A CASE STUDY OF RENTAL STORES

TSAF ASSIGNMENT

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MODULE TITLE: **TIME SERIES ANALYSIS AND FORECASTING (TSAF)**

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A CASE STUDY OF RENTAL STORES

A soft copy of all the required documentations, findings and outputs for this report is provided in the CD.

Question (a)

Plot the original time series and state the time-series component that present in the data.

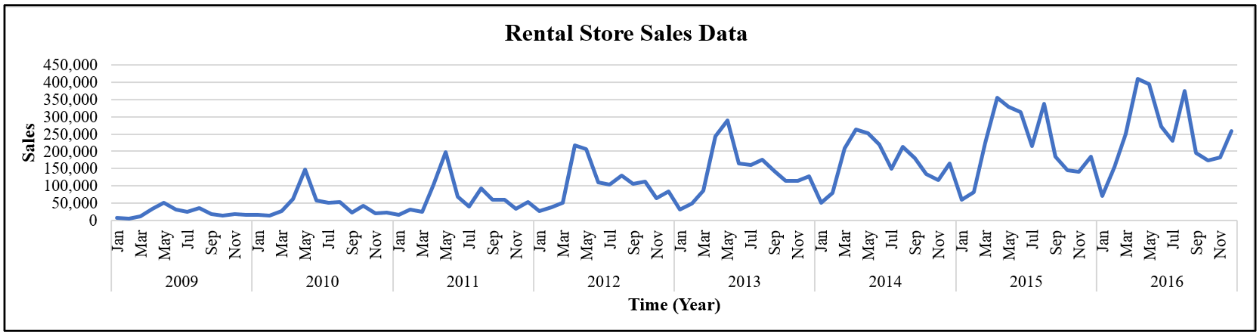

Figure 1 Time series graph for sales volume data

The time series graph is plotted from the year 2009 till 2016 for months January till December to show the monthly sales volume of John's business. The sales volume is at its highest in May and is at its lowest in January from the year 2009 till 2016. This shows that there is a strong **seasonal pattern** in the data due to similar regular wave-like patterns for each year. The time series plot shows a **steady upward trend**, where there is an increase in the sales volume from year 2009 till 2016. The sales volume is the highest in 2016 compared to other years. There is a steady increase in the sales volume from January to May and a gradual decrease from May till December for all the years except 2016, where it showed an increase in sales volume from November to December.

Question (b)

Conduct the autocorrelation analysis (ACF) to explore the data pattern for the sales volume. Provide detailed descriptions of findings from the autocorrelation analysis.

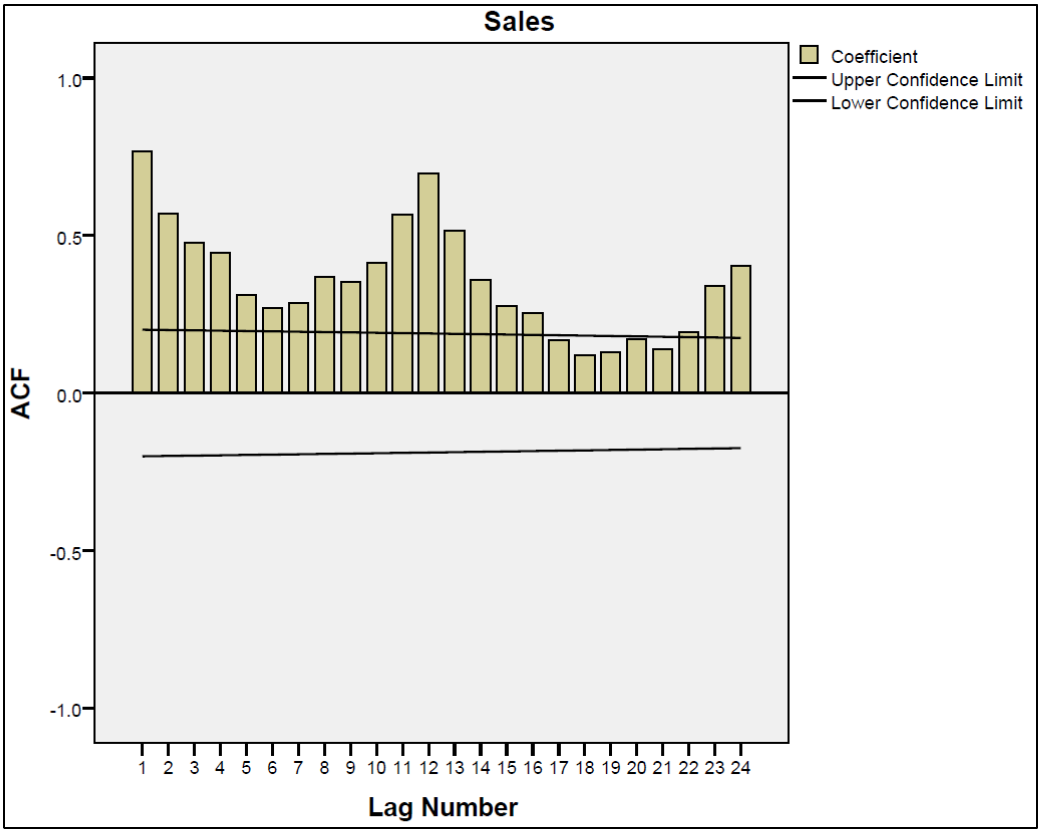

Figure 2 ACF plot for sales volume data

The graph above represents the correlogram for Autocorrelation Function (ACF) of the sales volume which was generated using IBM SPSS Statistics 22 software for 24 lags. The data is **non-stationary** as lags do not diminish to zero rapidly after the first two or three lags. Since the lags do not drop to zero quickly as the lag increases and the lags are not within the upper and lower confidence limits, the series is **not random**. There may be **seasonal pattern** at the multiples of the seasonal lag of 12 as the time series plot in (a) shows a strong seasonality in the data for every year. The time series plot in (a) shows an upward trend in the data which might be present in the ACF plot. The data has **trend pattern** which is increasing and decreasing. The first lag value is close to 1 and the successive lags up to lag 6 are dropping towards zero showing a decreasing trend. An increasing trend is observed from lag 6 till lag 12. This pattern is repeated in same manner for the next 12 lags in the second period.

Question (c)

Choose any THREE of the appropriate methods to forecast the sales volume.

The 3 methods chosen are multiplicative decomposition, moving average and Winters smoothing. As the ACF plot for time series is non-stationary, differencing is applied to make the data stationary before forecast using these methods are done.

The first method chosen is **multiplicative decomposition**. As the time series plot of the sales show seasonal pattern, multiplicative decomposition is a better method to opt for than additive decomposition. In the multiplicative model the time series is expressed as product of trend-cycle, seasonal and irregular components. This model is commonly applied if the effects of seasonality is multiplicative, that is increasing or decreasing over time and the multiplicative seasonal factor is a relative value of the original series. The model is preferred when seasonal swing about the trend is changing which is portrayed on the sales volume data (Eurostat, 2017; Damrongkulkamjorn & Churueang, 2017). Previous studies have reported that multiplicative decomposition technique has been profoundly used for forecasting tasks in business sector, such as sales projection or financial forecasting and hydropower consumption forecasting. It has been proved that this method produces smaller mean absolute percentage error (MAPE) compared to other seasonal models (Wang et al., 2011; Deng & Jirutitijaroen, 2017). For this study, ForecastX tool is used to perform forecast using multiplicative decomposition as this method yielded lesser error compared to additive decomposition.

The second method chosen is **Winters smoothing** (also known as Holt-Winters exponential smoothing). This classical technique is used by many companies to produce demand forecasts when their sales data contain trend and seasonal pattern. Holt-Winters is popular because it is simple, has low data storage requirement and is easily automated. Holt-Winters can also adapt to trend and seasonal changes in sales. Besides, Winters smoothing is extended to show multiplicative and additive seasonality (Gelper, Fried & Croux, 2009; Goodwin, 2010).The Holt-Winters model has been found to outperform other models in forecasting sales accurately based on previous literature studies (Alon, Qi & Sadowski, 2001). The three smoothing constants are used to update the level, slope and seasonal components which are vital when the trend and seasonality are changing at different rate over time. Compared to other exponential smoothing methods, Holt-Winters was found to be the better one with minimal error. For this study, the alpha (α), beta (β) and gamma (γ) values are determined using the default parameter settings in the ForecastX tool because this gives the best Holt-Winters model. According to John Galt, if the components change rapidly, large smoothing constants should be used and for stable components, the smoothing constant should be close to zero (John Galt Solutions, 2017). The default setting recommended the constant values which are $\alpha = 0.02$, $\beta = 0.88$, and $\gamma = 0.16$ with additive seasonal pattern for Holt-Winters smoothing.

The third method chosen is **Moving Average (MA)**. The simple moving average which is equally weighted is frequently used to estimate the current level of a time series by averaging the results of recent sales history to determine the projected forecast for future observations (Johnston et al., 1999). This method is easy to use, quick, relatively inexpensive and works better for short range forecasts of products which makes it one of the commonly used methods in sales forecasting (Lee et al., 2012). Shorter-period MA reacts faster to recent demand changes than do long-period MA. The ideal number of preceding periods should be chosen by selecting the number of periods which yield the least error. The forecast was done using ForecastX tool for different periods of MA from 3, 6, and 9 periods. But it was found that 3-MA yielded the least MAPE error, thus this was chosen to forecast the sales volume in this study (John Galt Solutions, 2017).

Question (d)

Perform the forecast using the methods you selected in part (c) for the period within January 2009 through December 2016 (only for those periods which are possible). Then, provide graph to compare between observed and forecast values for each method.

The forecast values were generated for all three methods using ForecastX tool enabled in Microsoft Excel and the graphs were generated using Microsoft Excel. The tables containing the actual values and forecast values from January 2009 through December 2016 are displayed here. The graphs show a comparison between the observed and forecast values for each method.

Multiplicative decomposition

Table 1 Actual and forecast values (2009-2016) using multiplicative decomposition

Dates	Actual Value	Forecast Value (Multiplicative Decomposition)
Jan-2009	6,028.00	*6,028.00*
Feb-2009	5,927.00	*5,927.00*
Mar-2009	10,515.00	*10,515.00*
Apr-2009	32,276.00	*32,276.00*
May-2009	51,920.00	*51,920.00*
Jun-2009	31,294.00	*31,294.00*
Jul-2009	23,573.00	*21,967.62*
Aug-2009	36,465.00	*31,297.25*
Sep-2009	18,959.00	*20,680.74*
Oct-2009	13,918.00	*20,114.48*
Nov-2009	17,987.00	*17,581.20*
Dec-2009	15,294.00	*24,434.08*
Jan-2010	16,850.00	*12,412.07*
Feb-2010	12,753.00	*18,785.11*
Mar-2010	26,901.00	*33,997.73*
Apr-2010	61,494.00	*75,079.70*
May-2010	147,862.00	*97,684.70*
Jun-2010	57,990.00	*54,299.36*
Jul-2010	51,318.00	*43,558.46*
Aug-2010	53,599.00	*61,024.82*

Sep-2010	23,038.00	*39,308.08*
Oct-2010	41,396.00	*36,743.06*
Nov-2010	19,330.00	*28,902.73*
Dec-2010	22,707.00	*36,216.82*
Jan-2011	15,395.00	*17,307.33*
Feb-2011	30,826.00	*25,494.39*
Mar-2011	25,589.00	*47,670.11*
Apr-2011	103,184.00	*105,989.31*
May-2011	197,608.00	*137,132.45*
Jun-2011	68,600.00	*77,903.99*
Jul-2011	39,909.00	*63,913.56*
Aug-2011	91,368.00	*89,194.72*
Sep-2011	58,781.00	*57,699.15*
Oct-2011	59,679.00	*56,526.27*
Nov-2011	33,443.00	*44,049.56*
Dec-2011	53,719.00	*54,024.94*
Jan-2012	27,773.00	*27,230.91*
Feb-2012	36,653.00	*41,251.60*
Mar-2012	51,157.00	*75,920.34*
Apr-2012	217,509.00	*169,628.62*
May-2012	206,229.00	*222,229.13*
Jun-2012	110,081.00	*125,535.24*
Jul-2012	102,893.00	*101,469.54*
Aug-2012	128,857.00	*140,843.21*
Sep-2012	104,776.00	*90,989.07*
Oct-2012	111,036.00	*83,964.64*
Nov-2012	63,701.00	*63,695.52*
Dec-2012	82,657.00	*79,943.05*
Jan-2013	31,416.00	*39,725.22*
Feb-2013	48,341.00	*59,375.27*
Mar-2013	85,651.00	*108,066.85*
Apr-2013	242,673.00	*234,343.30*
May-2013	289,554.00	*301,542.04*
Jun-2013	164,373.00	*171,049.05*
Jul-2013	160,608.00	*139,080.37*
Aug-2013	176,096.00	*194,802.08*

Sep-2013	142,363.00	*129,108.52*
Oct-2013	114,907.00	*121,063.72*
Nov-2013	113,552.00	*87,796.80*
Dec-2013	127,042.00	*105,384.91*
Jan-2014	51,604.00	*50,976.70*
Feb-2014	80,366.00	*74,098.58*
Mar-2014	208,938.00	*133,787.28*
Apr-2014	263,830.00	*290,411.14*
May-2014	252,216.00	*369,570.19*
Jun-2014	219,566.00	*205,807.52*
Jul-2014	149,082.00	*165,983.94*
Aug-2014	213,888.00	*229,502.18*
Sep-2014	178,947.00	*146,219.94*
Oct-2014	133,650.00	*135,268.14*
Nov-2014	116,946.00	*102,420.90*
Dec-2014	164,154.00	*127,115.39*
Jan-2015	58,843.00	*62,918.05*
Feb-2015	82,386.00	*94,475.96*
Mar-2015	224,803.00	*171,708.38*
Apr-2015	354,301.00	*368,662.38*
May-2015	328,263.00	*469,676.19*
Jun-2015	313,647.00	*261,131.88*
Jul-2015	214,561.00	*209,638.16*
Aug-2015	337,192.00	*293,855.64*
Sep-2015	183,482.00	*189,837.89*
Oct-2015	144,618.00	*173,785.33*
Nov-2015	139,750.00	*129,426.20*
Dec-2015	184,546.00	*155,336.74*
Jan-2016	71,043.00	*73,943.47*
Feb-2016	152,930.00	*107,788.69*
Mar-2016	250,559.00	*192,630.04*
Apr-2016	409,567.00	*415,297.50*
May-2016	394,747.00	*532,129.13*
Jun-2016	272,874.00	*299,286.03*
Jul-2016	230,303.00	*224,771.22*
Aug-2016	375,402.00	*311,820.92*

Sep-2016	195,409.00	*198,749.90*
Oct-2016	173,518.00	*179,886.08*
Nov-2016	181,702.00	*131,388.77*
Dec-2016	258,713.00	*157,157.40*

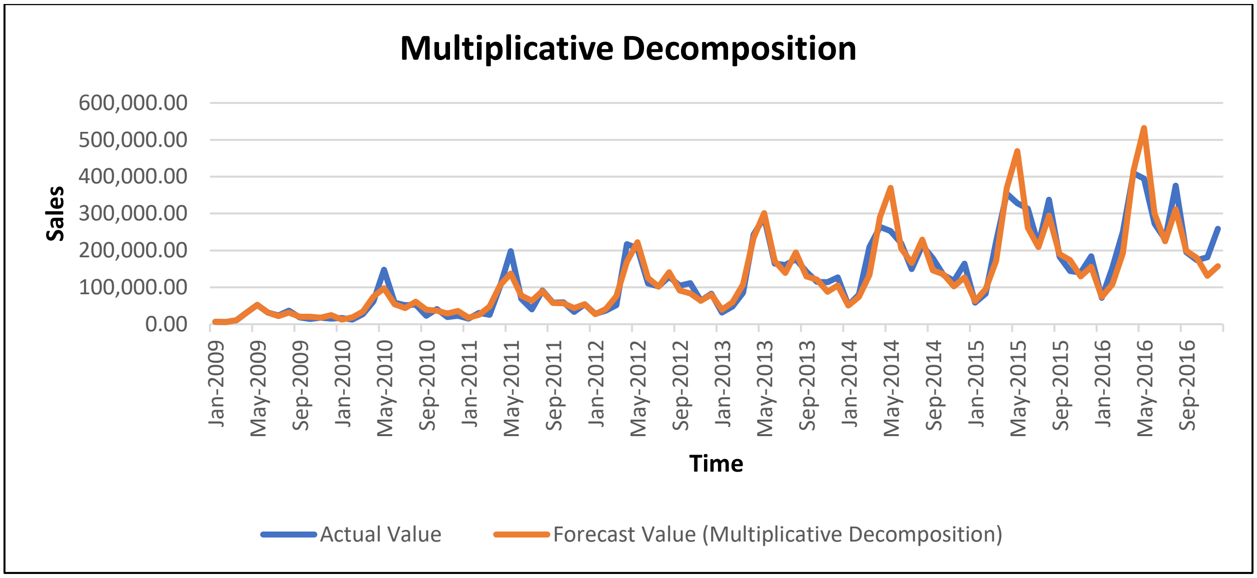

Figure 3 Time series graph using multiplicative decomposition

When multiplicative decomposition method was used to forecast the sales data, the time series plot for forecast shows similar seasonal pattern and upward trend as was observed in the actual time series plot. But, the forecasted sales volume is slightly over-forecasted than the actual sales.

Holt-Winters Smoothing

Table 2 Actual and forecast values (2009-2016) using Holt-Winters smoothing

Dates	**Actual Value**	**Forecast Value (Holt-Winters)**
Jan-2009	6,028.00	*5,979.07*
Feb-2009	5,927.00	*5,943.21*
Mar-2009	10,515.00	*11,641.90*
Apr-2009	32,276.00	*35,723.92*
May-2009	51,920.00	*62,834.56*
Jun-2009	31,294.00	*32,243.46*
Jul-2009	23,573.00	*24,379.38*
Aug-2009	36,465.00	*36,703.43*

Sep-2009	18,959.00	*17,676.68*
Oct-2009	13,918.00	*15,380.50*
Nov-2009	17,987.00	*16,172.25*
Dec-2009	15,294.00	*14,726.82*
Jan-2010	16,850.00	*31,952.28*
Feb-2010	12,753.00	*31,527.63*
Mar-2010	26,901.00	*35,821.65*
Apr-2010	61,494.00	*57,612.19*
May-2010	147,862.00	*78,240.62*
Jun-2010	57,990.00	*57,717.37*
Jul-2010	51,318.00	*50,081.54*
Aug-2010	53,599.00	*63,021.33*
Sep-2010	23,038.00	*45,225.16*
Oct-2010	41,396.00	*40,160.53*
Nov-2010	19,330.00	*43,850.50*
Dec-2010	22,707.00	*40,825.62*
Jan-2011	15,395.00	*44,043.54*
Feb-2011	30,826.00	*40,066.21*
Mar-2011	25,589.00	*52,948.41*
Apr-2011	103,184.00	*85,368.86*
May-2011	197,608.00	*163,113.11*
Jun-2011	68,600.00	*81,641.46*
Jul-2011	39,909.00	*74,364.34*
Aug-2011	91,368.00	*77,116.51*
Sep-2011	58,781.00	*48,359.59*
Oct-2011	59,679.00	*63,951.43*
Nov-2011	33,443.00	*44,997.49*
Dec-2011	53,719.00	*47,559.45*
Jan-2012	27,773.00	*41,932.59*
Feb-2012	36,653.00	*54,948.21*
Mar-2012	51,157.00	*51,861.34*
Apr-2012	217,509.00	*123,869.17*
May-2012	206,229.00	*217,508.03*
Jun-2012	110,081.00	*94,090.20*
Jul-2012	102,893.00	*68,846.38*
Aug-2012	128,857.00	*115,280.73*

Sep-2012	104,776.00	*83,486.93*
Oct-2012	111,036.00	*86,841.59*
Nov-2012	63,701.00	*62,446.37*
Dec-2012	82,657.00	*81,016.37*
Jan-2013	31,416.00	*58,112.67*
Feb-2013	48,341.00	*67,719.74*
Mar-2013	85,651.00	*80,254.77*
Apr-2013	242,673.00	*234,583.49*
May-2013	289,554.00	*235,981.68*
Jun-2013	164,373.00	*137,642.97*
Jul-2013	160,608.00	*128,611.69*
Aug-2013	176,096.00	*157,628.39*
Sep-2013	142,363.00	*132,987.04*
Oct-2013	114,907.00	*139,013.21*
Nov-2013	113,552.00	*94,142.07*
Dec-2013	127,042.00	*113,622.40*
Jan-2014	51,604.00	*66,643.40*
Feb-2014	80,366.00	*83,124.96*
Mar-2014	208,938.00	*117,800.01*
Apr-2014	263,830.00	*276,496.78*
May-2014	252,216.00	*317,538.61*
Jun-2014	219,566.00	*194,171.63*
Jul-2014	149,082.00	*189,917.88*
Aug-2014	213,888.00	*206,028.43*
Sep-2014	178,947.00	*173,340.59*
Oct-2014	133,650.00	*150,272.38*
Nov-2014	116,946.00	*143,289.90*
Dec-2014	164,154.00	*156,716.71*
Jan-2015	58,843.00	*84,847.30*
Feb-2015	82,386.00	*111,657.30*
Mar-2015	224,803.00	*227,112.62*
Apr-2015	354,301.00	*293,774.39*
May-2015	328,263.00	*290,113.54*
Jun-2015	313,647.00	*247,157.11*
Jul-2015	214,561.00	*186,269.10*
Aug-2015	337,192.00	*246,061.27*

Sep-2015	183,482.00	*213,323.93*
Oct-2015	144,618.00	*170,807.28*
Nov-2015	139,750.00	*155,630.47*
Dec-2015	184,546.00	*198,951.83*
Jan-2016	71,043.00	*98,082.52*
Feb-2016	152,930.00	*122,405.91*
Mar-2016	250,559.00	*262,761.89*
Apr-2016	409,567.00	*384,246.13*
May-2016	394,747.00	*360,993.25*
Jun-2016	272,874.00	*342,931.97*
Jul-2016	230,303.00	*246,671.49*
Aug-2016	375,402.00	*360,132.21*
Sep-2016	195,409.00	*220,936.94*
Oct-2016	173,518.00	*181,392.13*
Nov-2016	181,702.00	*175,241.21*
Dec-2016	258,713.00	*220,052.64*

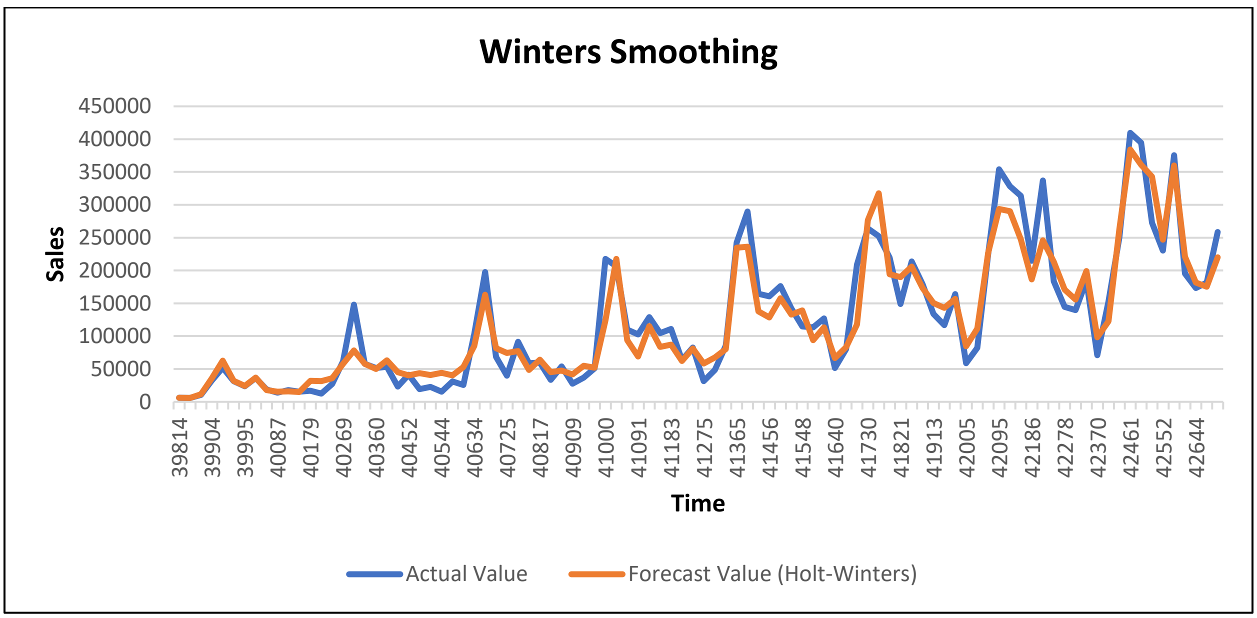

Figure 4 Time series graph using Holt-Winters smoothing

The patterns between the forecasted and actual values are similar using Holt-Winters smoothing method with seasonal and upward trend patterns, but the forecasted values are slightly under-forecasted than the actual values.

Moving Average

Table 3 Actual and forecast values (2009-2016) using 3-MA

Dates	Actual Value	Forecast Value (3-Moving Average)
Jan-2009	6,028.00	*20,475.75*
Feb-2009	5,927.00	*17,586.20*
Mar-2009	10,515.00	*15,254.36*
Apr-2009	32,276.00	*7,490.00*
May-2009	51,920.00	*16,239.33*
Jun-2009	31,294.00	*31,570.33*
Jul-2009	23,573.00	*38,496.67*
Aug-2009	36,465.00	*35,595.67*
Sep-2009	18,959.00	*30,444.00*
Oct-2009	13,918.00	*26,332.33*
Nov-2009	17,987.00	*23,114.00*
Dec-2009	15,294.00	*16,954.67*
Jan-2010	16,850.00	*15,733.00*
Feb-2010	12,753.00	*16,710.33*
Mar-2010	26,901.00	*14,965.67*
Apr-2010	61,494.00	*18,834.67*
May-2010	147,862.00	*33,716.00*
Jun-2010	57,990.00	*78,752.33*
Jul-2010	51,318.00	*89,115.33*
Aug-2010	53,599.00	*85,723.33*
Sep-2010	23,038.00	*54,302.33*
Oct-2010	41,396.00	*42,651.67*
Nov-2010	19,330.00	*39,344.33*
Dec-2010	22,707.00	*27,921.33*
Jan-2011	15,395.00	*27,811.00*
Feb-2011	30,826.00	*19,144.00*
Mar-2011	25,589.00	*22,976.00*
Apr-2011	103,184.00	*23,936.67*
May-2011	197,608.00	*53,199.67*
Jun-2011	68,600.00	*108,793.67*
Jul-2011	39,909.00	*123,130.67*
Aug-2011	91,368.00	*102,039.00*

Sep-2011	58,781.00	*66,625.67*
Oct-2011	59,679.00	*63,352.67*
Nov-2011	33,443.00	*69,942.67*
Dec-2011	53,719.00	*50,634.33*
Jan-2012	27,773.00	*48,947.00*
Feb-2012	36,653.00	*38,311.67*
Mar-2012	51,157.00	*39,381.67*
Apr-2012	217,509.00	*38,527.67*
May-2012	206,229.00	*101,773.00*
Jun-2012	110,081.00	*158,298.33*
Jul-2012	102,893.00	*177,939.67*
Aug-2012	128,857.00	*139,734.33*
Sep-2012	104,776.00	*113,943.67*
Oct-2012	111,036.00	*112,175.33*
Nov-2012	63,701.00	*114,889.67*
Dec-2012	82,657.00	*93,171.00*
Jan-2013	31,416.00	*85,798.00*
Feb-2013	48,341.00	*59,258.00*
Mar-2013	85,651.00	*54,138.00*
Apr-2013	242,673.00	*55,136.00*
May-2013	289,554.00	*125,555.00*
Jun-2013	164,373.00	*205,959.33*
Jul-2013	160,608.00	*232,200.00*
Aug-2013	176,096.00	*204,845.00*
Sep-2013	142,363.00	*167,025.67*
Oct-2013	114,907.00	*159,689.00*
Nov-2013	113,552.00	*144,455.33*
Dec-2013	127,042.00	*123,607.33*
Jan-2014	51,604.00	*118,500.33*
Feb-2014	80,366.00	*97,399.33*
Mar-2014	208,938.00	*86,337.33*
Apr-2014	263,830.00	*113,636.00*
May-2014	252,216.00	*184,378.00*
Jun-2014	219,566.00	*241,661.33*
Jul-2014	149,082.00	*245,204.00*
Aug-2014	213,888.00	*206,954.67*

Sep-2014	178,947.00	*194,178.67*
Oct-2014	133,650.00	*180,639.00*
Nov-2014	116,946.00	*175,495.00*
Dec-2014	164,154.00	*143,181.00*
Jan-2015	58,843.00	*138,250.00*
Feb-2015	82,386.00	*113,314.33*
Mar-2015	224,803.00	*101,794.33*
Apr-2015	354,301.00	*122,010.67*
May-2015	328,263.00	*220,496.67*
Jun-2015	313,647.00	*302,455.67*
Jul-2015	214,561.00	*332,070.33*
Aug-2015	337,192.00	*285,490.33*
Sep-2015	183,482.00	*288,466.67*
Oct-2015	144,618.00	*245,078.33*
Nov-2015	139,750.00	*221,764.00*
Dec-2015	184,546.00	*155,950.00*
Jan-2016	71,043.00	*156,304.67*
Feb-2016	152,930.00	*131,779.67*
Mar-2016	250,559.00	*136,173.00*
Apr-2016	409,567.00	*158,177.33*
May-2016	394,747.00	*271,018.67*
Jun-2016	272,874.00	*351,624.33*
Jul-2016	230,303.00	*359,062.67*
Aug-2016	375,402.00	*299,308.00*
Sep-2016	195,409.00	*292,859.67*
Oct-2016	173,518.00	*267,038.00*
Nov-2016	181,702.00	*248,109.67*
Dec-2016	258,713.00	*183,543.00*

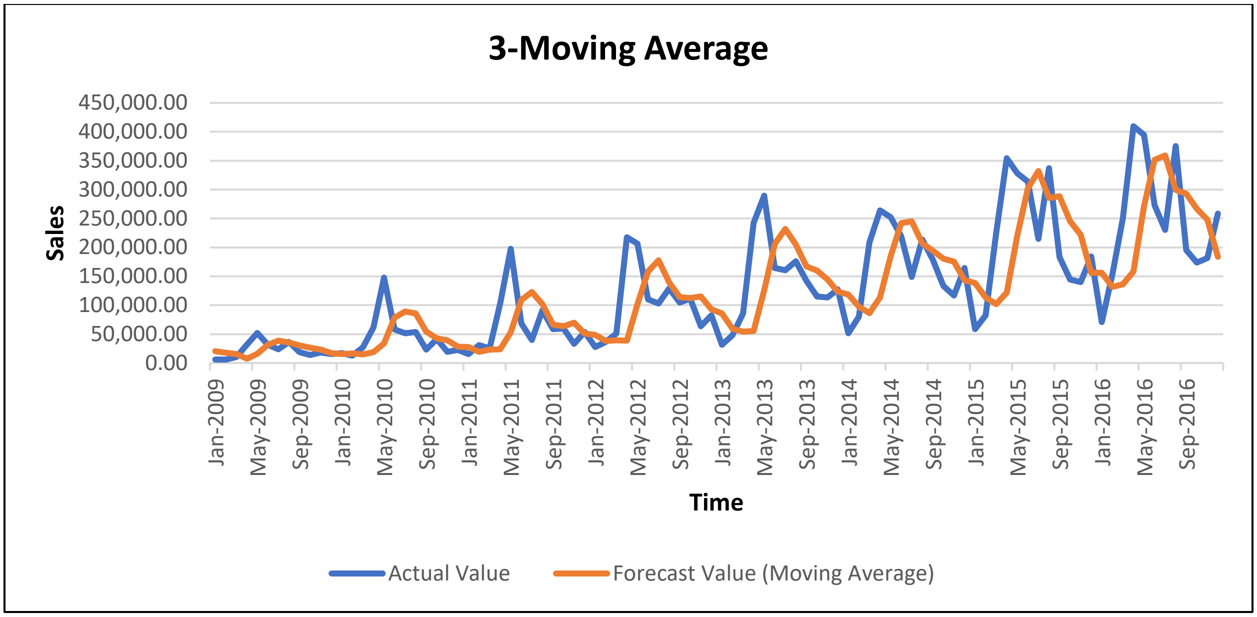

Figure 5 Time series graph using 3-MA

The forecast values follow a similar seasonal and trend pattern to the original data with the employment of 3-MA method but, the plot shows that the forecast values are largely under forecasted compared to the actual values.

Question (e)

Name two measures of forecast error that should be used in the above situations. Then, evaluate the statistics of forecast error for each method used in part (d). Identify which forecast method is more appropriate using the measure of forecast error and explain why.

The measures of forecast error such as Mean Absolute Percentage Error (MAPE), Mean Absolute Error (MAE), Root Mean Square Error (RMSE) and Mean Error (ME) were compared between the 3 methods and two of the best measures were chosen.

Table 4 Measures of forecast error for three forecast methods

Forecast Methods	Measure of Forecast Error			
	MAPE	**MAE**	**RMSE**	**ME**
Multiplicative Decomposition	**17.23%**	**19,988.91**	34,072.66	1,502.94
Holt-Winters Smoothing	**24.57%**	**20,718.45**	29,136.71	2,765.03
3-Moving Average	**50.68%**	**52,994.89**	75,268.84	4,066.30

MAPE and **MAE** were chosen as the measures of error to evaluate the prediction accuracy of the 3 methods. Previous studies have reported that MAPE and MAE are good measures of the deviations of the predicted values from the actual values. When comparing forecast methods on a single data set, the MAE is popular as it is easy to understand and compute. MAPE is a very common percentage error method used to compare between performances of time series models (Tseng, Yu & Tzeng, 2001; Tseng, Yu & Tzeng, 2002; Hyndman & Koehler, 2006).

The best forecast method that can be used to forecast the sales volume data is **multiplicative decomposition**. The MAPE and MAE values are the smallest compared the other two methods. This method is chosen as the ideal forecast technique because its measures of error are minimal.

<u>Question (f)</u>

Use the most appropriate forecast method which determined in part (e) to forecast the sales volume from January 2017 to December 2017.

The most appropriate method used to forecast the sales volume from January 2017 to December 2017 is multiplicative decomposition, as determined in part (e).

Table 5 Forecasted sales volume in 2017 using multiplicative decomposition

Time	Forecasted Sales Volume
Jan-2017	75,221.46
Feb-2017	108,745.47
Mar-2017	192,750.94
Apr-2017	412,588.74
May-2017	521,850.84
Jun-2017	287,216.42
Jul-2017	228,315.98
Aug-2017	315,378.86
Sep-2017	200,245.24
Oct-2017	180,677.14
Nov-2017	131,642.67
Dec-2017	157,290.61

Question (g)

Develop an ARIMA or SARIMA model using the autocorrelation analysis (ACF and PACF). Provide detailed descriptions of findings from the autocorrelation analysis (ACF and PACF). Find the measure of forecast error using the same forecast error as in part (e). Then, forecast the sales volume for January 2017 to December 2017.

ACF and PACF Behavior

Since the dataset was found to be non-stationary in (b), differencing was performed to make the dataset stationary. The plots below show the ACF and PACF after first differencing for non-seasonal data and first differencing for seasonal data.

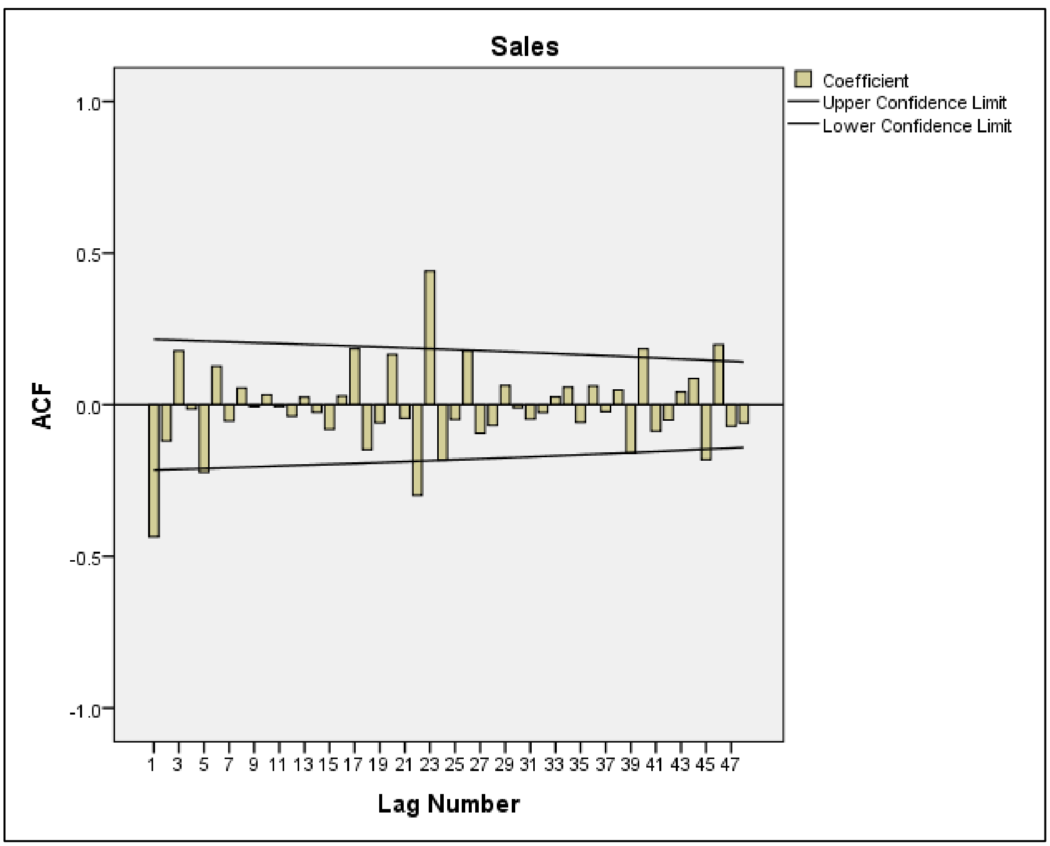

Figure 6 ACF plot after first differencing

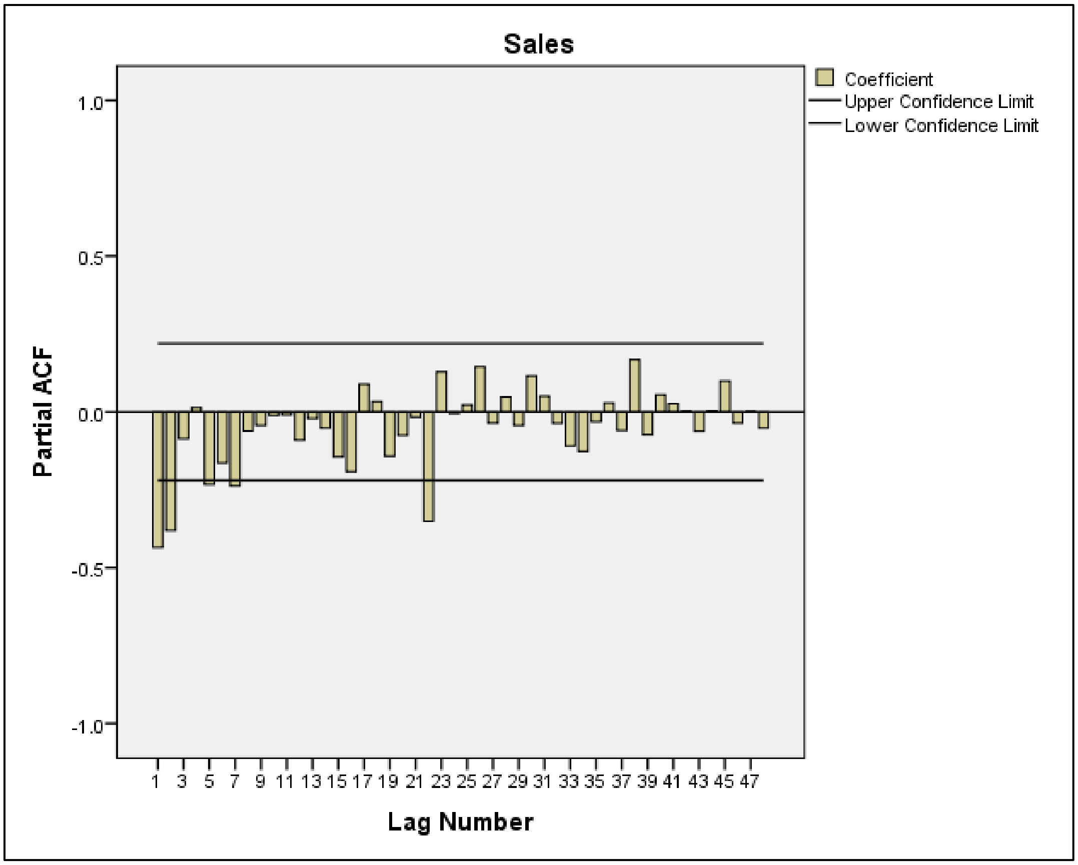

Figure 7 PACF plot after first differencing

The ACF looks to spike at lags 1 and 23 and cut off after lags 1 and 24. The PACF looks to die down at seasonal level with 12 observations per period. For non-seasonal data, the ACF shows cut off after lag 1. The PACF shows damped dying down in a sine-wave manner. Thus, these show that there is non-seasonal autoregressive order 1 and seasonal autoregressive order 1. The models look to be MA (1) x Seasonal MA $(1)_{12}$. This is suggesting a **SARIMA (0, 1, 1) (0, 1, 1)$_{12}$**.

<u>Forecast Error Measures</u>

The measures of forecast errors used which are similar to ones used in (e), MAPE and MAE, for the SARIMA model were generated using SPSS software.

Table 6 Measures of forecast error for SARIMA model

Forecast Model	**Measures of Forecast Error**	
	MAPE	**MAE**
SARIMA (0, 1, 1) (0, 1, 1)$_{12}$	24.396%	29,587.606

Hypothesis Testing

The significance of the SARIMA model is validated using hypothesis testing:

Table 7 SPSS output for SARIMA model

Model Statistics

Model	Model Fit statistics			Ljung-Box Q(18)		
	MaxAPE	MaxAE	Normalized BIC	Statistics	DF	Sig.
Sales-Model_1	97.767	244915.530	21.738	16.154	16	.442

ARIMA Model Parameters

					Estimate	SE
Sales-Model_1	Sales	Natural Logarithm	Difference		1	
			MA	Lag 1	.815	.071
			Seasonal Difference		1	
			MA, Seasonal	Lag 1	.357	.114

ARIMA Model Parameters

					t	Sig.
Sales-Model_1	Sales	Natural Logarithm	Difference			
			MA	Lag 1	11.409	.000
			Seasonal Difference			
			MA, Seasonal	Lag 1	3.123	.002

Ljung-Box Q

p-value = 0.442 > α = 0.05. This shows that the model is adequate.

MA Lag 1

H_0: $\theta_1 = 0$

H_1: $\theta_1 \neq 0$

p-value = 0.000 < α = 0.05. H_0 is rejected.

The coefficient of non-seasonal MA term (θ_1) is significant in the model.

Seasonal MA Lag 1

H_0: $\Theta_1 = 0$

H_1: $\Theta_1 \neq 0$

p-value = 0.002 < α = 0.05. H_0 is rejected.

The coefficient of seasonal MA term (Θ_1) is significant in the model.

This shows that the SARIMA model is significant. The SPSS Expert Modeler also generated SARIMA (0, 1, 1) $(0, 1, 1)_{12}$ as the best model for the sales volume dataset.

Model Equation

The coefficients for non-seasonal MA (θ_1) and seasonal MA (Θ_1) are significant, thus the model equation for SARIMA (0, 1, 1) $(0, 1, 1)_{12}$ is:

$\nabla_S^D \nabla^d Y_t = \delta + \theta_q(B)\, \Theta_Q(B^S)\, \varepsilon_t$

where S = 12, D = 1, d = 1, q = 1, Q = 1.

$(1 - B^S)^D (1 - B^d)\, Y_t = \delta + (1 - \theta_1 B)\,(1\ 1 - \Theta_1 B^S)\, \varepsilon_t$

Final expansion:

$Y_t = \delta + Y_{t-1} + Y_{t-12} - Y_{t-13} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \Theta_1\, \varepsilon_{t-12} + \theta_1\, \Theta_1\, \varepsilon_{t-13}$

$Y_t = \delta + Y_{t-1} + Y_{t-12} - Y_{t-13} + \varepsilon_t - 0.815\, \varepsilon_{t-1} - 0.357\, \varepsilon_{t-12} + (0.815)\,(0.357)\, \varepsilon_{t-13}$

$$\boxed{\mathbf{Y_t = \delta + Y_{t-1} + Y_{t-12} - Y_{t-13} + \varepsilon_t - 0.815\, \varepsilon_{t-1} - 0.357\, \varepsilon_{t-12} + 0.291\, \varepsilon_{t-13}}}$$

Forecast Values for SARIMA model

The values forecasted from January 2017 till December 2017 using SARIMA (0, 1, 1) $(0, 1, 1)_{12}$ is shown in the table and graph below.

Table 8 Forecasted sales volume using SARIMA model

Time	Forecasted Sales Volume
Jan-2017	89,551
Feb-2017	167,374
Mar-2017	320,077
Apr-2017	524,261
May-2017	506,163
Jun-2017	379,973
Jul-2017	301,743
Aug-2017	476,648
Sep-2017	265,665
Oct-2017	226,668
Nov-2017	227,302
Dec-2017	315,511

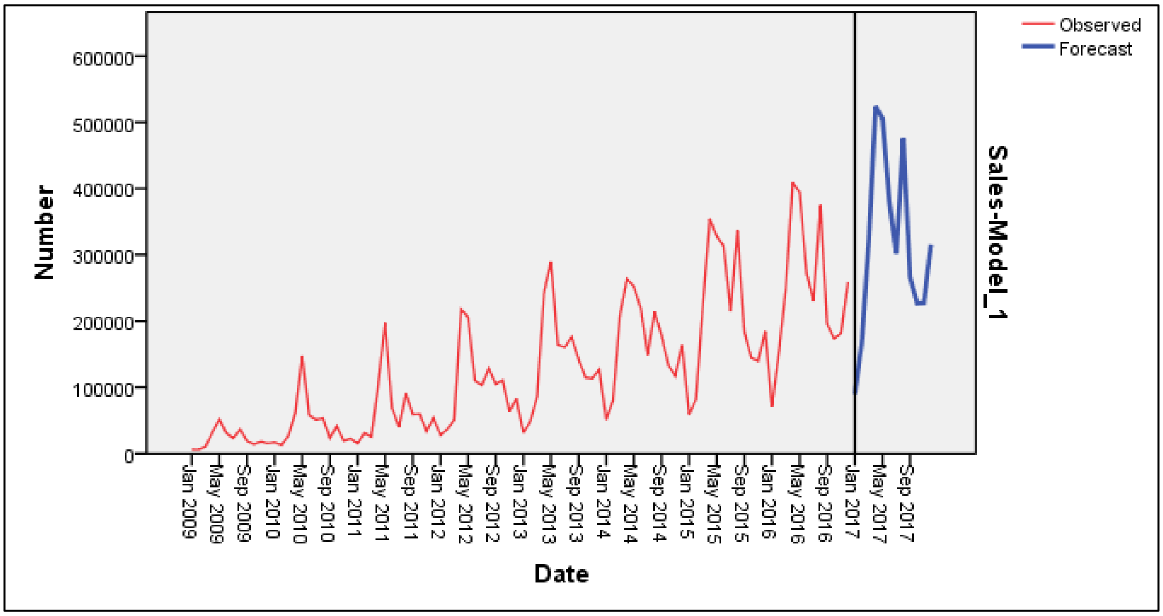

Figure 8 Time series graph for SARIMA model

Question (h)

Conclusion.

According to the case study, John would like to find a way of forecasting his monthly sales that will assist him in managing his rental stores business and to use in negotiating a loan repayment with his banker. The time series and seasonal models used to predict the forecast sales volume in the year 2017 can give John an insight into the future of his business and to tackle any issues quickly such as global downfall in sales. According to Forbes, digitalized retail is booming and this has a massive positive impact on sales of apparels. The sales of tuxedos using new techniques have resulted in massive profits for the business-makers. To improve his sales, John can promote his sales through websites and can even sell his apparels through online shopping sites which is cost-effective and highly profitable. Using the forecasted sales volume data, John now has a concrete statistical evidence that his business will improve in the future and this could be used as a leverage to negotiate a loan repayment with his investors and bankers. With new strategies such as online retail, customized tuxedo tailoring, discounted sales and door-to-door delivery, John's business is bound to expand to more rental stores.

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