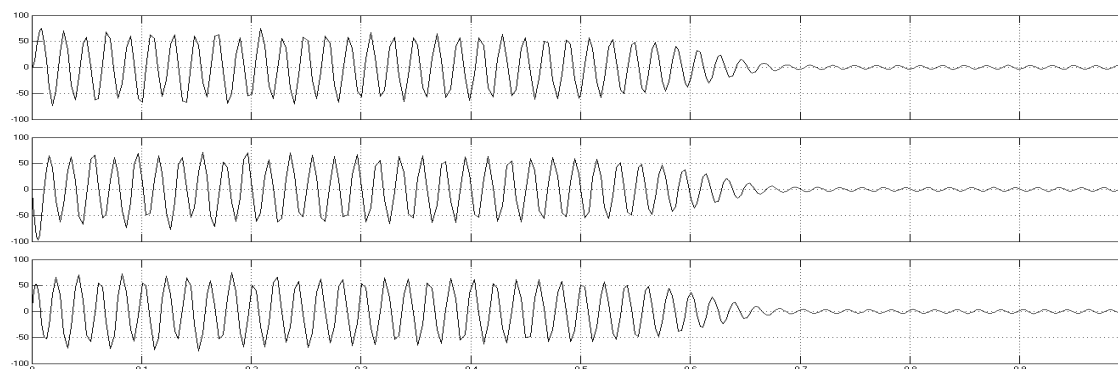


Figure 29: Stator Current Waveforms

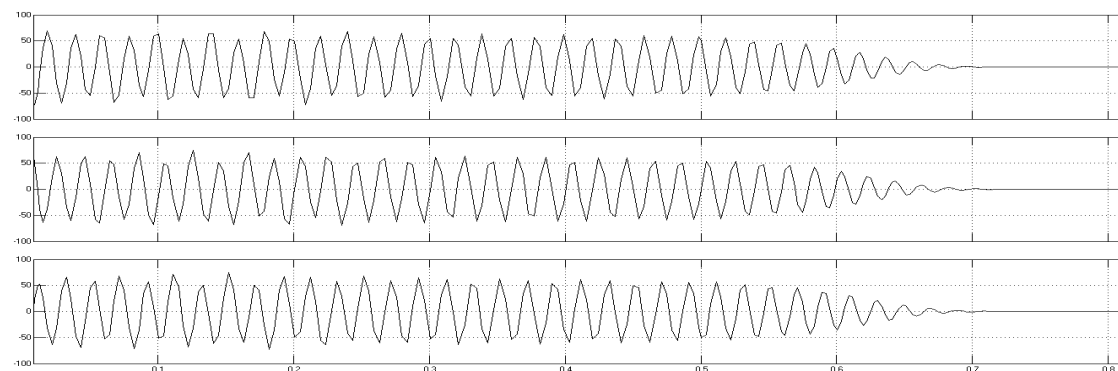


Figure 30: Rotor Current Waveforms

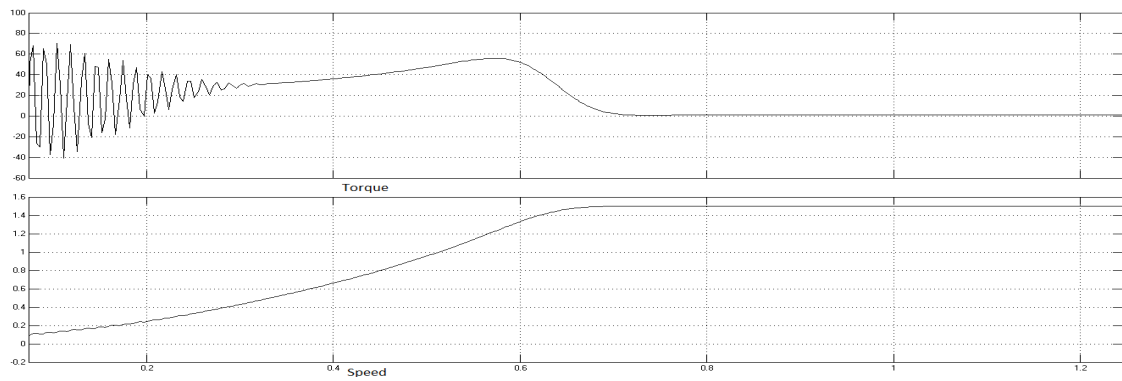


Figure 31: Torque and Speed Waveforms

### 6.2.2 Result for square wave input

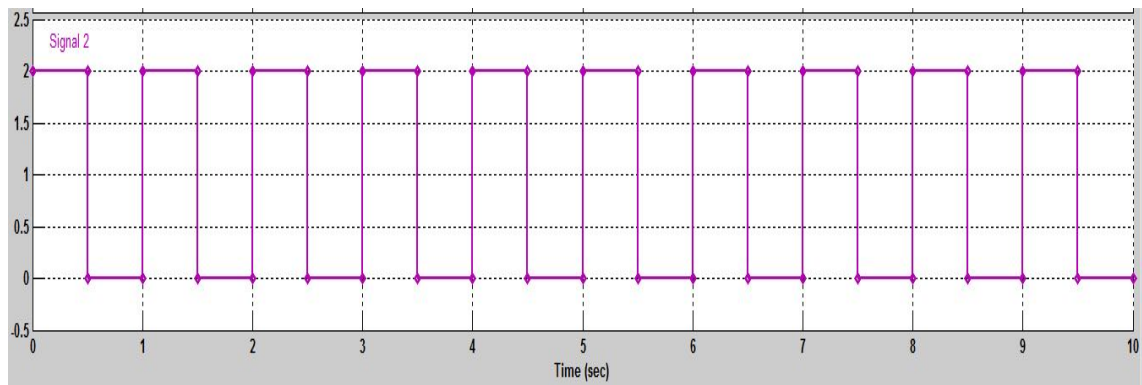


Figure 32: Square Wave Input

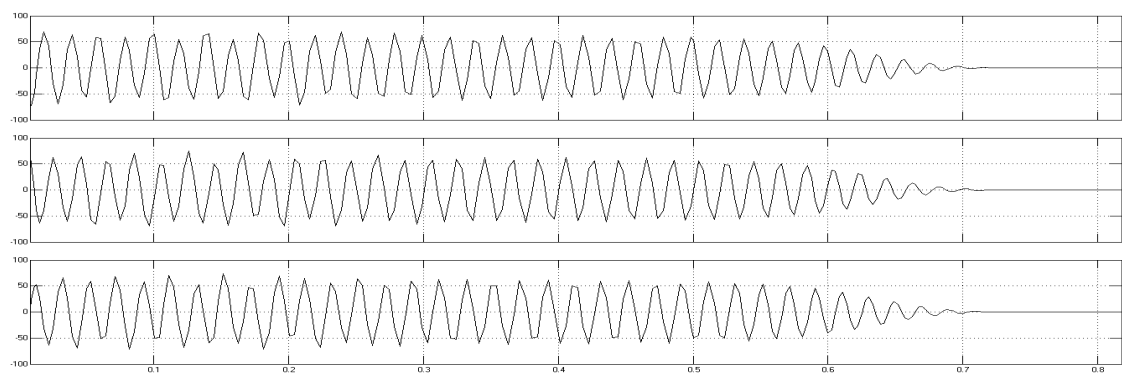


Figure 33: Stator Current Waveforms

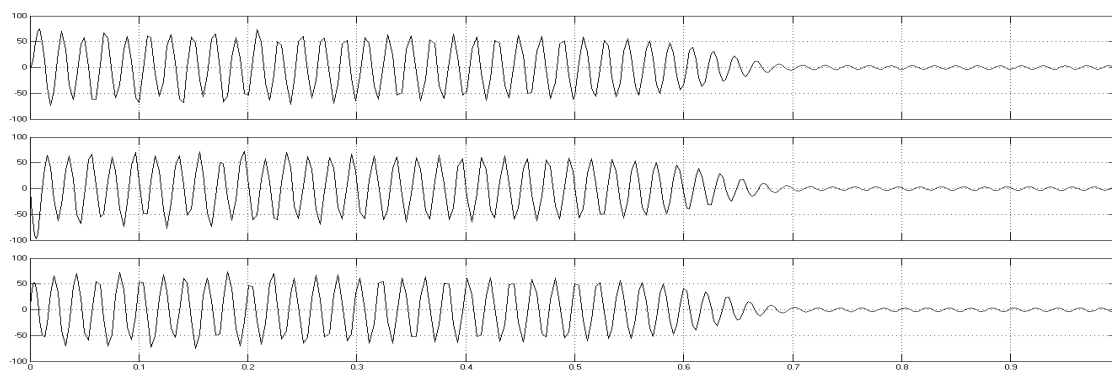


Figure 34: Rotor Current Waveforms

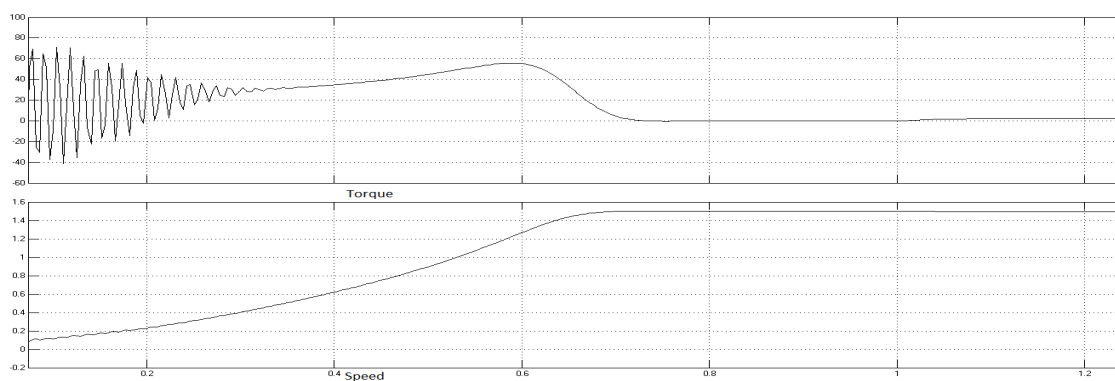


Figure 35: Torque and Speed Waveforms

### 6.2.3 Result for ramp plus step input

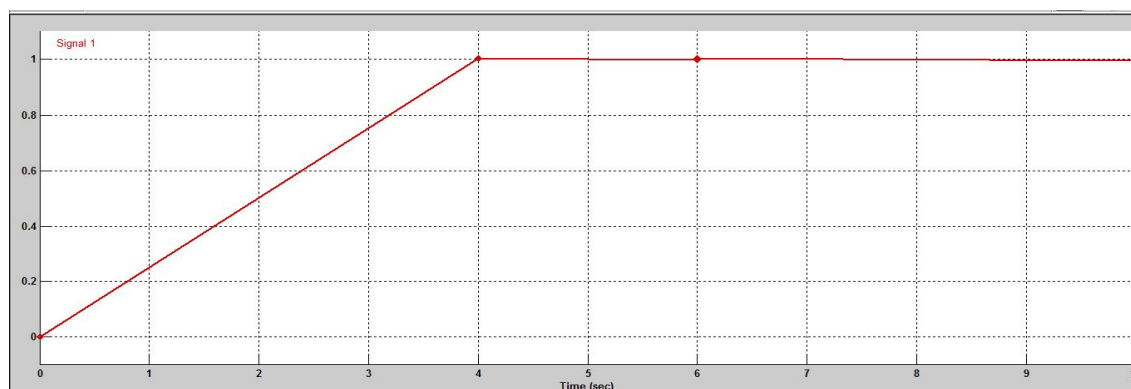


Figure 36: Ramp Plus Step Input

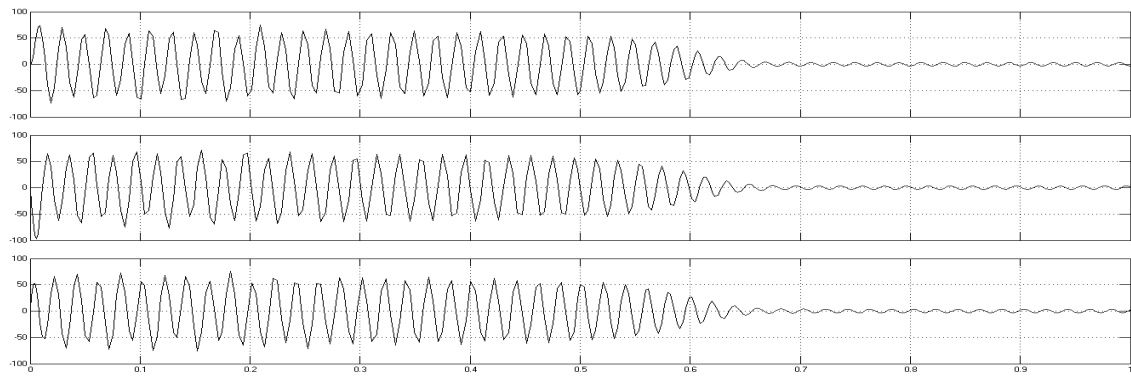


Figure 37: Stator Current Waveforms

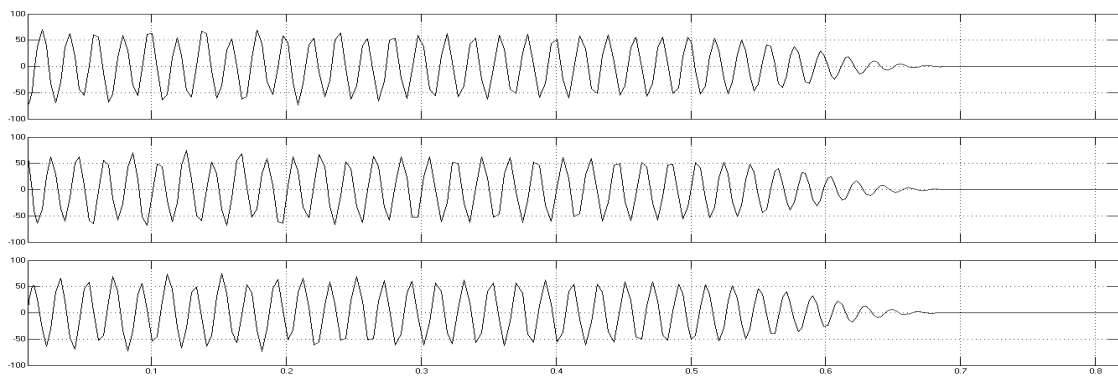


Figure 38: Rotor Current Waveforms

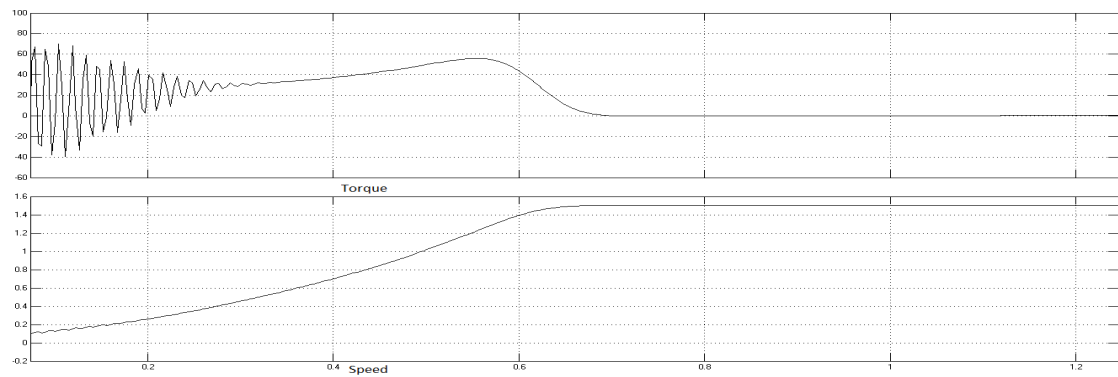


Figure 39: Torque and Speed Waveforms

### 6.3 Comparative analysis with Ideal Model

In this section we have discussed the induction motor speed, torque, and current characteristics with & without vector control method. The induction motor model used while obtaining characteristics of induction motor without vector control method is shown in following figure:

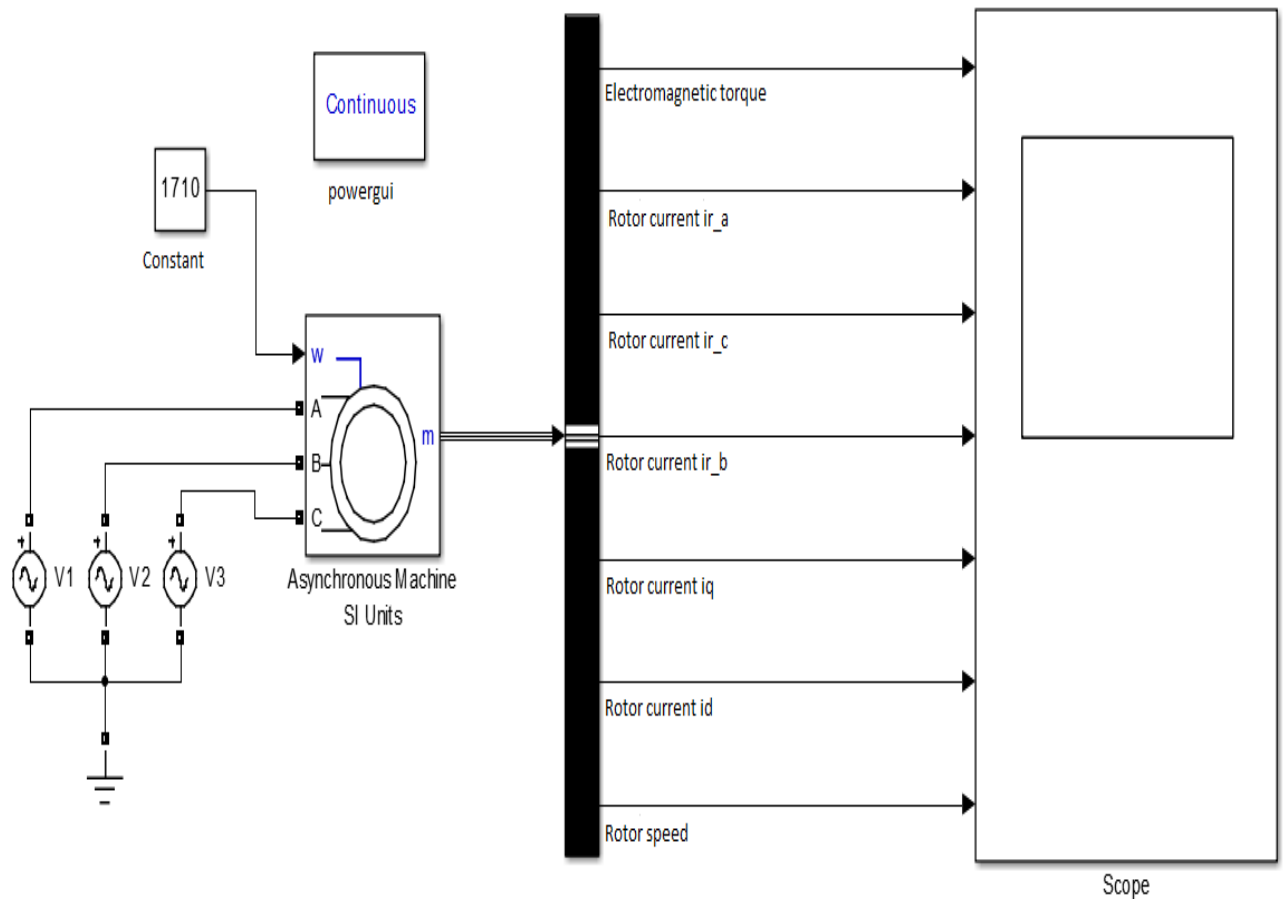


Figure 40: Ideal model of induction motor



In this model we varied the input in the form of speed and torque. For a speed 1710 rpm, the stator current, rotor current, torque, speed waveforms are as follow.

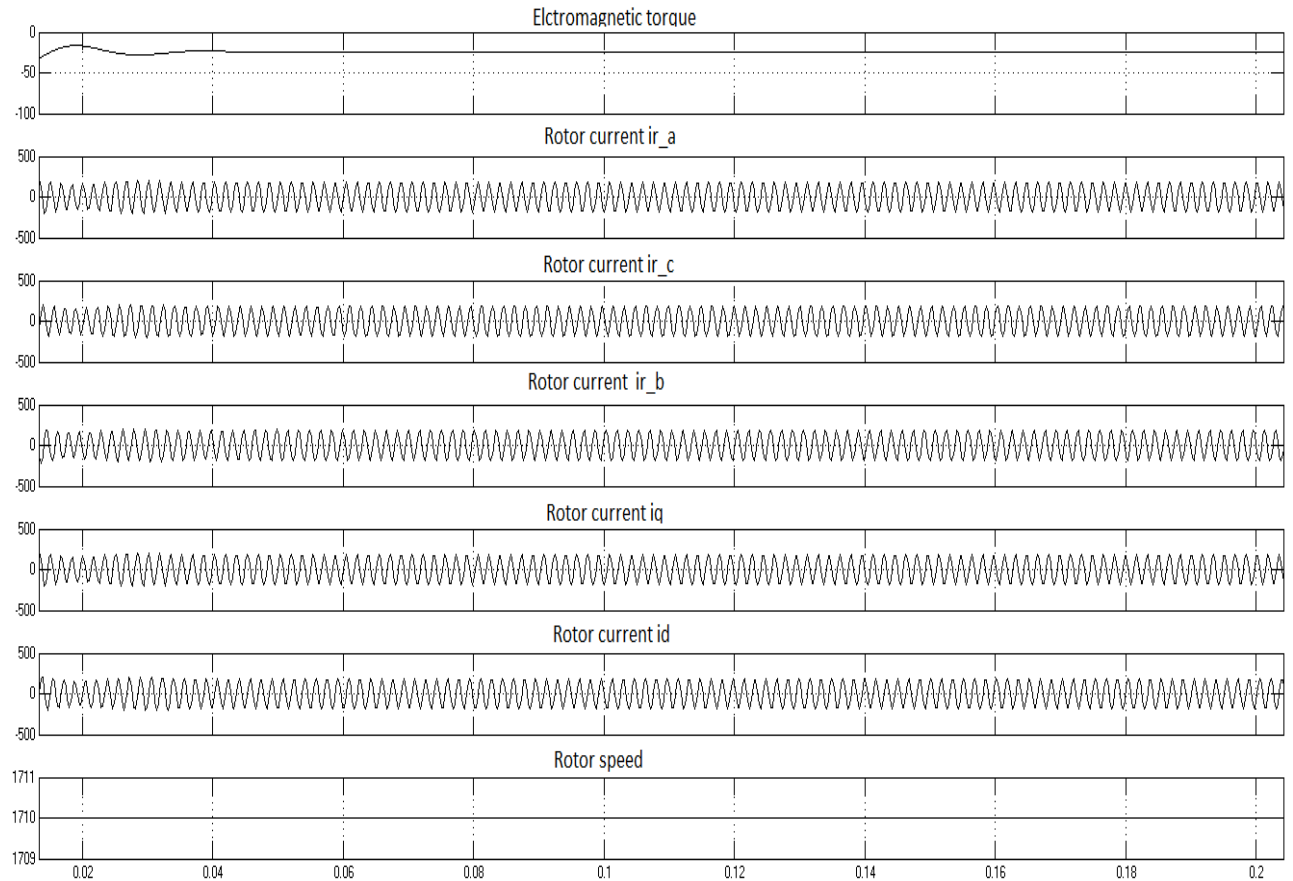


Figure 41: Waveforms of current, speed and torque

From comparative study we can say that performance characteristics of mathematical simulation is better i.e. vector control is superior than scalar control method. Transient response in case of mathematical simulation is better than ideal model.

## CONCLUSION

An implementation and dynamic modelling of a three-phase induction motor using Matlab/Simulink is presented in a step-by-step manner. The model was tested for 3 HP induction motor. The simulated machine has given a satisfactory response in terms of the torque and speed characteristics. This concludes that the Matlab/Simulink is a reliable and sophisticated way to analyse and predict the behaviour of induction motors using the theory of reference frames. By using this mathematical model we can vary the load torque according to the industrial application. Load torque defines current which is helpful to calculate copper losses from which required class of insulation can be identified.

When motor starts, initially pulsation occurs then immediately it goes to steady state condition and at no load output is zero. If the torque pulsation is very high then we can't use motor directly so change in parameter is required such as moment of inertia ( $J$ ). Moment of inertia is one of the important parameters in motor designing. From the value of moment of inertia we can obtain information such as energy stored, distortion, stabilizing time, settling time, pulsation losses, etc., which helps us to select material of IM. If we use the high quality material whose moment of inertia is low then we can obtain better response or if we use the low quality material whose moment of inertia is high then there will be large energy stored so it requires lot of time to dissipate energy hence rotor will take more time to reach stable state. Also, if we change the value of stator resistance then response also changes. If stator resistance is increased then torque pulsation also increases so settling time will be high and if stator resistance is decreased then torque pulsation also decreases, therefore, settling time will be less.

## **Future Scope**

- 1]Comparative analysis can be done for various sizes of motor.
- 2]This simulation model can be used for implement of hardware.
- 3]To study vector control this mathematical model can be coupled with PWM inverter.

Name:-Nimbalkar Gayatri Chandrakant.

Address:-A/p-Nimblak,

Tal-Phaltan,

Dist:-Satara-415523

Email:-Gayatri.nimbalkar1996@gmail.com

Date of Birth:-23/05/1996

Contact No:-9503485980

## References

- [1] “Mathematical Modelling of an 3-Phase Induction Motor”, Using MATLAB/Simulink Mr. Punit L. Ratnani<sup>1</sup>, Dr. A. G. Thosar
- [2] “ Induction Motor Modelling for Vector Control Purposes”, Helsinki University of Technology, Laboratory of Electromechanics , Report, Espoo 2000, 144 p.
- [3] “Comparative analysis of scalar and vector control of Induction motor through Modeling and Simulation”, published by Pabitra Kumar Behera<sup>1</sup>, Manoj Kumar Behera<sup>2</sup>, Amit Kumar Sahoo<sup>3</sup>
- [4] “Dynamic Simulation of a Three-Phase Induction Motor Using Matlab Simulink”, Adel Aktaibi and Daw Ghanim, graduate student members, IEEE, M. A. Rahman, life fellow, IEEE, Faculty of Engineering and Applied Science, Memorial University of Newfoundland
- [5] “ Modern power electronics and AC Drives”, by Bimal K. Bose
- [6] P. C. Krause, “Analysis of Electric Machinery”, McGraw-Hill Book Company, 1986.