وَمَا أُوتِيتُمْ مِنَ الْعِلْمِ إِلَّا قَلِيلًا

Analog IC Design

Lecture 05
MOSFET Small Signal Model

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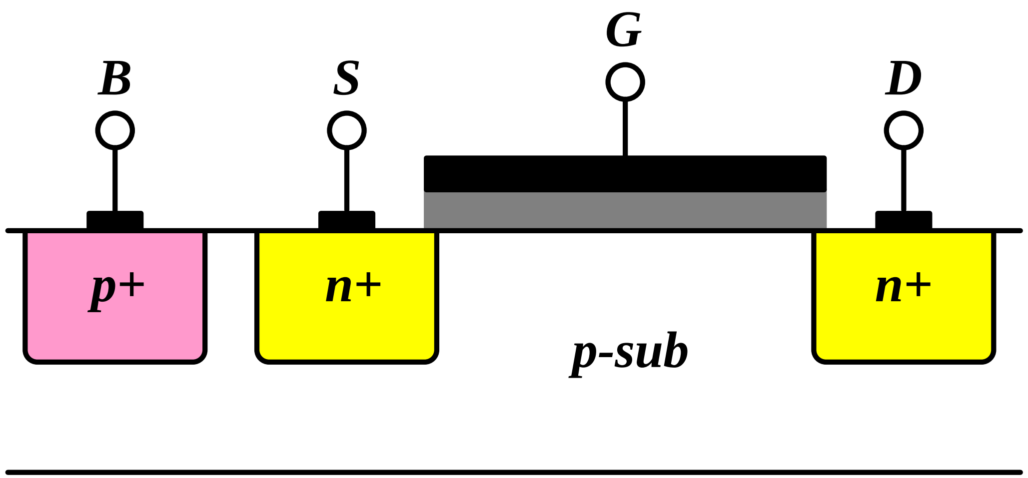
Outline

- Recapping previous key results
- The small signal approximation
- The transconductance (g_m)
- Body effect and body transconductance (g_{mb})
- Channel length modulation and output resistance ($r_o = \frac{1}{g_{ds}}$)
- Small signal model
- Short channel effects

N-Channel MOSFET Structure

- MOSFET: Metal-oxide-semiconductor field-effect transistor
- Three-terminal device: Gate (G), Source (S), and Drain (D)
- Substrate/Bulk/Body (S/B) can be treated as a fourth terminal



Regions of Operation Summary

OFF (Subthreshold)

$V_{GS} < V_{TH}$

ON

$V_{GS} > V_{TH}$

Triode

$V_{DS} < V_{ov}$

Or

$V_{GD} > V_{TH}$

$$I_D = \mu C_{ox} \frac{W}{L} \left(V_{ov} V_{DS} - \frac{V_{DS}^2}{2} \right)$$

Pinch-Off (Saturation)

$V_{DS} \geq V_{ov}$

Or

$V_{GD} \leq V_{TH}$

$$I_D = \frac{\mu C_{ox}}{2} \frac{W}{L} V_{ov}^2 (1 + \lambda V_{DS})$$

MOSFET in Saturation

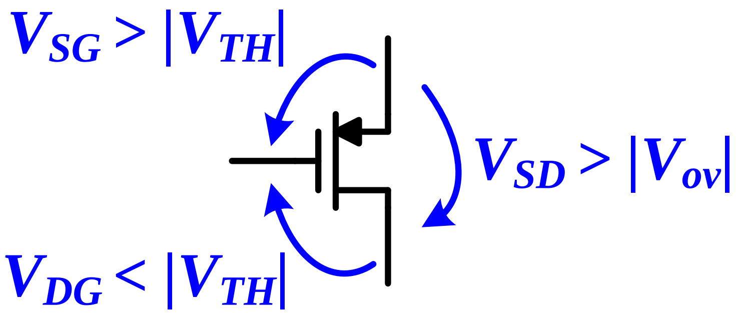
- The channel is pinched off if the difference between the gate and drain voltages is not sufficient to create an inversion layer

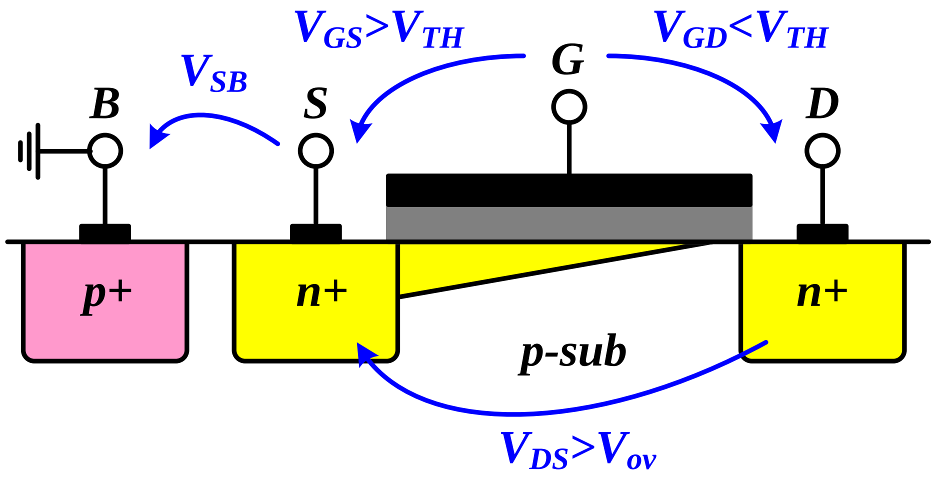
$$V_{GD} \le V_{TH} \quad OR \quad V_{DS} \ge V_{ov}$$

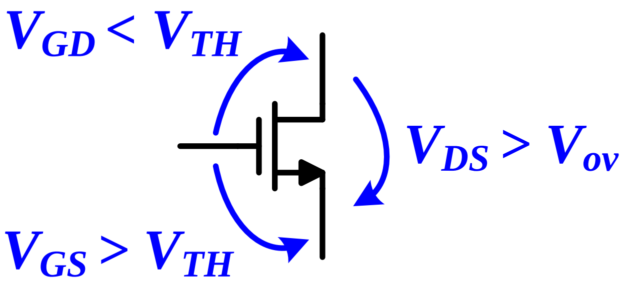
- Square-law (long channel MOS)

$$I_D = \frac{\mu_n C_{ox}}{2}\frac{W}{L} \cdot V_{ov}^2(1 + \lambda V_{DS})$$

$$V_{SB} \uparrow \;\Rightarrow\; V_{TH} \uparrow$$







Large Signal Model

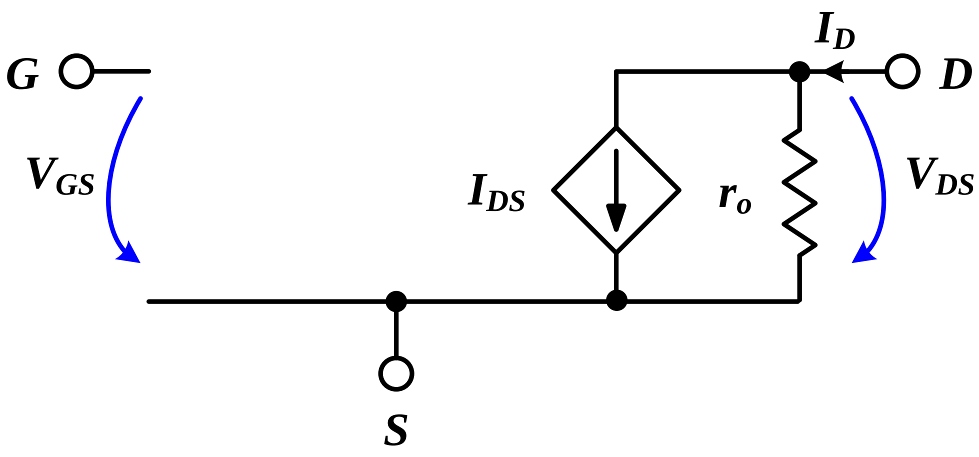
- ❑ The channel is pinched off if the difference between the gate and drain voltages is not sufficient to create an inversion layer

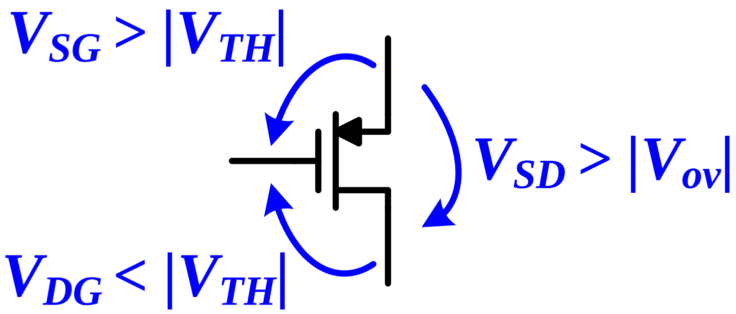
$$V_{GD} \leq V_{TH} \quad OR \quad V_{DS} \geq V_{ov}$$

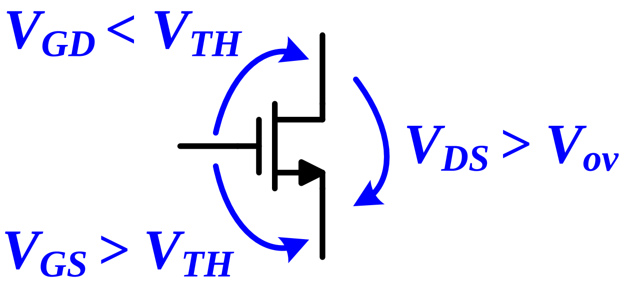
- ❑ Square-law (long channel MOS)

$$I_D = \frac{\mu_n C_{ox}}{2} \frac{W}{L} \cdot V_{ov}^2 (1 + \lambda V_{DS})$$

$$V_{SB} \uparrow \;\; \Rightarrow \;\; V_{TH} \uparrow$$



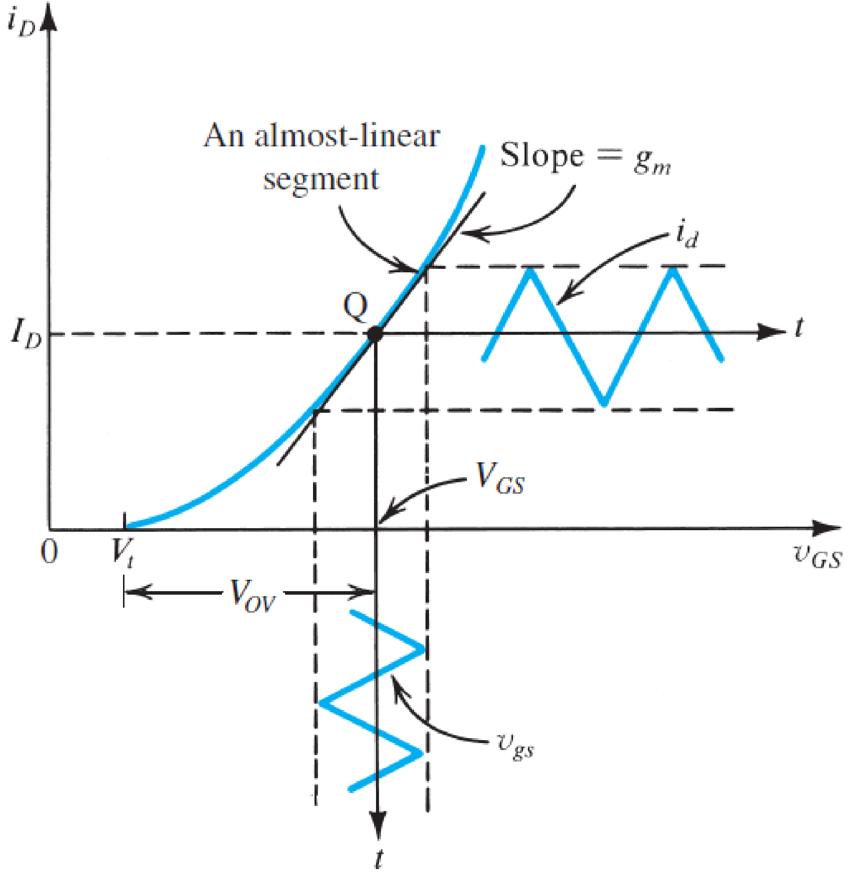




Small Signal Approximation

- The transistor is a VCCS
- Transconductance: how well it converts the voltage to a current

$$g_m = \frac{\Delta I_D}{\Delta V_{GS}} = \frac{\partial I_D}{\partial V_{GS}}$$

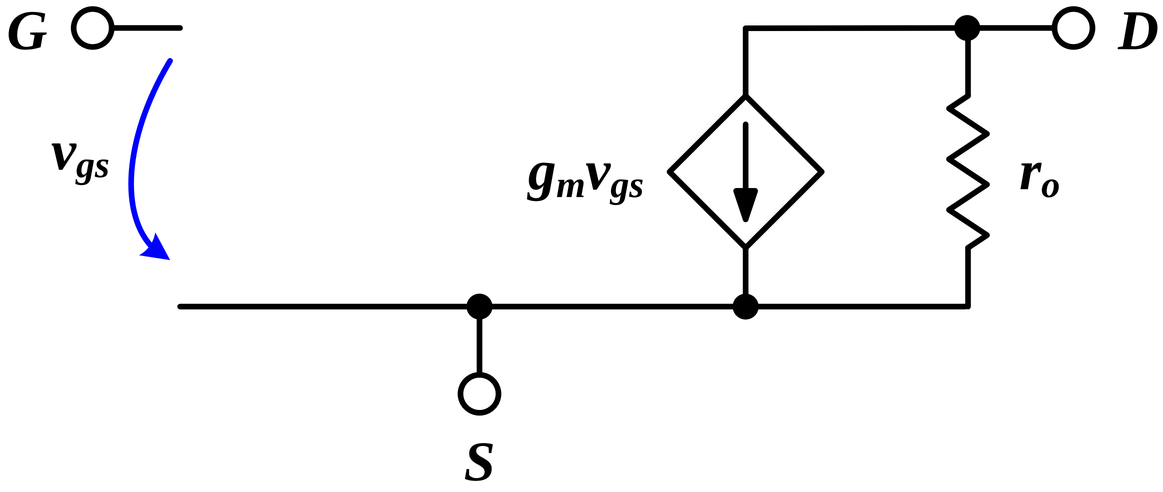


[Sedra/Smith, 2015]

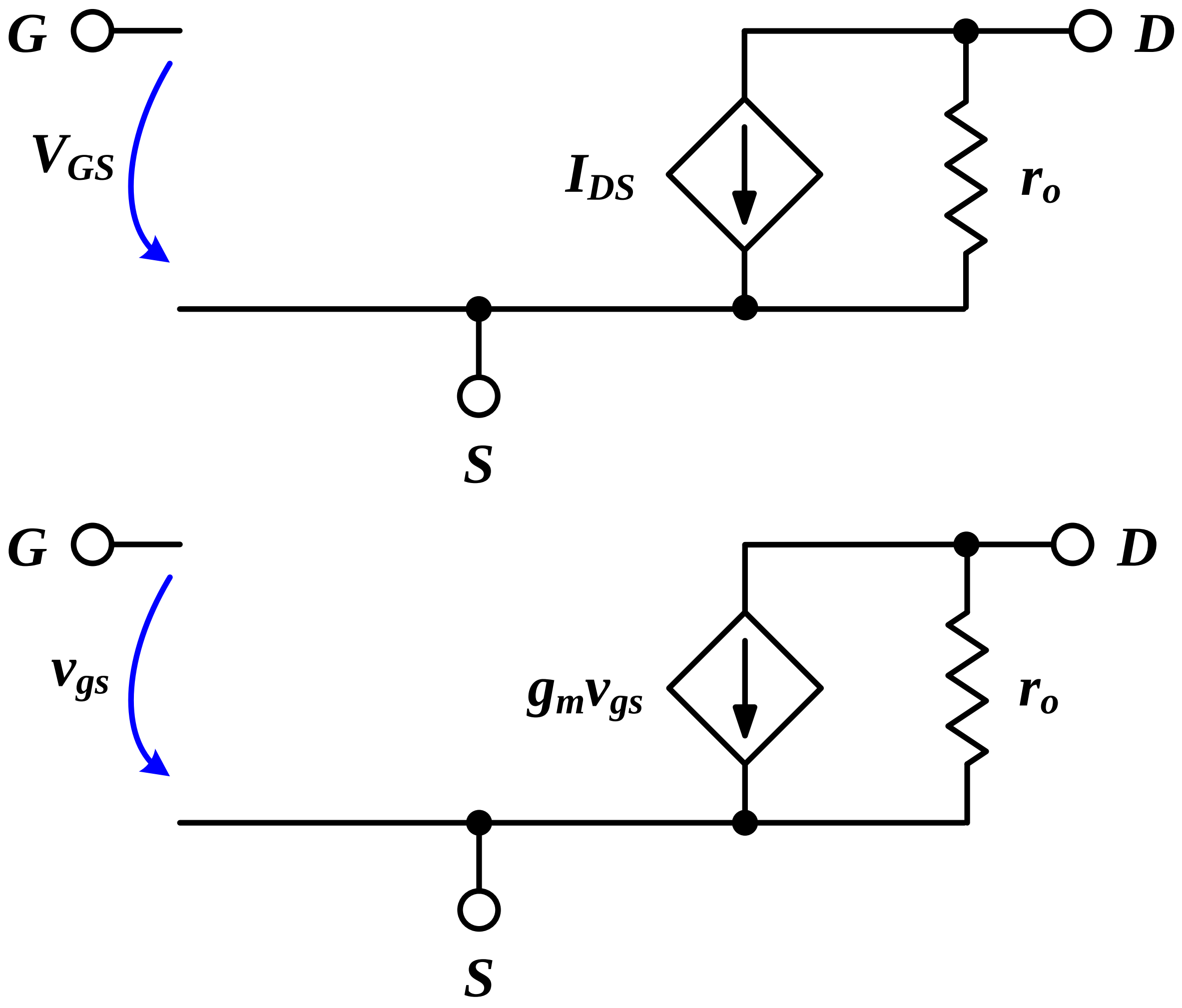
Small Signal Model

$$g_m = \frac{\Delta I_D}{\Delta V_{GS}} = \frac{\partial I_D}{\partial V_{GS}}$$

$$r_o = \frac{\Delta V_{DS}}{\Delta I_D} = \frac{1}{\frac{\partial I_D}{\partial V_{DS}}}$$



Large Signal vs Small Signal Model



Transconductance

- The transistor is a VCCS
- Transconductance: how well it converts the voltage to a current

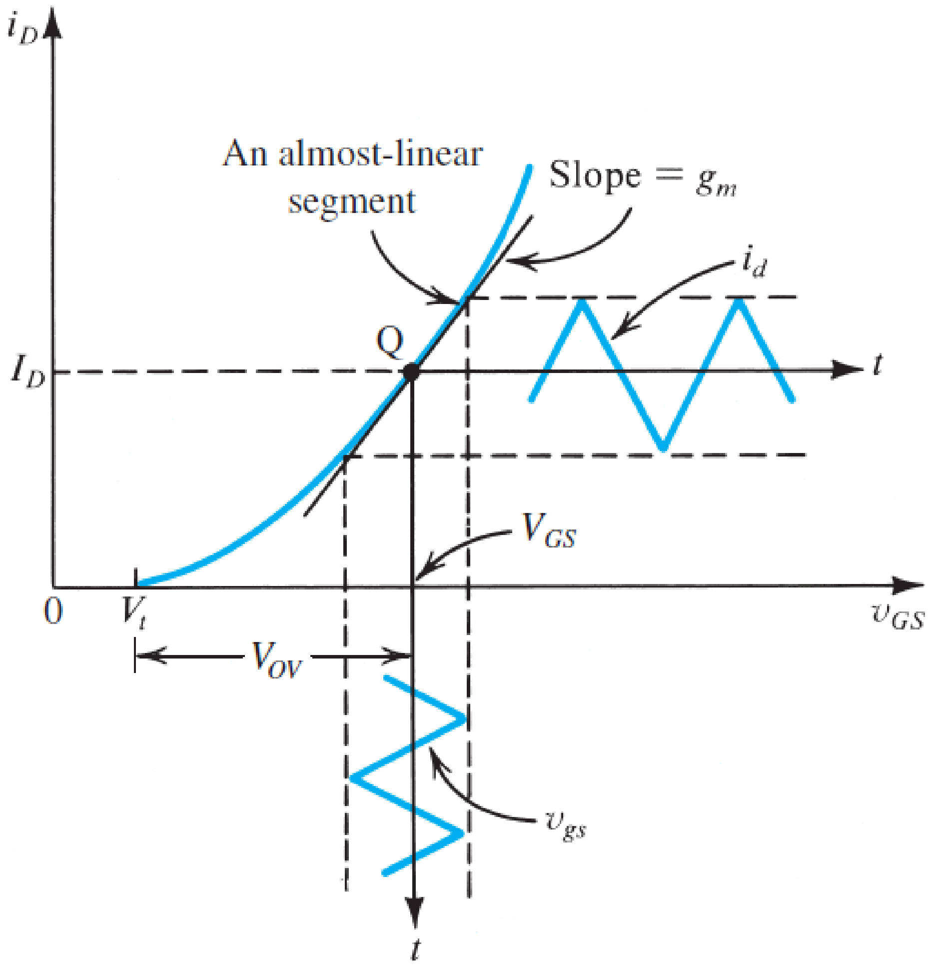
$$I_D \approx \frac{\mu_n C_{ox}}{2}\frac{W}{L}\cdot V_{ov}^2$$

$$g_m = \frac{\Delta I_D}{\Delta V_{GS}} = \frac{\partial I_D}{\partial V_{GS}} = \frac{\partial I_D}{\partial V_{ov}}$$

$$= \mu C_{ox}\frac{W}{L}V_{ov}$$

$$= \sqrt{\mu C_{ox}\frac{W}{L}\cdot 2I_D}$$

$$= \frac{2I_D}{V_{ov}}$$



[Sedra/Smith, 2015]

Transconductance

$$I_D \approx \frac{\mu_n C_{ox}}{2}\frac{W}{L}\cdot V_{ov}^2$$

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \mu C_{ox}\frac{W}{L}V_{ov} = \sqrt{\mu C_{ox}\frac{W}{L}\cdot 2I_D} = \frac{2I_D}{V_{ov}}$$

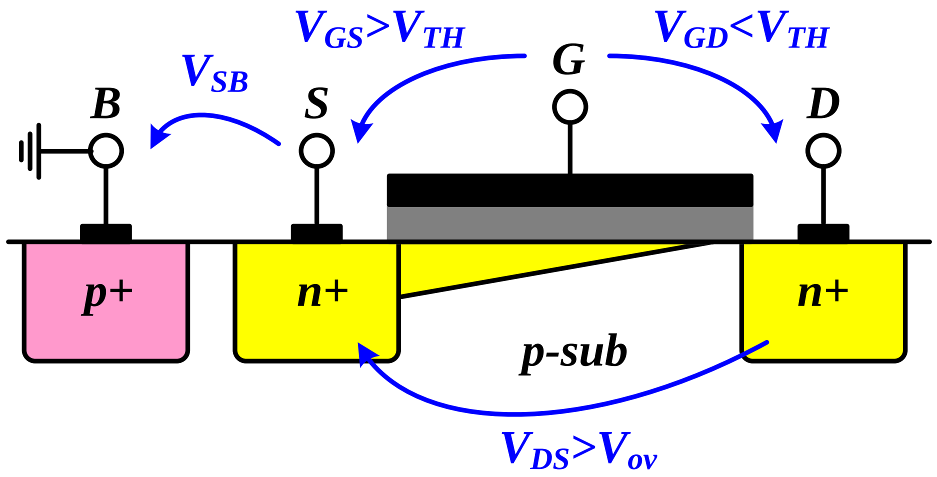
W/L constant	V_{ov} constant	I_D constant
$g_m \propto V_{ov}$	$g_m \propto W/L$	$g_m \propto \sqrt{W/L}$
$g_m \propto \sqrt{I_D}$	$g_m \propto I_D$	$g_m \propto 1/V_{ov}$

Body Effect

- V_{SB} affects the charge required to invert the channel
  - Increasing V_S or decreasing V_B increases V_{TH}

$$V_{TH} = V_{TH0} + \gamma\left(\sqrt{2\Phi_F + V_{SB}} - \sqrt{|2\Phi_F|}\right)$$

  - Φ_F = surface potential at threshold
    - Depends on doping level and intrinsic carrier concentration n_i
  - γ = body effect coefficient
    - Depends on C_{ox} and doping



Bulk Transconductance

- The bulk behaves as a second gate that changes the output current

$$g_{mb} = \frac{\partial I_D}{\partial V_{BS}} = \eta g_m$$

η is typically $0.1 \rightarrow 0.25$

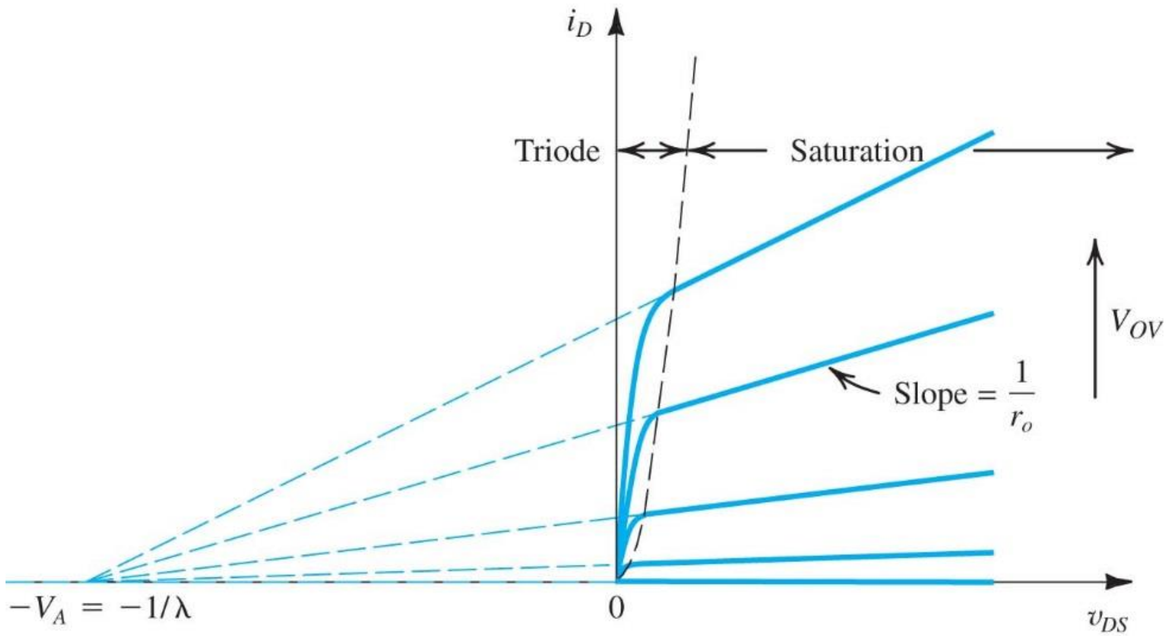
Channel Length Modulation (CLM)

- The VCCS is not ideal: There is some dependence on V_{DS}

$$r_o = \frac{\Delta V_{DS}}{\Delta I_D} = \frac{1}{\partial I_D / \partial V_{DS}} = \frac{1}{g_{ds}} = \frac{V_A}{I_{DS}} = \frac{1}{\lambda I_{DS}}$$

V_A: Early voltage ($V_A \propto L$) $\leftrightarrow$ λ: Channel length modulation coefficient ($\lambda \propto 1/L$)

$$I_D = I_{DS} + \frac{V_{DS}}{r_o} = I_{DS}\left(1 + \frac{V_{DS}/I_{DS}}{r_o}\right) = \frac{\mu C_{ox}}{2}\frac{W}{L}V_{ov}^2(1 + \lambda V_{DS})$$



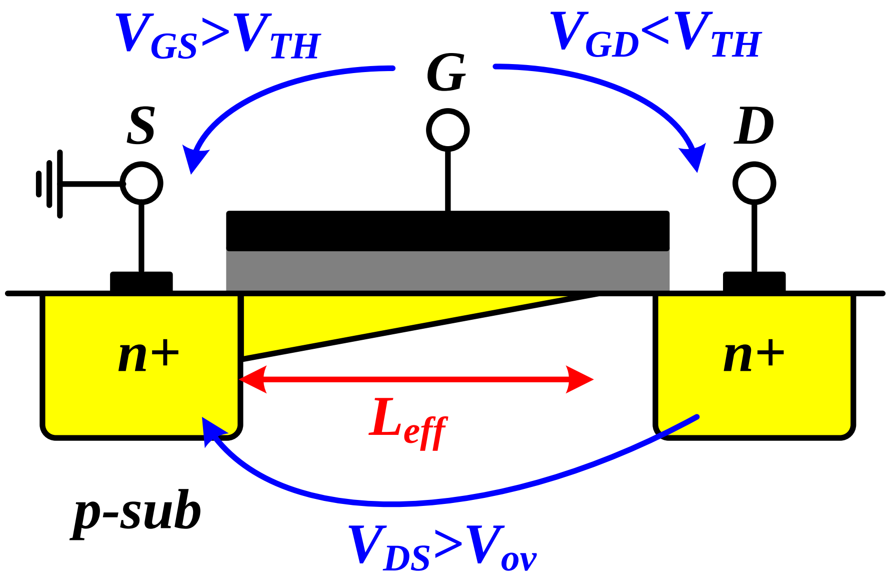
[Sedra/Smith, 2015]

Channel Length Modulation (CLM)

- L_{eff} decreases with V_{DS} → Shorter L gives more current
- V_A: Early voltage ($V_A \propto L$)
- λ: Channel length modulation coefficient ($\lambda \propto 1/L$)

$$I_D = \frac{\mu C_{ox}}{2}\frac{W}{L}V_{ov}^2(1+\lambda V_{DS}) \qquad r_o = \frac{V_A}{I_{DS}} = \frac{1}{\lambda I_{DS}}$$

- V_A increases with V_{DS}: higher r_o as we go deeper into saturation

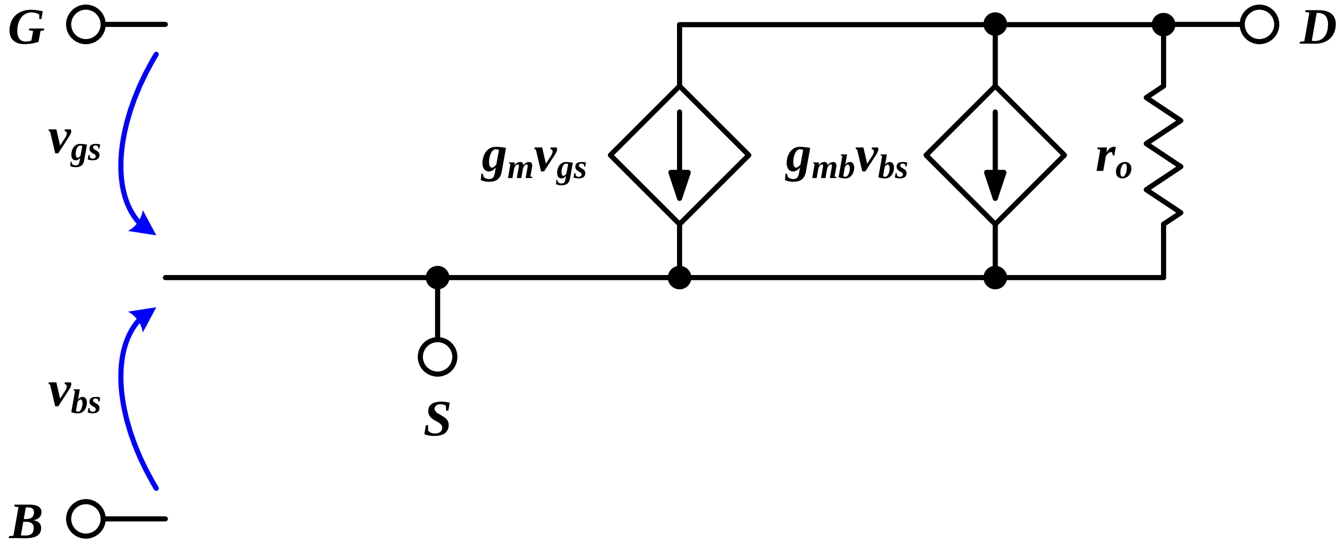


Low-Frequency Small-Signal Model

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \mu C_{ox} \frac{W}{L} V_{ov} = \sqrt{\mu C_{ox} \frac{W}{L} \cdot 2I_D} = \frac{2I_D}{V_{ov}}$$

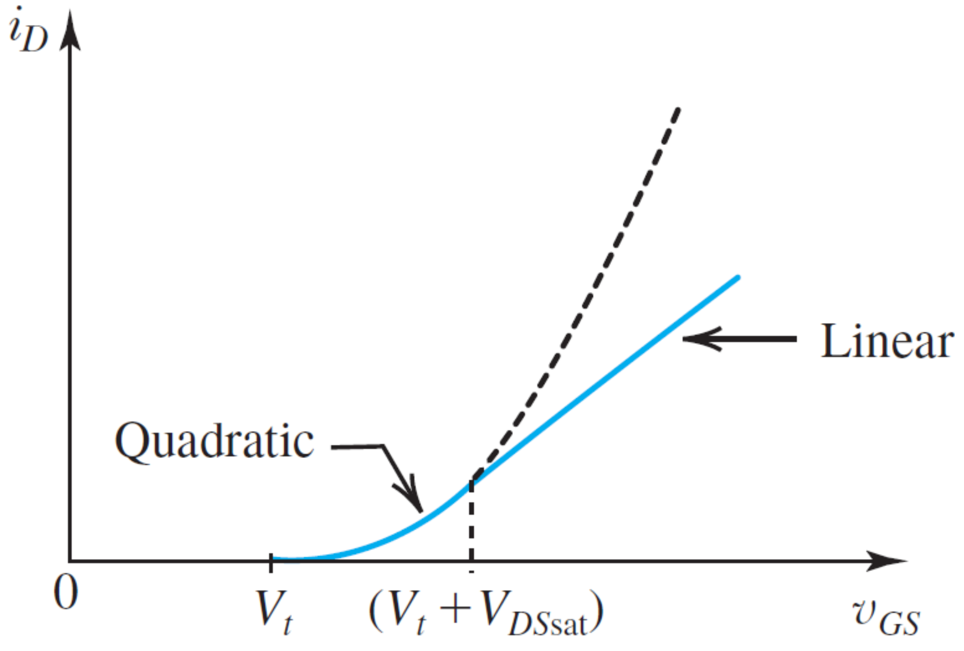
$$g_{mb} = \eta g_m \qquad \eta \approx 0.1 - 0.25$$

$$r_o = \frac{1}{\partial I_D / \partial V_{DS}} = \frac{V_A}{I_D} = \frac{1}{\lambda I_D} \qquad V_A \propto L \leftrightarrow \lambda \propto \frac{1}{L} \qquad V_{DS} \uparrow V_A \uparrow$$



Short Channel Effects: Velocity Saturation

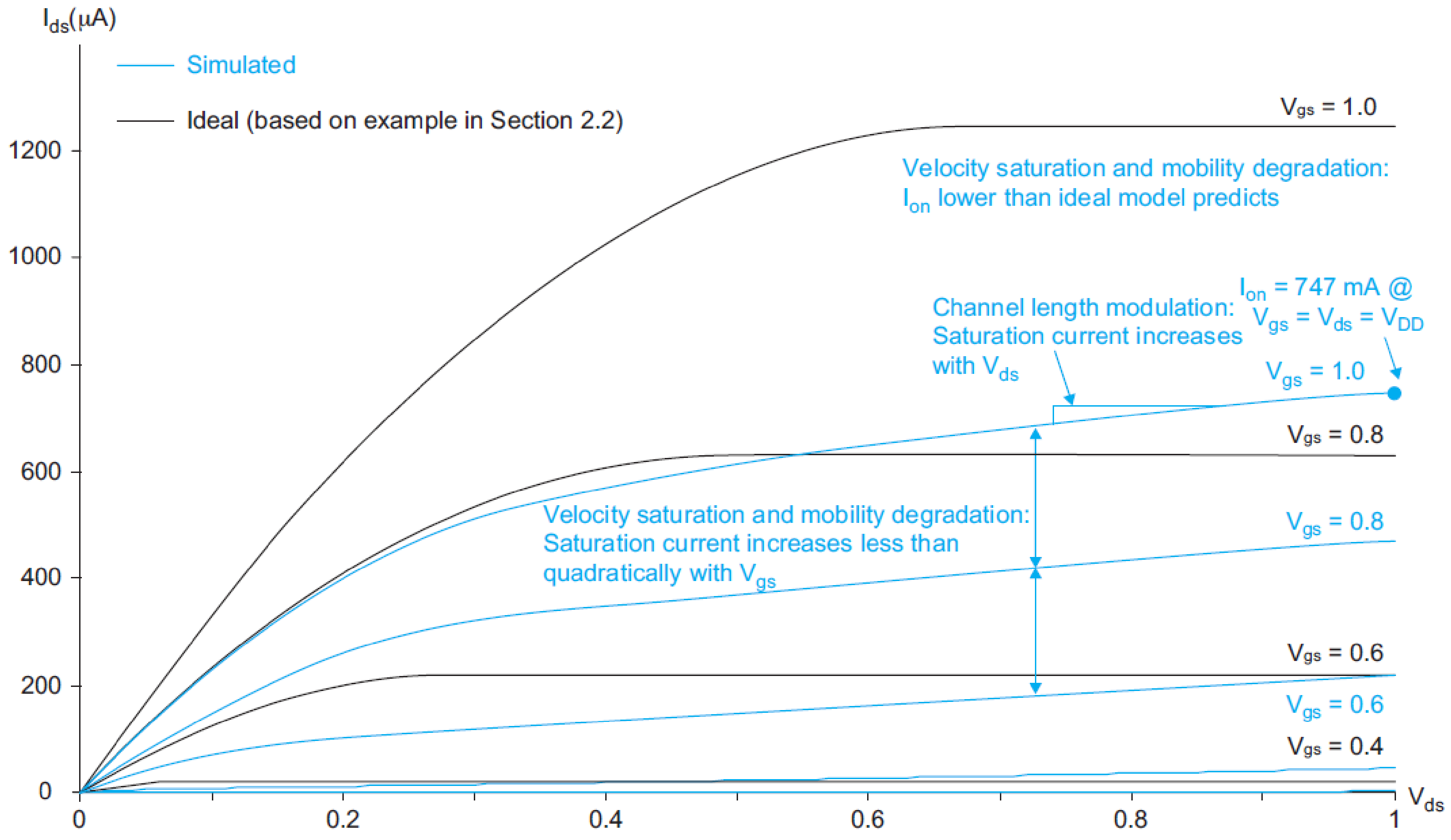
- ID-VGS quadratic: $g_m = \frac{\partial I_D}{\partial V_{GS}}$ = linear → g_m increases with V_{GS}
- ID-VGS linear: $g_m = \frac{\partial I_D}{\partial V_{GS}} =$ constant → g_m saturates



[Sedra/Smith, 2015]

Short Channel Effects: CLM and DIBL

- ID-VDS horizontal: no V_{DS} dependence → ideal current source
- More slope → more V_{DS} dependence → smaller r_o → smaller gain



[Weste & Harris]

Thank you!

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