وَمَا أُوتِيتُمْ مِنَ الْعِلْمِ إِلَّا قَلِيلًا

Analog IC Design

Lecture 16
OTA Frequency Compensation

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Outline

- Recapping previous key results
- Frequency compensation
- Compensation of single-stage OTAs
- Compensation of two-stage OTA
  - Miller compensation
  - The feedforward zero

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- ❑ Recapping previous key results
- ❑ Frequency compensation
- ❑ Compensation of single-stage OTAs
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MOSFET in Saturation

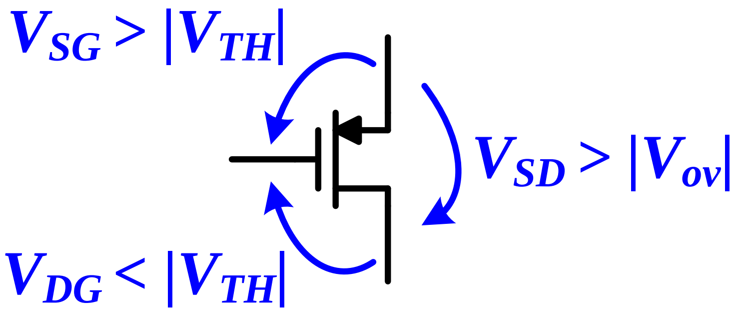
- The channel is pinched off if the difference between the gate and drain voltages is not sufficient to create an inversion layer

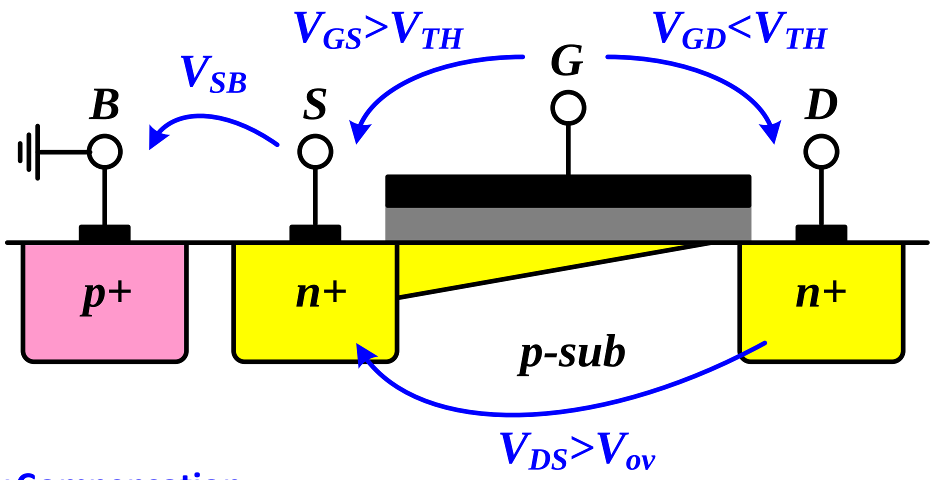
$$V_{GD} \leq V_{TH} \quad or \quad V_{DS} \geq V_{ov}$$

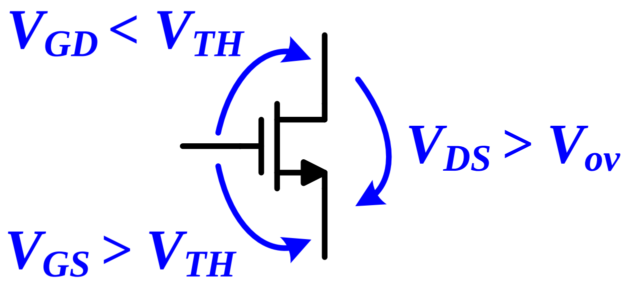
- Square-law (long channel MOS)

$$I_D = \frac{\mu_n C_{ox}}{2} \frac{W}{L} \cdot V_{ov}^2 (1 + \lambda V_{DS})$$

$$V_{SB} \uparrow \quad \Rightarrow \quad V_{TH} \uparrow$$







Regions of Operation Summary

OFF (Subthreshold)

$V_{GS} < V_{TH}$

ON

$V_{GS} > V_{TH}$

Triode

$V_{DS} < V_{ov}$

Or

$V_{GD} > V_{TH}$

$$I_D = \mu C_{ox}\frac{W}{L}\left(V_{ov}V_{DS} - \frac{V_{DS}^2}{2}\right)$$

Pinch-Off (Saturation)

$V_{DS} \geq V_{ov}$

Or

$V_{GD} \leq V_{TH}$

$$I_D = \frac{\mu C_{ox}}{2}\frac{W}{L}V_{ov}^2(1 + \lambda V_{DS})$$

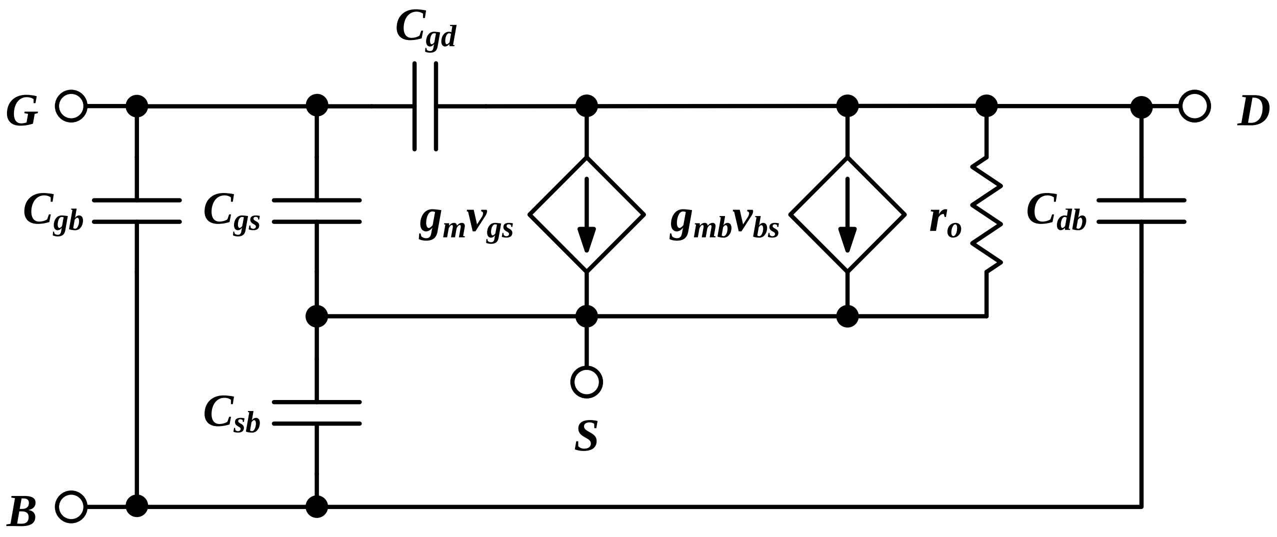
High Frequency Small Signal Model

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \mu C_{ox}\frac{W}{L}V_{ov} = \sqrt{\mu C_{ox}\frac{W}{L}\cdot 2I_D} = \frac{2I_D}{V_{ov}}$$

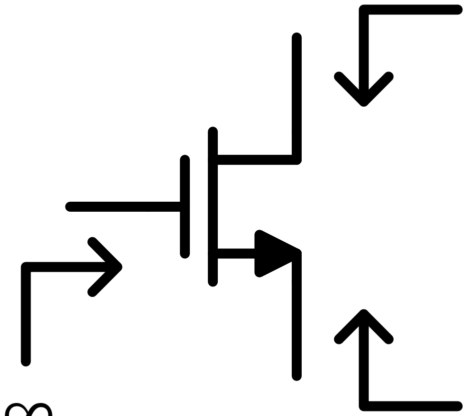
$$g_{mb} = \eta g_m \qquad \eta \approx 0.1 - 0.25$$

$$r_o = \frac{1}{\partial I_D/\partial V_{DS}} = \frac{V_A}{I_D} = \frac{1}{\lambda I_D} \qquad V_A \propto L \leftrightarrow \lambda \propto \frac{1}{L} \qquad V_{DS}\uparrow V_A\uparrow$$

$$C_{gb} \approx 0 \qquad C_{gs} \gg C_{gd} \qquad C_{sb} > C_{db}$$



Rin/out Shortcuts Summary



$$r_o[1 + (g_m + g_{mb})R_S]$$

H.I.N.

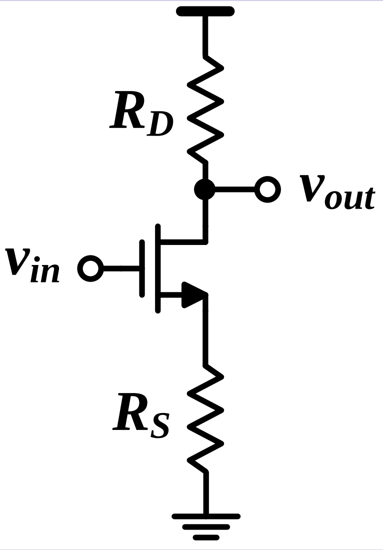
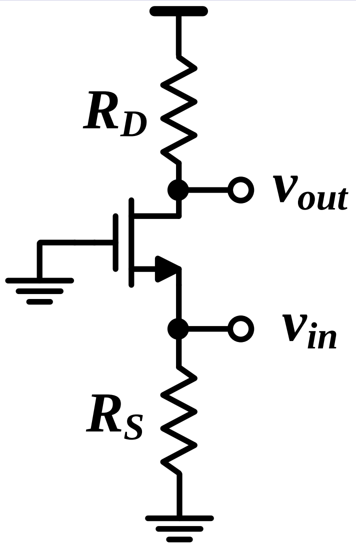
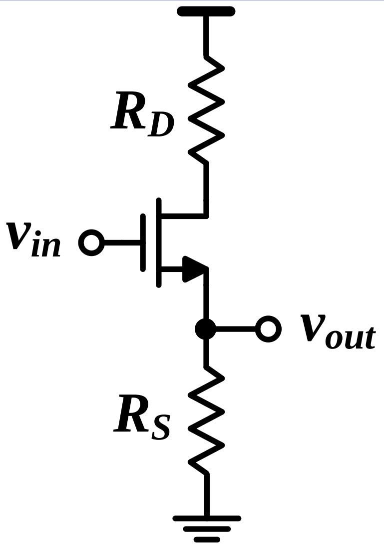
$$\frac{1}{g_m + g_{mb}}\left(1 + \frac{R_D}{r_o}\right)$$

L.I.N.

∞

At low frequencies **ONLY**

Summary of Basic Topologies

	CS	CG	CD (SF)
			
	Voltage & current amplifier	Voltage amplifier Current buffer	Voltage buffer Current amplifier
Rin	∞	$R_S \parallel \frac{1}{g_m + g_{mb}}\left(1 + \frac{R_D}{r_o}\right)$	∞
Rout	$R_D \parallel r_o[1 + (g_m + g_{mb})R_S]$	$R_D \parallel r_o$	$R_S \parallel \frac{1}{g_m + g_{mb}}\left(1 + \frac{R_D}{r_o}\right)$
Gm	$\frac{-g_m}{1 + (g_m + g_{mb})R_S}$	$g_m + g_{mb}$	$\frac{g_m}{1 + R_D/r_o}$

Differential Amplifier

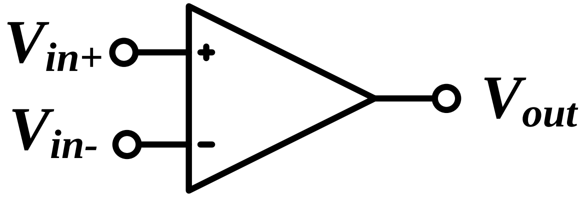
	Pseudo Diff Amp	Diff Pair (w/ ideal CS)	Diff Pair (w/ R_{SS})
A_{vd}	$-g_m R_D$	$-g_m R_D$	$-g_m R_D$
A_{vCM}	$-g_m R_D$	0	$\frac{-g_m R_D}{1+2(g_m+g_{mb})R_{SS}}$
A_{vd}/A_{vCM}	1	∞	$2(g_m+g_{mb})R_{SS} \gg 1$

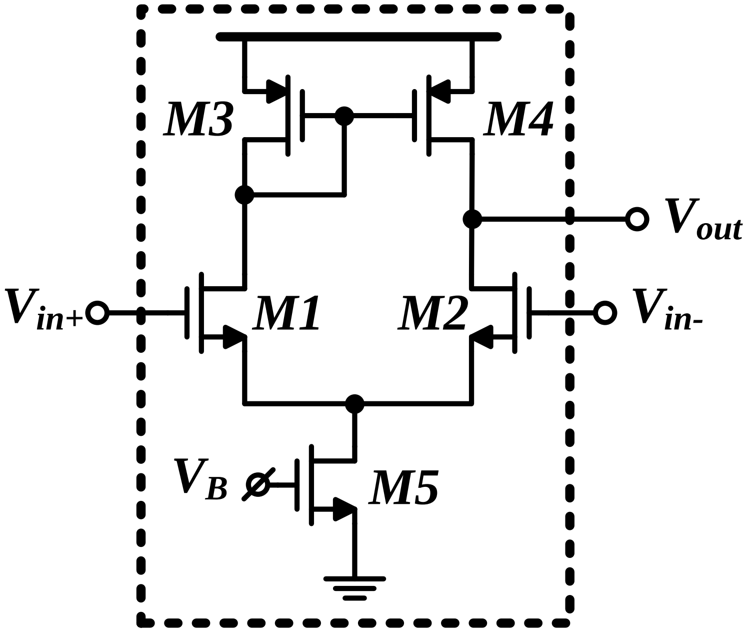
$$A_{vCM2d} = \frac{v_{od}}{v_{iCM}} \approx \frac{\Delta R_D}{2R_{SS}} + \frac{\Delta g_m R_D}{2g_{m1,2}R_{SS}}$$

$$CMRR = \frac{A_{vd}}{A_{vCM2d}}$$

Op-Amp

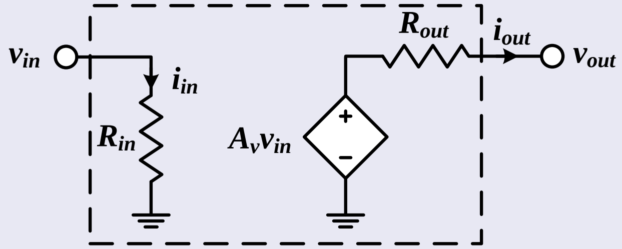
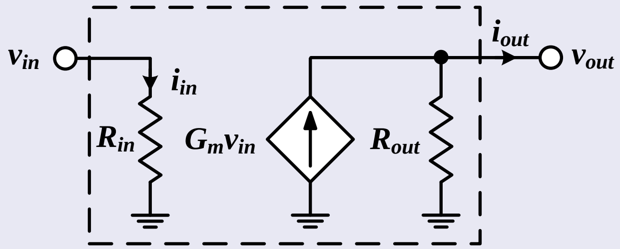
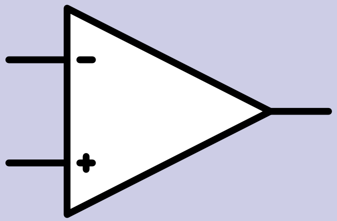
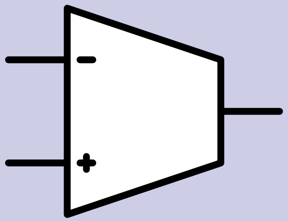
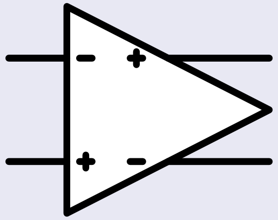
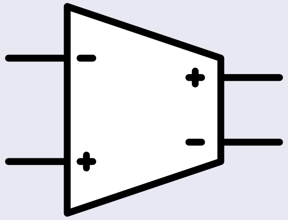
- An op-amp is simply a high gain differential amplifier
  - The gain can be increased by using cascodes and multi-stage amplification
- The diff amp is a key block in many analog and RF circuits
  - DEEP understanding of diff amp is ESSENTIAL





Op-Amp vs OTA

- In short, an OTA is an op-amp without an output stage (buffer)
- Some designers just use op-amp name and symbol for both

	Op-amp	OTA
Rout	**LOW**	**HIGH**
Model		
Diff input, SE output		
Fully diff		

V-star (V^*)

- V-star (V^*) is inspired by V_{ov} but calculated from actual simulation data

$$g_m = \frac{2I_D}{V^*} \leftrightarrow V^* = \frac{2I_D}{g_m} = \frac{2}{g_m/I_D}$$

- Figures-of-merit in terms of V^*

$$g_m r_o = \frac{2I_D}{V^*} \cdot \frac{1}{\lambda I_D} = \frac{2}{\lambda V^*}$$

$$f_T = \frac{g_m}{2\pi C_{gg}} = \frac{1}{2\pi} \cdot \frac{2I_D}{V^*} \cdot \frac{1}{C_{gg}}$$

$$\frac{g_m}{I_D} = \frac{2}{V^*}$$

- The boundary between weak and strong inversion ($n = 1.2 \rightarrow 1.5$)

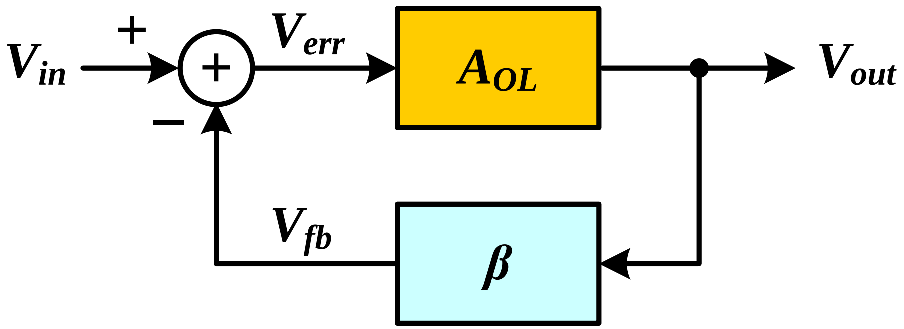
$$V_{ov}(SI) = V^*(WI) = 2nV_T \approx 60 \rightarrow 80mV$$

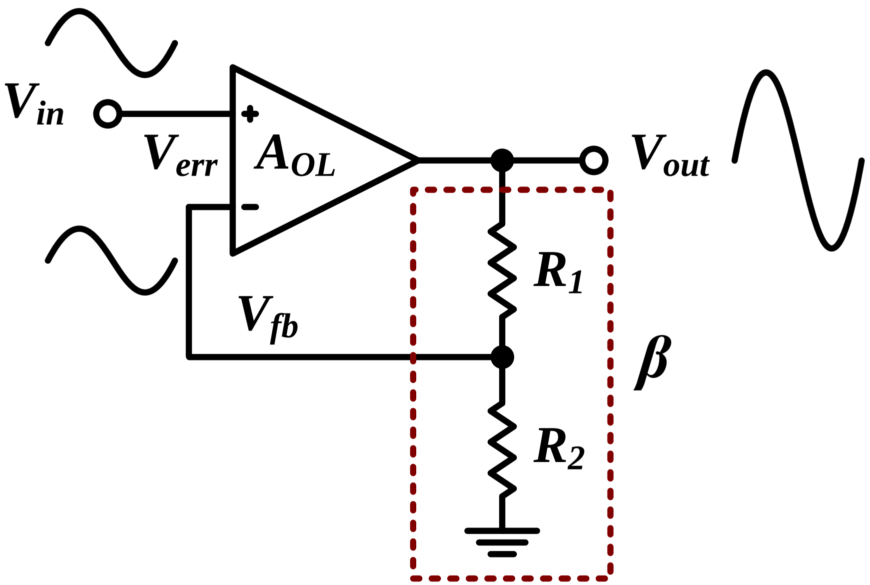
Negative Feedback

$$\beta = \frac{R_2}{(R_1 + R_2)}$$

$$A_{CL} = \frac{V_{out}}{V_{in}} = \frac{A_{OL}}{1 + \beta A_{OL}} = \frac{A_{OL}}{1 + \beta A_{OL}} \approx \frac{1}{\beta} = \frac{R_1 + R_2}{R_2} = 1 + \frac{R_1}{R_2}$$

$$\omega_{p,CL} = (1 + \beta A_{OLo})\omega_{P,OL}$$





Outline

- ❑ Recapping previous key results
- ❑ Frequency compensation
- ❑ Compensation of single-stage OTAs
- ❑ Compensation of two-stage OTA
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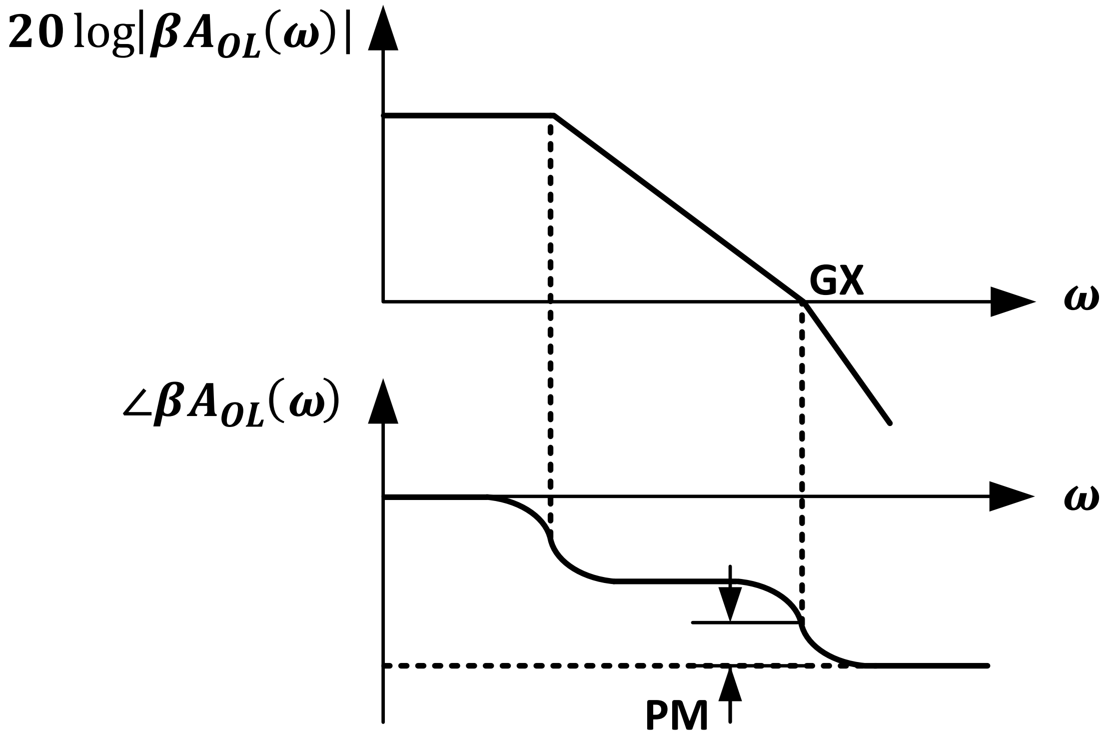
  - The feedforward zero

Phase Margin and the Ultimate GBW

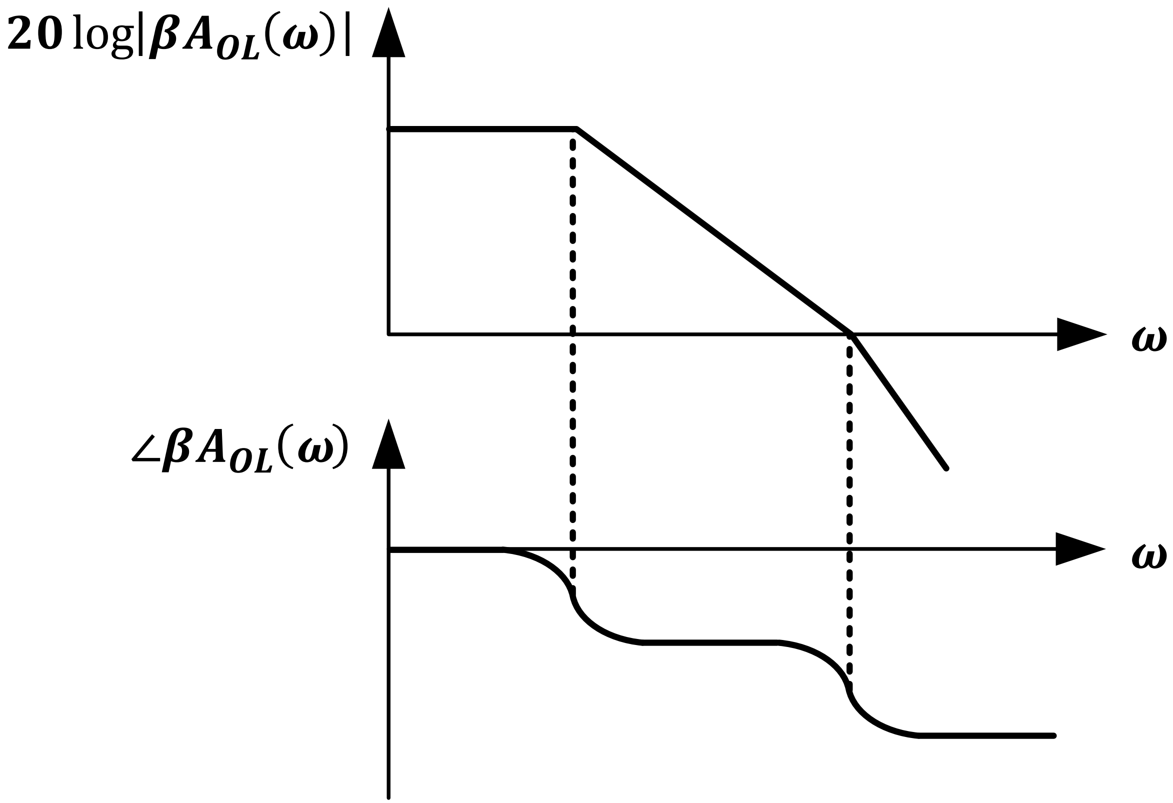
- If $\omega_{p2} = \omega_u$: PM = 45°
  - Typically inadequate (peaking/ringing)
- Thus ω_{p2} should be $> \omega_u$ → $\omega_{p1} \ll \omega_u < \omega_{p2}$
  - ω_{p1} defines OL BW and ω_{p2} defines ultimate GBW (max CL BW)

Frequency domain peaking
→ noise amplification
Time domain ringing
→ poor settling time



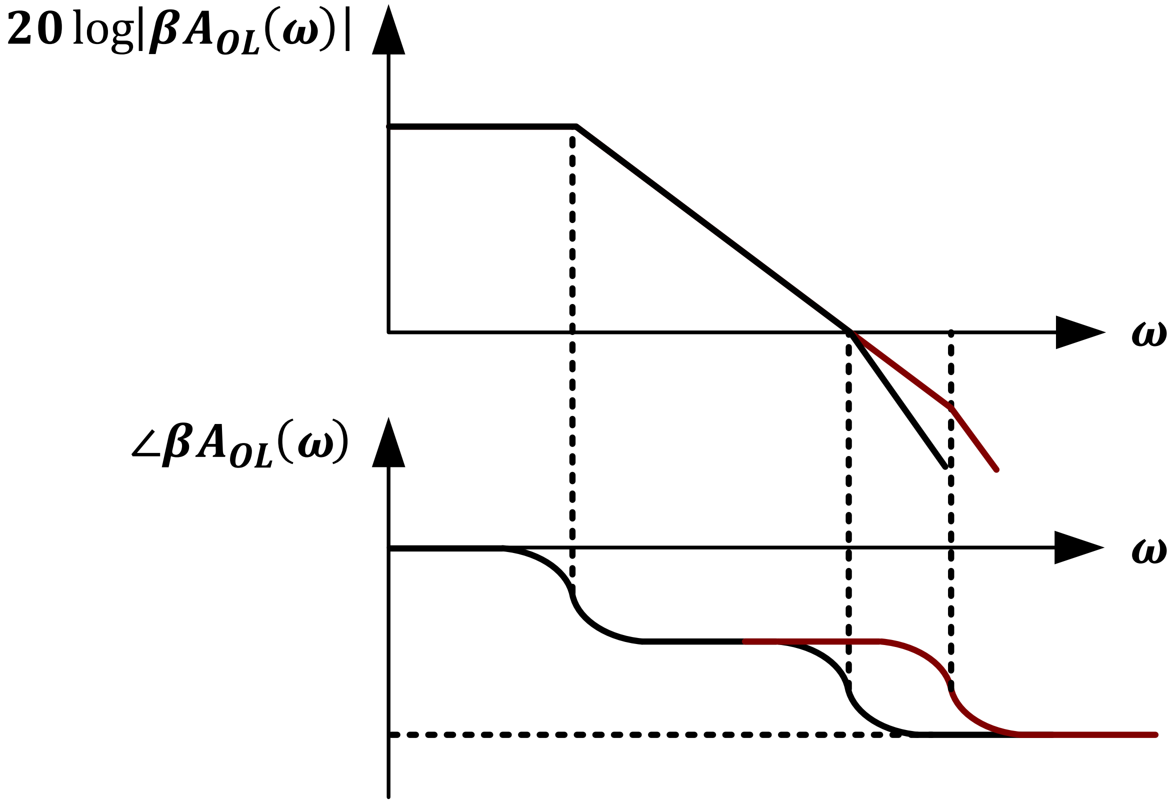
Frequency Compensation

- Frequency compensation: Modify the system to achieve a specific PM to control frequency domain peaking and time domain ringing
- GX < PX is not enough: We actually need $\omega_u < \omega_{p2}$



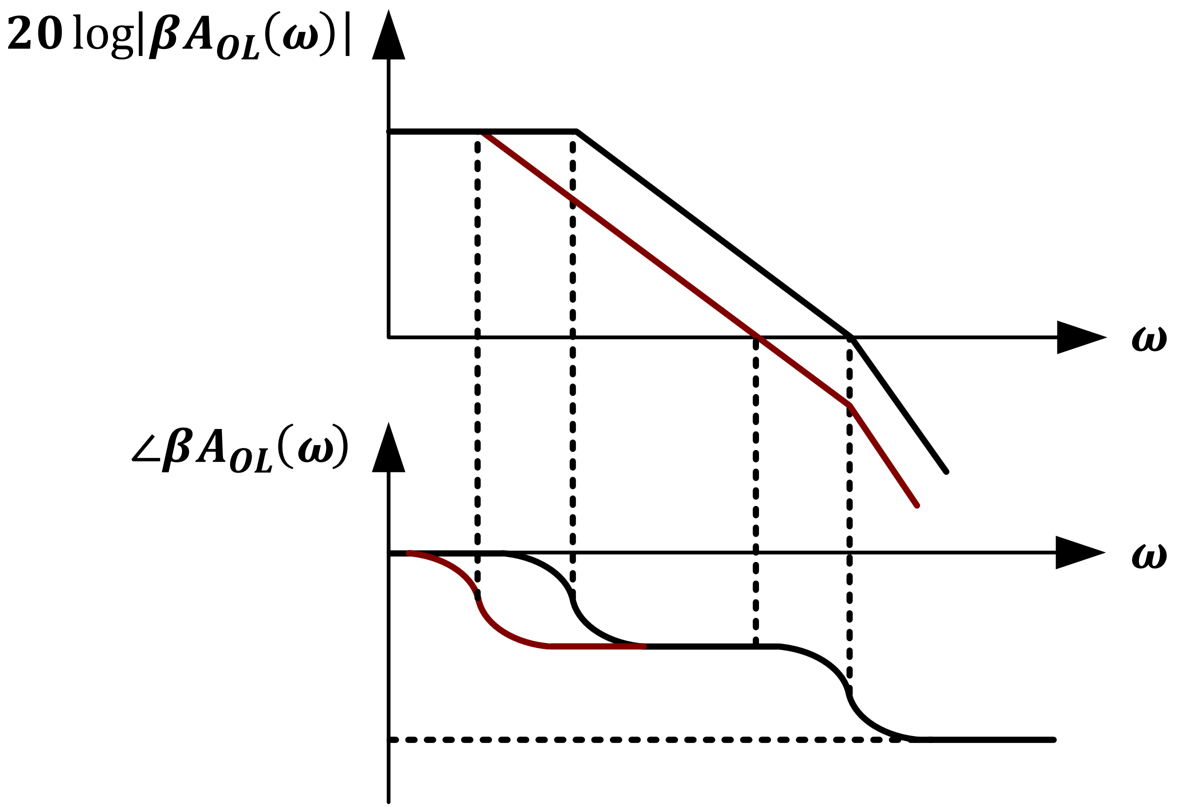
Frequency Compensation

- We need GX < PX (actually $\omega_u < \omega_{p2}$)
  - Push ω_{p2} **outwards**: lower resistance/capacitance
    - Not always feasible for free



Frequency Compensation

- We need GX < PX (actually $\omega_u < \omega_{p2}$)
  - Push GX / ω_u / ω_{p1} **inwards**: lower GBW



Compensation of Popular OTA Topologies

- Single-stage OTAs
  - 5T OTA
  - Telescopic cascode OTA
  - Folded cascode OTA
- Two-stage OTA
- Three-stage OTA
- Gain boosted OTA

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5T OTA

- $\omega_{p1} \ll \omega_u < \omega_{p2}$
- The H.I.N. sets BW_{OL}

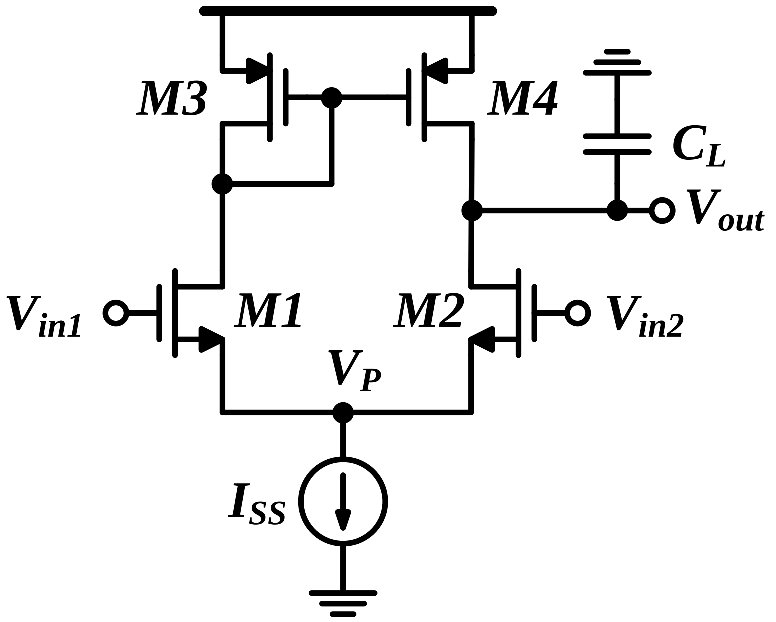
$$\omega_{p1} = \omega_{pout} \approx \frac{1}{R_{out}C_{out}}$$

- The first non-dominant pole (mirror node) sets the ultimate GBW
  - Ultimate CL bandwidth (buffer)

$$\omega_{p2} = \omega_{pM} \approx \frac{g_{m3}}{C_M} \sim \frac{\omega_T}{3}$$

- LHP zero:

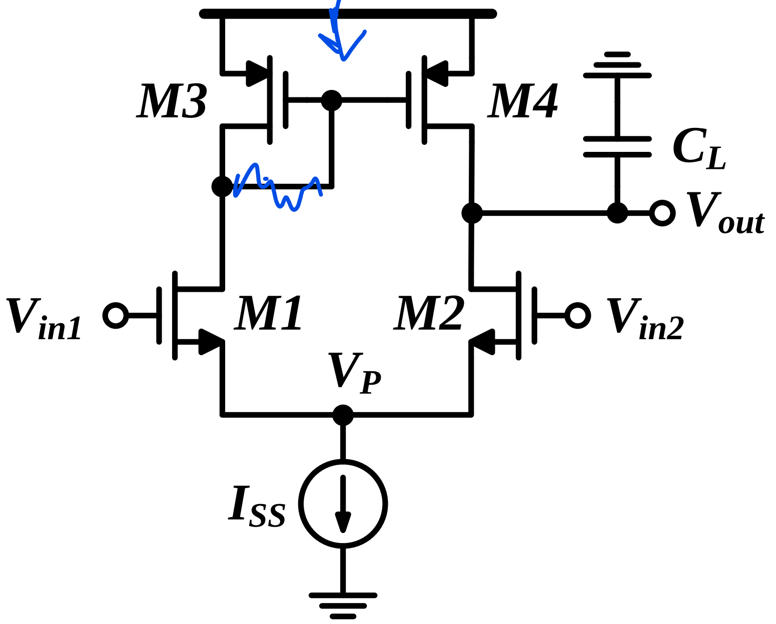
$$\omega_z = 2\omega_{p2}$$



5T OTA

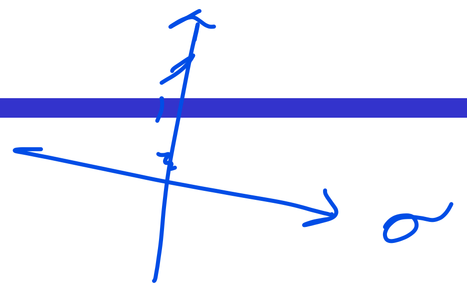
- ❑ Fully differential?

floating voltage,



5T OTA

s-plane



- Fully differential with mismatch?

$$H(s) = \frac{A_o/2}{1+\frac{s}{\omega_{p1}}} + \frac{A_o/2}{1+\frac{s}{\omega_{p1}+\Delta\omega}}$$

multiple paths

two poles

one zero

↓

contributes

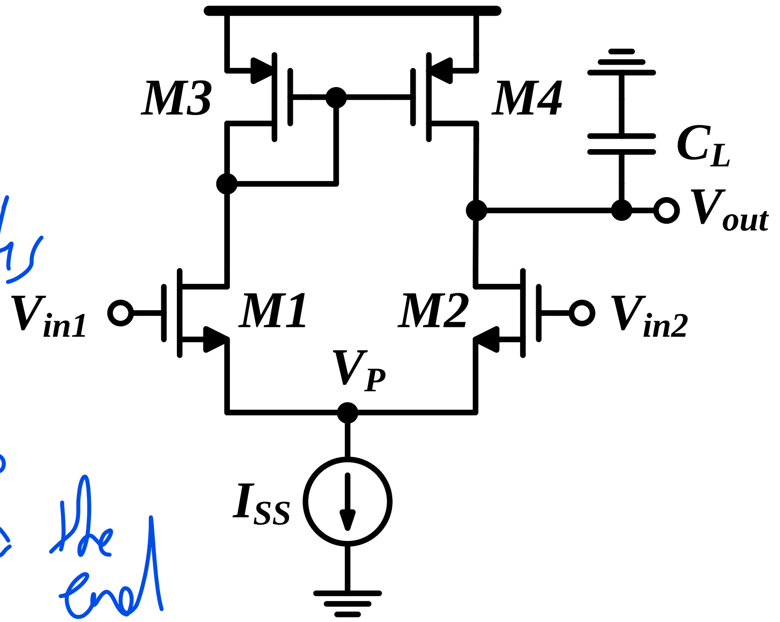
to

one pole

@ the end

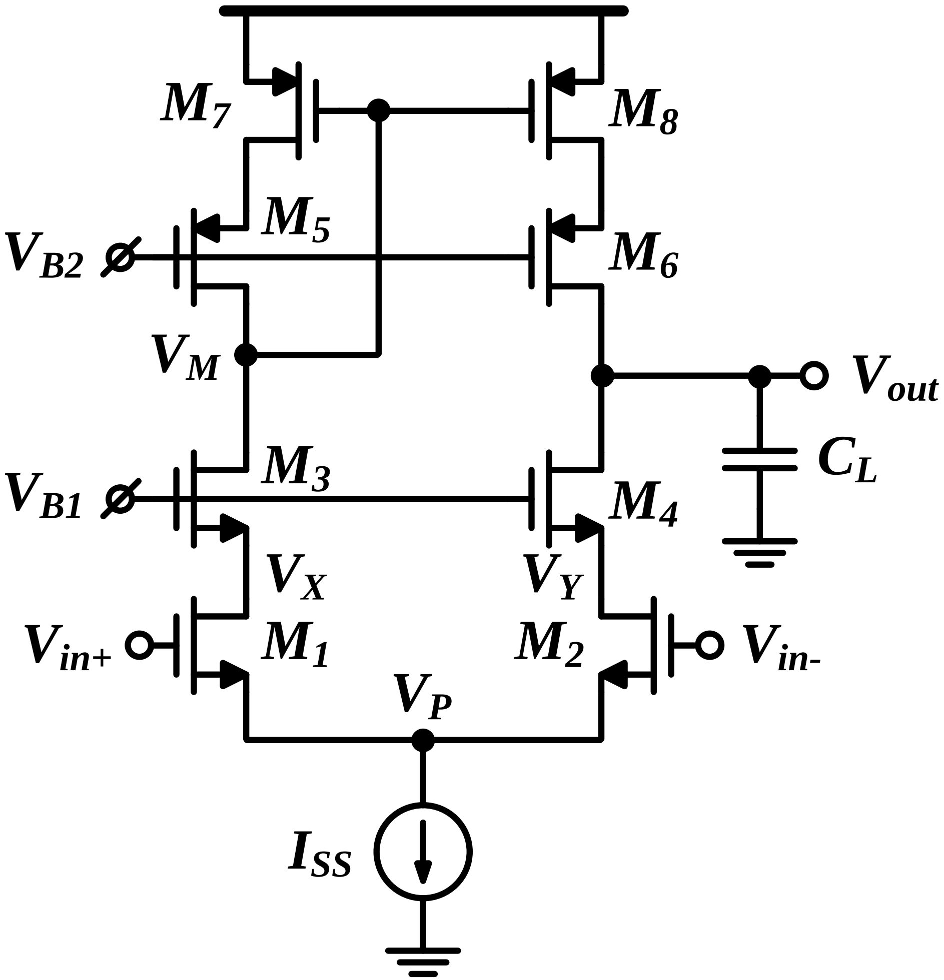
$$= \frac{A_o\left\{1+\frac{s}{2}\left(\frac{1}{\omega_{p1}}+\frac{1}{\omega_{p1}+\Delta\omega}\right)\right\}}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p1}+\Delta\omega}\right)}$$

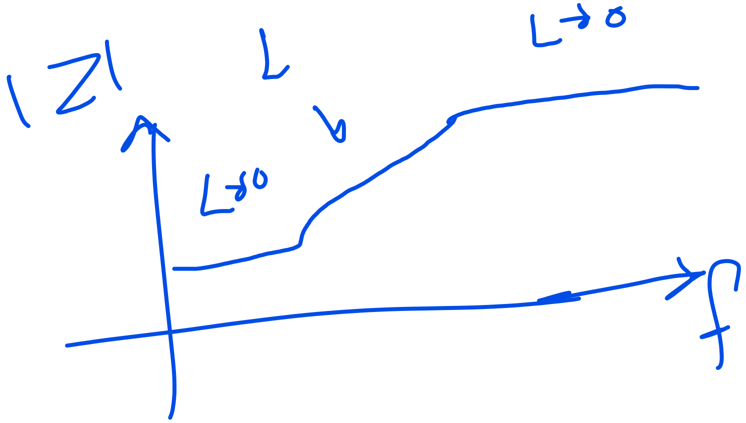
$$= \frac{A_o\left(1+\frac{s}{\omega_z}\right)}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p1}+\Delta\omega}\right)}$$



Telescopic Cascode

- Higher DC gain, but limited swing and additional poles





$Z_L = j\omega L = Z_x$

$L = \frac{Z_x}{j\omega}$

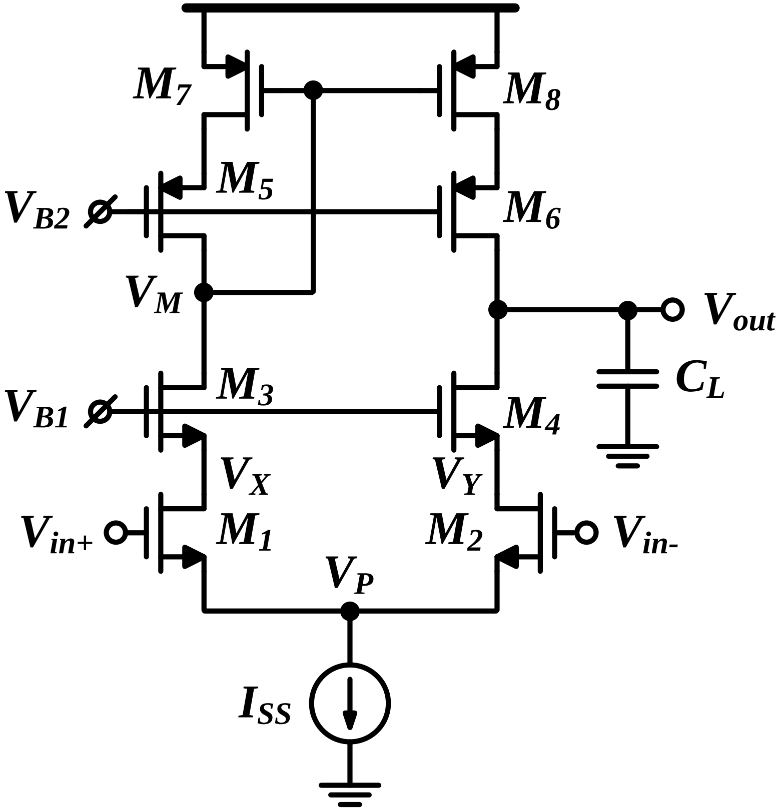
Telescopic Cascode: Poles

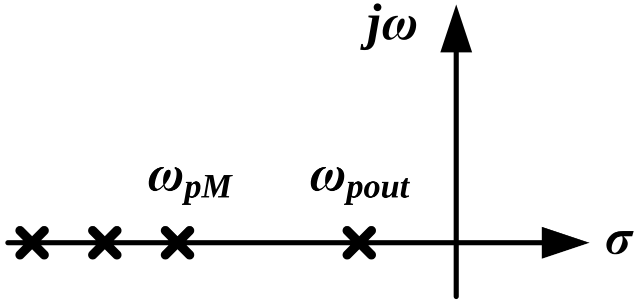
- $\omega_{p1} \ll \omega_u < \omega_{p2}$
- The H.I.N. sets BW_{OL}

$$\omega_{p1} = \omega_{pout} = \frac{1}{R_{out} C_{out}}$$

- V_X and V_Y contribute a single pole
- Non-dominant poles considerations:
  - C_{gs} is larger than other caps
  - PMOS contributes larger capacitances (lower ID/W)
- The first non-dominant pole (mirror node) sets the ultimate GBW

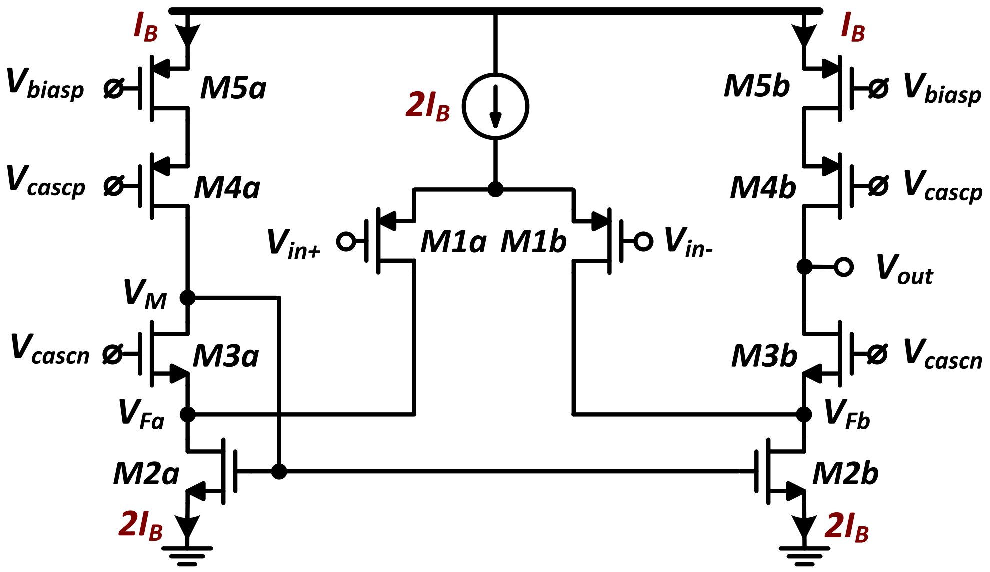
$$\omega_{p2} = \omega_{pM} \approx \frac{g_{m7}}{C_M} \sim \frac{\omega_T}{4}$$

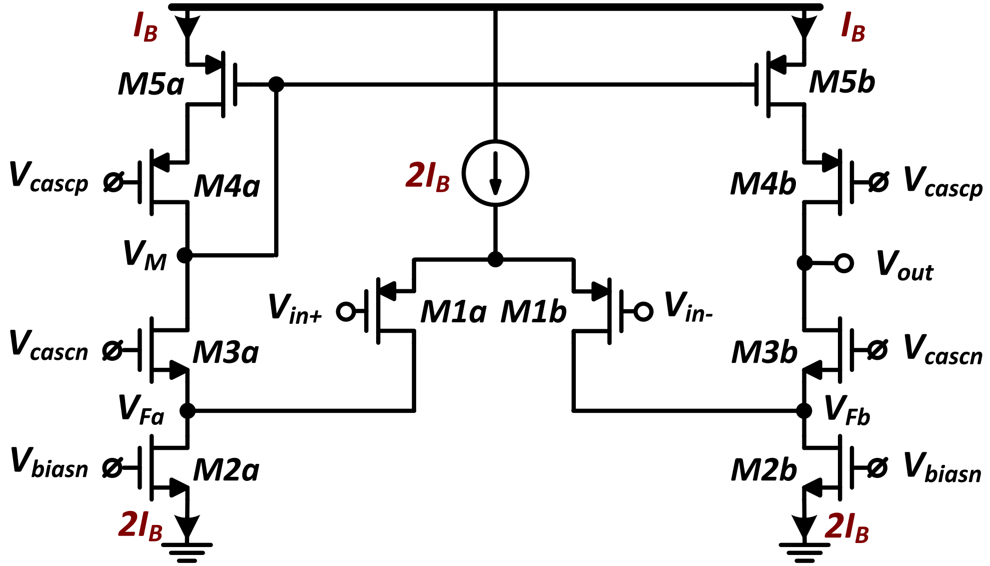




Folded Cascode

- Two possible implementations for SE output
- Compared to telescopic cascode:
  - More power (~2x)
  - Lower gain ($r_{o1}||r_{o2}$)
  - More nodes/poles
  - More complex
  - But input and output ranges decoupled





Folded Cascode: Poles

- ❑ $\omega_{p1} \ll \omega_u < \omega_{p2}$
- ❑ The H.I.N. sets BW_{OL}

$$\omega_{p1} = \omega_{pout} = \frac{1}{R_{out}C_{out}}$$

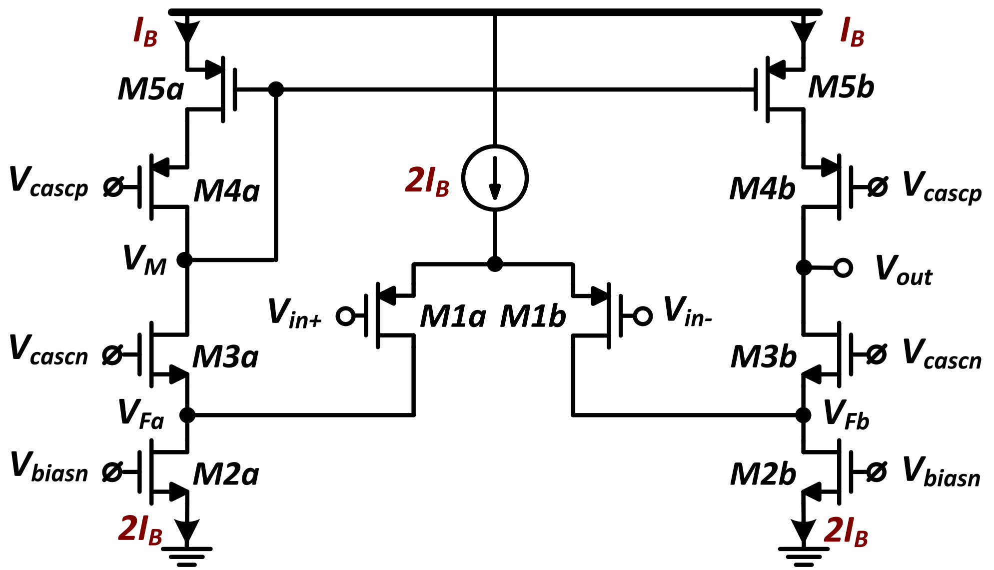
- ❑ The first non-dominant pole sets the ultimate GBW
  - Mirror node (V_M)

$$\omega_{p2} = \omega_{pM} \approx \frac{g_{m5}}{C_M}$$

  - **Or** folding node (V_F)

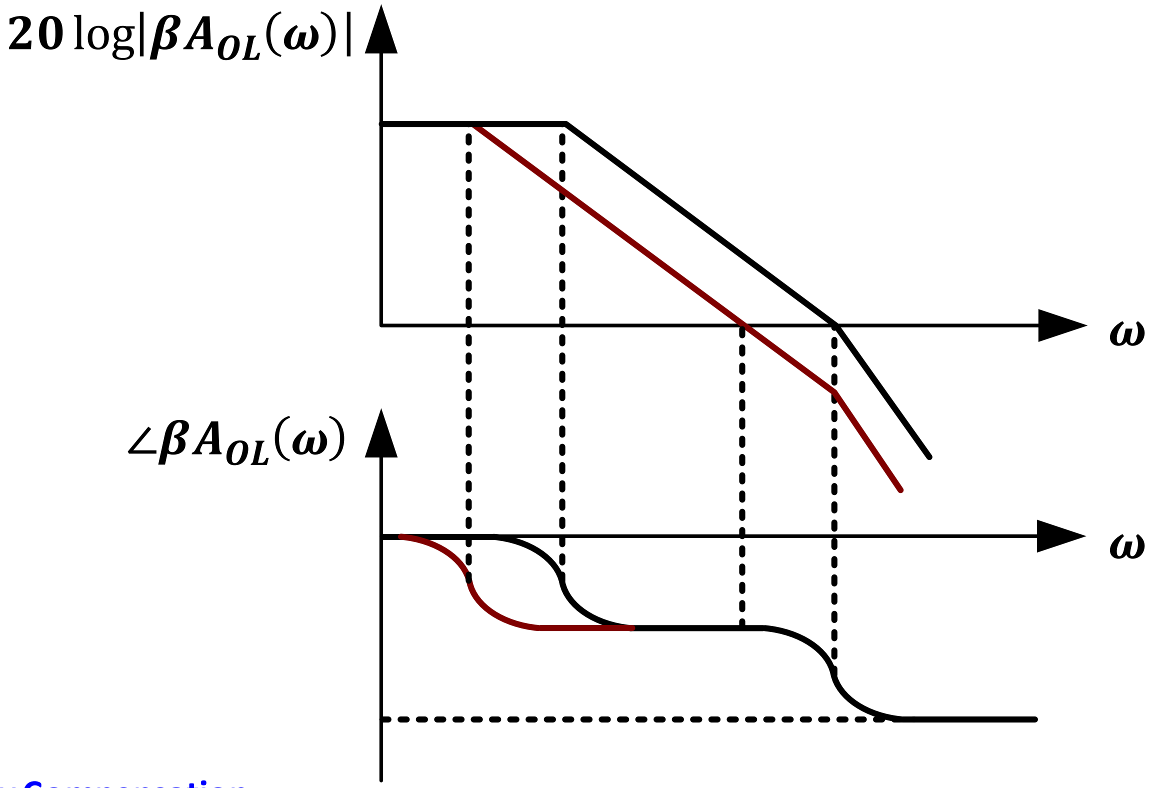
$$\omega_{p2} = \omega_{pF} \approx \frac{g_{m3}}{C_F}$$

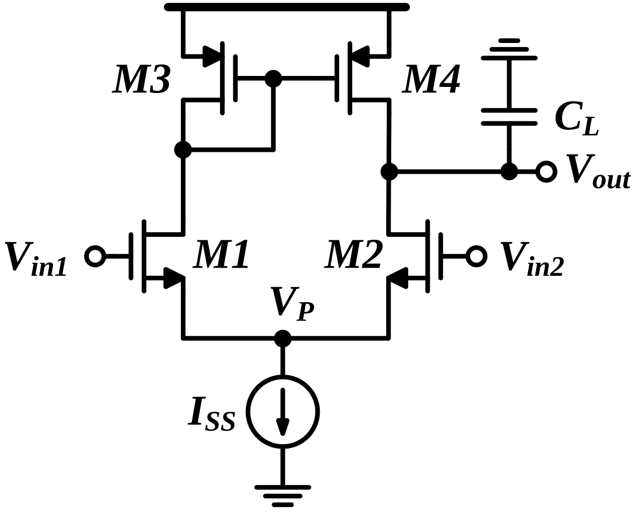
- ❑ V_{Fa} and V_{Fb} contribute a single pole
- ❑ Considerations:
  - C_{gs} larger than other caps
  - PMOS larger than NMOS
  - M2 has large capacitance (double the current)



Single-Stage OTAs: Compensation

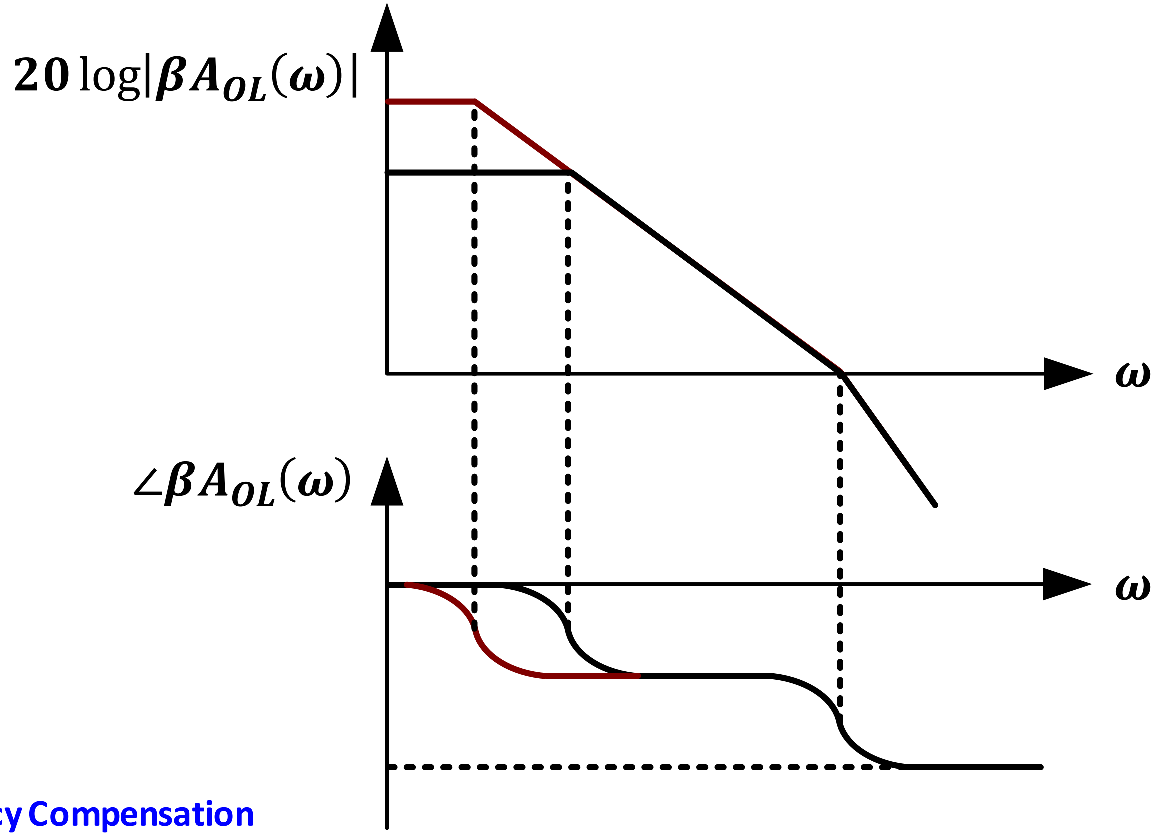
- Push GX / ω_u / ω_{p1} **inwards**: lower GBW
  - Increase C_L
  - **Single-stage OTAs are compensated by large load capacitance**

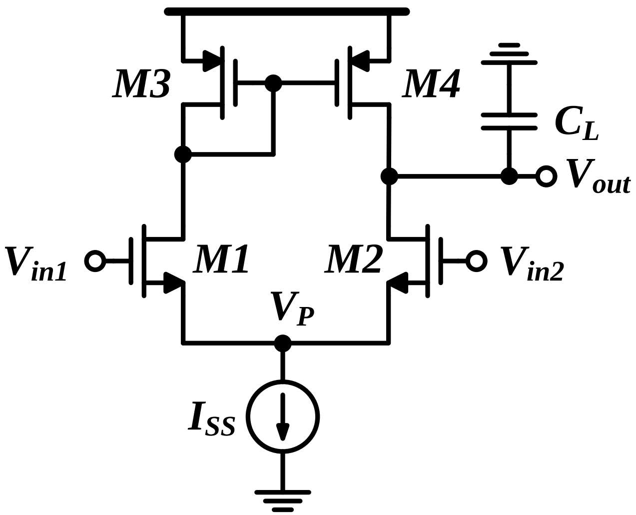




Single-Stage OTAs: Compensation

- Push GX / ω_u / ω_{p1} **inwards**: lower GBW
  - Increase C_L
  - **Single-stage OTAs are compensated by large load capacitance**
- Increasing R_{out} does not affect PM





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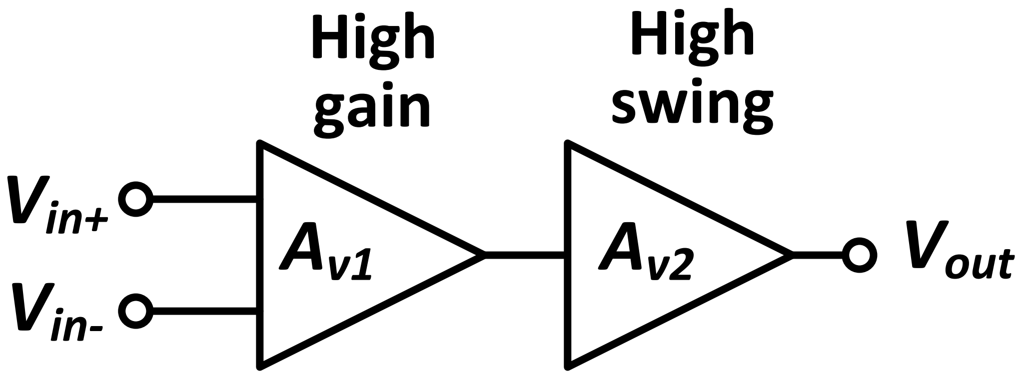
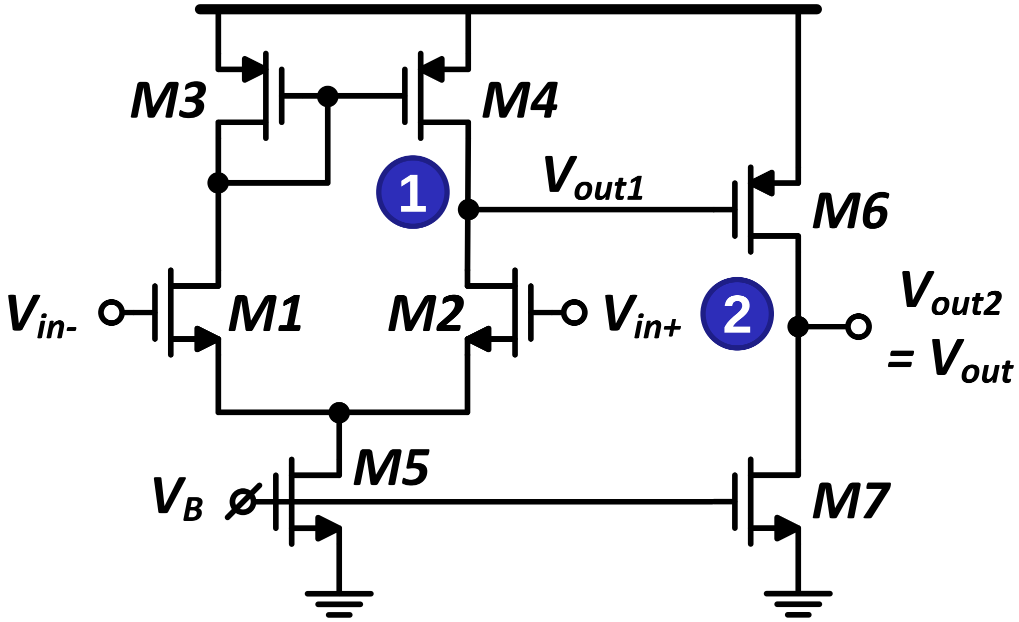
Two-Stage OTA

- Isolates the gain and swing requirements
  - But much more power consumption
  - And complicates stability requirements
- Three-stage OTA exists, but quite difficult to stabilize
- First stage can be 5T-OTA or cascode
- Second stage is typically a simple common-source
  - Allows maximum output swing



Two-Stage OTA: Poles

- Two gain stages → Two H.I.N.s
  - Two dominant poles!
- Internal pole: $\omega_{p1} = \frac{1}{R_{out1}C_1}$
- Output pole: $\omega_{p2} = \frac{1}{R_{out2}C_2}$



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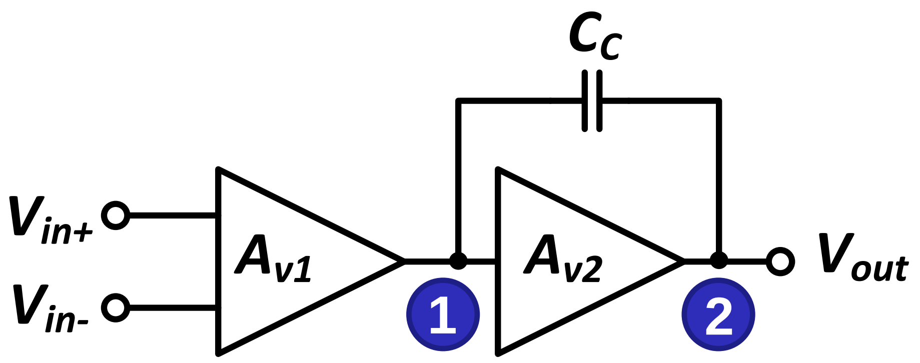
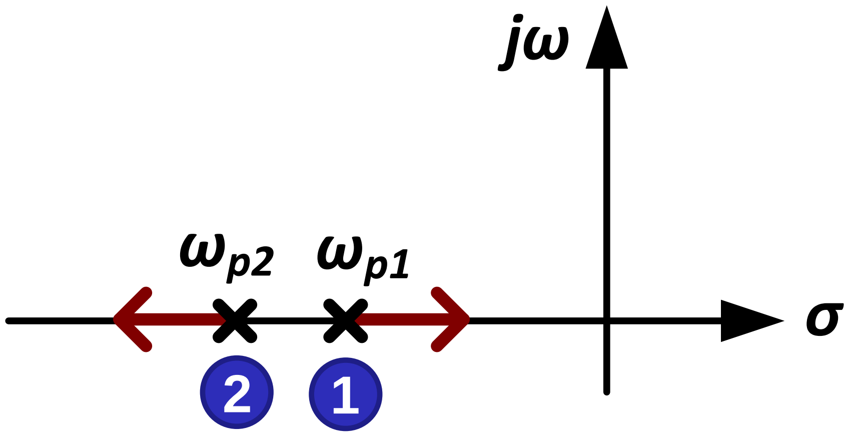
Two-Stage OTA: Miller Compensation

- Exploit Miller capacitance multiplication
- Pole splitting
  - Push ω_{p1} inwards → push GX inwards
  - Push ω_{p2} outwards → push PX outwards





CS HFR: Reminder

- Surprisingly, exact analysis gives a quite complex expression
  - See [Johns & Martin 2012] or [Razavi 2017]
- If dominant pole approximation is applied

$$\omega_{pd} \approx \frac{1}{b_1} = \frac{1}{R'_{sig}\left[C_{gs} + C_{gd}(1 + g_m R_{out})\right] + R_{out}\left(C_{out} + C_{gd}\right)}$$

  - Same result as OCTC (both based on same approximation)
- Additionally, dominant pole approx gives an expression for ω_{pnd}

$$\omega_{pnd} \approx \frac{1}{b_2 \omega_{p1}} = \frac{b_1}{b_2} = \frac{g_m C_{gd}}{C_{gd}\left(C_{gs} + C_{out}\right) + C_{gs} C_{out}}$$

  - If a large cap is connected parallel to C_{gd}: $\omega_{pnd} \approx \frac{g_m}{C_{gs}+C_{out}}$
    - Can be derived intuitively without analysis (how?)

Two-Stage OTA: Miller Compensation

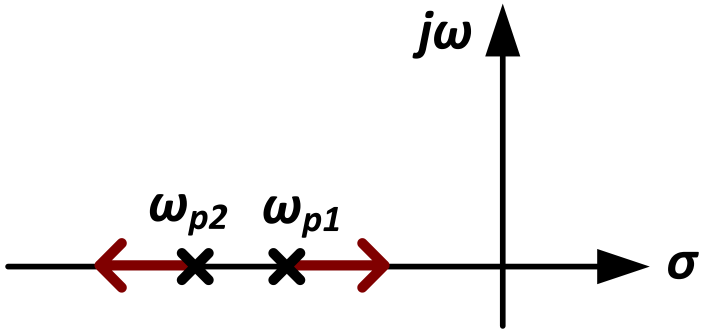
- Before compensation

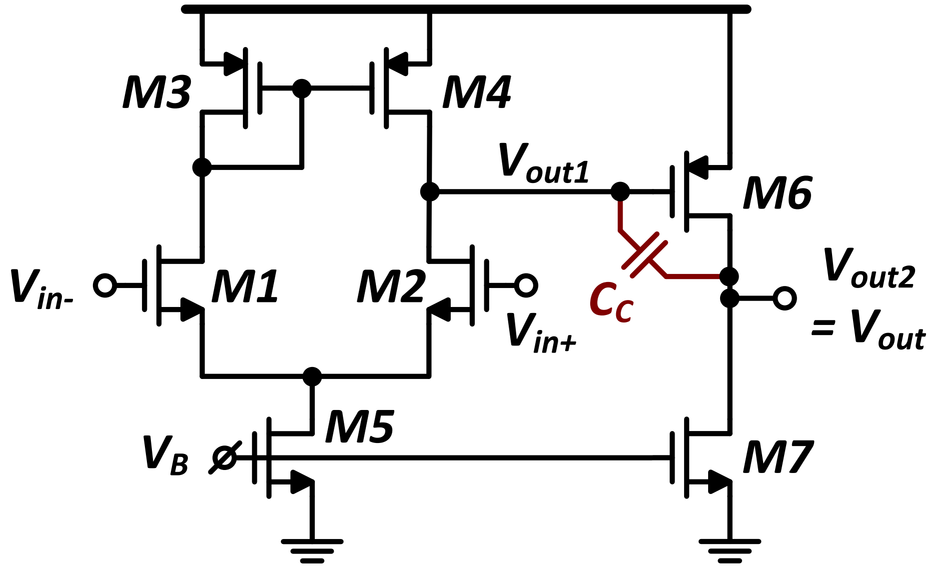
$$\omega_{p1} = \frac{1}{R_{out1}C_1} \quad \& \quad \omega_{p2} = \frac{1}{R_{out2}C_2}$$

- After compensation: Pole splitting

$$\omega_{p1} \approx \frac{1}{R_{out1}[(G_{m2}R_{out2})C_C + C_1] + R_{out2}(C_2 + C_C)} \approx \frac{1}{R_{out1}(G_{m2}R_{out2})C_C}$$

$$\omega_{p2} \approx \frac{G_{m2}C_C}{C_C(C_1 + C_2) + C_1C_2} \approx \frac{G_{m2}}{C_1 + C_2}$$





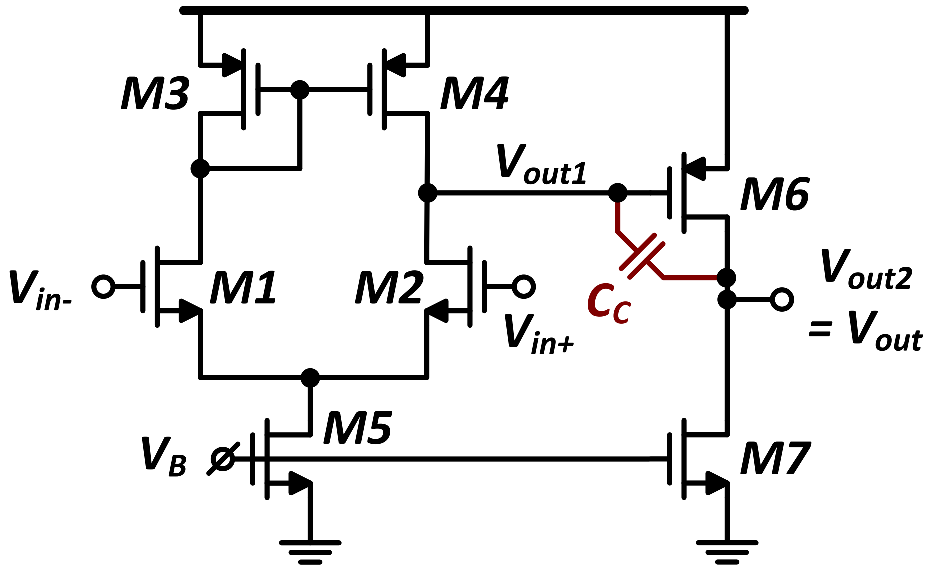
Stability Requirement: $\boldsymbol{\omega_u}$ and $\boldsymbol{\omega_{p2}}$

$$\omega_{p1} \approx \frac{1}{R_{out1}(G_{m2}R_{out2})C_C}$$

$$\omega_{p2} \approx \frac{G_{m2}}{C_1 + C_2} \approx \frac{G_{m2}}{C_L}$$

$$GBW \approx G_{m1}R_{out1}G_{m2}R_{out2} \cdot \frac{1}{R_{out1}(G_{m2}R_{out2})C_C}$$

$$GBW = \omega_u \approx \frac{G_{m1}}{C_C}$$



- **For critical damped response: $\boldsymbol{\zeta = 1}$, $\boldsymbol{Q = 0.5}$, and $\boldsymbol{PM \approx 76^o}$**

$$\omega_{p2} \approx 4\omega_u \rightarrow \frac{G_{m2}}{C_L} \approx \frac{4G_{m1}}{C_C}$$

Bias Current in 1st and 2nd Stages

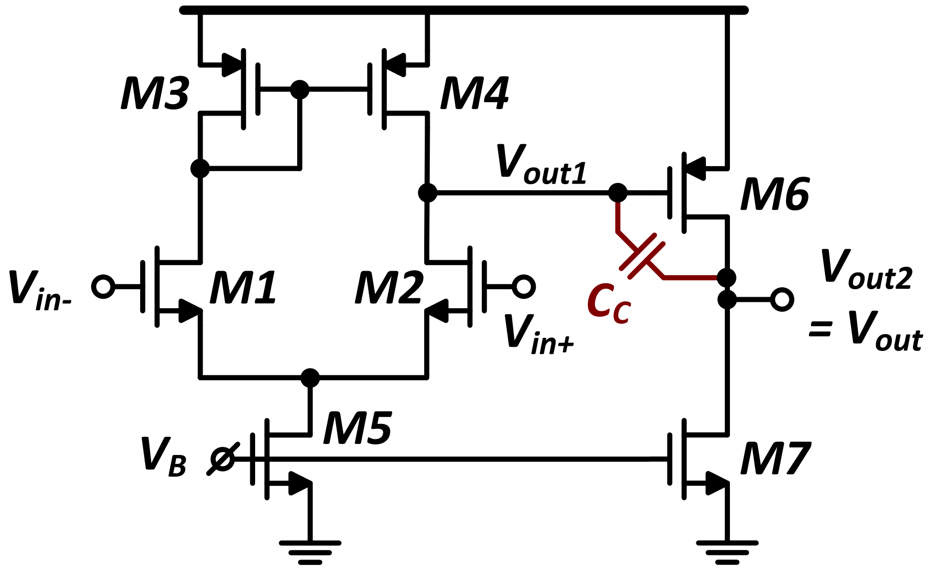
$$\frac{G_{m2}}{C_L} \approx \frac{4G_{m1}}{C_C}$$

- Assume $C_L = 4pF \; and \; C_C = C_L/2 = 2pF$

$$\frac{G_{m2}}{G_{m1}} = 4 \times \frac{4}{2} = 8$$

- If both stages use the same gm/ID
  - Note that $I_{B1} = 2 \times I_{D1,2}$

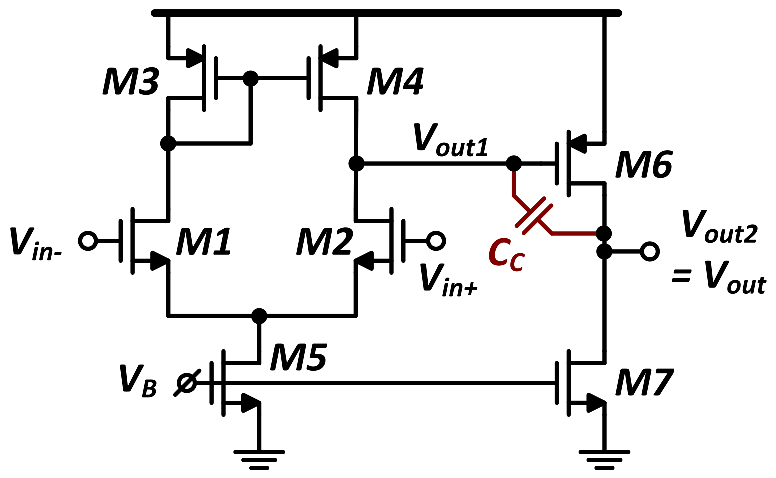
$$\frac{I_{B2}}{I_{B1}} = 4$$

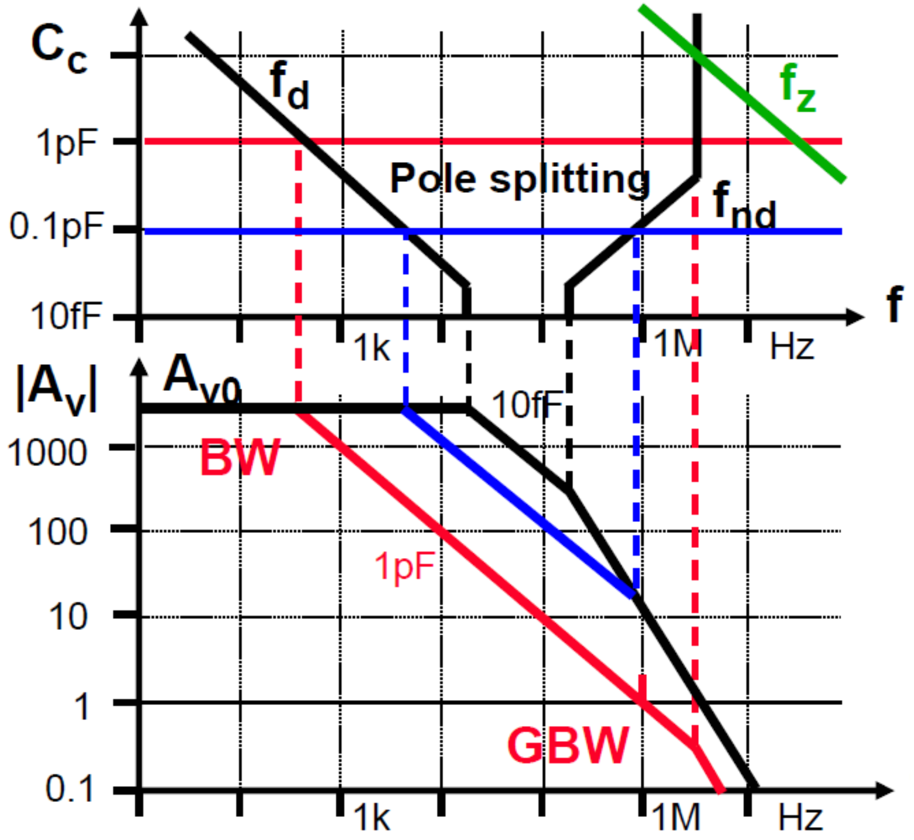


- 80% of the power is consumed in the second stage to achieve stability
  - 80% of bias current do not contribute (directly) to GBW
  - Miller OTA is very energy inefficient!

Miller OTA: Pole Splitting with C_C

- Too large C_C does not give more pole splitting: just smaller GBW
  - Usually we choose $C_1 < C_C < C_L$
  - Reasonable starting point: $C_C \approx (0.3 \rightarrow 0.5) \times C_L$

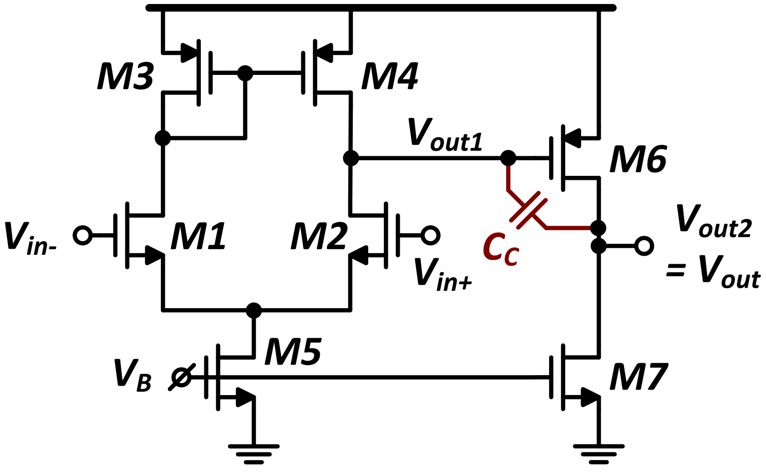


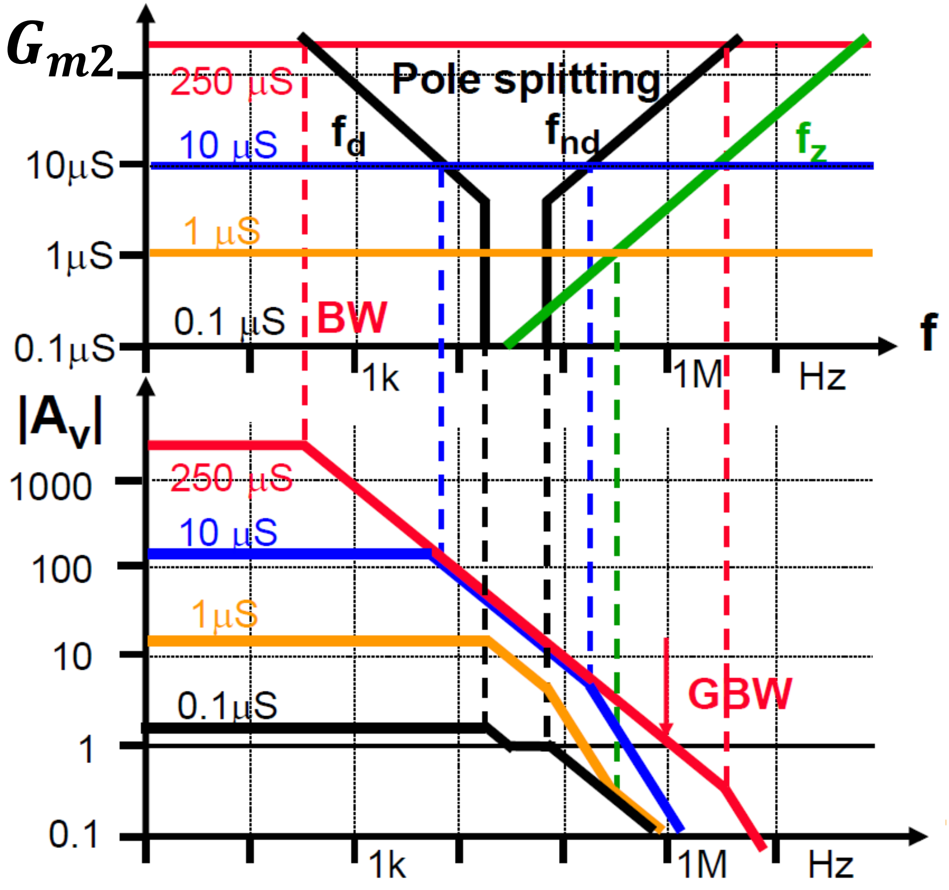


[Sansen, Essentials, 2006]

Miller OTA: Pole Splitting with G_{m2}

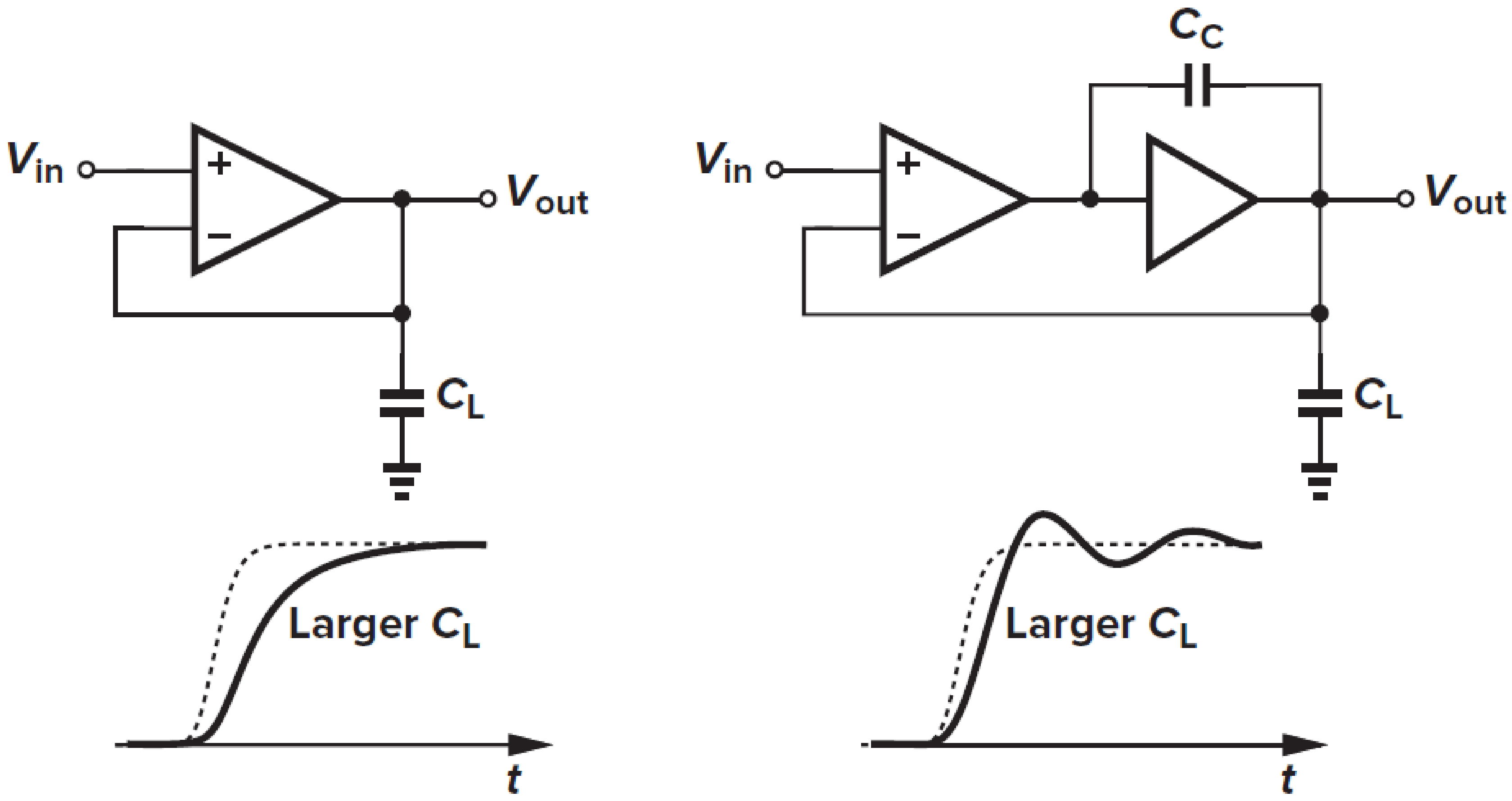
- Increasing G_{m2} works even better than increasing C_C
  - But more power consumption in the 2nd stage





[Sansen, Essentials, 2006]

Single vs Two-Stage OTA: Sensitivity to C_L



[Razavi, 2017]

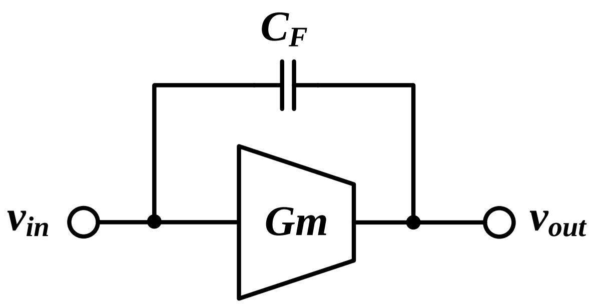
Outline

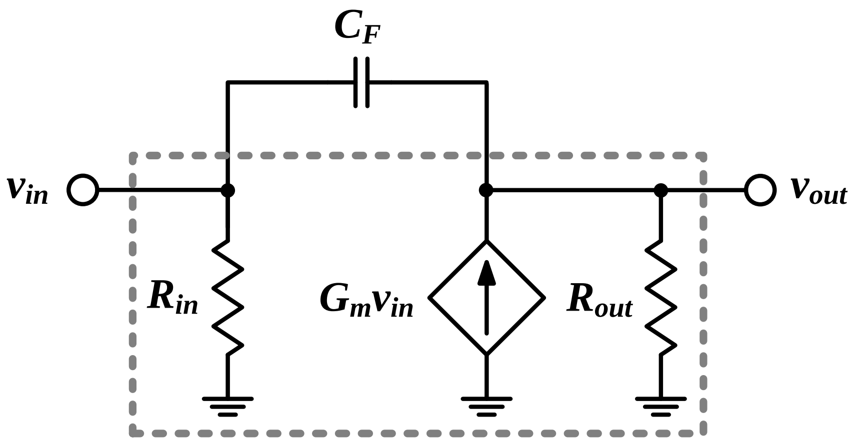
- Recapping previous key results
- Frequency compensation
- Compensation of single-stage OTAs
- Compensation of two-stage OTA
  - Miller compensation
  - The feedforward zero

The Feedforward Zero: Reminder

- $v_{out} = 0 \;\rightarrow i_{out} = 0$
- $v_{in}sC_F = -G_m v_{in}$

$$\boxed{s_z = -\frac{G_m}{C_F}}$$



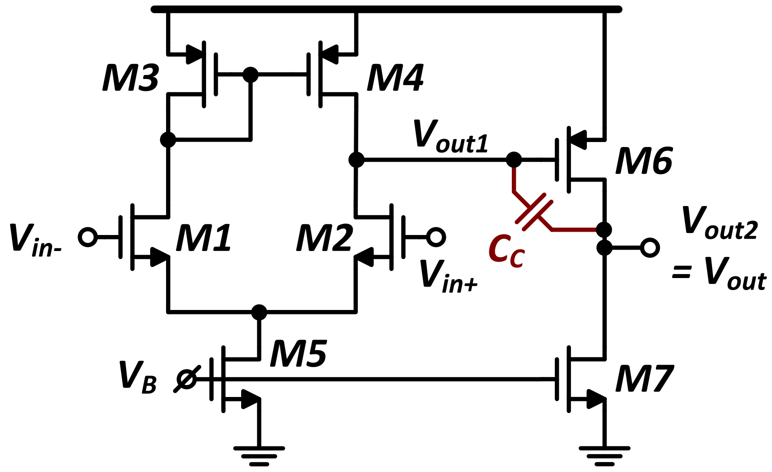


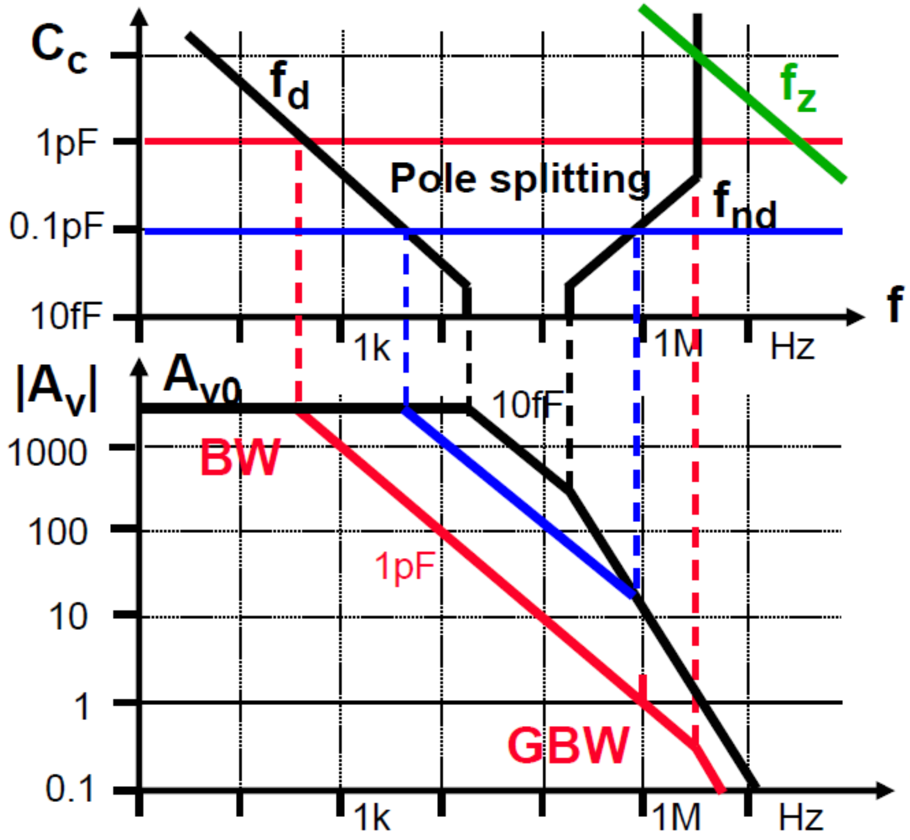
- LHP zero if G_m is +ve (e.g. CD)
- **RHP zero if G_m is –ve (e.g. CS)**
  - Mag inc and phase drops
  - Very bad for FB loop stability

Miller OTA: The RHP Zero

- RHP zero is bad for both magnitude and phase
  - Pushes GX outwards and pushes PX inwards
  - Increasing C_C may hurt stability!

$$\omega_z = \frac{g_{m6}}{C_C + C_{gd}} \approx \frac{G_{m2}}{C_C}$$





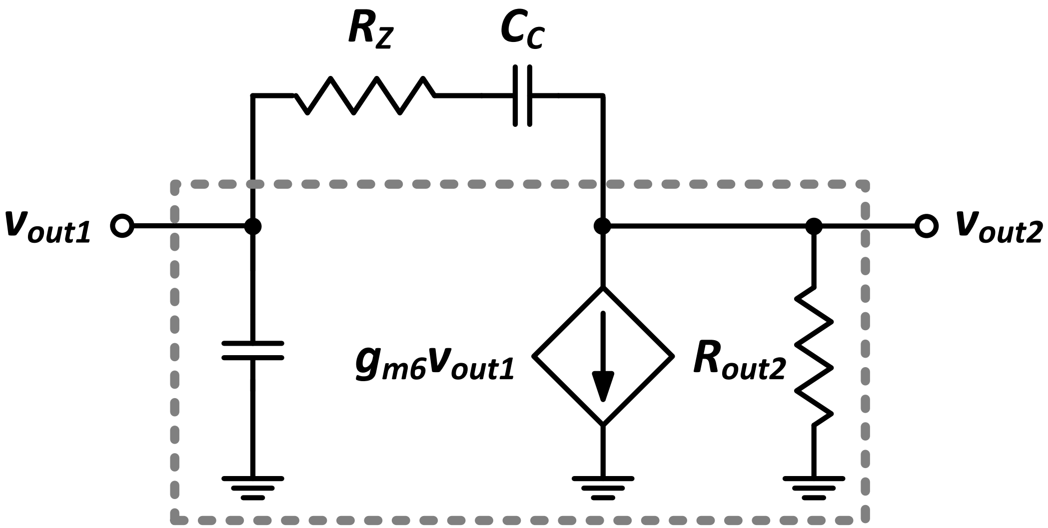
[Sansen, Essentials, 2006]

Handling the RHP Zero

- Add a resistance to control the value of the zero (other tricks exist)

$$\frac{v_{out1}}{R_Z + \frac{1}{s_Z C_C}} = g_{m6} v_{out1}$$

$$s_Z = \frac{1}{C_C\left(\frac{1}{g_{m6}} - R_Z\right)}$$



Miller Zero Placement

- Place the zero at ∞

$$\omega_Z = \frac{1}{C_C\left(\frac{1}{G_{m2}} - R_Z\right)} \rightarrow R_Z = \frac{1}{G_{m2}}$$

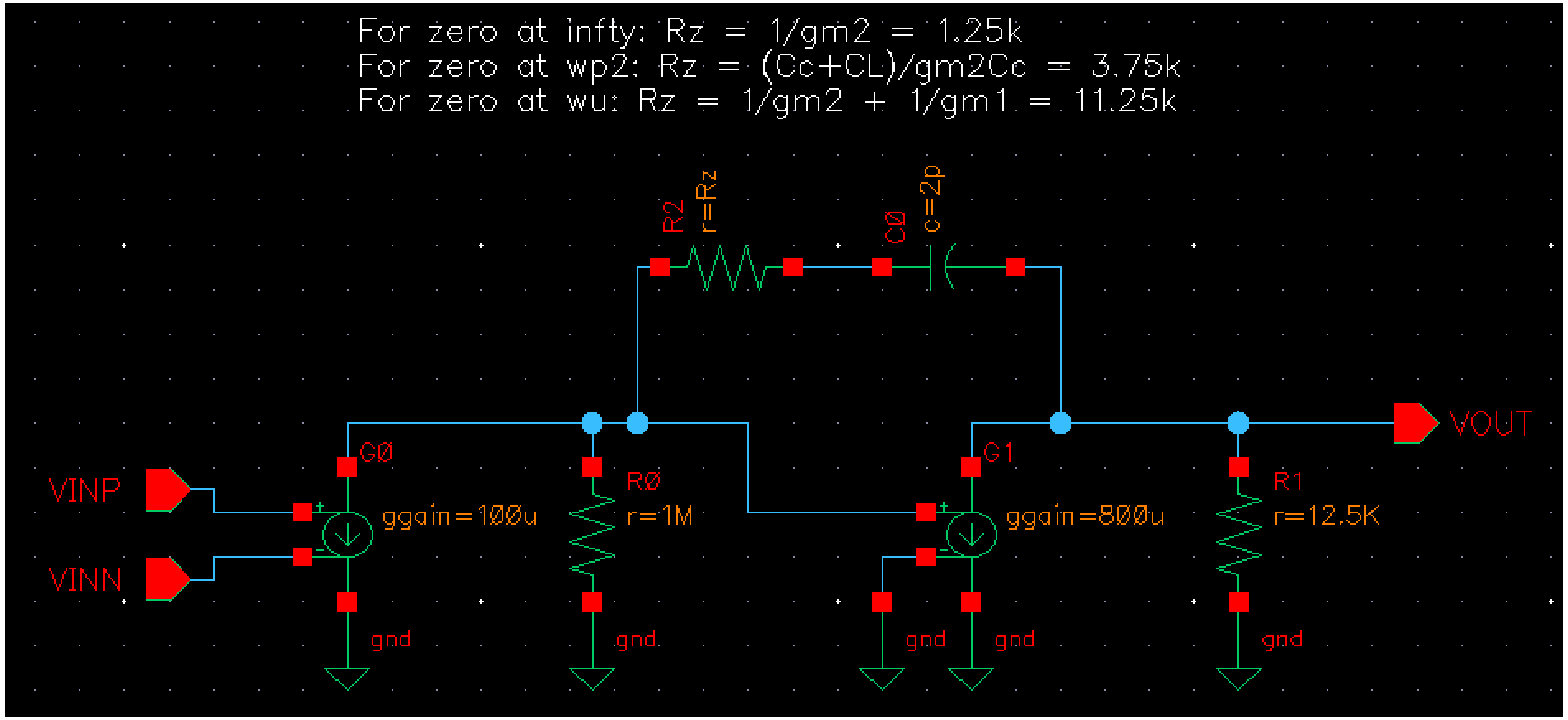
- Some designers try to move the zero to the LHP to cancel the first non-dominant pole and/or improve the PM

$$\frac{1}{C_C\left(R_Z - \frac{1}{G_{m2}}\right)} = \frac{G_{m2}}{C_L} \rightarrow R_z = \frac{C_L + C_C}{G_{m2} C_C}$$

  - Practically never achieved due to variations
  - Actually does not lead to faster response (why?)
- Pushing the LHP zero to lower frequencies is even worse
  - Very poor settling time (a.k.a. pole-zero doublet)
  - Noise amplification

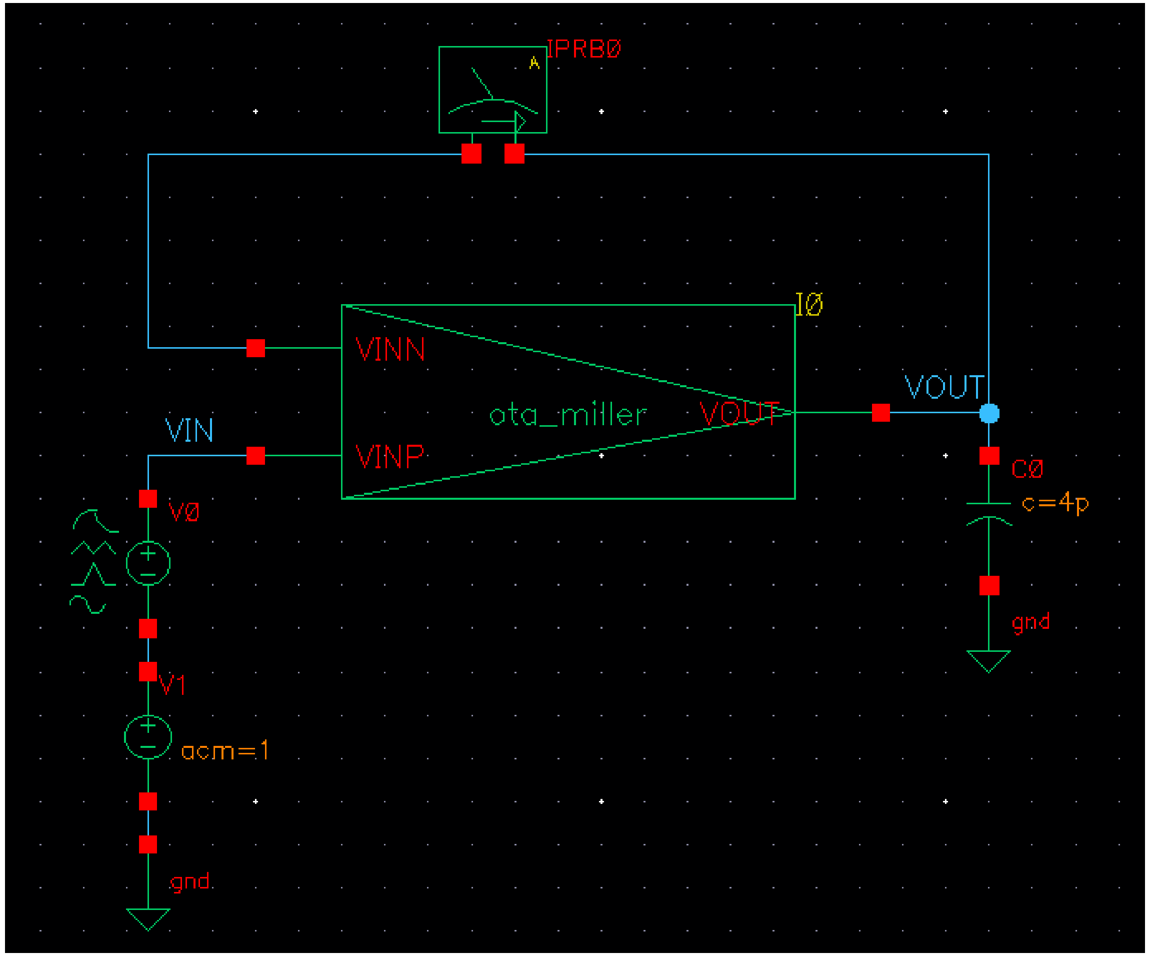
Miller OTA Behavioral Model

- Try three different strategies for Miller zero placement
  - ∞
  - Cancel ω_{p2}
  - At ω_u (this is a designer mistake rather than a valid strategy!)

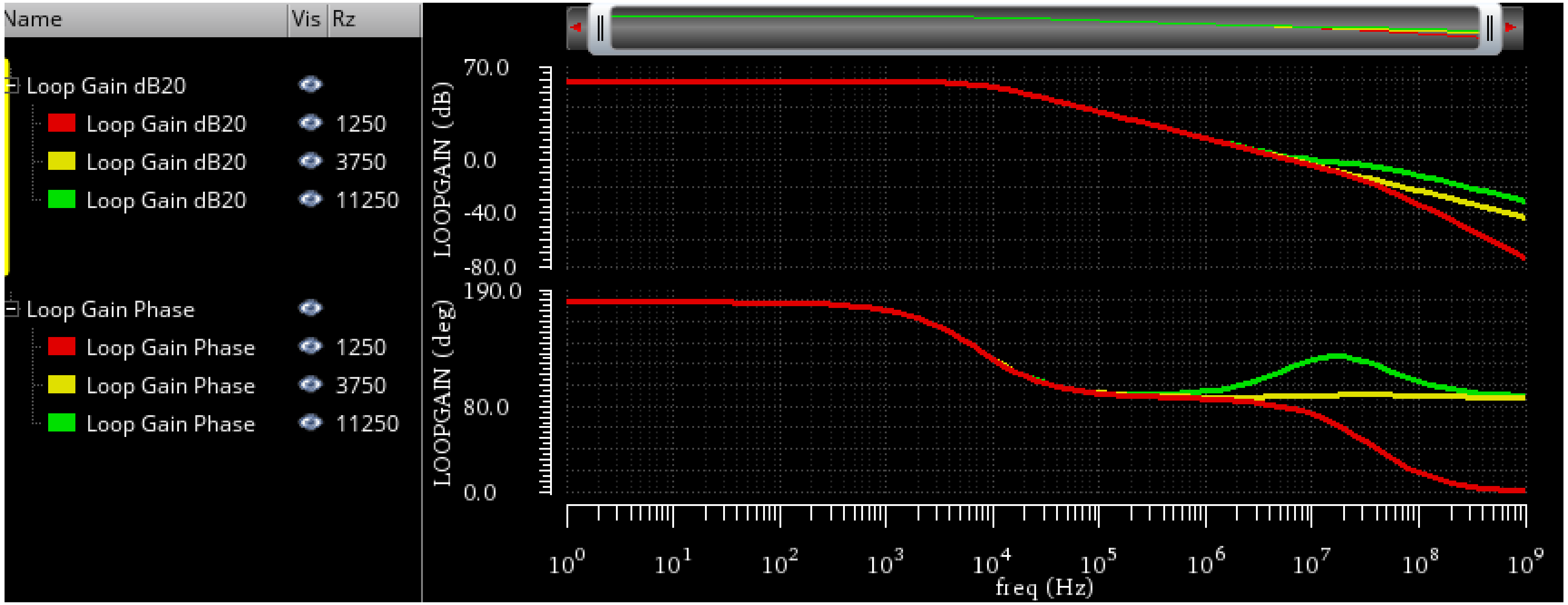


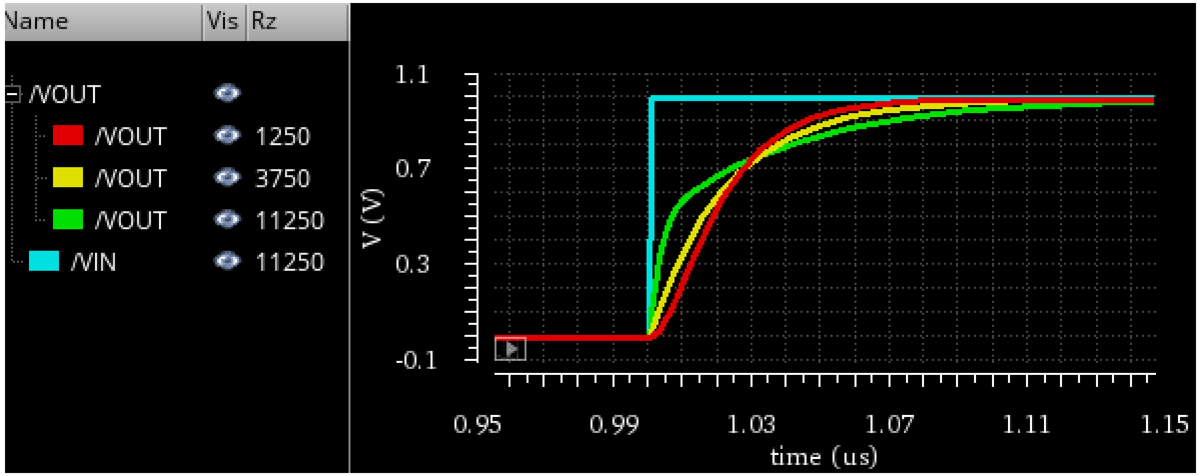
TB for LG and Step Response

- ❑ Verify stability, ac response, and transient response in unity-gain buffer configuration.

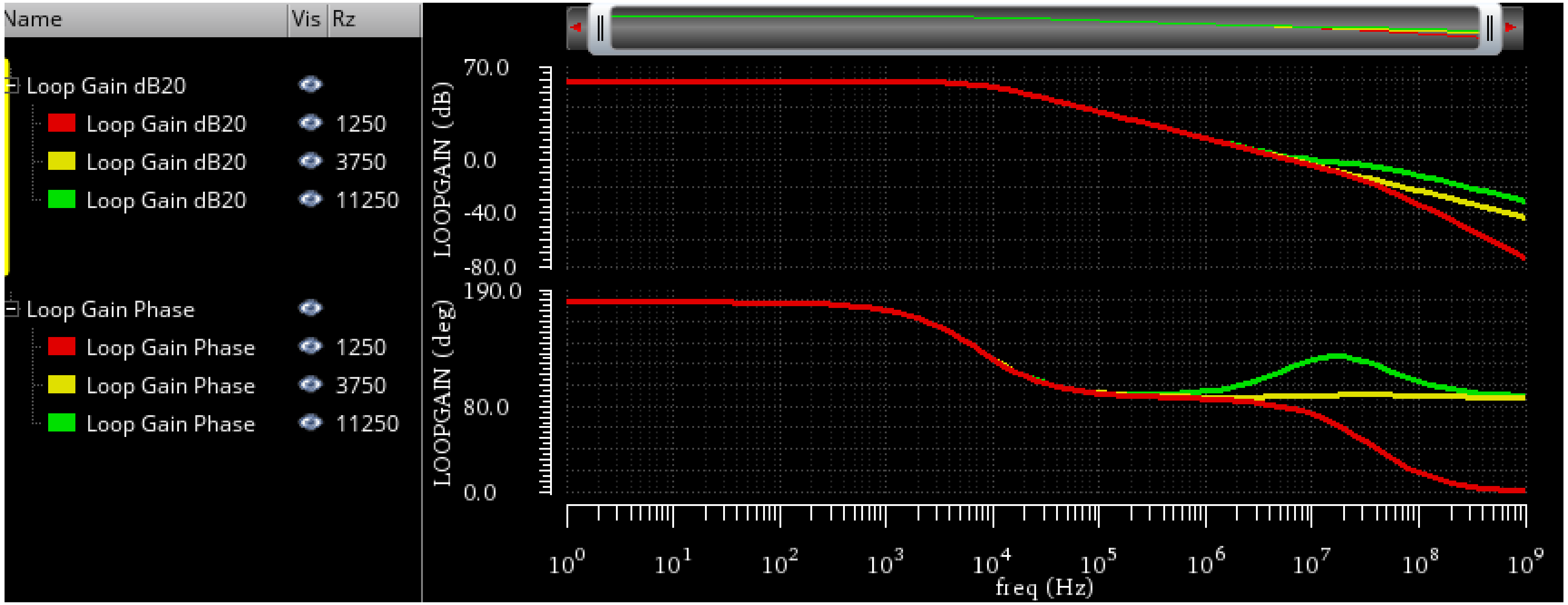


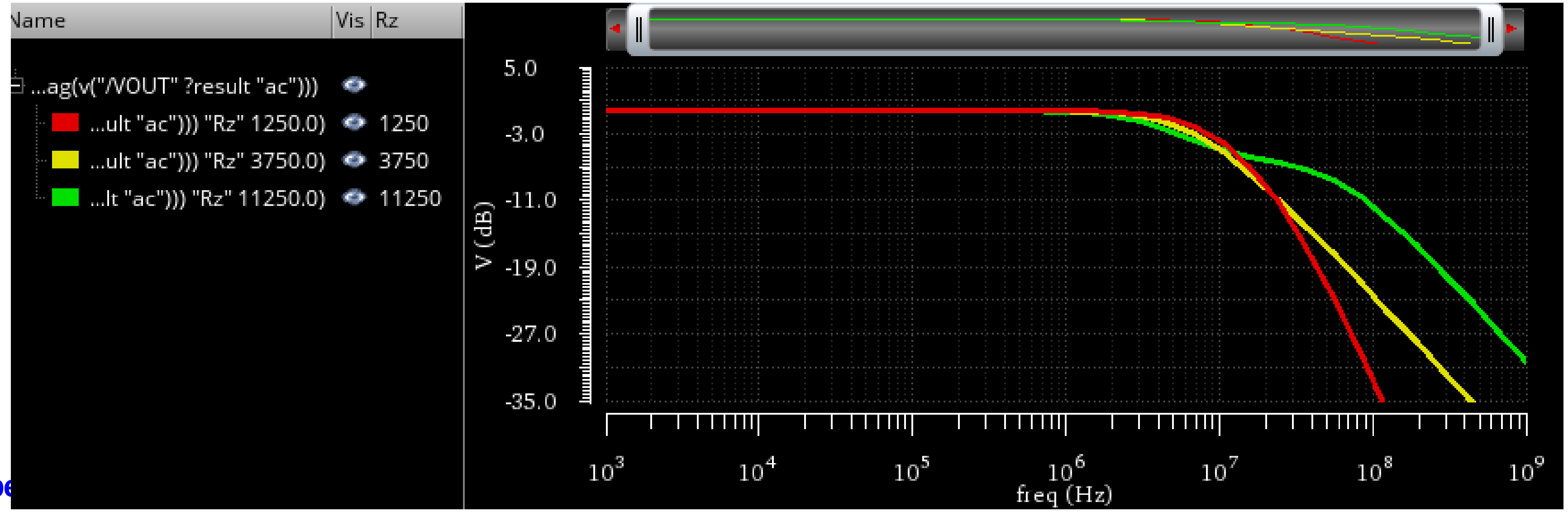
LG and Step Response





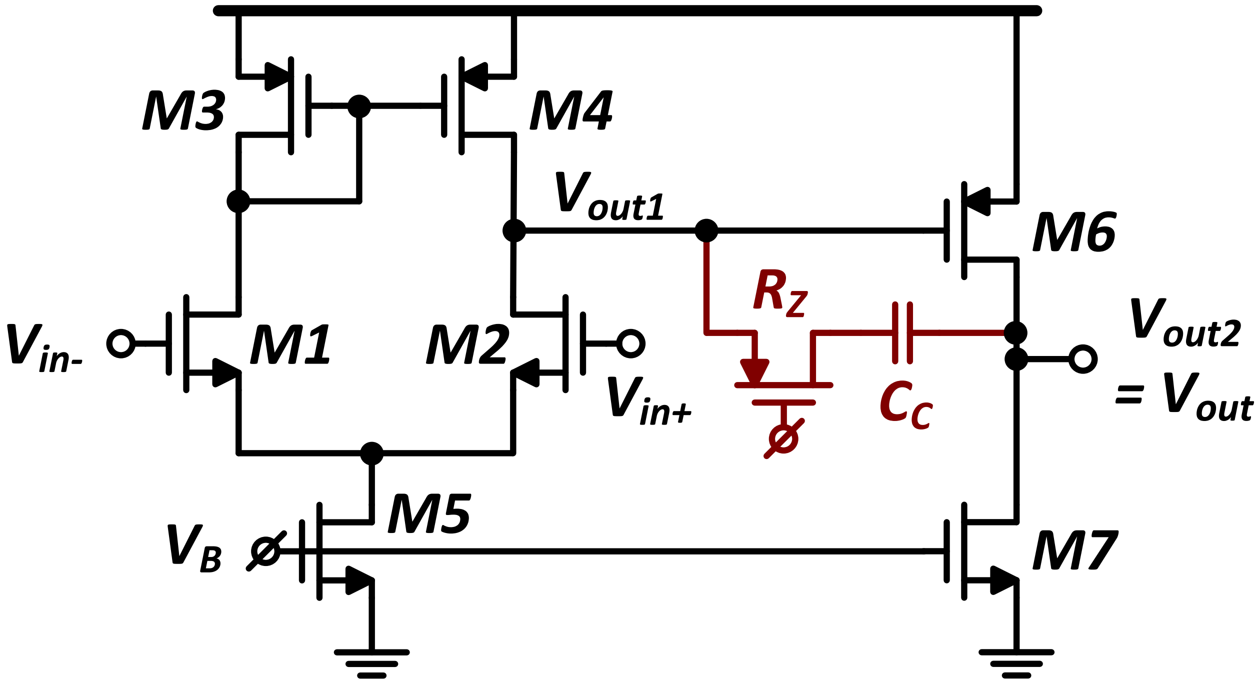
LG and CL Gain





R_Z Implementation

- R_Z can be implemented as a simple resistor or using a transistor in the linear region
  - Clever biasing can make it track variations (maintain $R_Z \approx \frac{1}{g_{m6}}$)
  - Why R_Z is implemented as PMOS?



Thank you!

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