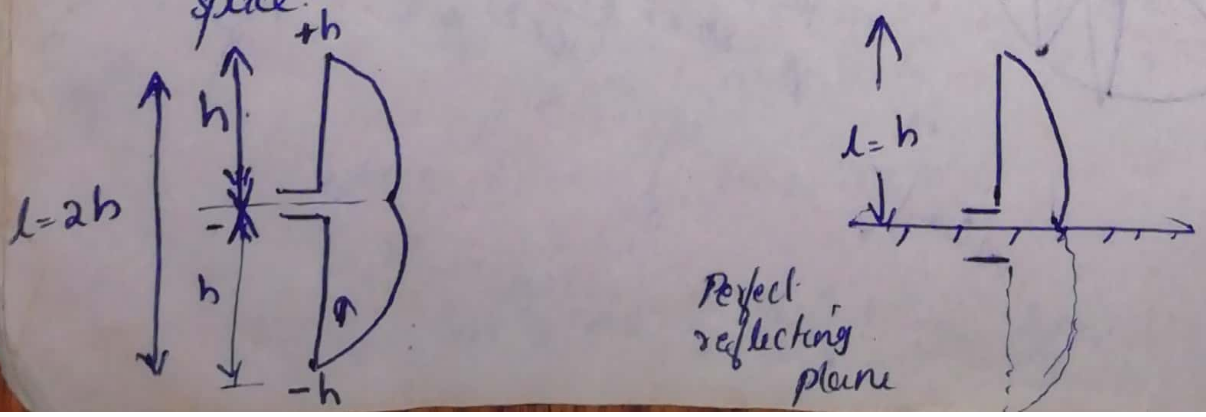
Monopoles and Dipoles

Dipole is usually fed at the centre having max. current at the centre. ie max. radiation in the plane normal to the axis. The current amplitude will be minimum at the ends.

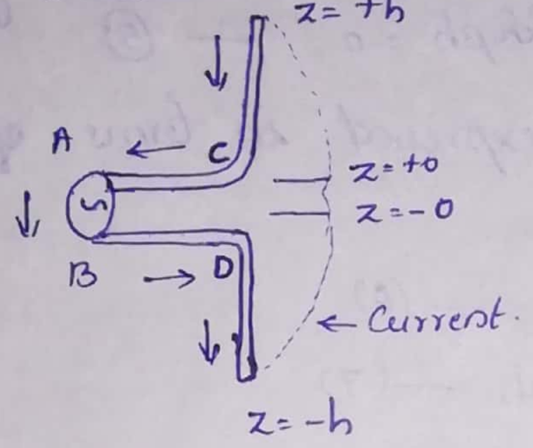
Half-wave dipole :- It has the physical length of $\lambda/2$ in free space at the freq. of operation. It is the fundamental radio antenna, made usually of metal rod or thin wire.

Quarter wave Monopole antenna :- A Monopole antenna consists of one-half of a dipole, usually a short vertical antenna mounted above the earth or a ground (reflecting plane. ~~This is also known as quarter wave antenna~~

It is fed in the lower end, which is near a conductive surface which works as a reflector. The current in the reflected image has the same direction and phase that the current in the real antenna. The set quarter wave plus image forms a half wave dipole that radiates only the upper half of space.



Asymtotic Current Distribution in Dipole:



Let the dipole antenna be fed by a two wire transmission line and the generator is connected at the terminals AB.

In each antenna arm the current is sinusoidal.

$$I(z) = A' \cos\beta z + B' \sin\beta z \quad \dots \quad z > +0$$
$$I(z) = A'' \cos\beta z + B'' \sin\beta z \quad \dots \quad z < -0 \qquad \text{— (1)}$$

At the ends of dipole the current must vanish as there is no conductor where the current can flow.

when $z = \pm h$

$$I(+h) = I(-h) = 0 \qquad \text{— (2)}$$

Since the figure is symmetrical and if the generator is balanced w.r.t. the feeder line at A, B the currents in AC and BD are equal and opposite. Thus the antenna current entering at C must be equal to that leaving D.

$$I(+0) = I(-0) \qquad \text{— (3)}$$

Applying (2) in (1)

$$A' \cos\beta(+h) + B' \sin\beta(+h) = 0$$

$$\cancel{A'' \cos\beta}\ A' \cos\beta h + B' \sin\beta h = 0 \qquad \text{— (4)}$$

$|||^{rly}$ $A''\cos\beta(-h) + B''\sin\beta(-h) = 0$. $\sin(-\theta) = -\sin\theta$

$A''\cos\beta h - B''\sin\beta h = 0$. — (5) $\cos(-\theta) = \cos\theta$

A' and B' can be expressed in terms of a new constant I_1

$$A' = I_1 \sin\beta h \quad \text{— (6)}$$

$$B' = -I_1 \cos\beta h \quad \text{— (7)}$$

$$A'' = I_2 \sin\beta h \quad \text{— (8)}$$

$$B'' = +I_2 \cos\beta h \quad \text{— (9)}$$

using (6) to (9) in (1)

$$I(z) = I_1 \sin\beta h \cos\beta z - I_1 \cos\beta h \sin\beta z$$

$$= I_1 \sin(\beta h - \beta z)$$

$$I(z) = I_1 \sin\beta(h-z) \quad [z > +0]$$

Similarly

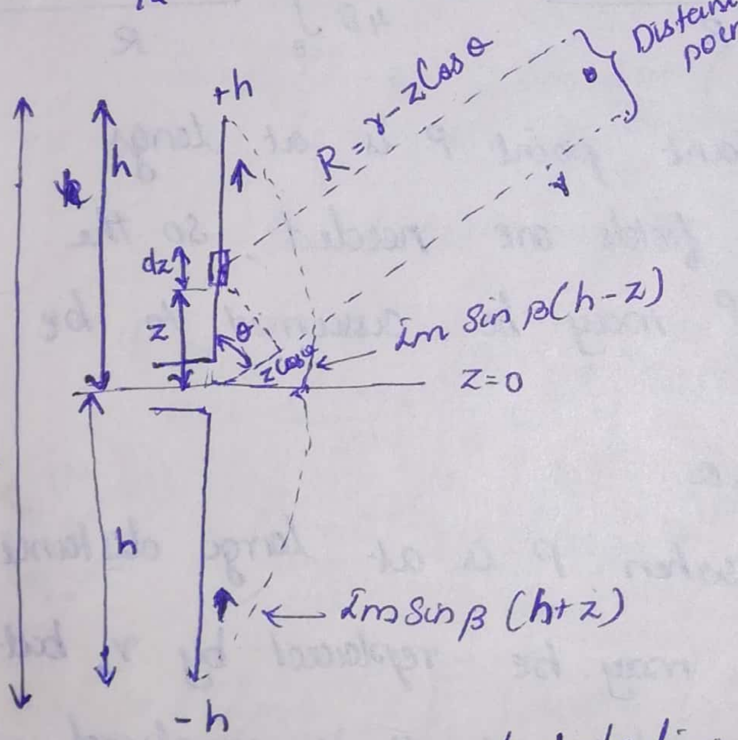
$$I(z) = I_2 \sin\beta(h+z) \quad [z < -0]$$

Let $I_1 = I_2 = I_m$.

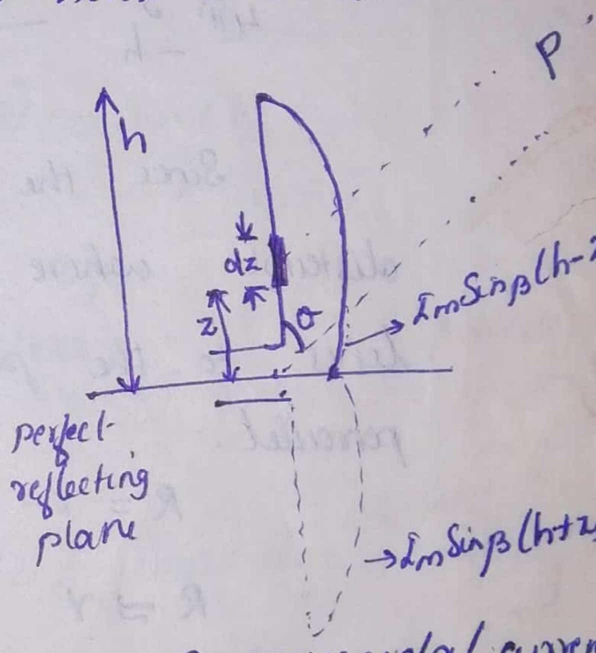
$$\therefore \boxed{\begin{aligned} I = I(z) &= I_m \sin\beta(h-z) \quad z > +0 \\ I = I(z) &= I_m \sin\beta(h+z) \quad z < -0. \end{aligned}}$$

Radiation from a Half wave dipole or quarterwave monopole.

A $\lambda/2$ antenna is the fundamental radio antenna of metal rod or thin wire which has a physical length of halfwavelength in free space at the freq. of operation. A $\lambda/2$ antenna is also known as Hertz antenna.



Sinusoidal current distribution assumed in centre fed dipole antenna



A sinusoidal current distribution assumed in $\lambda/4$ monopole

The sinusoidal current distribution is given by

$$I = I_m \sin\beta(h-z) \quad \text{for } z>0$$

$$I = I_m \sin\beta(h+z) \quad \text{for } z<0.$$

Vector potential at a distant point P due to current element $I\,dz$ is given by

$$dA_z = \frac{\mu I e^{-j\beta R}}{4\pi R} \cdot dz$$

where $R \rightarrow$ distance between $I\,dz$ to distant point

The total vector potential due to all such current element at distant point P is given by,

$$\int dA_z = \int_{-h}^{0} \frac{\mu}{4\pi R} I\, e^{-j\beta R}\, dz + \int_{0}^{h} \frac{\mu}{4\pi R} I\, e^{-j\beta R}\, dz$$

$$A_z = \frac{\mu}{4\pi} \int_{-h}^{0} \frac{I_m \sin\beta(h+z)\, e^{-j\beta R}}{R}\, dz + \frac{\mu}{4\pi} \int_{0}^{h} \frac{I_m \sin\beta(h-z)\, e^{-j\beta R}}{R}\, dz$$

Since the distant point P is at large distance where the fields are needed, so the lines to the point P may be assumed to be parallel.

$$R = r - z\cos\theta$$

$R \rightleftharpoons r$ when P is at large distance.

The denominator R may be replaced by r, but not in the numerator, because R is involved in the phase factor and hence the difference between R and r is important.

Then A_z can be written as

$$A_z = \frac{\mu}{4\pi} \int_{-h}^{0} \frac{I_m \sin\beta(h+z)\, e^{-j\beta(r - z\cos\theta)}}{r}\, dz + \frac{\mu}{4\pi} \int_{0}^{h} \frac{I_m \sin\beta(h-z)}{r}\, e^{-j\beta(r - z\cos\theta)}\, dz$$

$$= \frac{\mu}{4\pi} I_m e^{-j\beta r}\left[\int_{-h}^{0} \frac{\sin\beta(h+z)}{r} e^{j\beta z\cos\theta} dz + \int_{0}^{h} \frac{\sin\beta(h-z)}{r} e^{j\beta z\cos\theta} dz\right]$$

For $\lambda/2$ antenna, $l = 2h = \lambda/2$.

$$h = \lambda/4$$

$$\sin\beta(h+z) = \sin\frac{2\pi}{\lambda}\left(\frac{\lambda}{4}+z\right) = \sin\left(\frac{\pi}{2}+\beta z\right)$$
$$= \cos\beta z$$

$$\sin\beta(h-z) = \sin\frac{2\pi}{\lambda}\left(\frac{\lambda}{4}-z\right) = \sin\left(\frac{\pi}{2}-\beta z\right)$$
$$= \cos(\beta z)$$

$$\therefore \sin\beta(h+z) = \sin\beta(h-z) = \cos\beta z$$

Hence

$$A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi r}\left[\int_{-h}^{0} \cos\beta z \cdot e^{j\beta z\cos\theta} \cdot dz + \int_{0}^{h} \cos\beta z \cdot e^{j\beta z\cos\theta} \cdot dz\right]$$

Reversing the order of integration

$$= \frac{\mu I_m e^{-j\beta r}}{4\pi r}\left[\int_{0}^{h} \cos\beta z\, e^{-j\beta z\cos\theta} \cdot dz + \int_{0}^{h} \cos\beta z \cdot e^{j\beta z\cos\theta} \cdot dz\right]$$

$$= \frac{\mu I_m e^{-j\beta r}}{4\pi r}\left[\int_{0}^{h} \cos\beta z \left[e^{-j\beta z\cos\theta} + e^{j\beta z\cos\theta}\right] \cdot dz\right]$$

$$\frac{e^{jx}+e^{-jx}}{2} = \cos x$$

$$= \frac{\mu I_m e^{-j\beta r}}{4\pi r}\left[\int_{0}^{h} \cos\beta z \cdot (2)\cos(\beta z\cos\theta) \cdot dz\right]$$

$$2\cos A\cos B = \cos(A+B) + \cos(A-B)$$

$$= \frac{\mu I_m e^{-j\beta r}}{4\pi r}\left[\int_{0}^{h} \left[\cos(\beta z+\beta z\cos\theta) + \cos(\beta z - \beta z\cos\theta)\right] dz\right]$$

$$= \frac{\mu I_m e^{-j\beta r}}{4\pi r}\left[\left\{\int_{0}^{h} \cos\beta z(1+\cos\theta) + \cos\beta z(1-\cos\theta)\right\} \cdot dz\right]$$

$$= \frac{\mu I_m e^{-j\beta r}}{4\pi r}\left[\frac{\sin\{\beta z(1+\cos\theta)\}}{\beta(1+\cos\theta)} + \frac{\sin\beta z(1-\cos\theta)}{\beta(1-\cos\theta)}\right]_0^h$$

$$= \frac{\mu I_m e^{-j\beta r}}{4\pi r\beta}\left[\frac{(1-\cos\theta).\sin\{\beta z(1+\cos\theta)\} + (1+\cos\theta).\sin\beta z(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}\right]_0$$

$h = \lambda/4$

$$= \frac{\mu I_m e^{-j\beta r}}{4\pi r\beta}\left[\frac{(1-\cos\theta)\sin\left\{\frac{2\pi}{\lambda}.\frac{\lambda}{4}(1+\cos\theta)\right\} + (1+\cos\theta)\sin\left\{\frac{2\pi}{\lambda}.\frac{\lambda}{4}(1-\cos\theta)\right\}}{1-\cos^2\theta}\right]$$

$$= \frac{\mu I_m e^{-j\beta r}}{4\pi r\beta}\left[\frac{(1-\cos\theta).\sin(\pi/2 + \pi/2\cos\theta) + (1+\cos\theta)\sin(\pi/2 - \pi/2\cos\theta)}{\sin^2\theta}\right]$$

$$= \frac{\mu I_m e^{-j\beta r}}{4\pi r\beta}\left[\frac{(1-\cos\theta)\cos(\pi/2\cos\theta) + (1+\cos\theta)\cos(\pi/2\cos\theta)}{\sin^2\theta}\right]$$

$$= \frac{\mu I_m e^{-j\beta r}}{4\pi r\beta}\left[\cancel{\frac{\cos(\pi/2\cos\theta).[1+\cos^2\theta]-\cos\theta}{\sin^2\theta}}\right]$$

$$= \frac{\mu I_m e^{-j\beta r}}{4\pi r\beta}\left[\frac{\cos(\pi/2\cos\theta).[1-\cos\theta+1+\cos\theta]}{\sin^2\theta}\right]$$

$$= \frac{\mu I_m e^{-j\beta r}}{4\pi r\beta}\left[\frac{\cos(\pi/2.\cos\theta).\ 2}{\sin^2\theta}\right]$$

$$= \frac{\mu I_m e^{-j\beta r}}{2\pi r\beta}\left[\frac{\cos(\pi/2.\cos\theta)}{\sin^2\theta}\right]$$

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From ~~Maxwell's~~ eqn. $B = \nabla \times A = \mu H$

$$\mu H_\phi = (\nabla \times A)_\phi$$

$$= \frac{1}{r}\left[\frac{\partial}{\partial r} A_\theta . r - \frac{\partial}{\partial \theta} . A_r\right]$$

$$= \frac{1}{r}\left[\frac{\partial}{\partial r} A_\theta . r\right]$$ when only radiation field is considered

$$= \frac{1}{r}\frac{\partial}{\partial r}(-A_z \sin\theta) r$$

$$= -\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\mu I_m e^{-j\beta r}}{2\pi\beta r} . \frac{\cos(\pi/2 . \cos\theta) . \sin\theta . r}{\sin^2\theta}\right).$$

$$= \frac{-1}{r}\frac{\partial}{\partial r}\left[\frac{\mu I_m e^{-j\beta r} . \cos(\pi/2 . \cos\theta)}{2\pi\beta . \sin\theta}\right].$$

$$= \frac{-1}{r}\frac{\mu I_m . \cos(\pi/2 . \cos\theta)}{2\pi\beta \sin\theta} . e^{-j\beta r} . (-j\beta)$$

$$\mu H_\phi = \frac{j \mu I_m . e^{-j\beta r} . \cos(\pi/2 . \cos\theta)}{2\pi r \sin\theta}.$$

$$H_\phi = \frac{j I_m . e^{-j\beta r} . \cos(\pi/2 . \cos\theta)}{2\pi r \sin\theta}.$$

$|e^{j\theta}| = 1$

$|x+jy| = \sqrt{x^2+y^2}$

$$|H_\phi| = \frac{I_m}{2\pi r}\frac{\cos(\pi/2 . \cos\theta)}{\sin\theta} \; A/m^2$$

The above expression represents magnetic field intensity for a half wave dipole or a quarter wave monopole

The electric field expression for the radiation field can be obtained from

$$\frac{E_\theta}{H_\phi} = \eta = 120\pi .$$

$$|E_\theta| = 120\pi |H_\phi|$$

$$= 120\pi \cdot \frac{I_m}{2\pi r} \frac{\cos(\pi/2 \cdot \cos\theta)}{\sin\theta}$$

$$\boxed{|E_\theta| = \frac{60 I_m}{r} \frac{\cos(\pi/2 \cdot \cos\theta)}{\sin\theta}} \quad V/m$$

This is the expression for the electric field intensity for the radial field of a $\lambda/2$ antenna or a $\lambda/4$ monopole antenna.

The average value of power can be written as

$$P_{av} = \frac{E_\theta}{\sqrt{2}} \cdot \frac{H_\phi}{\sqrt{2}} = \frac{1}{2} E_\theta \cdot H_\phi$$

$$= \frac{1}{2} \frac{60 I_m}{r} \frac{\cos(\pi/2 \cdot \cos\theta)}{\sin\theta} \cdot \frac{I_m}{2\pi r} \cdot \frac{\cos(\pi/2 \cdot \cos\theta)}{\sin\theta}$$

$$= \frac{15 I_m^2}{\pi r^2} \cdot \left[\frac{\cos \cdot (\pi/2 \cdot \cos\theta)}{\sin\theta}\right]^2 .$$

Since $I_{rms} = \frac{I_m}{\sqrt{2}}$.

$$P_{av} = \frac{15(I_{rms} \cdot \sqrt{2})^2}{\pi r^2} \cdot \frac{\cos^2(\pi/2 \cdot \cos\theta)}{\sin^2\theta}$$

$$= \frac{30 I_{rms}^2}{\pi r^2} \cdot \frac{\cos^2(\pi/2 \cdot \cos\theta)}{\sin^2\theta} \, //$$

The total power radiated by a half wave dipole is obtained by integrating P_{av} over the surface of a sphere.

$$W = \int_{\phi=0}^{2\pi} \int_0^{\pi} P_{av} \cdot r^2 \sin\theta \cdot d\theta \cdot d\phi$$

$$= \int_0^{2\pi} \int_0^{\pi} \frac{30 I_{rms}^2}{\pi r^2} \frac{\cos^2\left(\frac{\pi}{2} \cdot \cos\theta\right)}{\sin^2\theta} \cdot r^2 \sin\theta \, d\theta \, d\phi$$

$$= \frac{2\pi \cdot 30 I_{rms}^2}{\pi} \int_0^{\pi} \frac{\cos^2\left(\frac{\pi}{2} \cdot \cos\theta\right)}{\sin\theta} \cdot d\theta.$$

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$$= 60 I_{rms}^2 \cdot \int_0^{\pi} \frac{\cos^2\left(\frac{\pi}{2} \cdot \cos\theta\right)}{\sin\theta} \cdot d\theta.$$

The integral in the above expression can be evaluated numerically to give a value of 1.219

$\therefore W = 1.219 \times 60 I_{rms}^2$.

$\therefore W = 73.14 I_{rms}^2$.

$\therefore$ The total power radiated by a $\lambda/2$ antenna is $W = 73.14 I_{rms}^2$.

$$R_r = \frac{W}{I_{rms}^2} = 73.14 \approx 73\,\Omega$$

ie Radiation resistance of a centre fed half wave dipole is $73\,\Omega$.

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For a quarterwave monopole antenna, the radiation resistance is half of dipoles' radiation resistance.

ie $\frac{73.14}{2} \simeq 36.57\,\Omega$.

To evaluate $\int_0^{\pi} \frac{\cos^2(\frac{\pi}{2}\cdot\cos\theta)}{\sin\theta}\,d\theta$.

$2\cos^2\theta = 1+\cos 2\theta$

$= \frac{1}{2}\int_0^{\pi} \frac{1+\cos(\pi\cdot\cos\theta)}{\sin\theta}\,d\theta$

put $t = \cos\theta$

$\frac{dt}{d\theta} = -\sin\theta$. ie $dt = -\sin\theta\cdot d\theta$.

$d\theta = -\frac{dt}{\sin\theta} = \frac{-dt}{\sqrt{1-\cos^2\theta}}$

$= \frac{-dt}{\sqrt{1-t^2}}$

when $\theta=\pi$, $t=\cos\pi = -1$

$\theta = 0$, $t = \cos 0 = 1$

$\therefore \frac{1}{2}\int_0^{\pi} \frac{1+\cos(\pi\cdot\cos\theta)}{\sin\theta}\,d\theta = \frac{1}{2}\int_1^{-1} \frac{1+\cos\pi t}{\sqrt{1-t^2}}\cdot\frac{-dt}{\sqrt{1-t^2}}$

$= -\frac{1}{2}\int_1^{-1} \frac{1+\cos\pi t}{1-t^2}\,dt$

$= \frac{1}{2}\int_{-1}^{1} (1+\cos\pi t)\cdot\frac{1}{2}\left[\frac{1}{(1+t)}+\frac{1}{(1-t)}\right]dt$

$\frac{1}{1+t}+\frac{1}{1-t} = \frac{1-t+1+t}{1-t^2} = \frac{2}{1-t^2}$

put ~~$t = -x$~~, ~~dt~~ $= -dx$.

$$= \frac{1}{4}\left[\int_{-1}^{1} \frac{1+\cos\pi t}{1+t}\,dt + \int_{-1}^{1} \frac{1+\cos\pi t}{1-t}\,dt\right]$$

put $t = -x$

$dt = -dx$

when $t = +1$ $\quad x = -1$

$t = -1$ $\quad x = 1$

substituting this in the 2^{nd} integral

$$= \frac{1}{4}\left[\int_{-1}^{1} \frac{1+\cos\pi t}{1+t}\,dt + \int_{1}^{-1} \frac{1+\cos(-\pi x)}{1-(-x)}\,(-dx)\right]$$

$\cos(-\theta) = \cos\theta$

$$= \frac{1}{4}\left[\int_{-1}^{1} \frac{1+\cos\pi t}{1+t}\,dt - \int_{1}^{-1} \frac{1+\cos\pi x}{1+x}\,dx\right]$$

$$= \frac{1}{4}\left[\int_{-1}^{1} \frac{1+\cos\pi t}{1+t}\,dt + \int_{-1}^{1} \frac{1+\cos\pi x}{1+x}\,dx\right]$$

Now 1^{st} integral $= 2^{nd}$ integral.

$$\therefore \text{ integral} = 2\times\frac{1}{4}\int_{-1}^{1} \frac{1+\cos\pi x}{1+x}\,dx$$

put $1+x = \frac{y}{\pi}$ or $\pi + \pi x = y$

$\pi x = y - \pi$

$dx = \frac{dy}{\pi}$

when $x = -1$, $y = 0$

$x = 1$, $y = 2\pi$

$$\therefore I = \frac{1}{2}\int_{0}^{2\pi} \frac{1+\cos\pi\left(\frac{y-\pi}{\pi}\right)}{y/\pi}\cdot\frac{dy}{\pi}$$

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$$= \frac{1}{2}\int_0^{2\pi} \frac{1+\cos(y-\pi)}{y}\,.dy$$

$\cos-(\pi-y)$
$= \cos-(-y)$
$= \cos y.$

$$= \frac{1}{2}\int_0^{2\pi} \frac{1-\cos y}{y}\,.dy$$

$\cos(y-\pi)$
$= -\cos y$

$$= \frac{1}{2}\int_0^{2\pi} \frac{1-\left(1-\frac{y^2}{2!}+\frac{y^4}{4!}-\frac{y^6}{6!}+\frac{y^8}{8!}-\cdots\right)}{y}\,.dy$$

$$= \frac{1}{2}\int_0^{2\pi}\left[\frac{y}{2!}-\frac{y^3}{4!}+\frac{y^5}{6!}-\cdots\right].dy$$

$$= \frac{1}{2}\left[\frac{y^2}{2\times 2!}-\frac{y^4}{4\times 4!}+\frac{y^6}{6\times 6!}-\cdots\right]_0^{2\pi}$$

$$= \frac{1}{2}\left[\frac{(2\pi)^2}{2\times 2!}-\frac{(2\pi)^4}{4\times 4!}+\cdots\right]$$

Taking 8 terms of the series

sum of $+^{ve}$ terms $= 26.878$

sum of $-^{ve}$ terms $= 24.44$

Difference $= 2.437$

$\therefore\ I = \frac{1}{2} \times 2.437 = 1.219$

$W = 60\, I_{rms}^2 \times 1.219$

$= 73.140$