

Analog IC Design

Lecture 15 Negative Feedback

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Outline

- ☐ Recapping previous key results
- ☐ General feedback system
- ☐ Loop gain
- ☐ Why negative feedback?
- ☐ Stability of feedback system
- ☐ Root locus and Bode plot
- ☐ Phase and gain margin

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MOSFET in Saturation

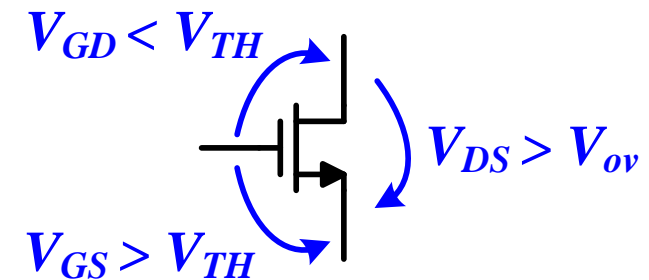
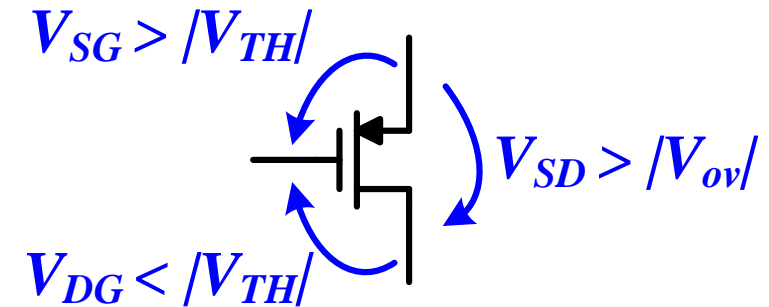
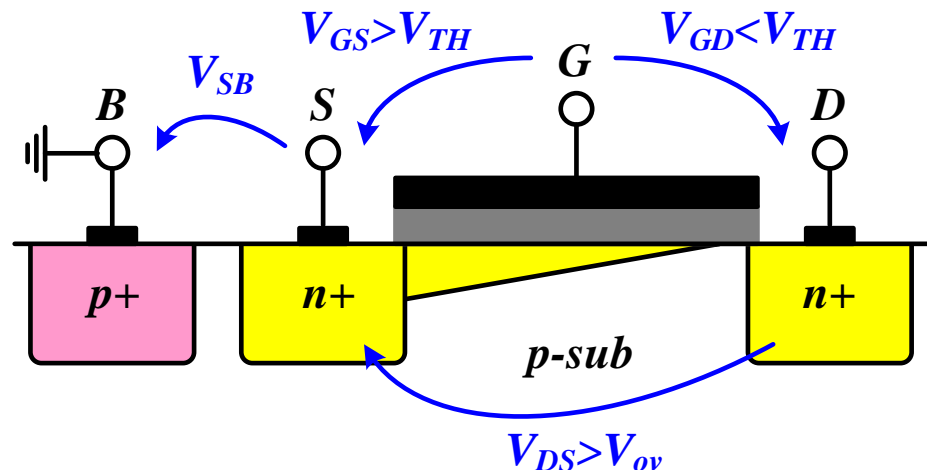
- ❑ The channel is pinched off if the difference between the gate and drain voltages is not sufficient to create an inversion layer

$$V_{GD} \leq V_{TH} \quad \text{OR} \quad V_{DS} \geq V_{ov}$$

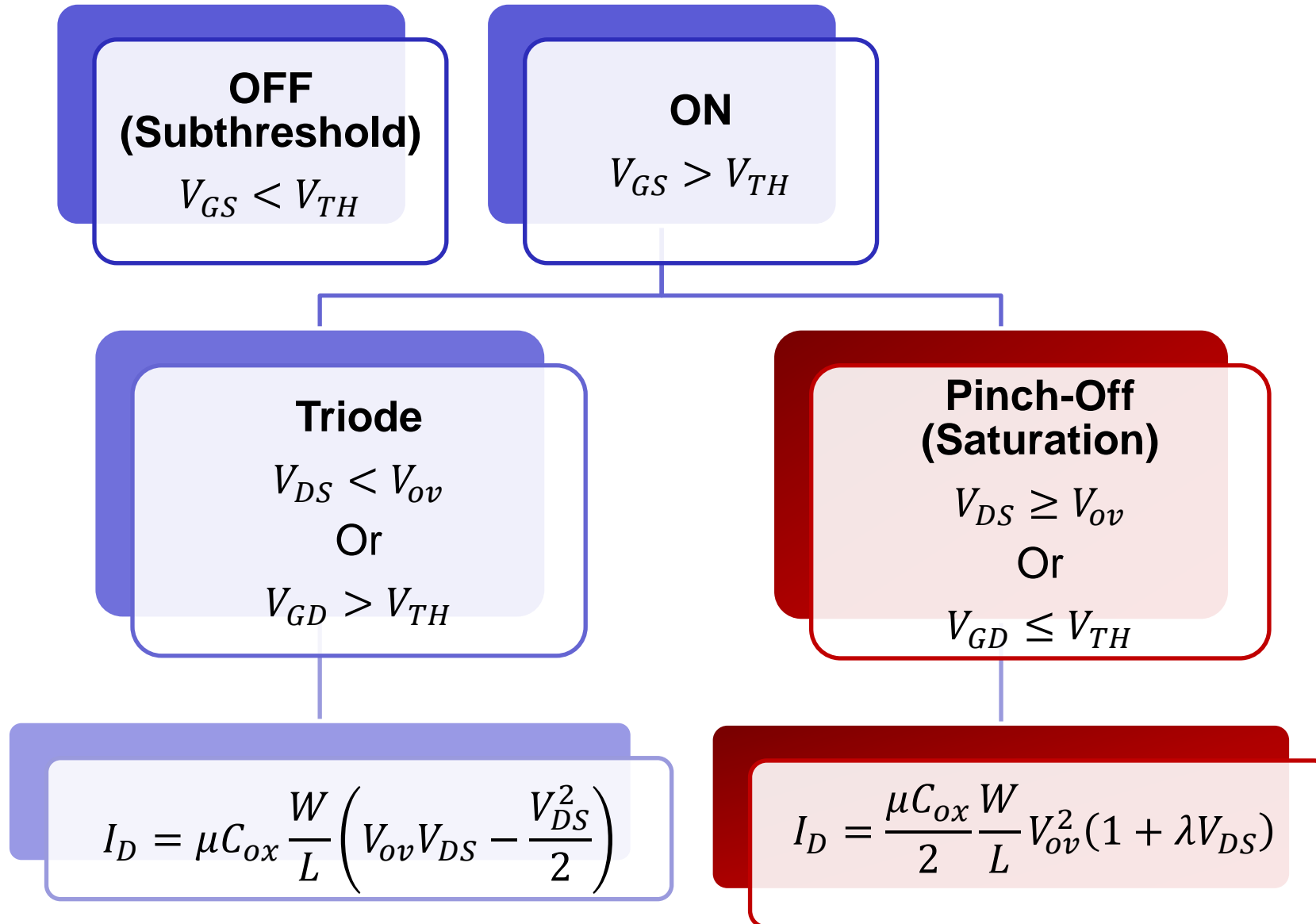
- ❑ Square-law (long channel MOS)

$$I_D = \frac{\mu_n C_{ox}}{2} \frac{W}{L} \cdot V_{ov}^2 (1 + \lambda V_{DS})$$

$$V_{SB} \uparrow \Rightarrow V_{TH} \uparrow$$



Regions of Operation Summary



High Frequency Small Signal Model

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \mu C_{ox} \frac{W}{L} V_{ov} = \sqrt{\mu C_{ox} \frac{W}{L} \cdot 2I_D} = \frac{2I_D}{V_{ov}}$$

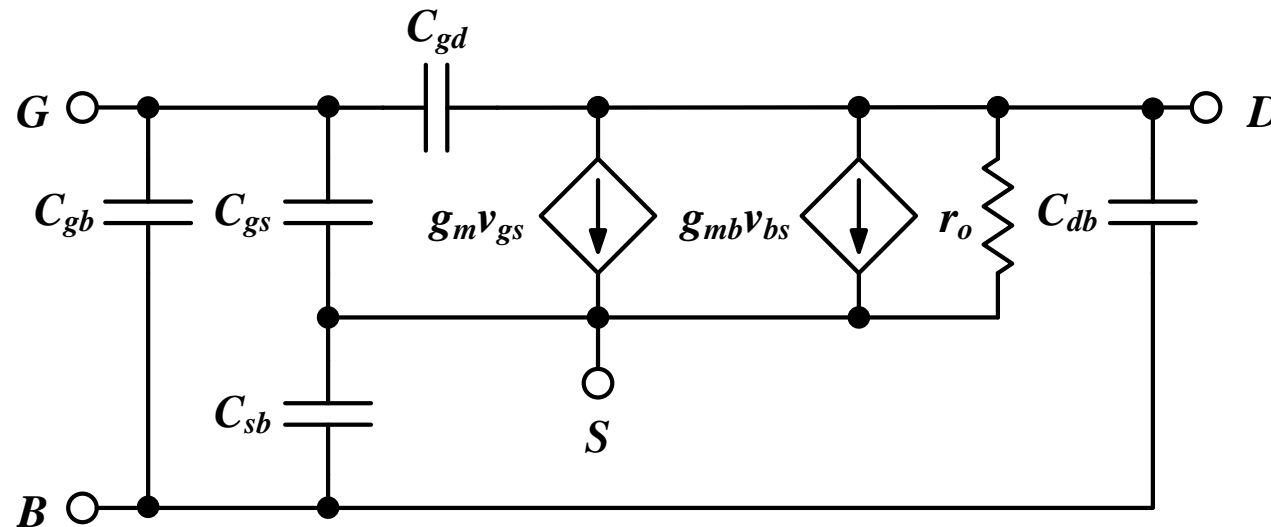
$$g_{mb} = \eta g_m \quad \eta \approx 0.1 - 0.25$$

$$r_o = \frac{1}{\partial I_D / \partial V_{DS}} = \frac{V_A}{I_D} = \frac{1}{\lambda I_D} \quad V_A \propto L \leftrightarrow \lambda \propto \frac{1}{L} \quad V_{DS} \uparrow V_A \uparrow$$

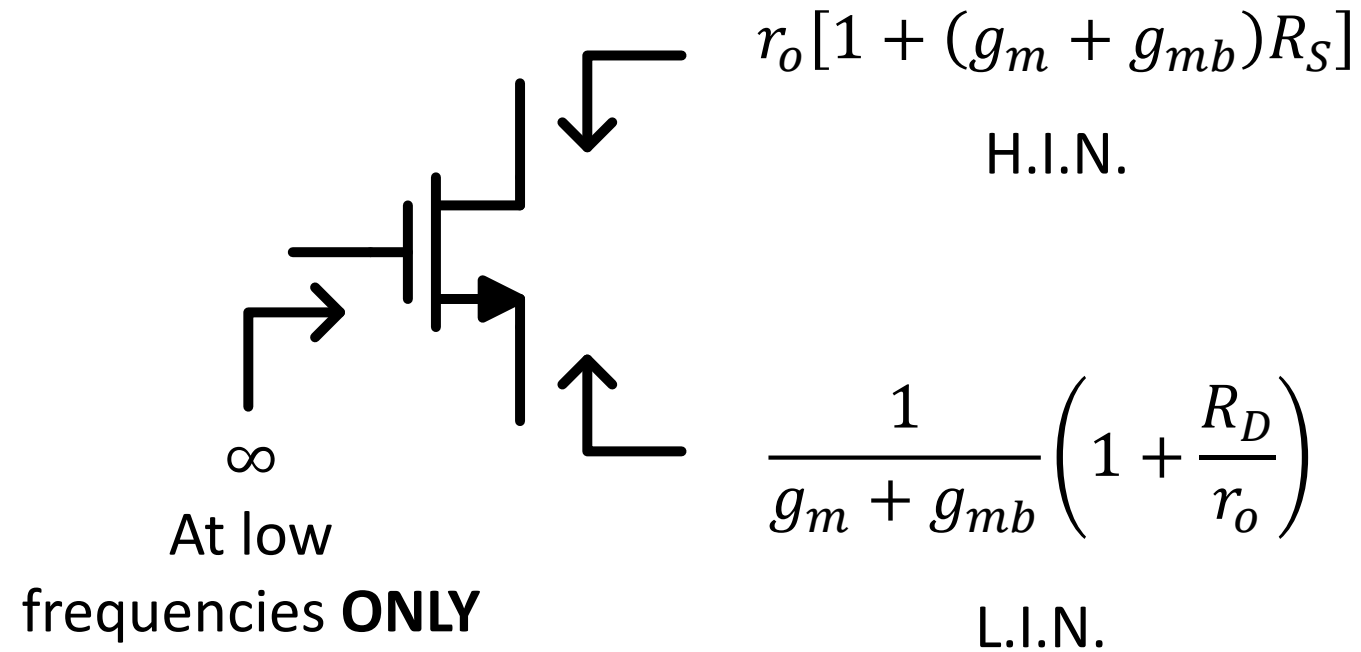
$$C_{gb} \approx 0$$

$$C_{gs} \gg C_{gd}$$

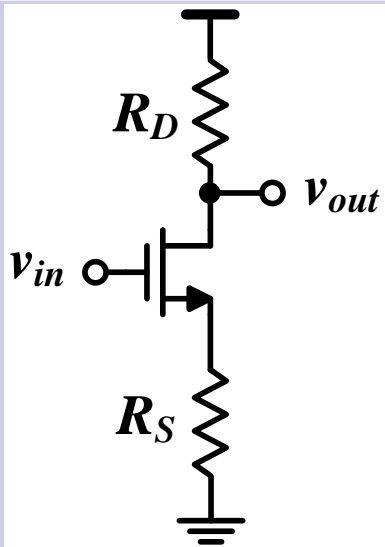
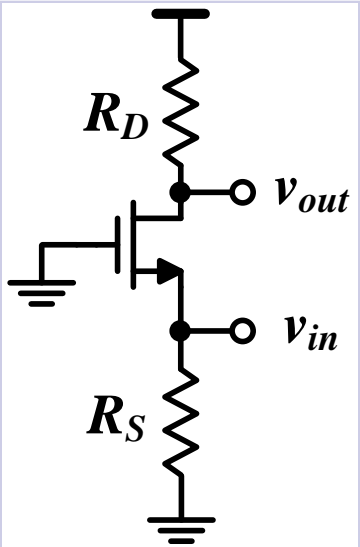
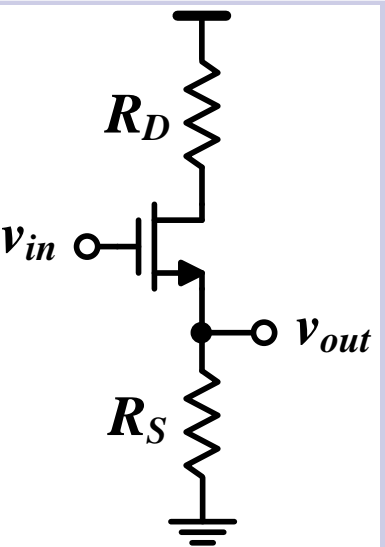
$$C_{sb} > C_{db}$$



Rin/out Shortcuts Summary



Summary of Basic Topologies

	CS	CG	CD (SF)
			
	Voltage & current amplifier	Voltage amplifier Current buffer	Voltage buffer Current amplifier
R_{in}	∞	$R_S \parallel \frac{1}{g_m + g_{mb}} \left(1 + \frac{R_D}{r_o} \right)$	∞
R_{out}	$R_D \parallel r_o [1 + (g_m + g_{mb})R_S]$	$R_D \parallel r_o$	$R_S \parallel \frac{1}{g_m + g_{mb}} \left(1 + \frac{R_D}{r_o} \right)$
G_m	$\frac{-g_m}{1 + (g_m + g_{mb})R_S}$	$g_m + g_{mb}$	$\frac{g_m}{1 + R_D/r_o}$

Differential Amplifier

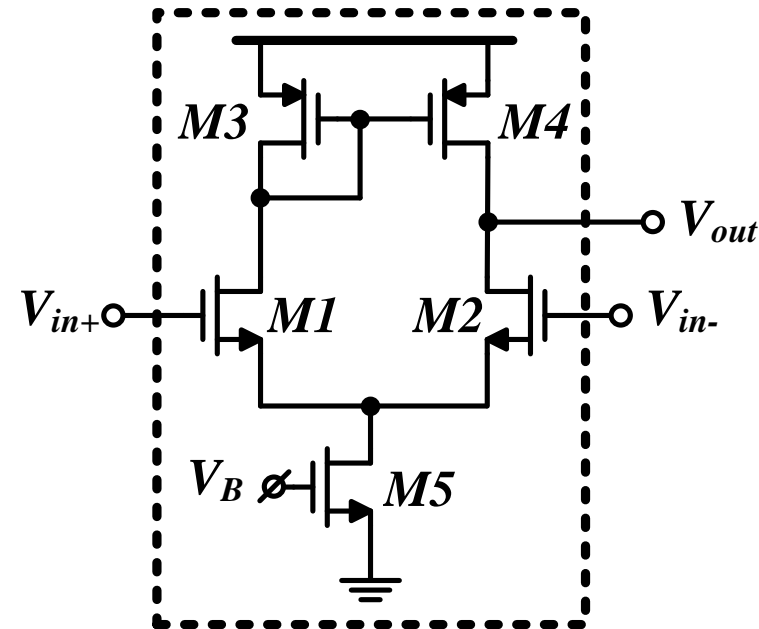
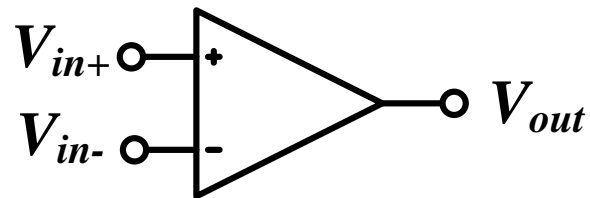
	Pseudo Diff Amp	Diff Pair (w/ ideal CS)	Diff Pair (w/ R_{SS})
A_{vd}	$-g_m R_D$	$-g_m R_D$	$-g_m R_D$
A_{vCM}	$-g_m R_D$	0	$\frac{-g_m R_D}{1 + 2(g_m + g_{mb})R_{SS}}$
A_{vd}/A_{vCM}	1	∞	$2(g_m + g_{mb})R_{SS} \gg 1$

$$A_{vCM2d} = \frac{v_{od}}{v_{iCM}} \approx \frac{\Delta R_D}{2R_{SS}} + \frac{\Delta g_m R_D}{2g_{m1,2}R_{SS}}$$

$$CMRR = \frac{A_{vd}}{A_{vCM2d}}$$

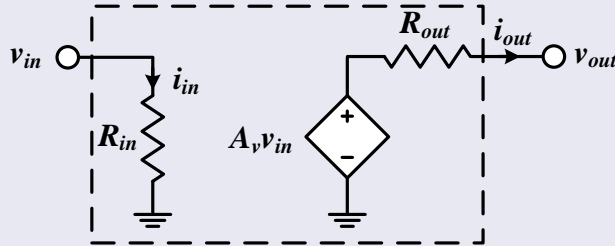
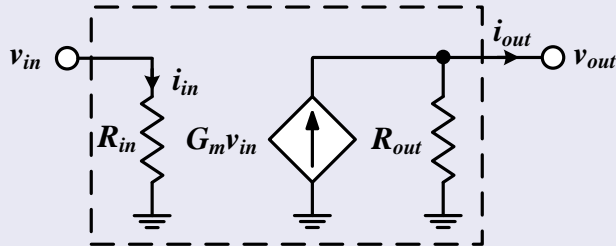
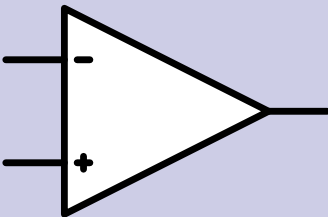
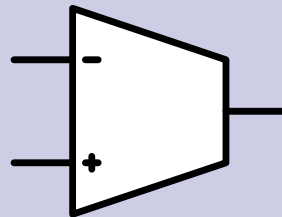
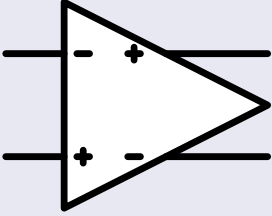
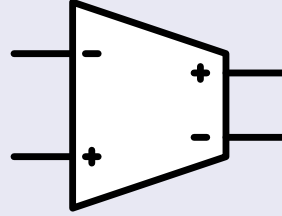
Op-Amp

- ❑ An op-amp is simply a high gain differential amplifier
 - The gain can be increased by using cascodes and multi-stage amplification
- ❑ The diff amp is a key block in many analog and RF circuits
 - DEEP understanding of diff amp is ESSENTIAL



Op-Amp vs OTA

- ❑ In short, an OTA is an op-amp without an output stage (buffer)
- ❑ Some designers just use op-amp name and symbol for both

	Op-amp	OTA
Rout	LOW	HIGH
Model		
Diff input, SE output		
Fully diff		

V-star (V^*)

- V-star (V^*) is inspired by V_{ov} but calculated from actual simulation data

$$g_m = \frac{2I_D}{V^*} \leftrightarrow V^* = \frac{2I_D}{g_m} = \frac{2}{g_m/I_D}$$

- Figures-of-merit in terms of V^*

$$g_m r_o = \frac{2I_D}{V^*} \cdot \frac{1}{\lambda I_D} = \frac{2}{\lambda V^*}$$

$$f_T = \frac{g_m}{2\pi C_{gg}} = \frac{1}{2\pi} \cdot \frac{2I_D}{V^*} \cdot \frac{1}{C_{gg}}$$

$$\frac{g_m}{I_D} = \frac{2}{V^*}$$

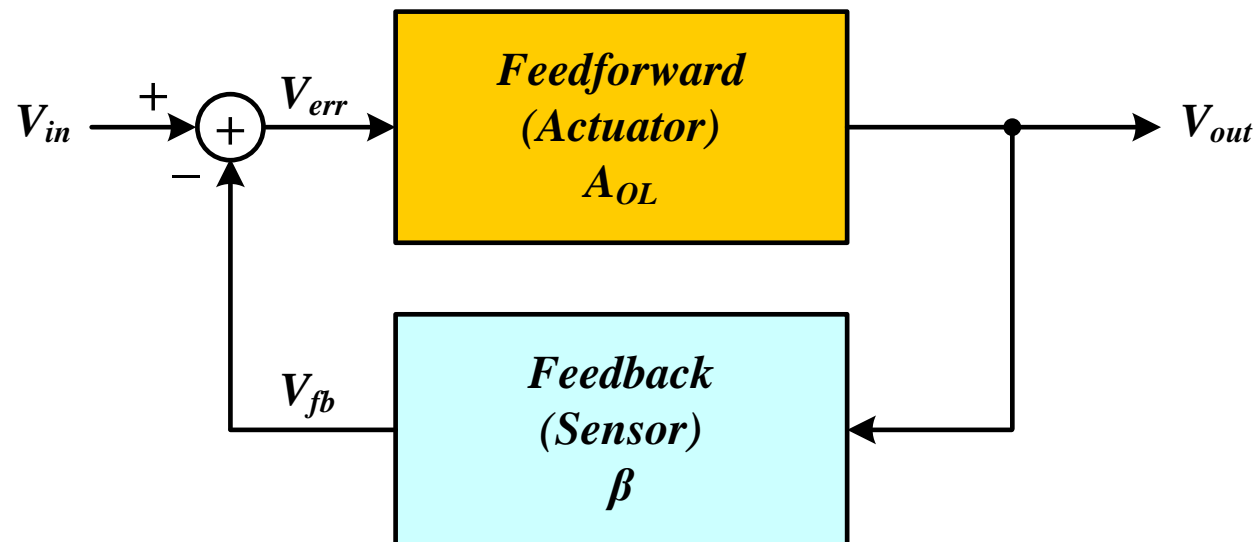
- The boundary between weak and strong inversion ($n = 1.2 \rightarrow 1.5$)

$$V_{ov}(SI) = V^*(WI) = 2nV_T \approx 60 \rightarrow 80mV$$

Outline

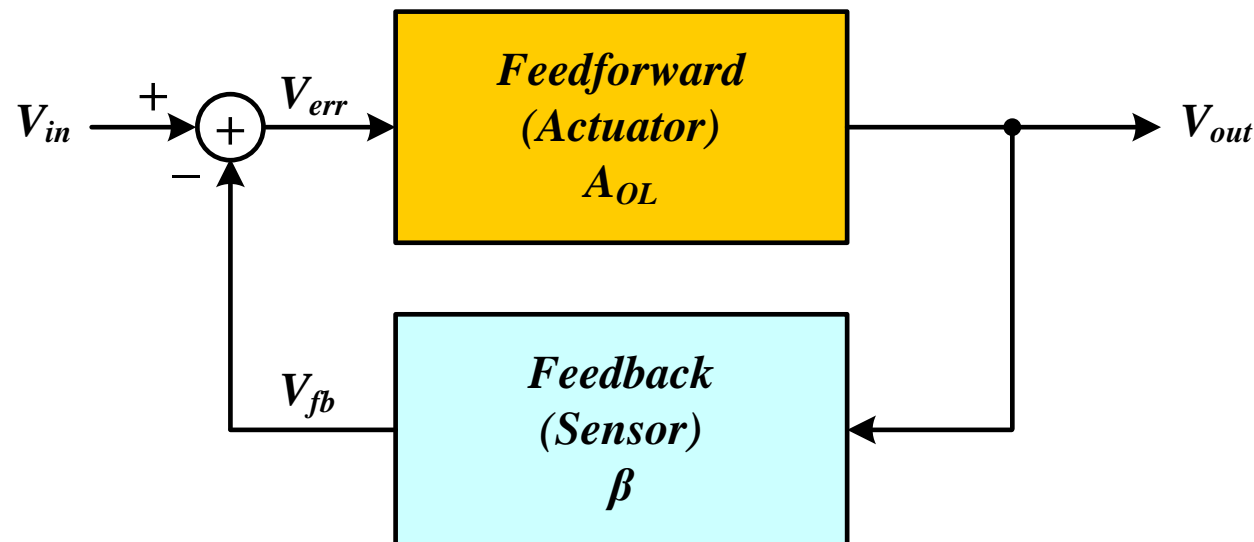
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- ☒ General feedback system
- ☐ Loop gain
- ☐ Why negative feedback?
- ☐ Stability of feedback system
- ☐ Root locus and Bode plot
- ☐ Phase and gain margin

General Feedback System



General Feedback System

- ❑ Error signal = $V_{err} = V_{in} - V_{fb}$
- ❑ Open loop (OL) gain = $A_{OL} = \frac{V_{out}}{V_{err}} \gg 1$
- ❑ Feedback factor = $\beta = \frac{V_{fb}}{V_{out}}$
- ❑ Closed loop (CL) gain = $A_{CL} = \frac{V_{out}}{V_{in}}$



Closed-loop Gain

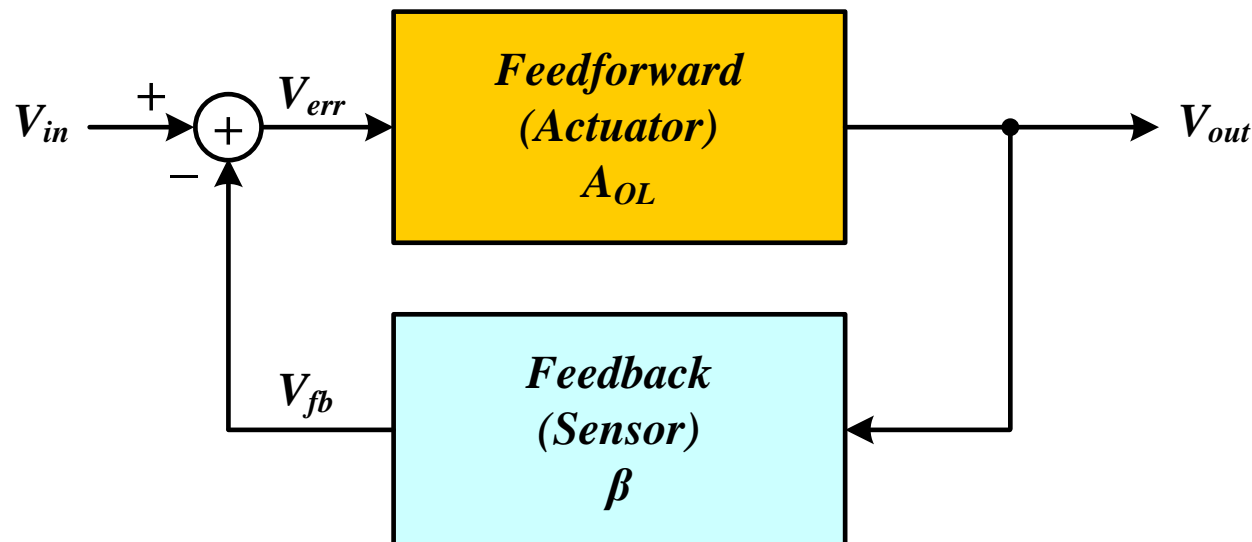
$$V_{out} = A_{OL}(V_{in} - V_{fb}) = A_{OL}(V_{in} - \beta V_{out})$$

$$A_{CL} = \frac{V_{out}}{V_{in}} = \frac{A_{OL}}{1 + \beta A_{OL}} = \frac{A_{OL}}{1 + LG}$$

□ Loop gain = $LG = \beta A_{OL} \gg 1$

$$A_{CL} \approx \frac{1}{\beta}$$

□ Closed-loop gain is independent of open-loop gain!



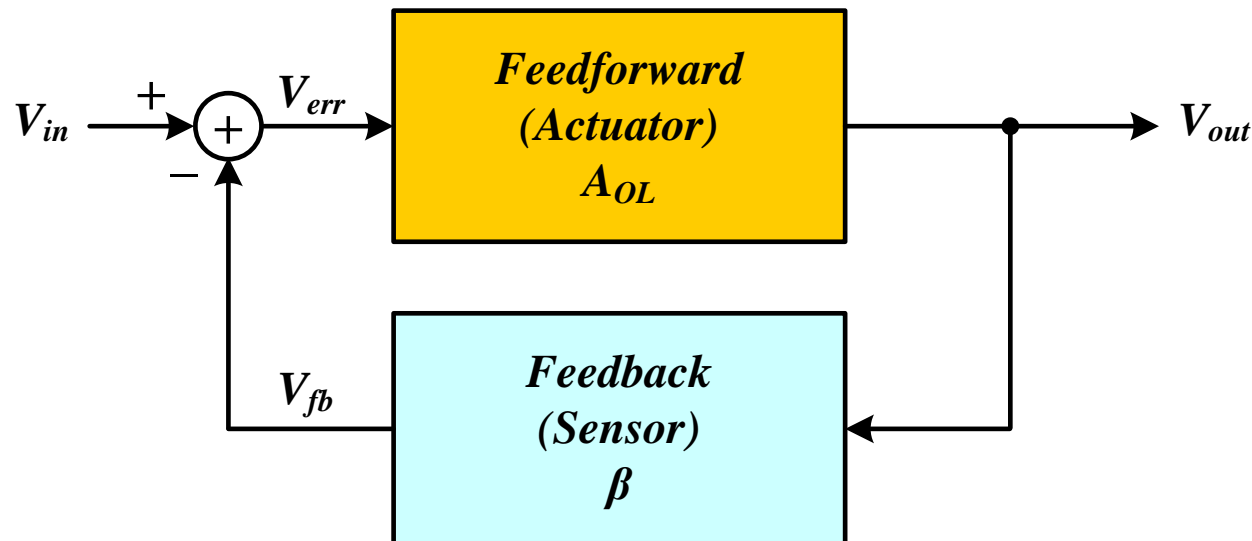
Error Signal

$$V_{err} = V_{in} - V_{fb} = V_{in} - \beta V_{out} = V_{in} - \beta A_{OL} V_{err}$$
$$V_{err} = \frac{V_{in}}{1 + \beta A_{OL}} = \frac{V_{in}}{1 + LG}$$

□ Loop gain = $LG = \beta A_{OL} \gg 1$

$$V_{err} = \frac{V_{in}}{1 + LG} \rightarrow 0$$

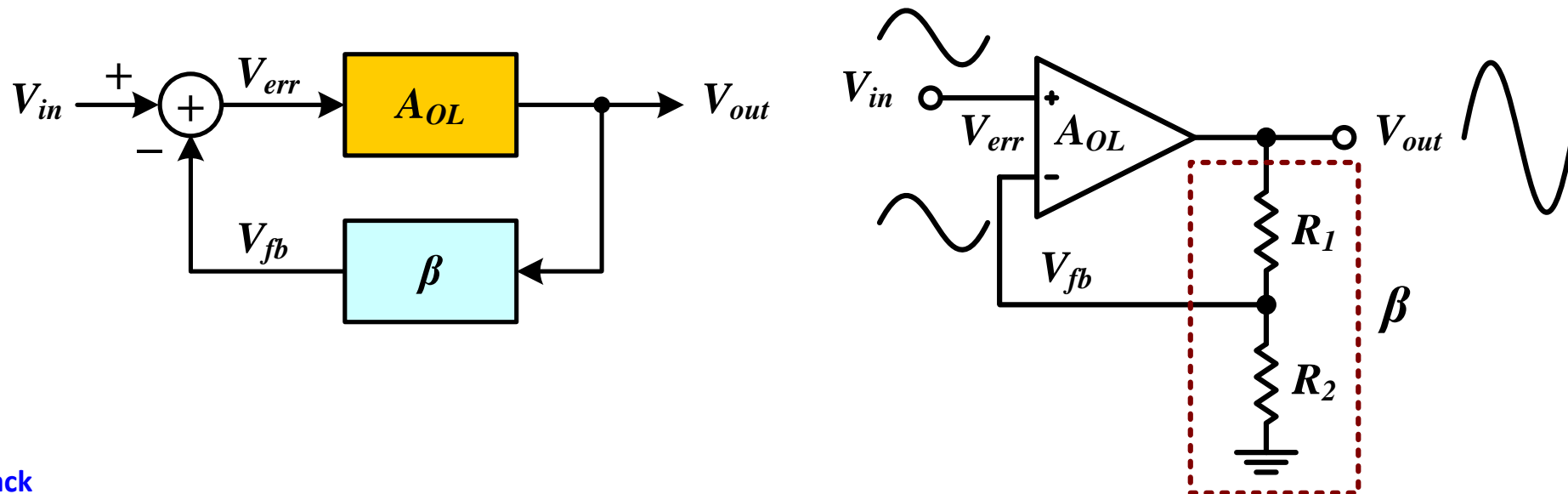
□ Negative feedback loop works to minimize the error signal



Feedback Example

- ❑ Op-amp functions: (1) subtraction and (2) amplification
- ❑ The network R_1 and R_2 functions: (1) sensing the output voltage and (2) providing a feedback factor $\beta = \frac{R_2}{(R_1 + R_2)}$

$$A_{CL} = \frac{V_{out}}{V_{in}} = \frac{A_{OL}}{1 + \beta A_{OL}} = \frac{A_{OL}}{1 + \frac{R_2}{(R_1 + R_2)} \cdot A_{OL}} \approx \frac{R_1 + R_2}{R_2} = 1 + \frac{R_1}{R_2}$$

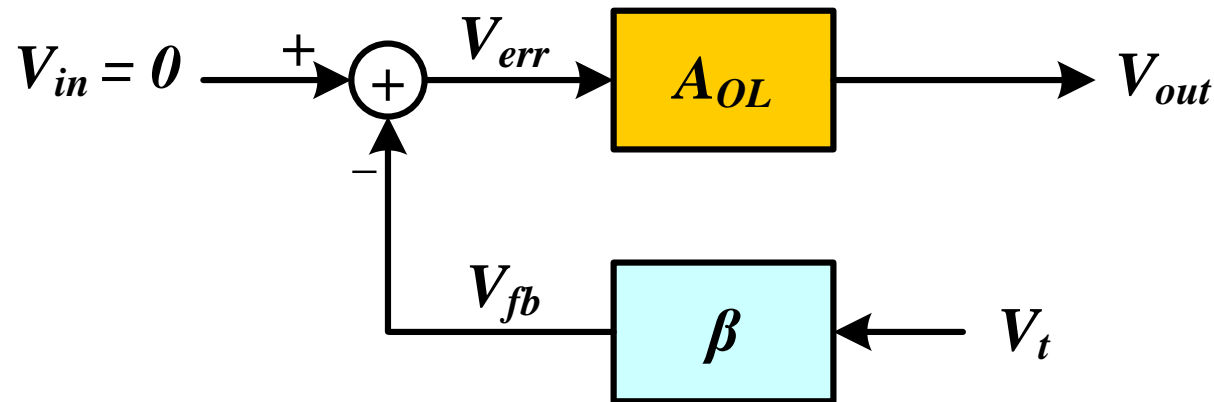


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- ☐ Recapping previous key results
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- ☐ **Loop gain**
- ☐ Why negative feedback?
- ☐ Stability of feedback system
- ☐ Root locus and Bode plot
- ☐ Phase and gain margin

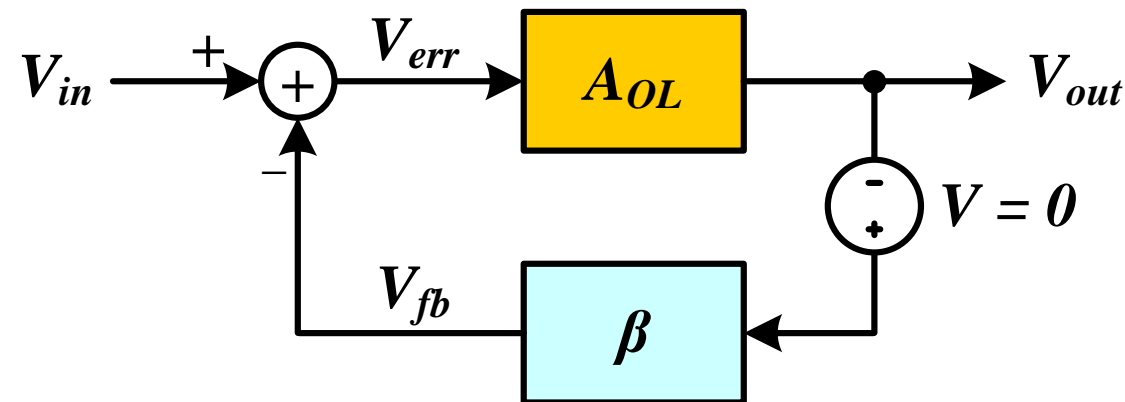
Loop Gain

- ❑ Deactivate the input → Break the loop → Apply a test source → Calculate the gain around the loop
- ❑ Loop gain = $LG = -\frac{V_{out}}{V_t} = \beta A_{OL}$
- ❑ A.k.a. loop transmission, return ratio ...
- ❑ But biasing/loading changes when we break the loop!
 - Make sure dc biasing is properly set
 - Add a dummy load



Loop Gain

- ❑ Modern circuit simulators can compute the loop gain without explicitly breaking the loop
 - Use stability (STB) analysis
 - Insert a 0V dc voltage source or iprobe in the loop
 - Polarity matters for Eldo, but not for Spectre
 - Loop gain = $LG = \beta A_{OL}$ is calculated by the simulator



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Why Negative Feedback?

- ❑ We use a very high gain amplifier (A_{OL}), but end up with a much smaller gain A_{CL}
$$= \frac{A_{OL}}{1 + \beta A_{OL}} \approx \frac{1}{\beta}$$
- ❑ We can design high gain amplifiers, but we really do not need all that gain
- ❑ High gain is the balance that we use to buy other useful properties
- ❑ Negative feedback useful properties
 1. **Gain Desensitization → Stable, linear, and accurate gain**
 2. Bandwidth Extension
 3. Modification of I/O Impedances

Gain Desensitization

$$A_{CL} = \frac{A_{OL}}{1 + \beta A_{OL}} = \frac{1}{\beta} \left(\frac{1}{\frac{1}{\beta A_{OL}} + 1} \right) \approx \frac{1}{\beta} \left(1 - \frac{1}{\beta A_{OL}} \right) = \frac{1}{\beta} \left(1 - \frac{1}{LG} \right)$$

- ❑ Static gain error

$$\epsilon_s = \frac{|A_{CL,ideal} - A_{CL,actual}|}{A_{CL,ideal}} \approx \frac{1}{\beta A_{OL}} = \frac{1}{LG}$$

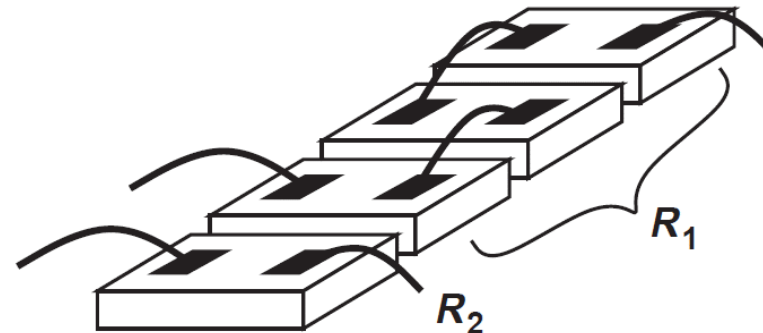
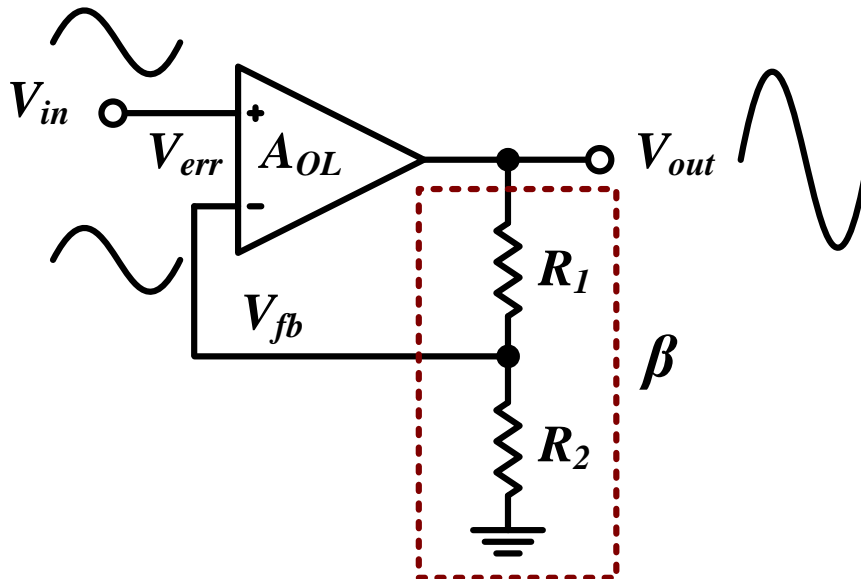
- ❑ A_{OL} varies due to PVT, load, and input signal variations
- ❑ A_{CL} almost independent of A_{OL} (if $LG \gg 1$)
 - Independent of PVT: stable and robust
 - Independent of load: stable and robust
 - Independent of input level: linear

Gain Desensitization

- ❑ In IC design, we cannot control absolute values due to PVT, load, and input signal variations
- ❑ But we can precisely control ratios of MATCHED components

$$A_{CL} = \frac{Y}{X} = \frac{A_{OL}}{1 + \beta A_{OL}} = \frac{A_{OL}}{1 + \frac{R_2}{(R_1 + R_2)} \cdot A_{OL}} \approx \frac{R_1 + R_2}{R_2} = 1 + \frac{R_1}{R_2}$$

- ❑ $R_1 = 3R$ and $R_2 = R \Rightarrow A_{CL} = 4 \Rightarrow$ Stable, linear, and **accurate**



Bandwidth Extension

- Assume the op-amp (OL) is a first order system

$$A_{OL}(s) = \frac{A_{OLo}}{1 + \frac{s}{\omega_{p,OL}}}$$

- The CL transfer function is also a first order system

$$A_{CL}(s) = \frac{A_{OL}(s)}{1 + \beta A_{OL}(s)} = \frac{\frac{A_{OLo}}{(1 + \beta A_{OLo})}}{1 + \frac{s}{(1 + \beta A_{OLo})\omega_{p,OL}}} = \frac{A_{CLo}}{1 + \frac{s}{\omega_{p,CL}}}$$

- But the pole is at a much higher frequency

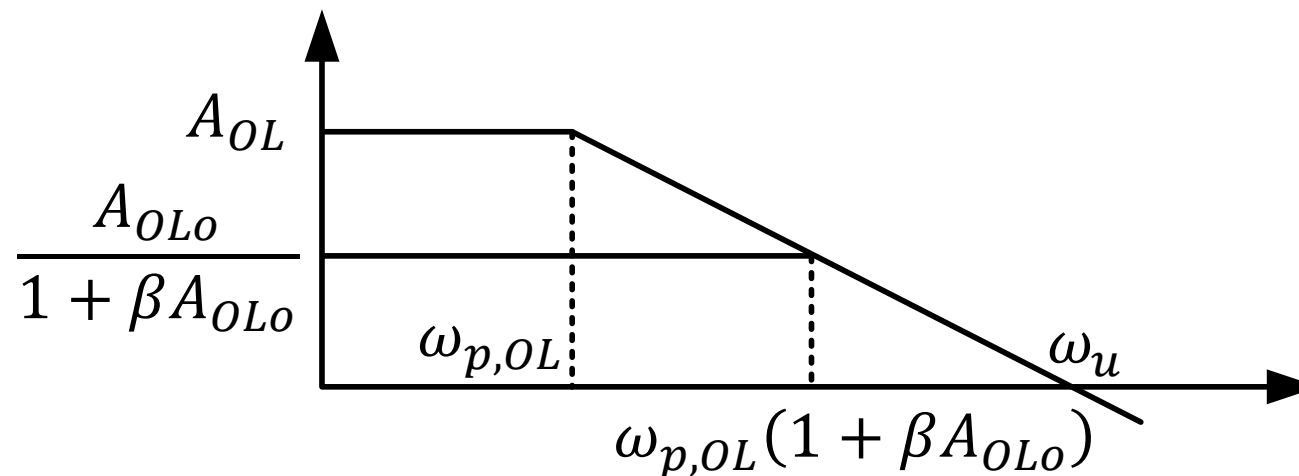
$$\omega_{p,CL} = (1 + LG_o)\omega_{p,OL}$$

- CL DC gain reduced by $(1 + LG_o)$
- CL bandwidth extended by $(1 + LG_o)$
- GBW (and UGF) remains constant

Bandwidth Extension

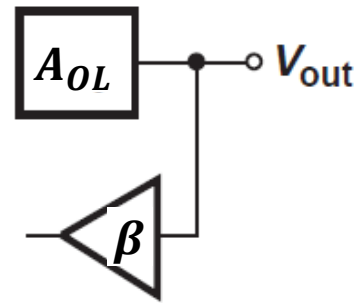
- ❑ CL DC gain reduced by $(1 + LG_o)$
- ❑ CL bandwidth extended by $(1 + LG_o)$
- ❑ GBW (and UGF) remains constant

$$A_{CL}(s) = \frac{\frac{A_{OLo}}{(1 + LG_o)}}{1 + \frac{s}{(1 + LG_o)\omega_{p,OL}}} = \frac{A_{CLo}}{1 + \frac{s}{\omega_{p,CL}}}$$

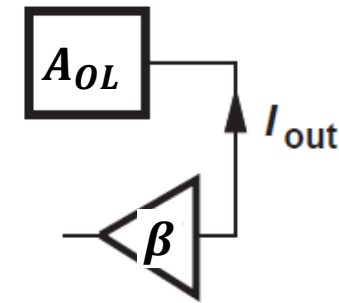


Modification of I/O Impedances

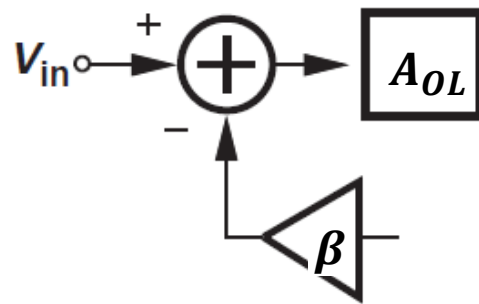
- ❑ Shunt sensing/mixing \rightarrow R decreases
- ❑ Series sensing/mixing \rightarrow R increases



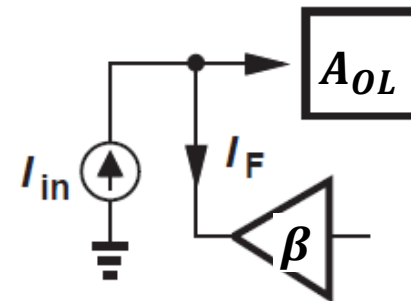
Output impedance falls by $1 + \text{loop gain}$.



Output impedance rises by $1 + \text{loop gain}$.



Input impedance rises by $1 + \text{loop gain}$.



Input impedance falls by $1 + \text{loop gain}$.

The Price We Pay

- ❑ Negative feedback useful properties
 1. Gain Desensitization → Stable, linear, and accurate gain
 2. Bandwidth Extension
 3. Modification of I/O Impedances
- ❑ The price we pay to buy these useful properties
 1. Gain reduction
 2. The risk of instability

Outline

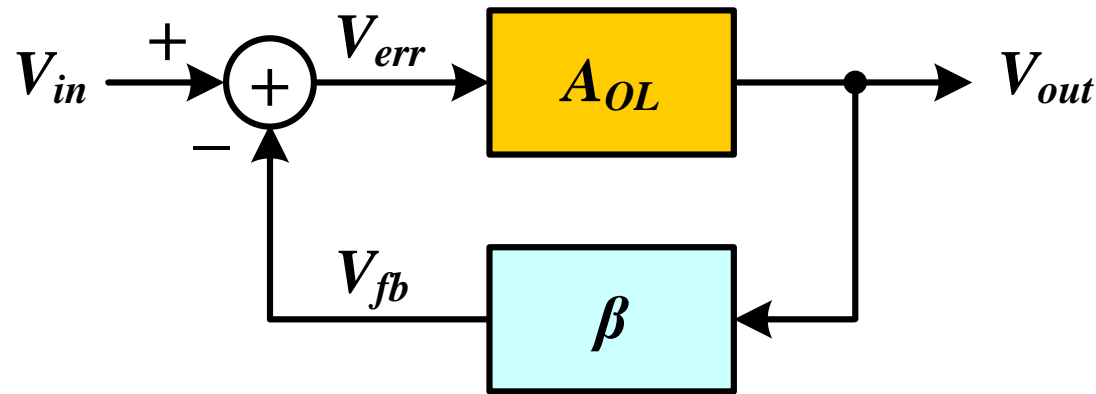
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Stability of Feedback System

$$A_{CL}(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{A_{OL}(s)}{1 + \beta A_{OL}(s)}$$

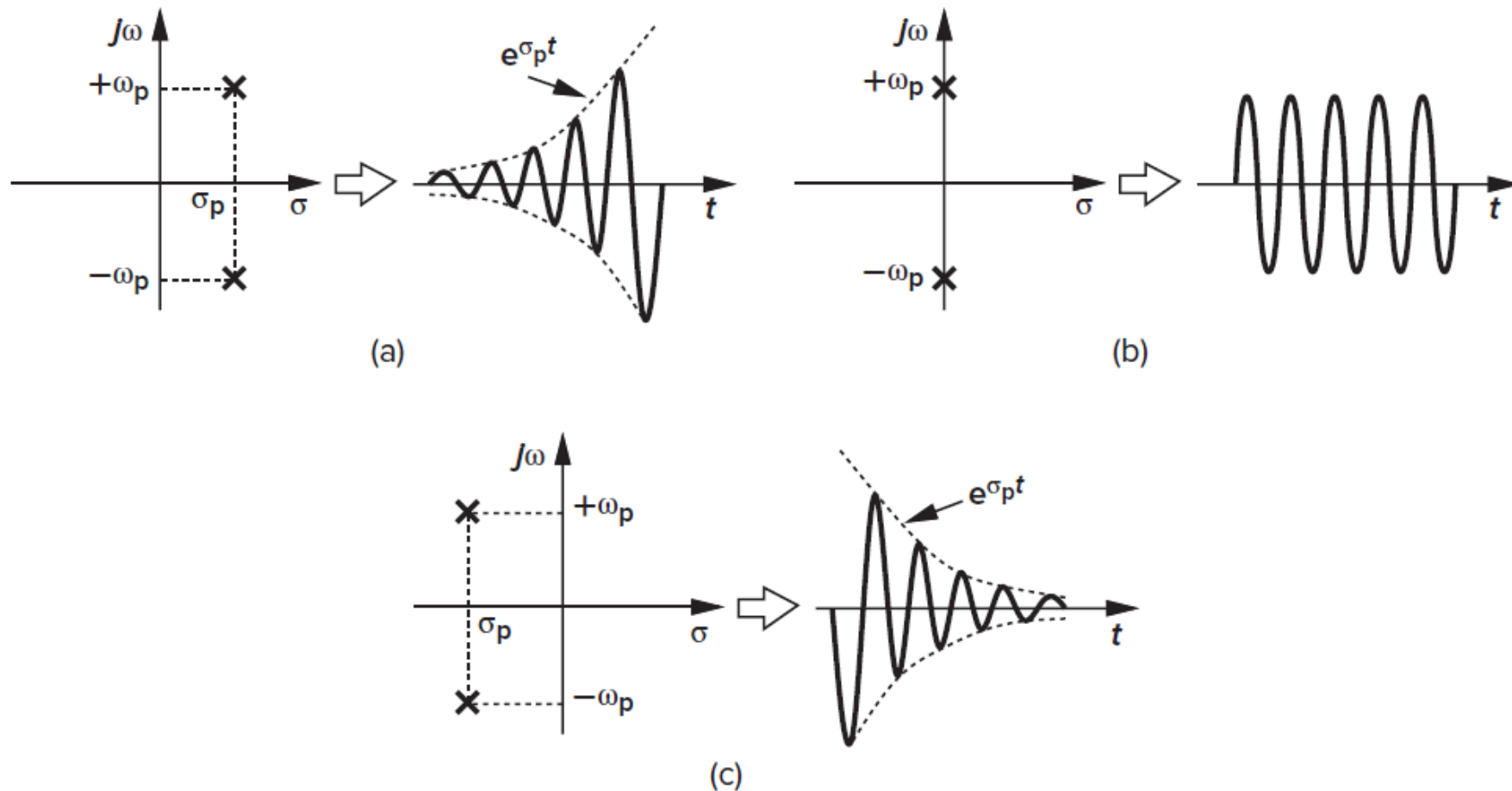
□ Barkhausen's Oscillation Criteria

$$|\beta A_{OL}(s)| = 1$$
$$\angle \beta A_{OL}(s) = -180$$



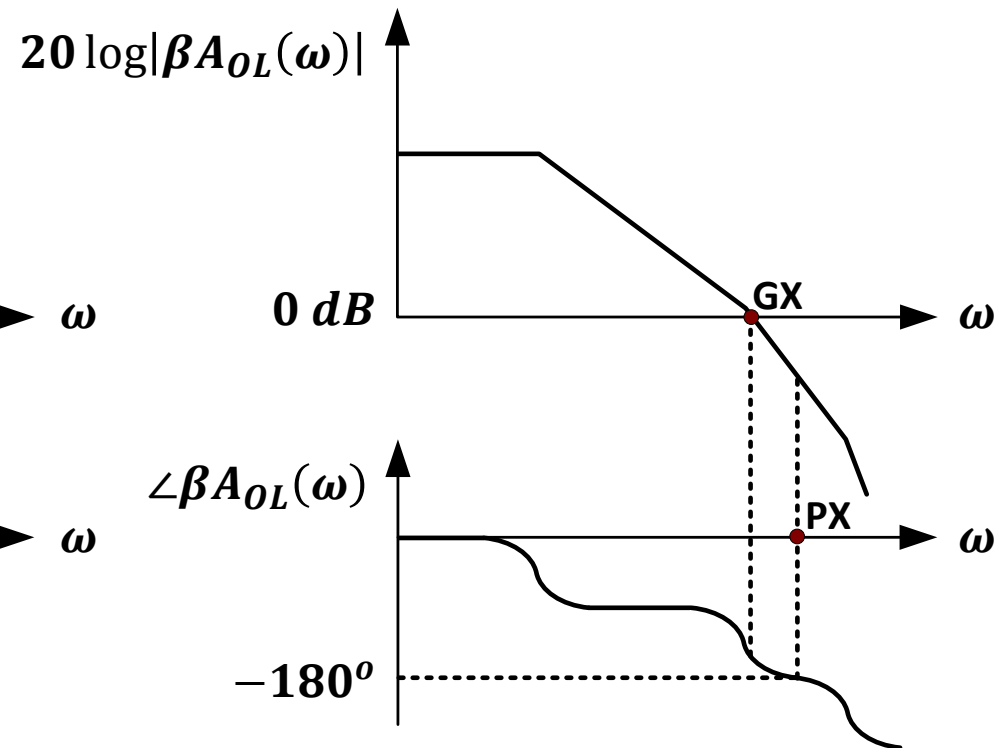
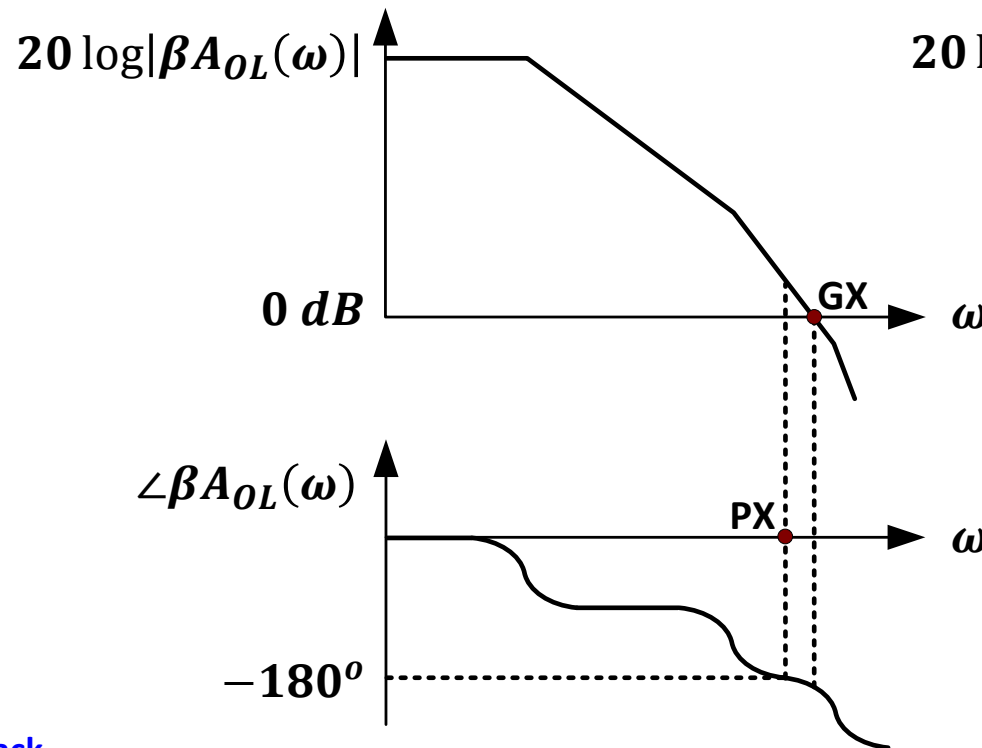
Stable vs Unstable System: Pole-Zero Plot

Laplace domain	Time domain
$\frac{1}{s - a}$	e^{at}



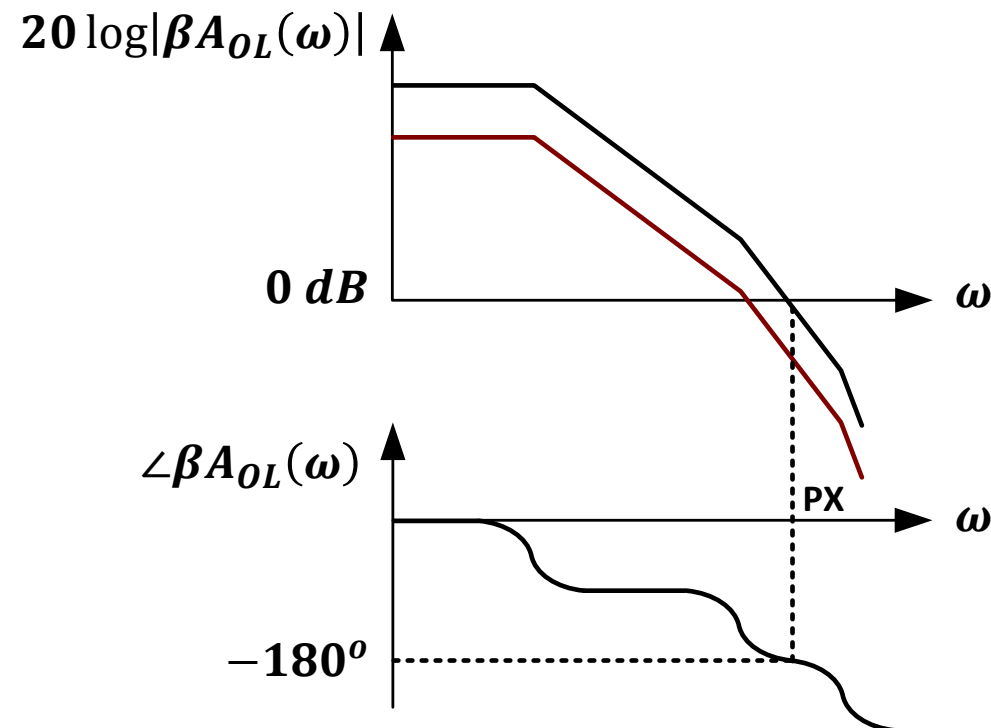
Stable vs Unstable System: Bode Plot

- Gain crossover frequency (GX): @ $|\beta A_{OL}(s)| = 1$
 - Same as ω_u
- Phase crossover frequency (PX): @ $\angle \beta A_{OL}(s) = -180^\circ$
- For a stable system: $GX < PX$



Effect of Feedback Factor (β)

- We assume β is independent of frequency
 - $\angle\beta A_{OL}$ is independent of $\beta \rightarrow$ PX is independent of β
- Increasing β shifts mag up \rightarrow GX increases \rightarrow bad for stability
- Worst-case stability corresponds to $\beta = 1 \rightarrow \beta A_{OL} = A_{OL}$
 - Unity-gain feedback \rightarrow buffer \rightarrow smallest CL gain

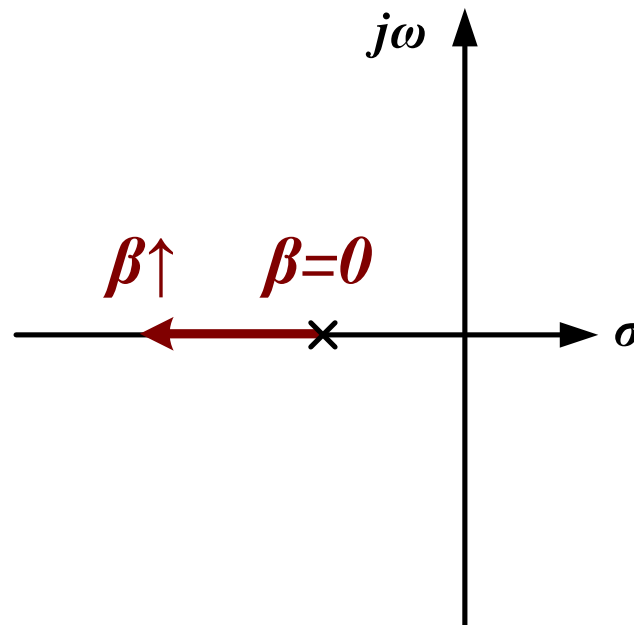


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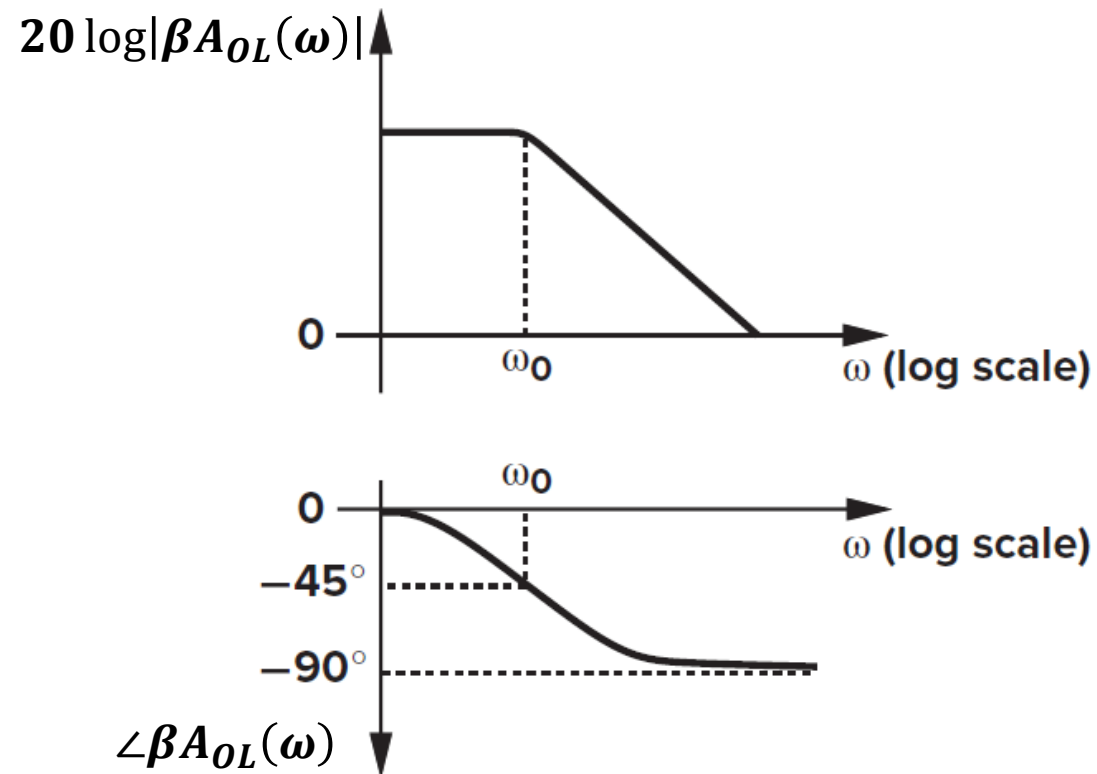
Single-Pole System: Root Locus

- Note that β does not affect poles of LG (assuming β is real!)
 - But it affects poles of A_{CL} : Roots of the characteristic equation $(1 + \beta A_{OL})$
- The locus exists on real axis to the left of an odd number of poles and zeros.
- The locus starts at the open-loop poles and ends at the open-loop zeros or at infinity.
- For first-order system, pole always in LHP: Unconditionally stable



Single-Pole System: Bode Plot

$$A_{CL}(s) = \frac{A_{OL}(s)}{1 + \beta A_{OL}(s)} = \frac{\frac{A_{OLO}}{(1 + \beta A_{OLO})}}{1 + \frac{s}{(1 + \beta A_{OLO})\omega_{p,OL}}} = \frac{A_{CLO}}{1 + \frac{s}{\omega_{p,CL}}}$$

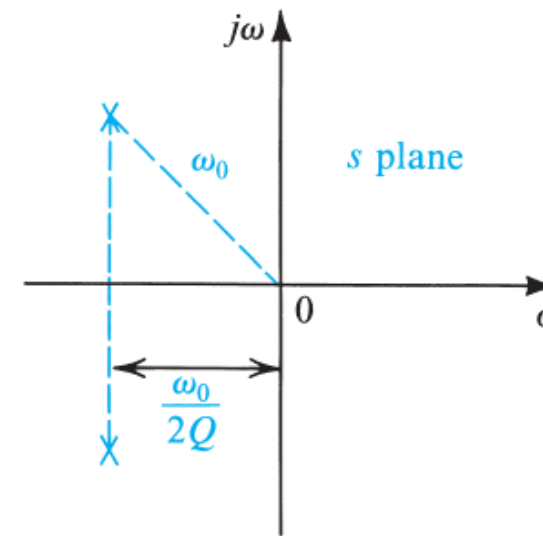
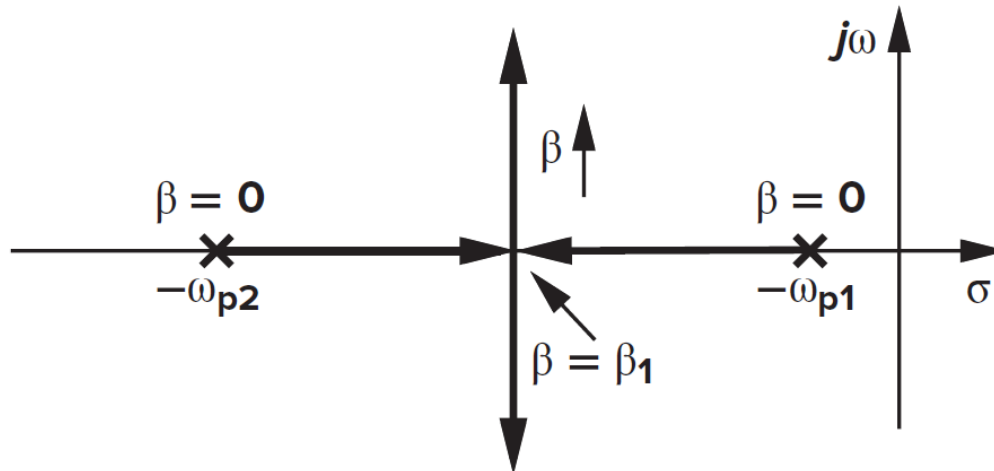


Two-Pole System: Root Locus

□ Poles always in LHP: Unconditionally stable

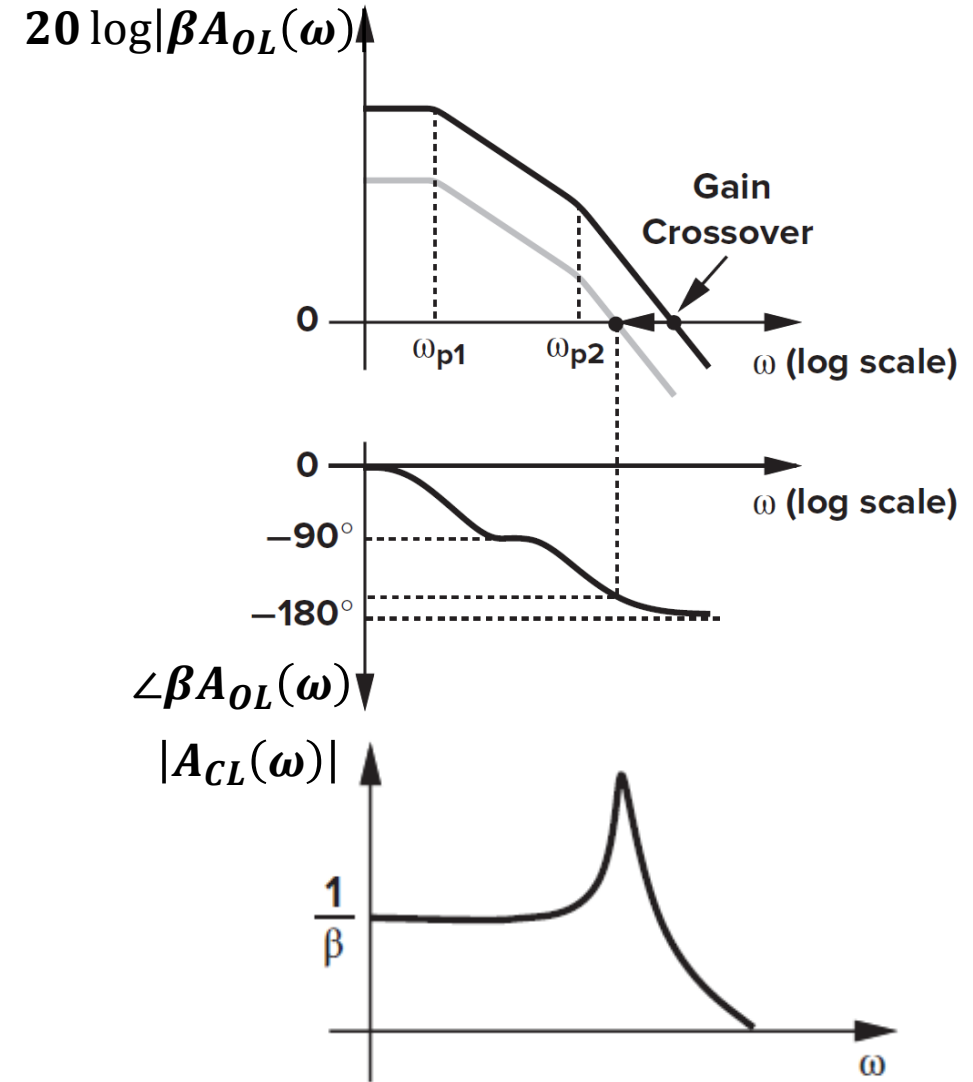
$$A_{CL}(s) = \frac{A_{CLo}}{1 + \frac{1}{Q} \frac{s}{\omega_o} + \frac{s^2}{\omega_o^2}} = \frac{A_{CLo}}{1 + 2\zeta \frac{s}{\omega_o} + \frac{s^2}{\omega_o^2}}$$

- $\zeta > 1$ ($Q < 0.5$): Overdamped (real and distinct CL poles)
- $\zeta = 1$ ($Q = 0.5$): Critical damped (real and equal CL poles)
- $\zeta < 1$ ($Q > 0.5$): Underdamped (complex conjugate CL poles)



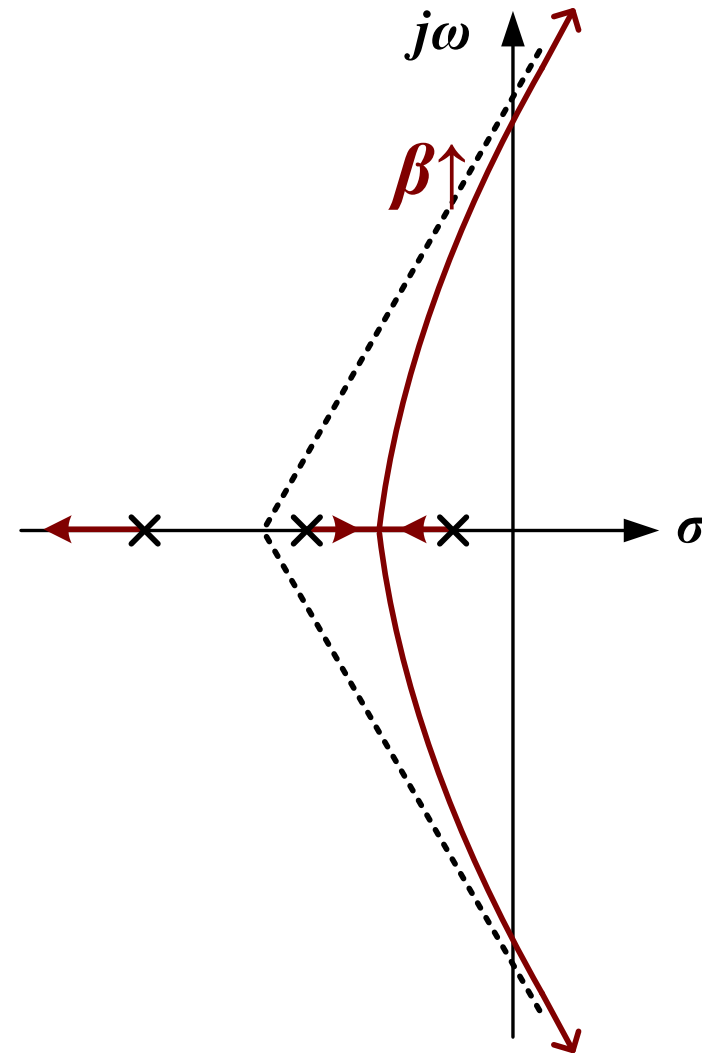
Two-Pole System: Bode Plot

- ❑ Phase shift always $< 180^\circ$
 - Unconditionally stable
- ❑ $\zeta > 1$ ($Q < 0.5$): Overdamped
- ❑ $\zeta = 1$ ($Q = 0.5$): Critical damped
- ❑ $\zeta < 1$ ($Q > 0.5$): Underdamped
 - Overshoot in step response
- ❑ $\zeta < 1/\sqrt{2} = 0.707$
 - $Q > 1/\sqrt{2} = 0.707$
 - Peaking in frequency response
 - Ringing in step response



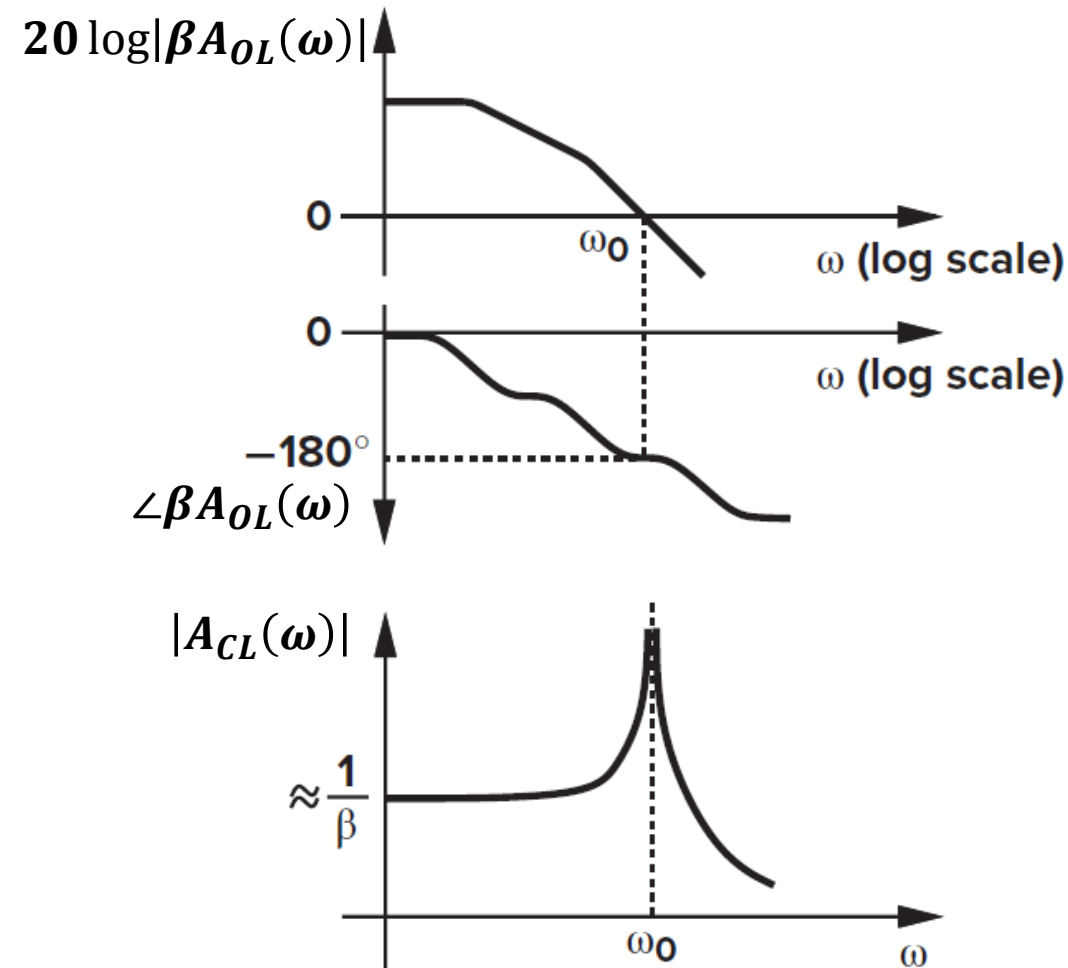
Three-Pole System: Root Locus

- ❑ Poles cross $j\omega$ axis at a specific value of $\beta = \beta_{crit}$
 - The system becomes unstable
- ❑ Pole are NOT always in LHP
 - Conditionally stable: $\beta < \beta_{crit}$



Three-Pole System: Bode Plot

- ❑ Instability (oscillation) condition can be satisfied



Stability Summary

- ❑ First-order system
 - Unconditionally stable
- ❑ Third order system
 - Conditionally stable: Set $\beta < \beta_{crit}$
- ❑ Second-order system
 - Unconditionally stable
 - But may suffer from CL peaking/ringing if close to oscillation condition
 - How much margin is needed?

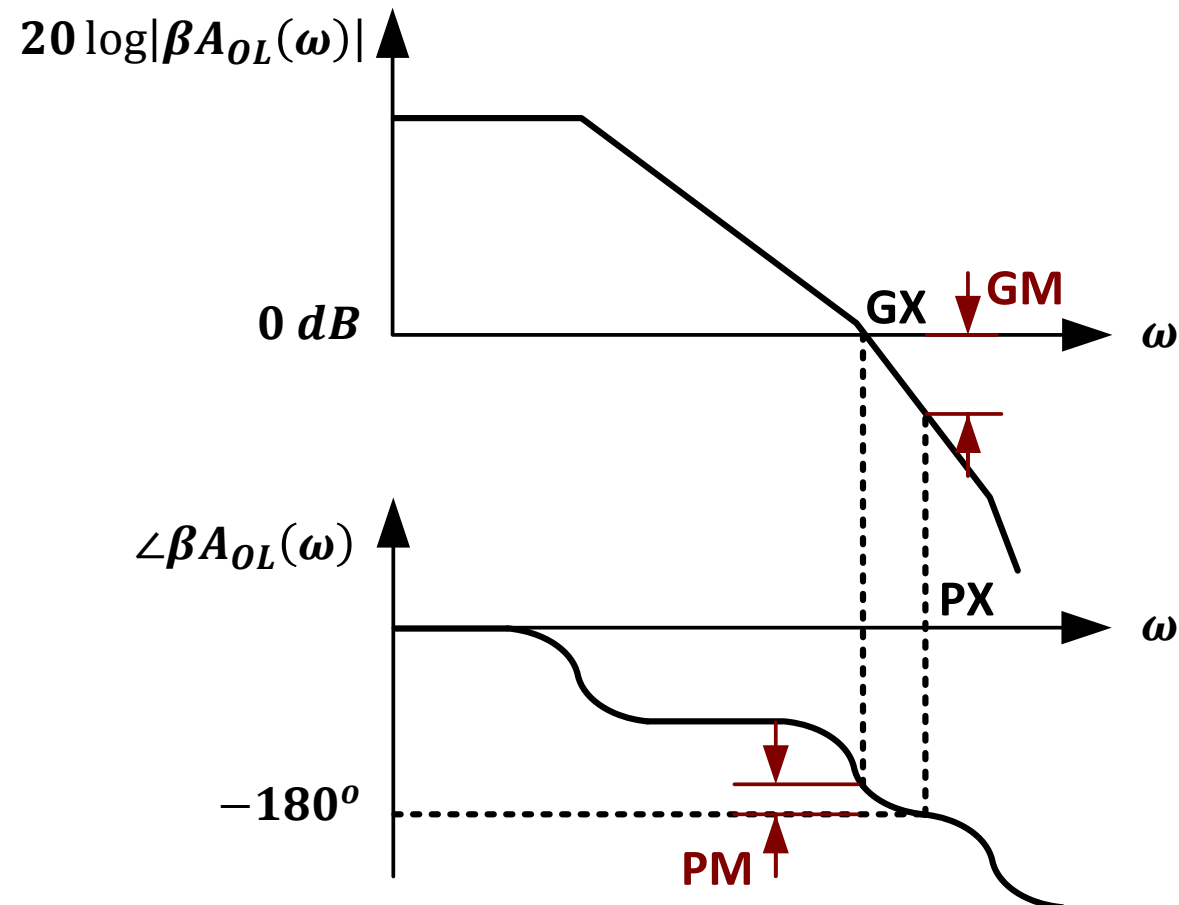
Outline

- ☐ Recapping previous key results
- ☐ General feedback system
- ☐ Loop gain
- ☐ Why negative feedback?
- ☐ Stability of feedback system
- ☐ Root locus and Bode plot
- ☐ Phase and gain margin

Phase and Gain Margin

$$PM = 180^\circ - |\angle \beta A_{OL}(GX)| = 180^\circ - \tan^{-1} \left(\frac{GX}{\omega_{p1}} \right) - \tan^{-1} \left(\frac{GX}{\omega_{p2}} \right)$$

$$GM = 0 - 20 \log |\beta A_{OL}(PX)|$$



Phase Margin (PM)

$$PM = 90^\circ - \tan^{-1} \left(\frac{GX}{\omega_{p2}} \right)$$

PM > 0 → stable

But low PM means:

→ frequency domain peaking

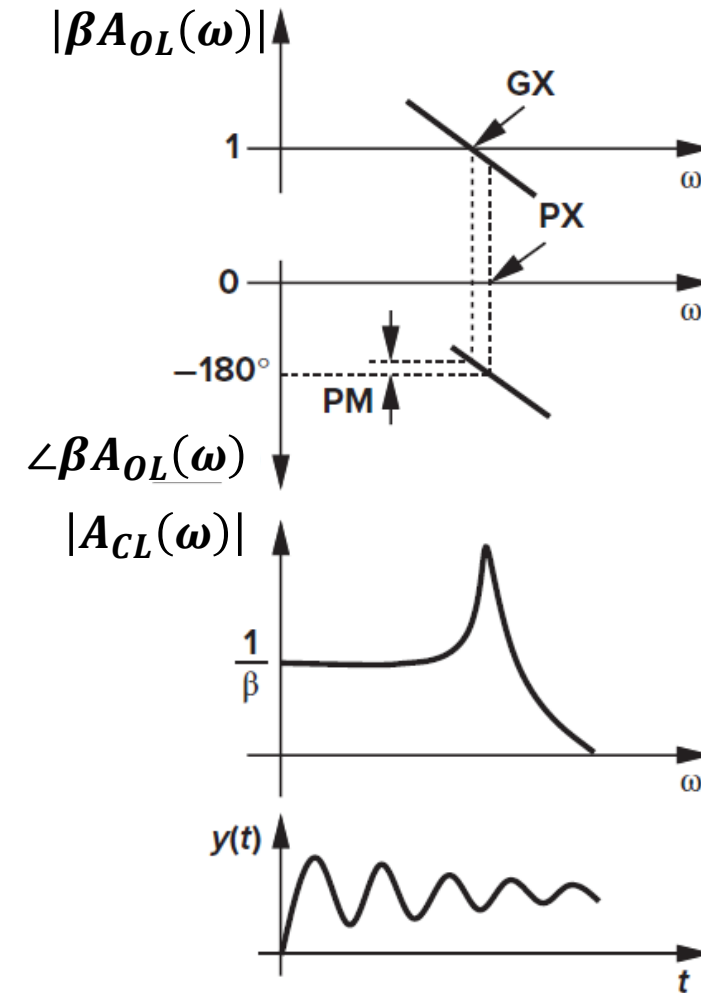
→ time domain ringing

Frequency domain peaking

→ noise amplification

Time domain ringing

→ poor settling time



Phase Margin (PM)

$$PM = 90^\circ - \tan^{-1} \left(\frac{GX}{\omega_{p2}} \right)$$

$PM > 0 \rightarrow$ stable

But low PM means:

\rightarrow frequency domain peaking

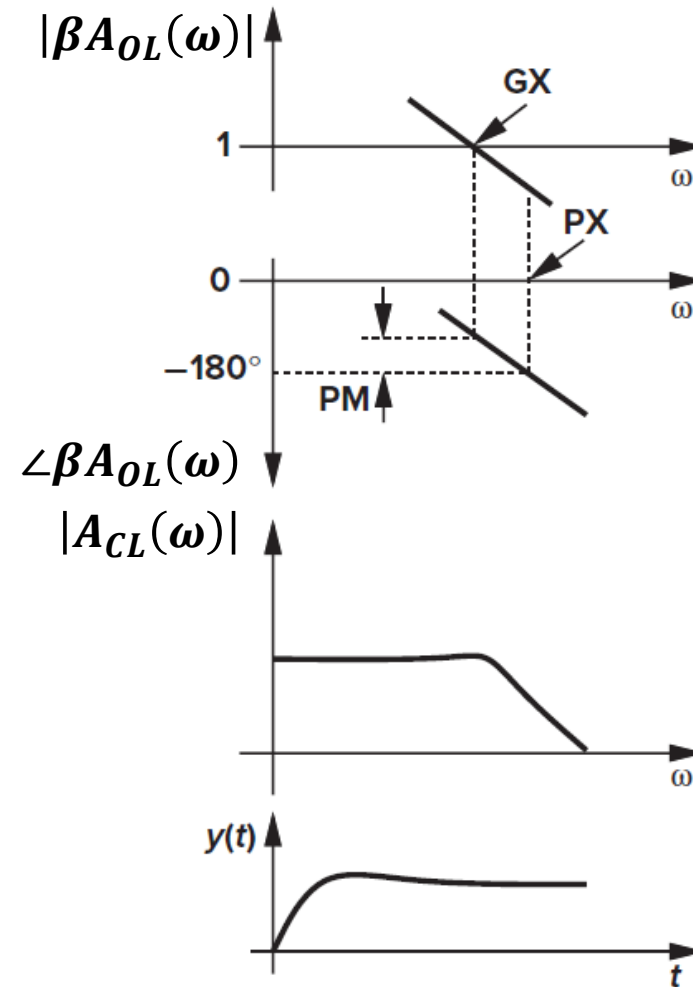
\rightarrow time domain ringing

Frequency domain peaking

\rightarrow noise amplification

Time domain ringing

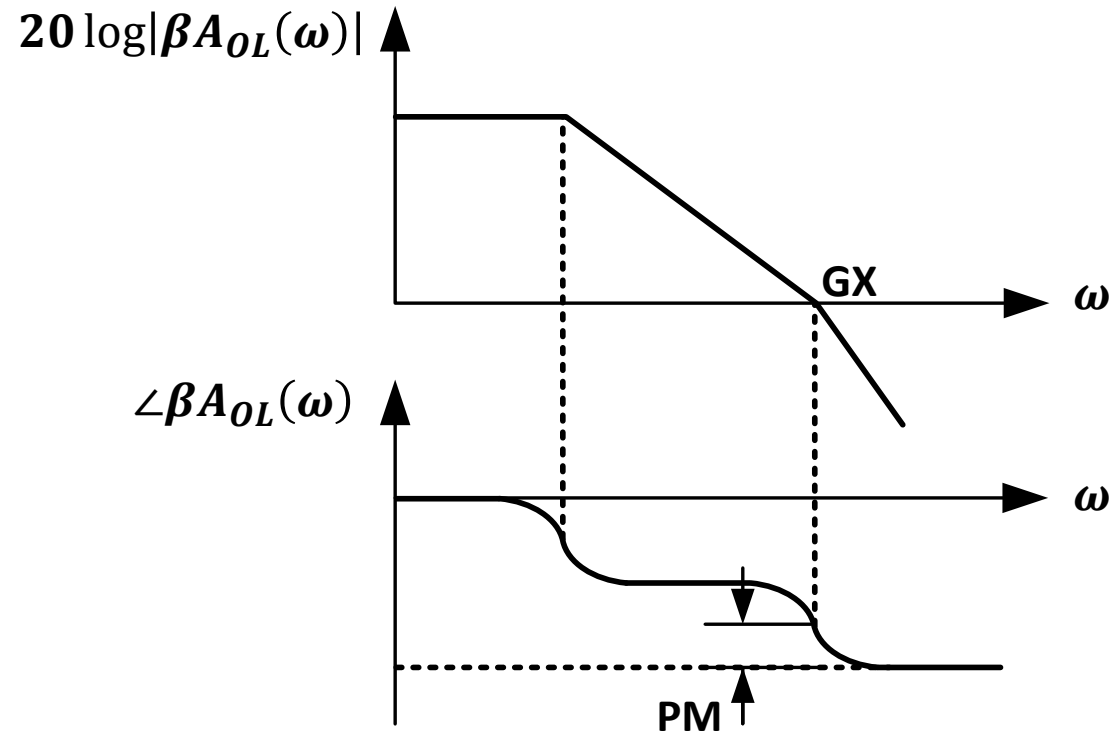
\rightarrow poor settling time



Phase Margin: Ultimate GBW

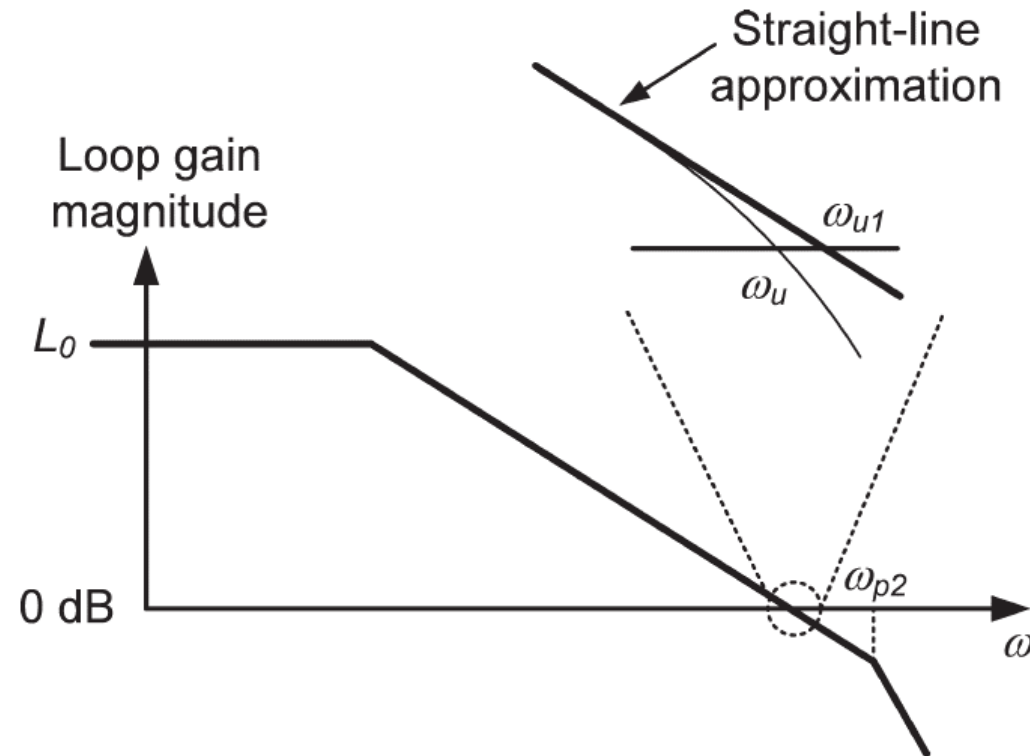
- If $\omega_{p2} = \omega_u$: PM = 45°
 - Typically inadequate (peaking/ringing)
- Thus ω_{p2} should be $> \omega_u \rightarrow \omega_{p1} \ll \omega_u < \omega_{p2}$
 - ω_{p1} defines OL BW and ω_{p2} defines ultimate GBW (max CL BW)

Frequency domain peaking
→ noise amplification
Time domain ringing
→ poor settling time



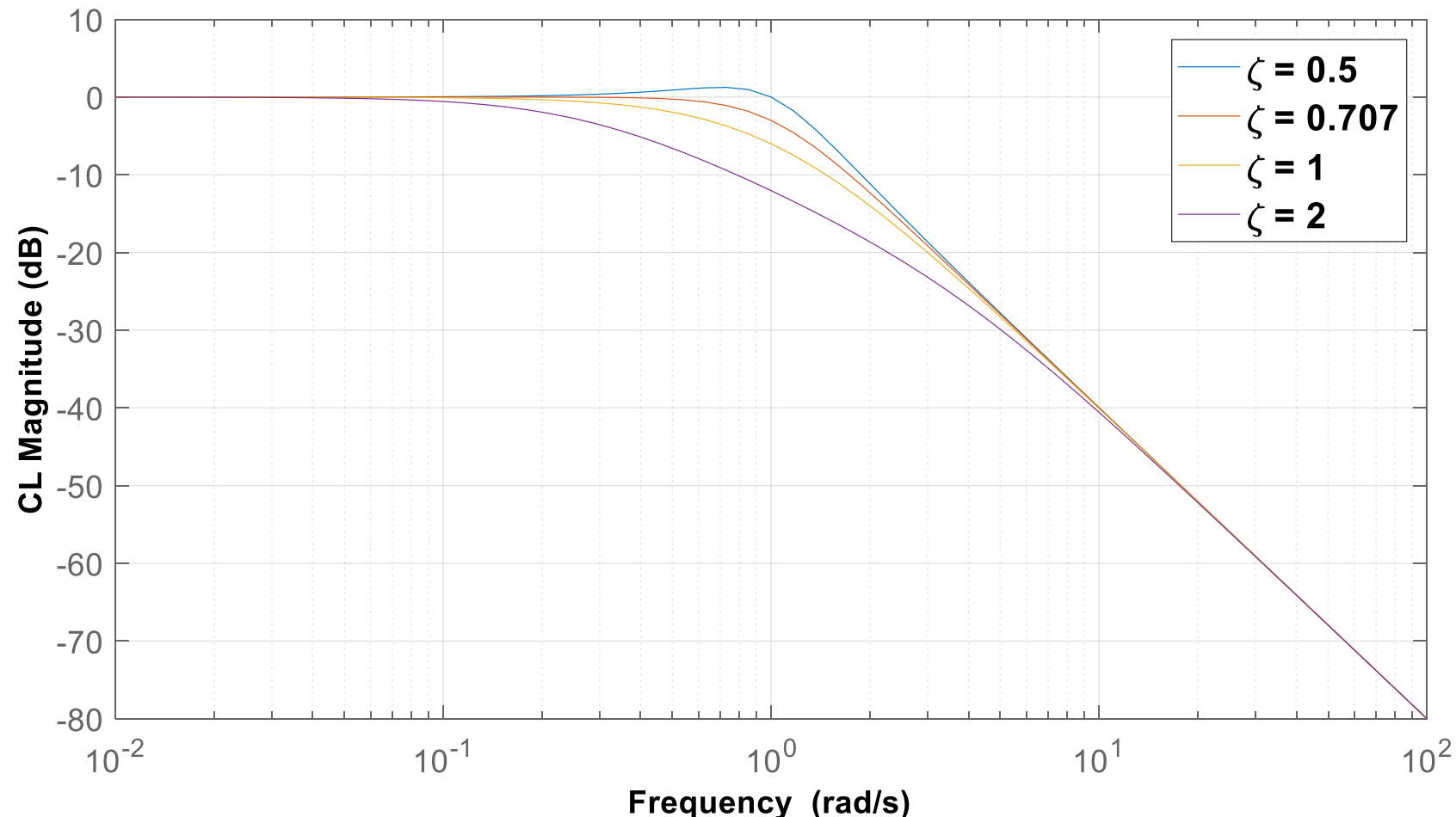
Asymptotic vs Actual UGF

- ❑ For $\omega < \omega_u$ the Bode plot is similar to a 1st order system
- ❑ For a true 1st order system: $\text{GBW} = \text{UGF} = \omega_u = \omega_{u1}$
- ❑ But the 2nd pole causes some bending below the asymptote
 - Actual ω_u slightly $< \omega_{u1}$: $\text{UGF} < \text{GBW}$



Optimum Phase Margin (Freq Response)

- Maximum CL BW without peaking occurs at $\zeta = Q = 0.707$
 - Maximally flat response



Optimum Phase Margin (Freq Response)

□ Maximum CL BW without peaking occurs at $\zeta = Q = 0.707$

- Maximally flat response
- $\omega_{p2} = 2\omega_{u1}$ and $PM \approx 65^\circ$

ω_{p2}/ω_{u1}	Q	ω_u/ω_{u1}	Phase margin ($^\circ$)
1	1	0.786	51.8
2	0.707	0.910	65.5
3	0.577	0.953	72.4
4	0.500	0.972	76.3
5	0.477	0.981	78.9
6	0.408	0.987	80.7
7	0.378	0.990	81.9
8	0.354	0.992	82.9
9	0.333	0.994	83.7
10	0.316	0.995	84.3
∞	—	1	90

Optimum Phase Margin (Freq Response)

- ❑ Maximum CL BW without peaking occurs at $\zeta = Q = 0.707$
 - Maximally flat response
 - $\omega_{p2} = 2\omega_{u1}$ and $PM \approx 65^\circ$
- ❑ But $\zeta < 1$: Underdamped system
 - Overshoot exists in transient response

Optimum Phase Margin (Tran Response)

□ Fastest settling without overshoot occurs at $\zeta = 1$ ($Q = 0.5$)

- Critical damped system
- $\omega_{p2} = 4\omega_{u1}$ and $PM \approx 76^\circ$

3x real or actual

ω_{p2}/ω_{u1}	Q	ω_u/ω_{u1}	Phase margin ($^\circ$)
1	1	0.786	51.8
2	0.707	0.910	65.5
3	0.577	0.953	72.4
4	0.500	0.972	76.3
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ω_{p2}
1st order system

Second-Order System Speed-up

- ❑ First order system corresponds to $\omega_{p2}/\omega_{u1} \rightarrow \infty$
 - Too much overdamping
- ❑ The settling of 2nd order system with optimum PM is faster
 - Critical damped system is faster than overdamped system
- ❑ But we must take some extra margin to account for variations

Dynamic settling error (ϵ_d)	t_s/τ ($\omega_{p2}/\omega_{u1} \rightarrow \infty$)	t_s/τ ($\omega_{p2}/\omega_{u1} = 4$)	Speedup (%)
10%	2.3	1.9	15.5
1%	4.6	3.3	27.9
0.1%	6.9	4.6	33.1
0.01%	9.2	5.9	36.2

Thank you!

References

- ❑ A. Sedra and K. Smith, “Microelectronic Circuits,” Oxford University Press, 7th ed., 2015
- ❑ B. Razavi, “Design Of Analog CMOS Integrated Circuit,” 2nd ed., McGraw-Hill, 2017.
- ❑ T. C. Carusone, D. Johns, and K. W. Martin, “Analog Integrated Circuit Design,” 2nd ed., Wiley, 2012.
- ❑ P. Jespers and B. Murmann, Systematic Design of Analog CMOS Circuits Using Pre-Computed Lookup Tables, Cambridge University Press, 2017.

Bandwidth Extension

- ❑ Cascade of feedback amplifiers provides the same gain and a much faster response
 - But power consumption and area doubled

