Analog IC Design

Lecture 08
Frequency Response (1)

Dr. Hesham A. Omran

Integrated Circuits Laboratory (ICL)
Electronics and Communications Eng. Dept.
Faculty of Engineering
Ain Shams University

Outline

- Recapping previous key results
- Bode plot review
- Where are the capacitors?
- Approximate analysis techniques
  - Short-circuit time constant (SCTC) and open-circuit time constant (OCTC) techniques
  - Dominant pole approximation
- IC amplifier frequency response
  - Unity gain frequency (UGF) and gain-bandwidth product (GBW)
- Calculating zeros and poles by inspection
  - Associating poles with nodes
- Miller's theorem

Outline

- Recapping previous key results
- Bode plot review
- Where are the capacitors?
- Approximate analysis techniques
  - Short-circuit time constant (SCTC) and open-circuit time constant (OCTC) techniques
  - Dominant pole approximation
- IC amplifier frequency response
  - Unity gain frequency (UGF) and gain-bandwidth product (GBW)
- Calculating zeros and poles by inspection
  - Associating poles with nodes
- Miller's theorem

MOSFET in Saturation

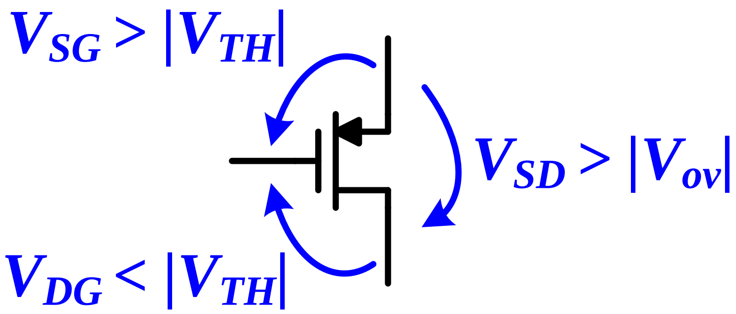
- The channel is pinched off if the difference between the gate and drain voltages is not sufficient to create an inversion layer

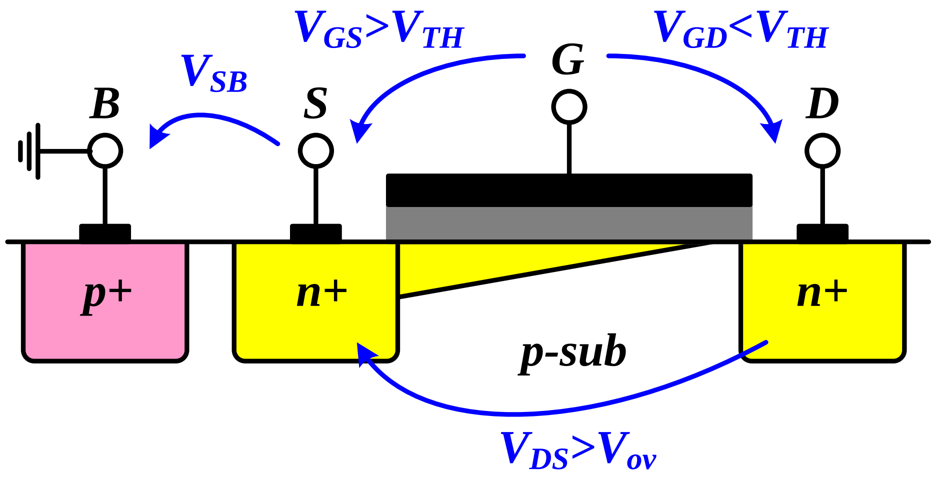
$$V_{GD} \leq V_{TH} \quad OR \quad V_{DS} \geq V_{ov}$$

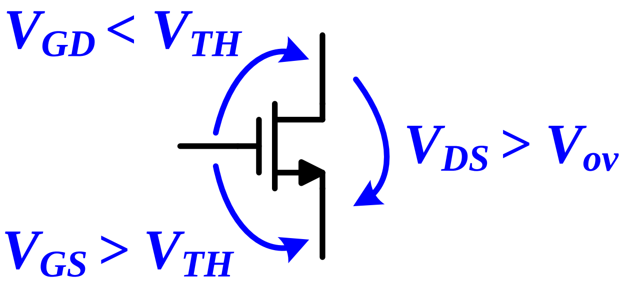
- Square-law (long channel MOS)

$$I_D = \frac{\mu_n C_{ox}}{2}\frac{W}{L} \cdot V_{ov}^2(1 + \lambda V_{DS})$$

$$V_{SB} \uparrow \;\Rightarrow\; V_{TH} \uparrow$$







Regions of Operation Summary

OFF (Subthreshold)

$V_{GS} < V_{TH}$

ON

$V_{GS} > V_{TH}$

Triode

$V_{DS} < V_{ov}$

Or

$V_{GD} > V_{TH}$

$$I_D = \mu C_{ox} \frac{W}{L}\left(V_{ov}V_{DS} - \frac{V_{DS}^2}{2}\right)$$

Pinch-Off (Saturation)

$V_{DS} \geq V_{ov}$

Or

$V_{GD} \leq V_{TH}$

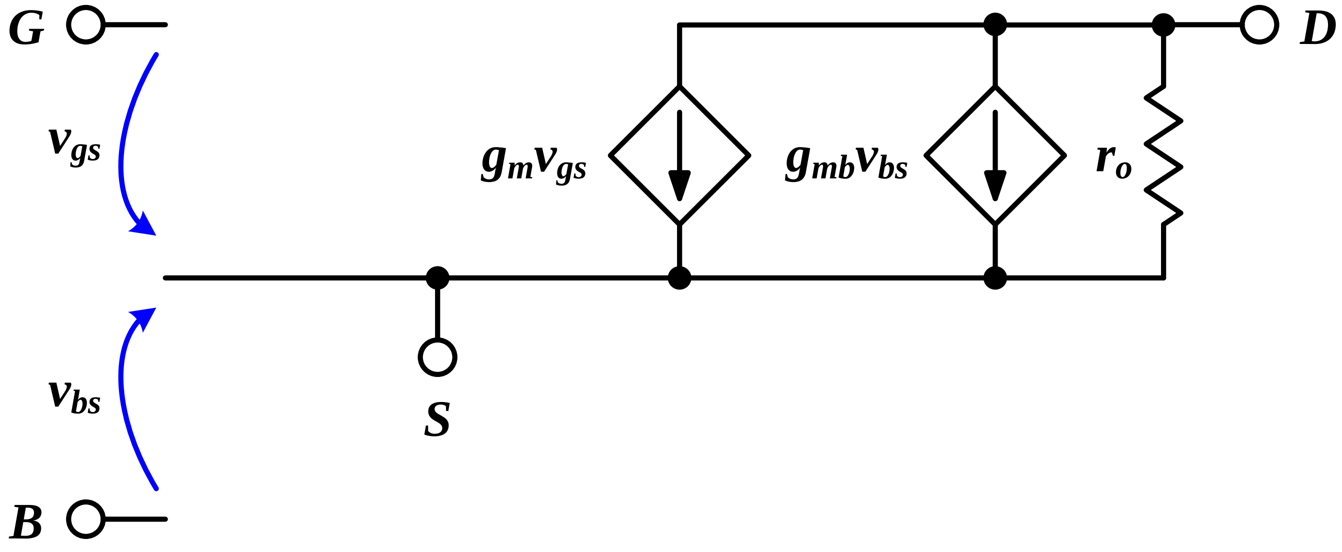
$$I_D = \frac{\mu C_{ox}}{2}\frac{W}{L}V_{ov}^2(1 + \lambda V_{DS})$$

Low-Frequency Small-Signal Model

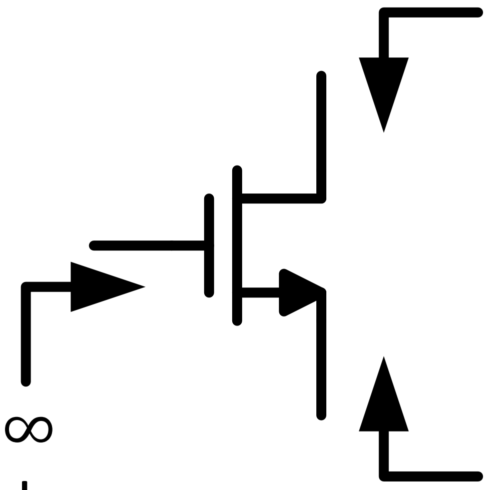
$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \mu C_{ox} \frac{W}{L} V_{ov} = \sqrt{\mu C_{ox} \frac{W}{L} \cdot 2I_D} = \frac{2I_D}{V_{ov}}$$

$$g_{mb} = \eta g_m \qquad \eta \approx 0.1 - 0.25$$

$$r_o = \frac{1}{\partial I_D / \partial V_{DS}} = \frac{V_A}{I_D} = \frac{1}{\lambda I_D} \qquad V_A \propto L \leftrightarrow \lambda \propto \frac{1}{L} \qquad V_{DS} \uparrow V_A \uparrow$$



Rin/out Shortcuts Summary



$$r_o[1 + (g_m + g_{mb})R_S]$$

H.I.N.

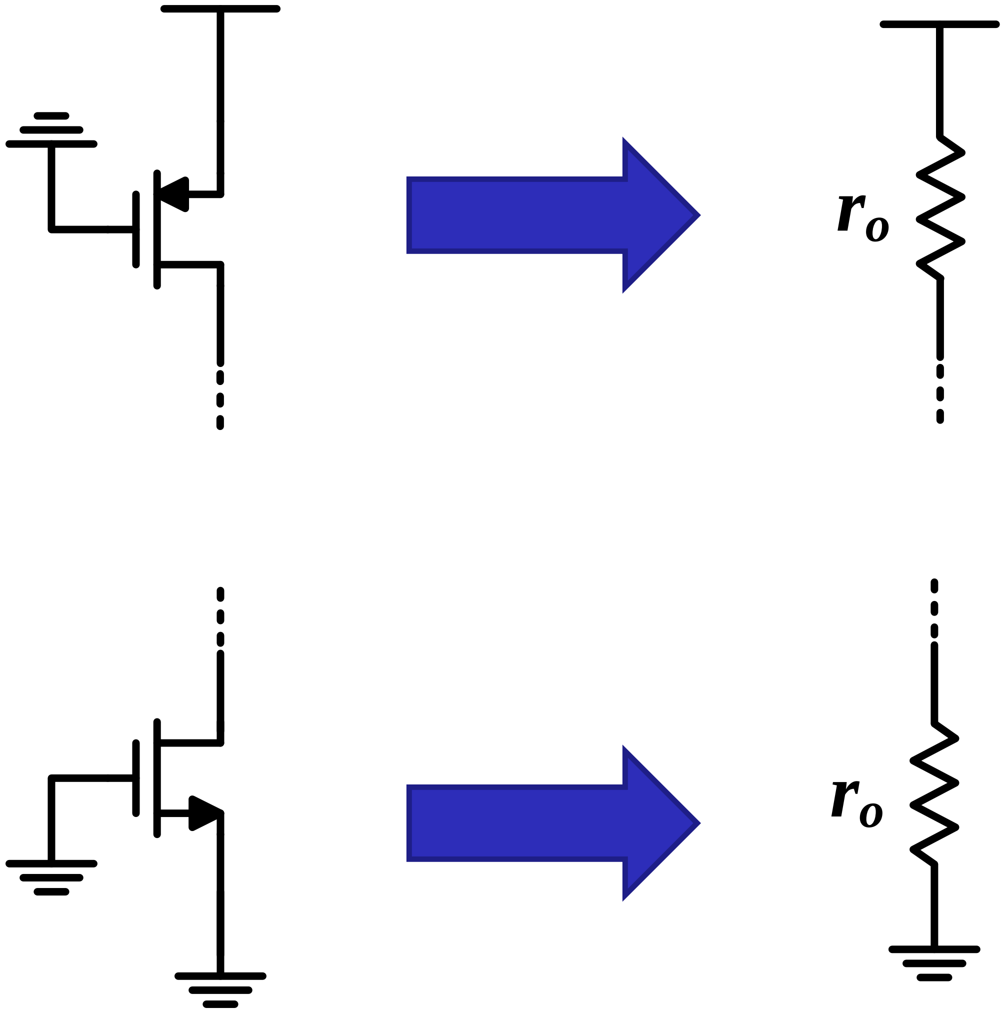
∞

At low frequencies **ONLY**

$$\frac{1}{g_m + g_{mb}}\left(1 + \frac{R_D}{r_o}\right)$$

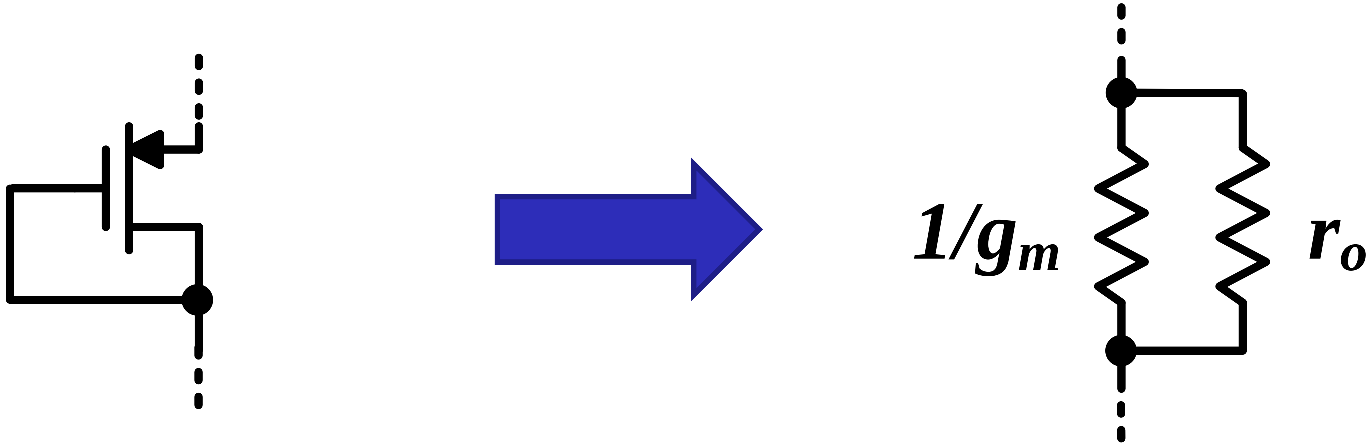
L.I.N.

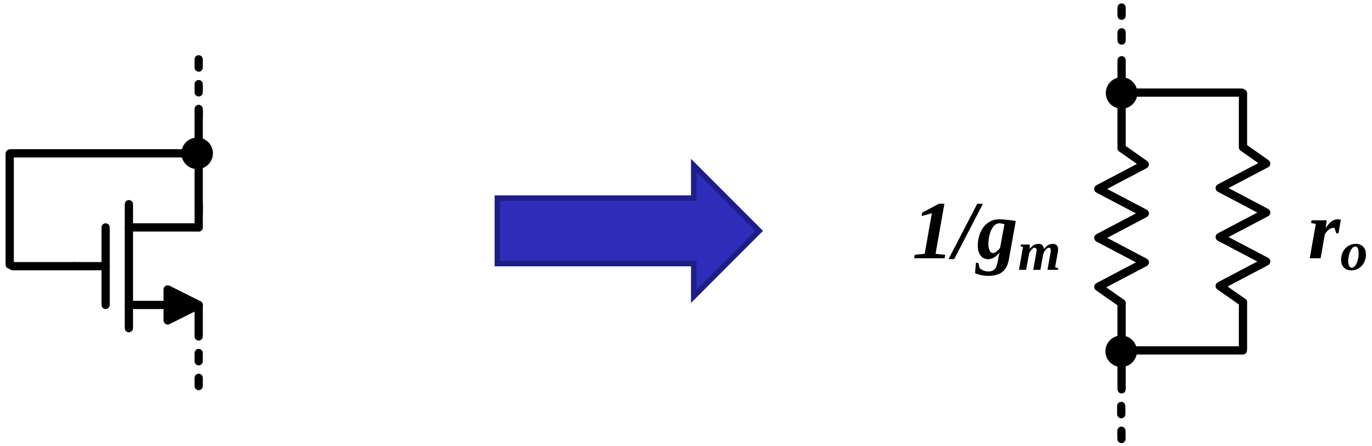
Active Load (Source OFF)



Diode Connected (Source Absorption)

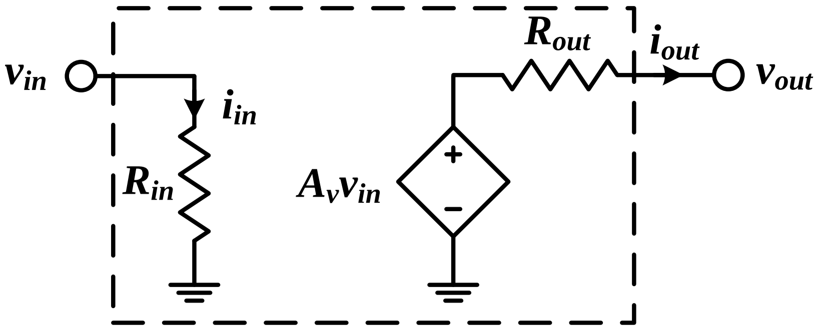
- Always in saturation ($V_{DS} = V_{GS} > V_{ov}$)
- Body effect: $g_m \rightarrow g_m + g_{mb}$ (if G is ac gnd)



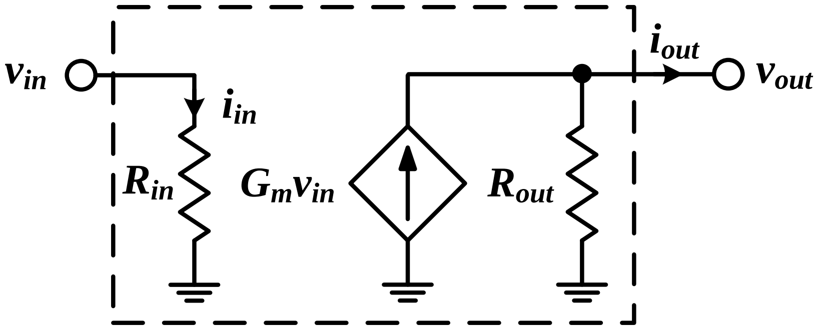


Why GmRout?

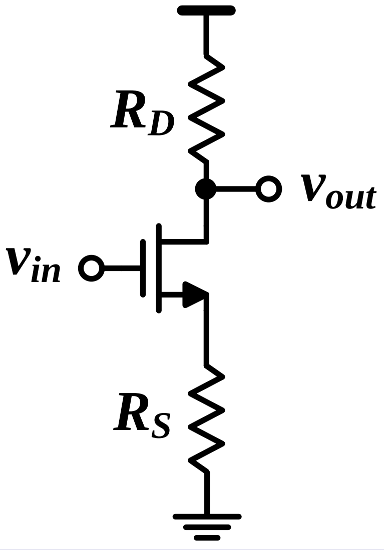
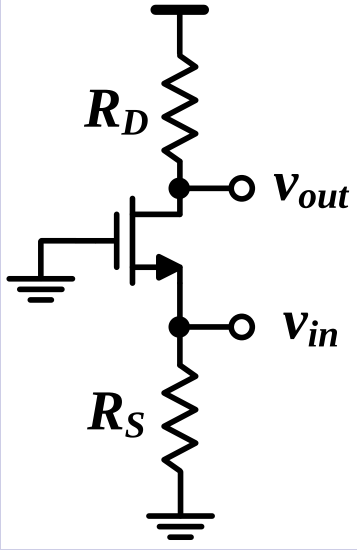
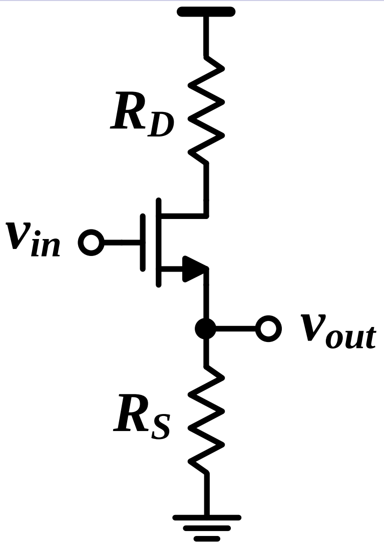
$$R_{in} = v_{in}/i_{in}$$
$$R_{out} = v_x/i_x \; @ \; v_{in} = 0$$
$$G_m = i_{out,sc}/v_{in}$$
$$A_v = G_m R_{out}$$
$$A_i = G_m R_{in}$$



- Divide and conquer
  - Rout simplified: vin=0
  - Gm simplified: vout=0
  - We already need Rin/out and Gm
  - We can quickly and easily get Rin/out from the shortcuts



Summary of Basic Topologies

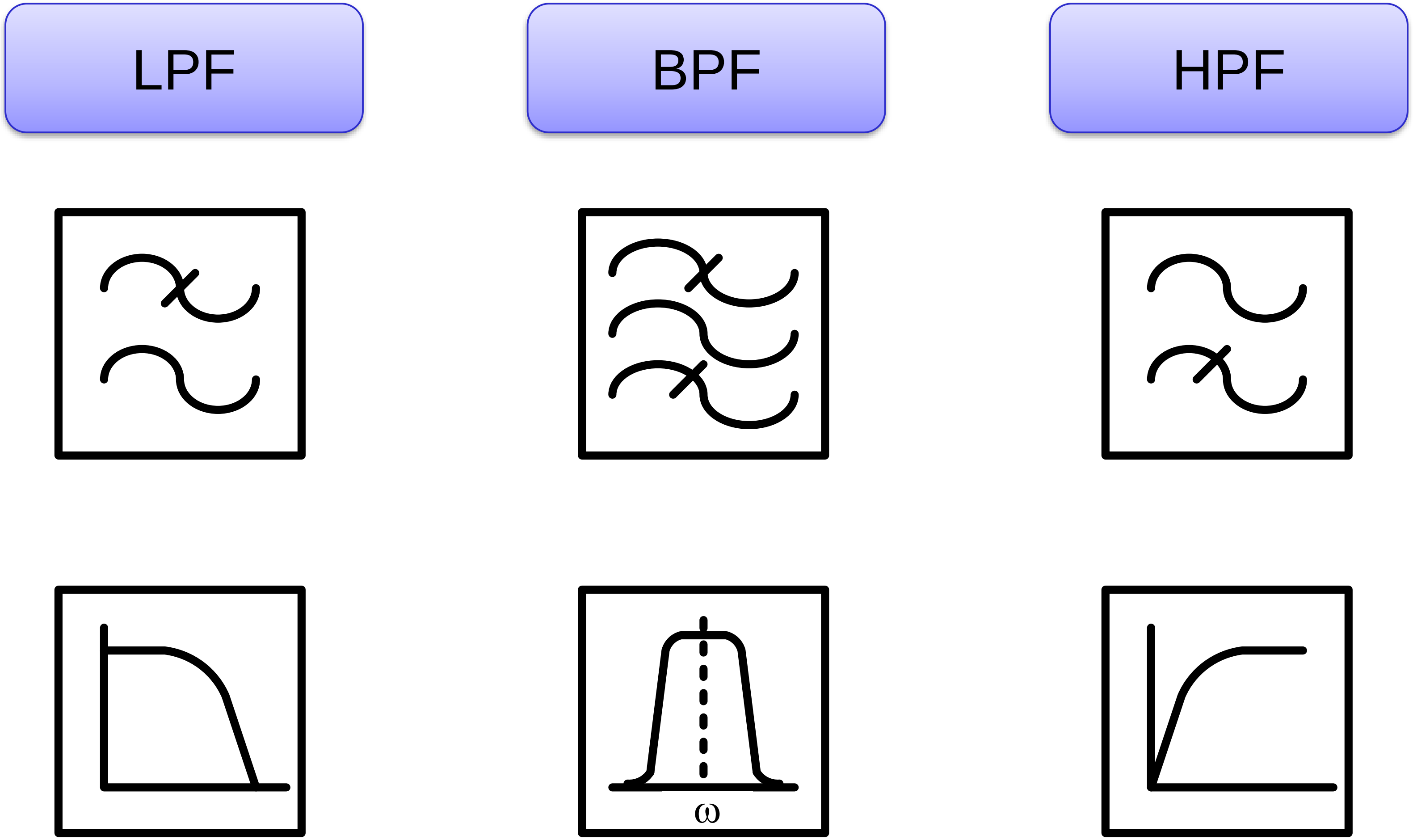
	CS	CG	CD (SF)
			
	Voltage & current amplifier	Voltage amplifier Current buffer	Voltage buffer Current amplifier
Rin	∞	$R_S \| \| \frac{1}{g_m + g_{mb}}\left(1 + \frac{R_D}{r_o}\right)$	∞
Rout	$R_D \| \| r_o[1 + (g_m + g_{mb})R_S]$	$R_D \| \| r_o$	$R_S \| \| \frac{1}{g_m + g_{mb}}\left(1 + \frac{R_D}{r_o}\right)$
Gm	$\frac{-g_m}{1 + (g_m + g_{mb})R_S}$	$g_m + g_{mb}$	$\frac{g_m}{1 + R_D/r_o}$

Outline

- ❑ Recapping previous key results
- ❑ Bode plot review
- ❑ Where are the capacitors?
- ❑ Approximate analysis techniques
  - Short-circuit time constant (SCTC) and open-circuit time constant (OCTC) techniques
  - Dominant pole approximation
- ❑ IC amplifier frequency response
  - Unity gain frequency (UGF) and gain-bandwidth product (GBW)
- ❑ Calculating zeros and poles by inspection
  - Associating poles with nodes
- ❑ Miller's theorem

Frequency Response

- Y-axis: magnitude of frequency response, x-axis: frequency



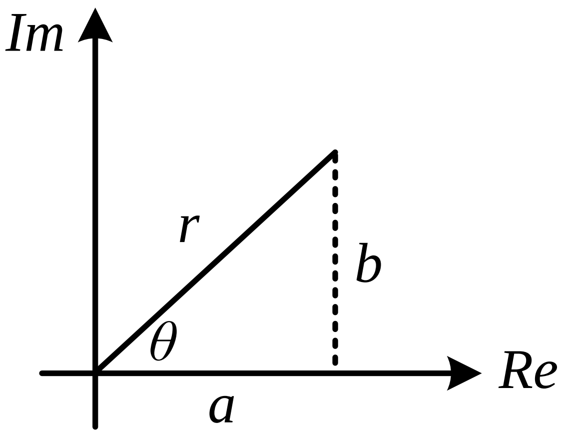
Poles and Zeros

- Transfer function

$$H(s) = \frac{N(s)}{D(s)}$$

- Zeros: roots of numerator => $N(s)$
- Poles: roots of denominator => $D(s)$
- Frequency response: $s \Rightarrow j\omega$

$$H(j\omega) = \frac{V_{out}(j\omega)}{V_{in}(j\omega)} = |H(j\omega)|e^{j\phi}$$



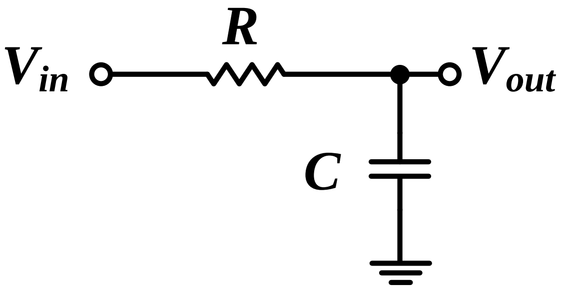
- Magnitude$(a + jb) = r = \sqrt{a^2 + b^2}$
- Phase$(a + jb) = \theta = \tan^{-1}\frac{b}{a}$

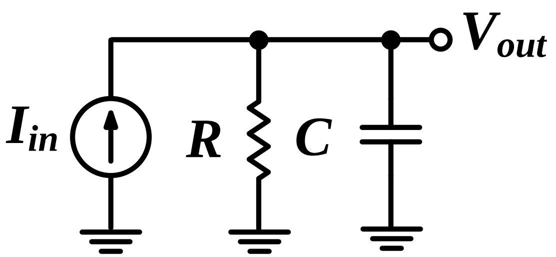
1st Order LPF

$$H(s)=\frac{v_{out}}{v_{in}}=\frac{1/sC}{R+1/sC}=\frac{1}{1+sRC}=\frac{1}{1+s\tau}$$

$$H(j\omega)=\frac{v_{out}}{v_{in}}=\frac{1/j\omega C}{R+1/j\omega C}=\frac{1}{1+j\omega RC}=\frac{1}{1+\frac{j\omega}{\omega_c}}$$

- ❑ $\tau=RC$: time constant
- ❑ $\omega_c=\frac{1}{\tau}=\frac{1}{RC}$: cutoff/corner frequency
- ❑ Poles: $s_p=-\frac{1}{\tau}=-\omega_c$, Zeros: ?
- ❑ $|H(j\omega)|=\frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_c}\right)^2}}$
- ❑ $P\big(H(j\omega)\big)=-\tan^{-1}\frac{\omega}{\omega_c}$





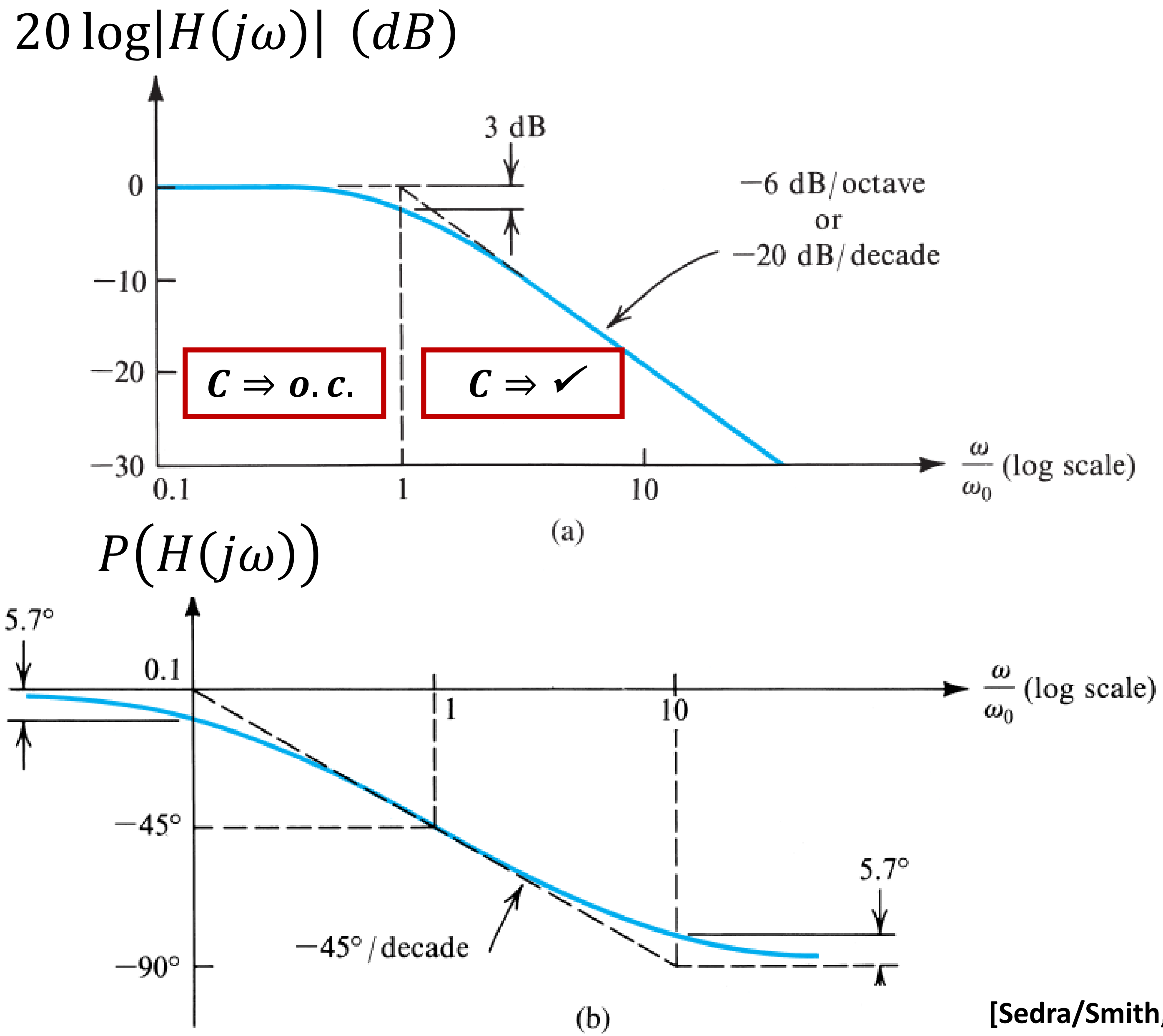
Bode Plot Rules

	Pole	Zero
Magnitude	-20 dB/decade Actual Mag @ pole: -3 dB	+20 dB/decade Actual Mag @ zero: +3 dB
Phase	-90^o Actual Phase @ pole: -45^o	LHP zero: +90^o Actual Phase @ zero: +45^o
		RHP zero: -90^o Actual Phase @ zero: -45^o

→ RHP: Right-half plane ($Re\{s\} > 0$)

→ LPH: Left-half plane ($Re\{s\} < 0$)

1^{st} Order LPF Bode Plot

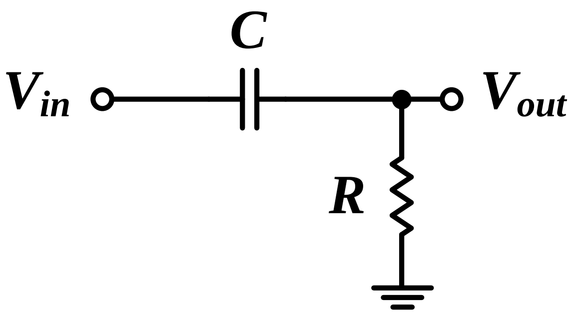


[Sedra/Smith, 2015]

1st Order HPF

$$H(s) = \frac{v_{out}}{v_{in}} = \frac{R}{R + 1/sC} = \frac{sRC}{1 + sRC} = \frac{s\tau}{1 + s\tau}$$

$$H(j\omega) = \frac{v_{out}}{v_{in}} = \frac{R}{R + 1/j\omega C} = \frac{j\omega RC}{1 + j\omega RC} = \frac{\frac{j\omega}{\omega_c}}{1 + \frac{j\omega}{\omega_c}}$$



- Poles: $s_p = -\frac{1}{\tau} = -\omega_c$
- Zeros: $s_z = 0$
- $|H(j\omega)| = \frac{\frac{\omega}{\omega_c}}{\sqrt{1+\left(\frac{\omega}{\omega_c}\right)^2}}$
- $P\big(H(j\omega)\big) = 90^o - \tan^{-1}\frac{\omega}{\omega_c}$

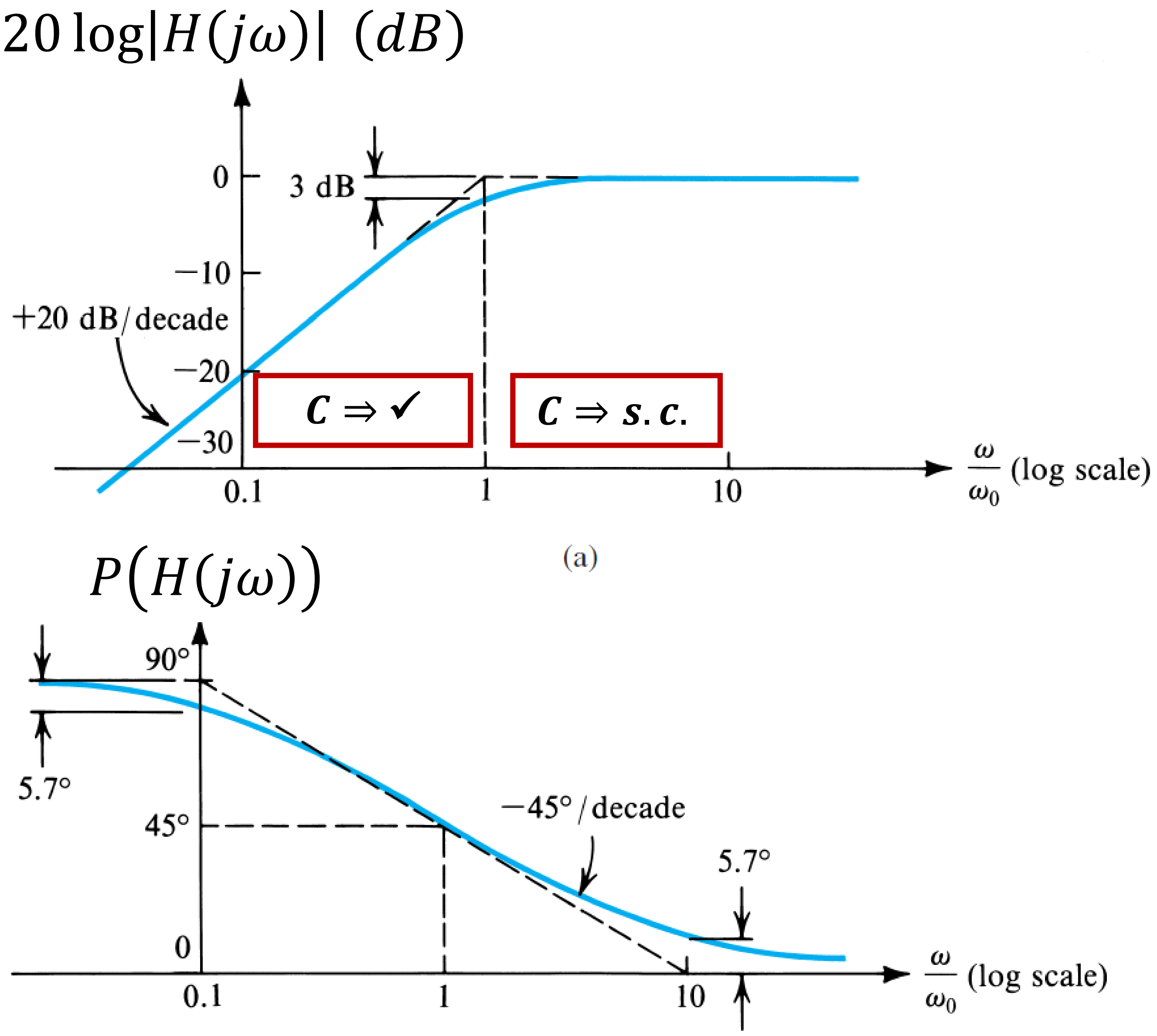
Bode Plot Rules

	Pole	Zero
Magnitude	-20 dB/decade Actual Mag @ pole: -3 dB	+20 dB/decade Actual Mag @ zero: +3 dB
Phase	-90^{o} Actual Phase @ pole: -45^{o}	LHP zero: $+90^{o}$ Actual Phase @ zero: $+45^{o}$
		RHP zero: -90^{o} Actual Phase @ zero: -45^{o}

→ RHP: Right-half plane ($Re\{s\} > 0$)

→ LPH: Left-half plane ($Re\{s\} < 0$)

1st Order HPF Bode Plot



(b)

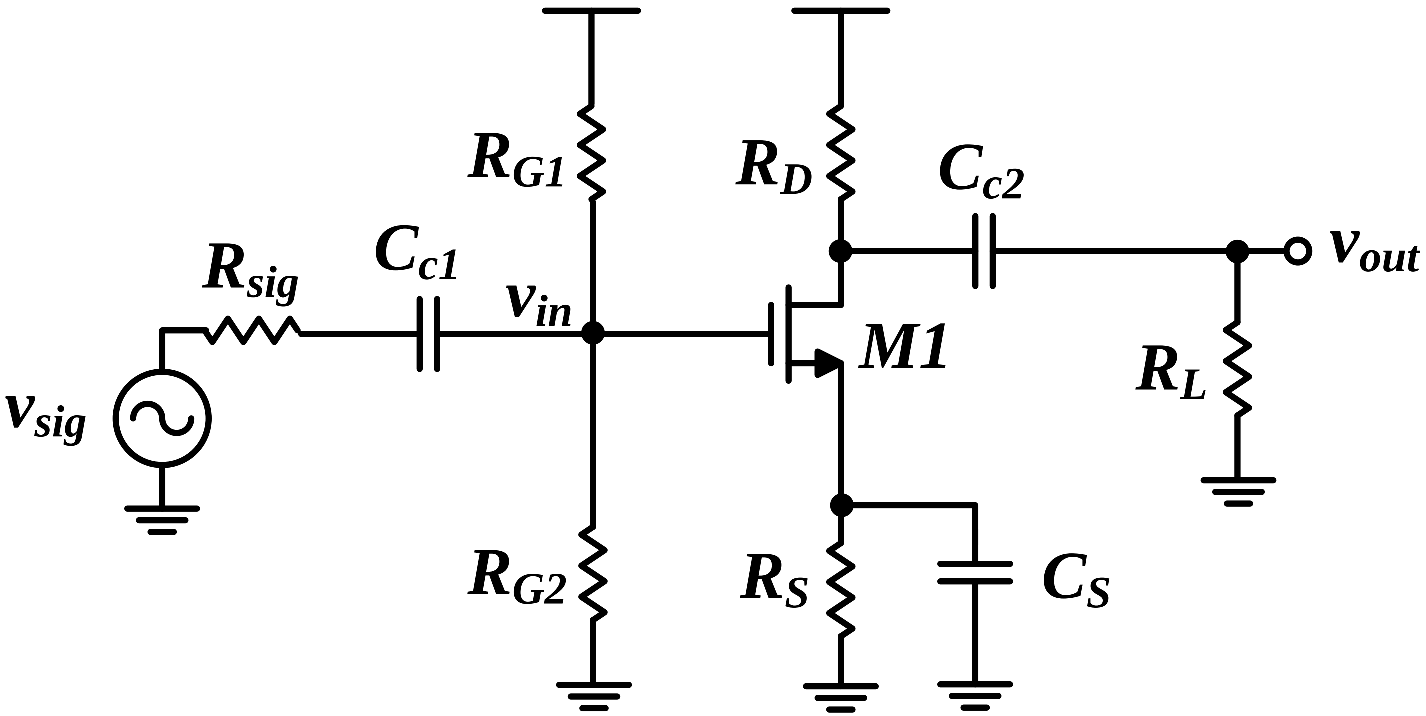
[Sedra/Smith, 2015]

Outline

- ❑ Recapping previous key results
- ❑ Bode plot review
- ❑ Where are the capacitors?
- ❑ Approximate analysis techniques
  - Short-circuit time constant (SCTC) and open-circuit time constant (OCTC) techniques
  - Dominant pole approximation
- ❑ IC amplifier frequency response
  - Unity gain frequency (UGF) and gain-bandwidth product (GBW)
- ❑ Calculating zeros and poles by inspection
  - Associating poles with nodes
- ❑ Miller's theorem

Where are the Capacitors?

- Coupling capacitors: C_{C1} and C_{C2} → act as HPF (affect LFR)
- Bypass capacitor: C_S
- Usually quite large $\sim \mu F$



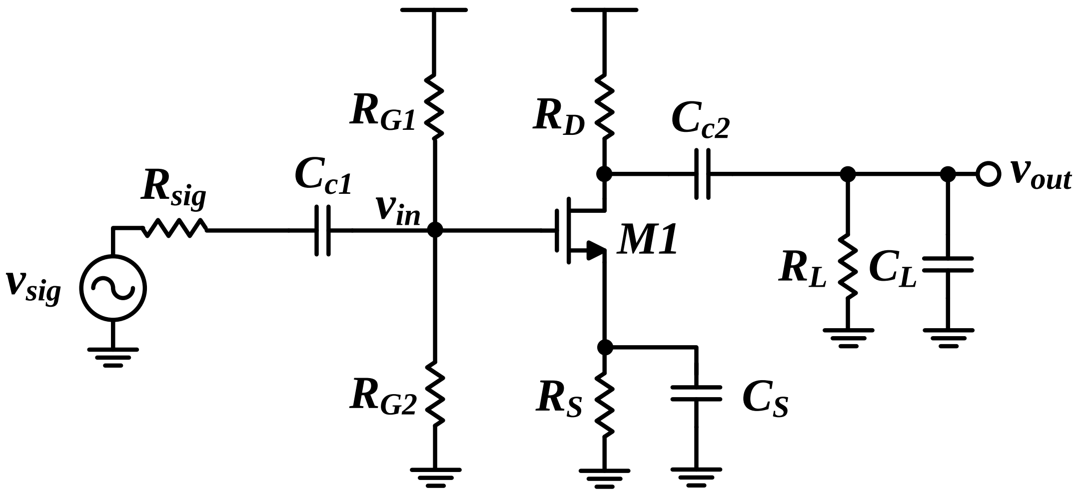
Effect of Bypass Capacitor

- Does C_S act as a LPF or a HPF?
  - By intuition, at high frequency C_S will increase the gain → HPF

$$G_m(s) = \frac{g_m}{1+g_m Z_S} \qquad Z_S = \frac{1}{1/R_S+sC_S} = \frac{R_S}{1+sR_SC_S}$$

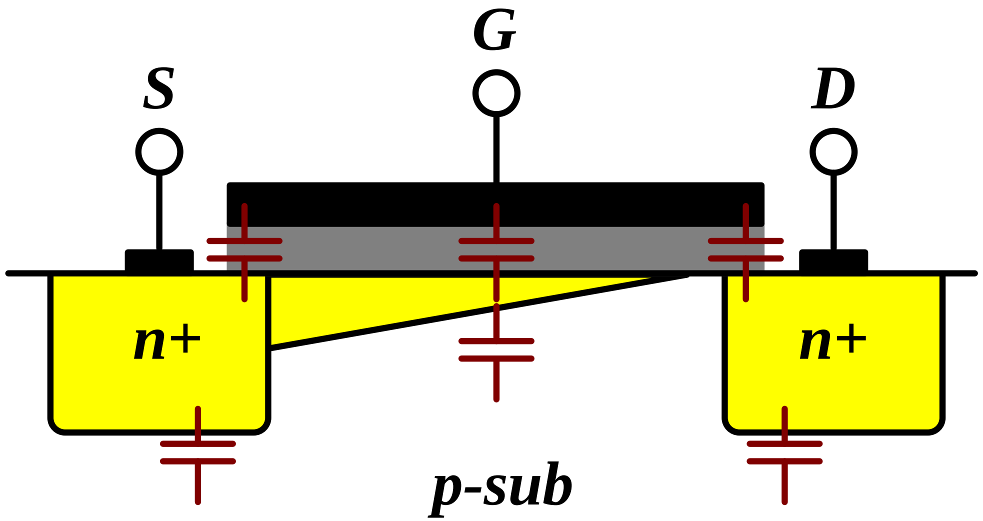
$$G_m(s) = \frac{g_m(1+sR_SC_S)}{(1+g_mR_S)\left(1+\frac{sR_SC_S}{1+g_mR_S}\right)}$$

$$s_z = -\frac{1}{R_SC_S} \; and \; s_p = -\frac{1+g_mR_S}{R_SC_S} \Rightarrow \omega_p > \omega_z \Rightarrow HPF$$



Where are the Capacitors?

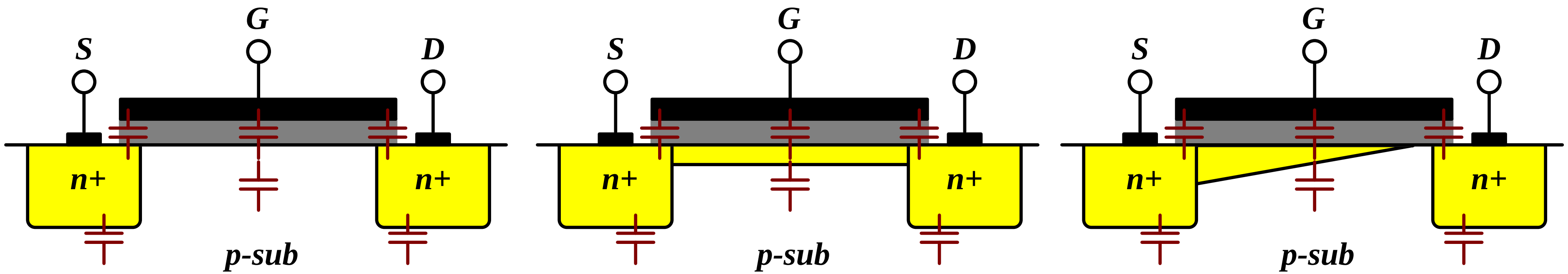
- Gate capacitance ($C_{gg} = C_{gb} + C_{gs} + C_{gd}$)
  - Intrinsic part fundamental to MOSFET operation
  - Parasitic part due to the overlap between gate and S/D (C_{ov})
- S/D capacitance ($C_{sb}\ and\ C_{db}$)
  - Parasitic capacitances due to reverse biased pn-junctions
    - Bottom-plate (C_j) and side-wall components (C_{jsw})
- Usually quite small $\sim fF$



MOSFET Capacitance

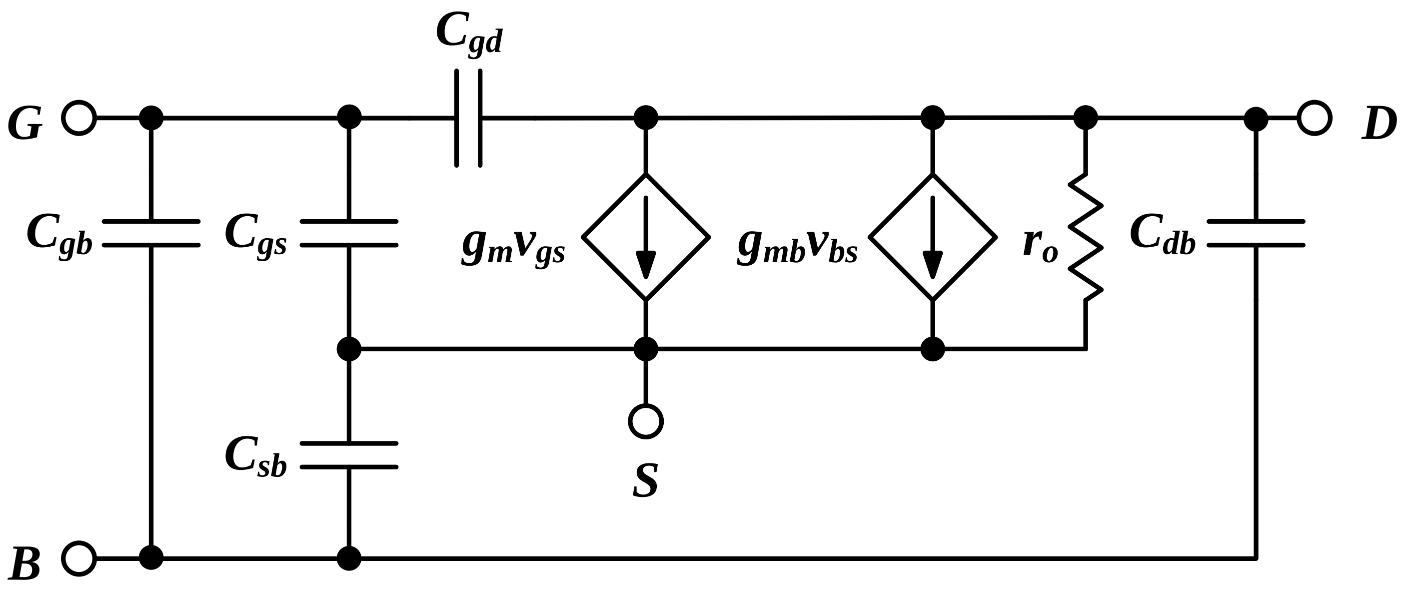
- C_{ov} per unit width, C_j per unit area, and C_{jsw} per unit perimeter

	Cutoff	Triode	Saturation
C_{gb}	$< WLC_{ox}$	0	$\mathbf{0}$
C_{gs}	WC_{ov}	$\frac{1}{2}WLC_{ox} + WC_{ov}$	$\boldsymbol{\frac{2}{3}WLC_{ox} + WC_{ov}}$
C_{gd}	WC_{ov}	$\frac{1}{2}WLC_{ox} + WC_{ov}$	$\boldsymbol{WC_{ov}}$
C_{sb}	$A_S C_j + P_S C_{jsw}$	$\left(A_S + \frac{WL}{2}\right) C_j + P_S C_{jsw}$	$\boldsymbol{(A_S + WL)C_j + P_S C_{jsw}}$
C_{db}	$A_D C_j + P_D C_{jsw}$	$\left(A_D + \frac{WL}{2}\right) C_j + P_D C_{jsw}$	$\boldsymbol{A_D C_j + P_D C_{jsw}}$



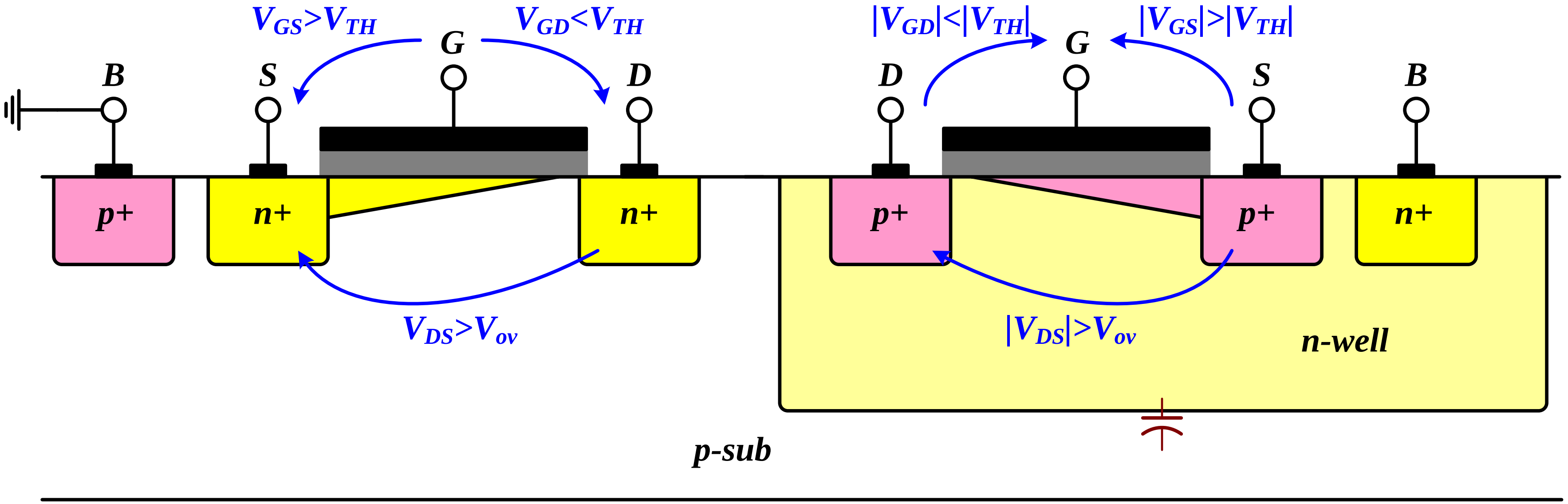
High Frequency Small Signal Model

- MOSFET capacitances act as LPF (affect HFR)
- In pinch-off saturation
  - $C_{gb} \approx 0$ (for SI only)
  - $C_{gs} \gg C_{gd}$ (not valid for $L \downarrow\downarrow$, why?)
  - $C_{sb} > C_{db}$



N-Well Capacitance

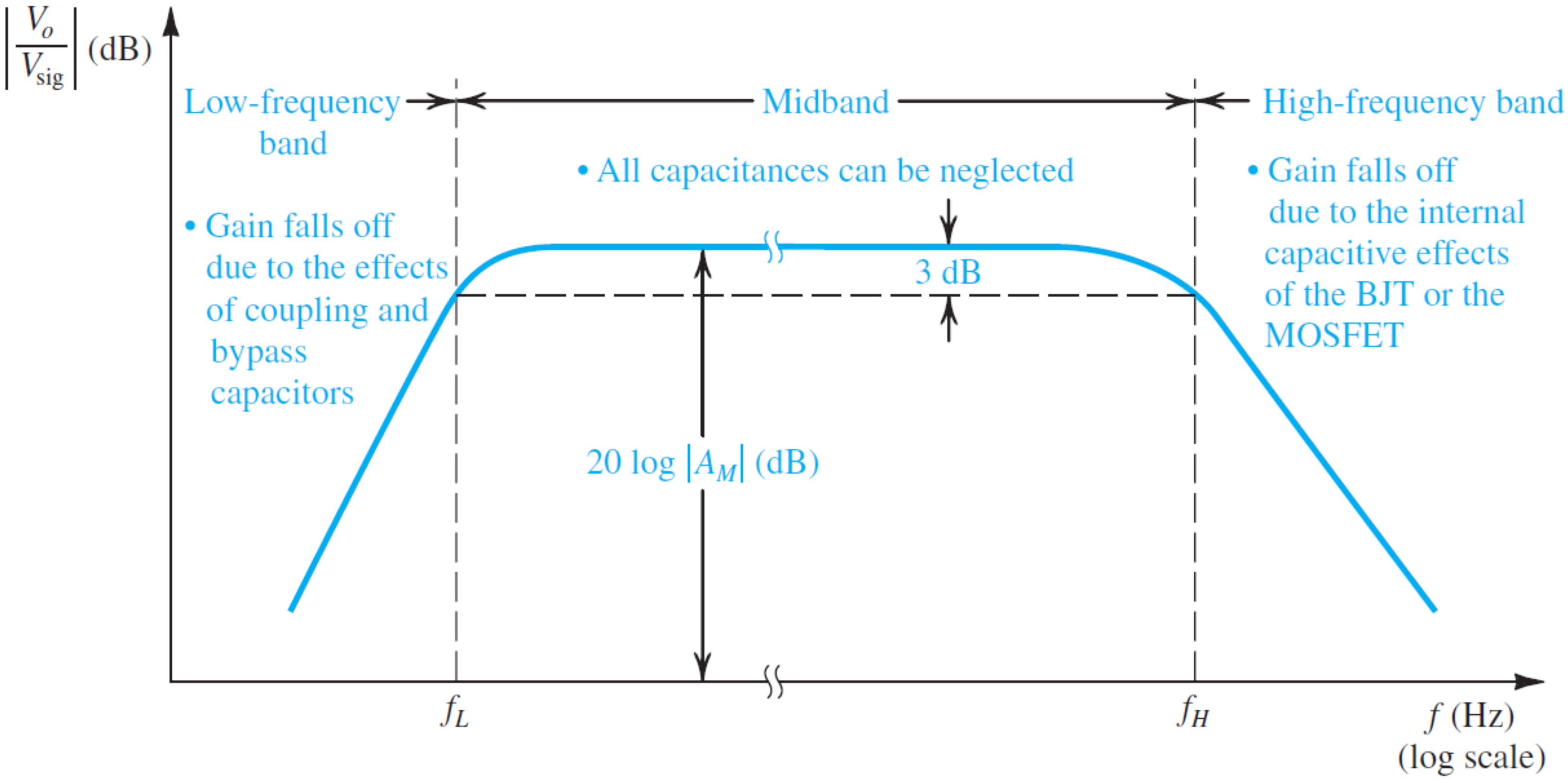
- There is an additional junction cap between n-well and p-sub
- If the n-well is tied to VDD this cap is ac shorted (why?)
- But if n-well is floating (e.g., PMOS S and B connected) this cap is not ac shorted
  - Usually not modeled in SPICE
  - Must be added manually $\sim 0.05fF/\mu m^2$



Outline

- ❑ Recapping previous key results
- ❑ Bode plot review
- ❑ Where are the capacitors?
- ❑ Approximate analysis techniques
  - Short-circuit time constant (SCTC) and open-circuit time constant (OCTC) techniques
  - Dominant pole approximation
- ❑ IC amplifier frequency response
  - Unity gain frequency (UGF) and gain-bandwidth product (GBW)
- ❑ Calculating zeros and poles by inspection
  - Associating poles with nodes
- ❑ Miller's theorem

Frequency Response



Coupling and bypass cap	✓	s.c.	s.c.
MOSFET and load cap	o.c.	o.c.	✓

[Sedra/Smith, 2015]

SCTC and OCTC Techniques

- Low-frequency range (LFR) => Not common in Analog IC design
  - Only consider one cap at a time → Assume other caps are s.c. → s.c. time constant (SCTC) technique
  - $\omega_{L,3dB} \approx \omega_{L1} + \omega_{L2} + \cdots$
  - Highest pole dominates (L.I.N. dominates)
- **High-frequency range (HFR) => More important in Analog ICs**
  - Only consider one cap at a time → Assume other caps are o.c. → o.c. time constant (OCTC) technique
  - $\omega_{H,3dB} \approx \omega_{H1} // \omega_{H2} // \cdots$
  - Lowest pole dominates (H.I.N. dominates)
- Both SCTC and OCTC provide good approx if one pole is dominant and poles are real

Dominant Pole Approximation

- Assume the poles are real and widely separated: $\omega_{p1} \ll \omega_{p2}$

$$A_v(s) \approx \frac{A_o}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)} = \frac{A_o}{1+\left(\frac{1}{\omega_{p1}}+\frac{1}{\omega_{p2}}\right)s+\frac{1}{\omega_{p1}\omega_{p2}}s^2}$$

$$\approx \frac{A_o}{1+\left(\frac{1}{\omega_{p1}}\right)s+\frac{1}{\omega_{p1}\omega_{p2}}s^2} = \frac{A_o}{1+b_1 s+b_2 s^2}$$

$$\omega_{p1} \approx \frac{1}{b_1} \quad \text{and} \quad \omega_{p2} \approx \frac{1}{b_2\omega_{p1}} = \frac{b_1}{b_2}$$

- OCTC provides an approx value for dominant pole only
- Dominant pole approx provides an approx value for both dominant and non-dominant poles

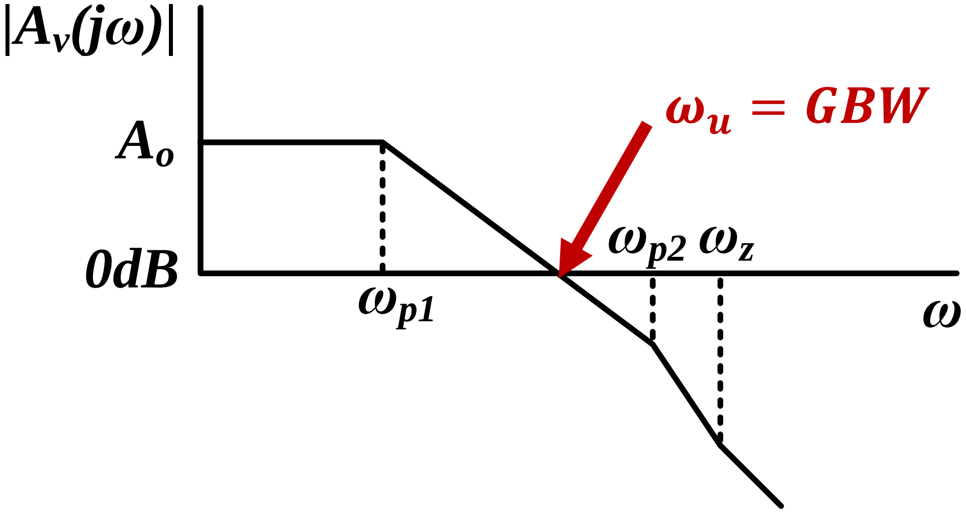
Outline

- Recapping previous key results
- Bode plot review
- Where are the capacitors?
- Approximate analysis techniques
  - Short-circuit time constant (SCTC) and open-circuit time constant (OCTC) techniques
  - Dominant pole approximation
- IC amplifier frequency response
  - Unity gain frequency (UGF) and gain-bandwidth product (GBW)
- Calculating zeros and poles by inspection
  - Associating poles with nodes
- Miller’s theorem

IC Amplifier Frequency Response

- $\boldsymbol{A_o}$ is the low-frequency gain (or DC gain) of the amplifier
- $\boldsymbol{\omega_{p1}}$ is the dominant pole $(\omega_{pd}) \approx$ 3dB bandwidth $= BW = \omega_{3dB}$
- $\boldsymbol{\omega_{p2}}$ is the non-dominant pole (ω_{pnd})
- Unity gain frequency (UGF, ω_u) is the frequency at which gain is unity (1 = 0dB)
- Gain-Bandwidth Product (GBW) $= Gain \times BW \approx A_o\omega_{p1}$
- Usually, we design the amplifier such that ω_{p2} and $\omega_z > \omega_u$

$$\boxed{A_v(s) = \frac{A_o\left(1+\frac{s}{\omega_z}\right)}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)}}$$



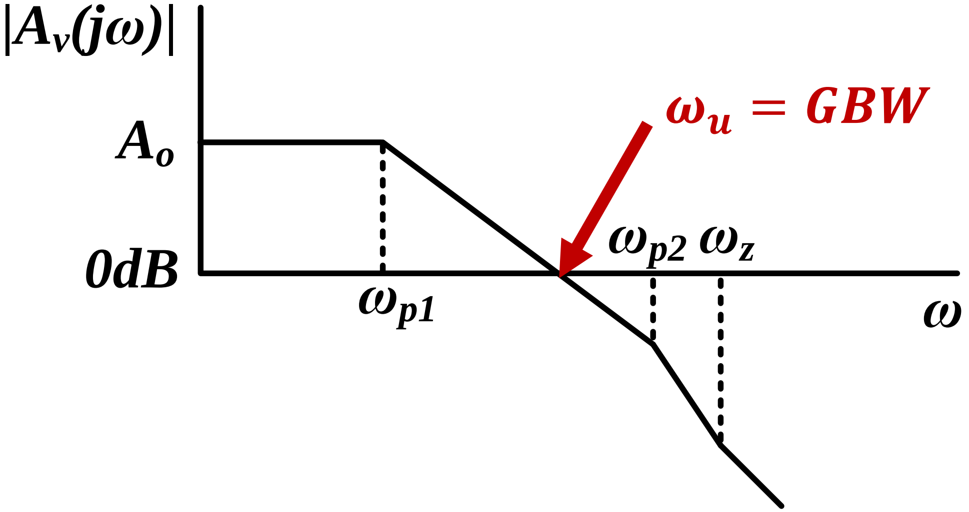
UGF and GBW

- At UGF $\omega = \omega_u$: $\omega \gg \omega_{p1}$ and $\omega \ll \omega_{p2}, \omega_z$

$$|A_v(\omega_u)| = \left| \frac{A_o\left(1 + \frac{j\omega_u}{\omega_z}\right)}{\left(1 + \frac{j\omega_u}{\omega_{p1}}\right)\left(1 + \frac{j\omega_u}{\omega_{p2}}\right)} \right| \approx \left| \frac{A_o}{j\omega_u/\omega_{p1}} \right| = 1 = 0\ dB$$

$$\boxed{UGF = \omega_u \approx A_o\omega_{p1} \approx Gain \times BW = GBW}$$

$$\boxed{A_v(s) = \frac{A_o\left(1 + \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{\omega_{p1}}\right)\left(1 + \frac{s}{\omega_{p2}}\right)}}$$



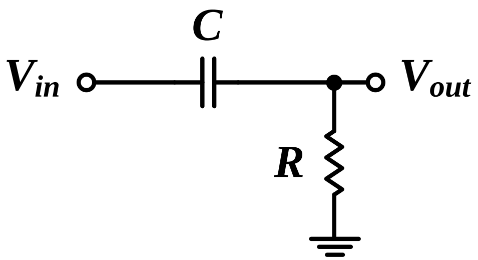
Outline

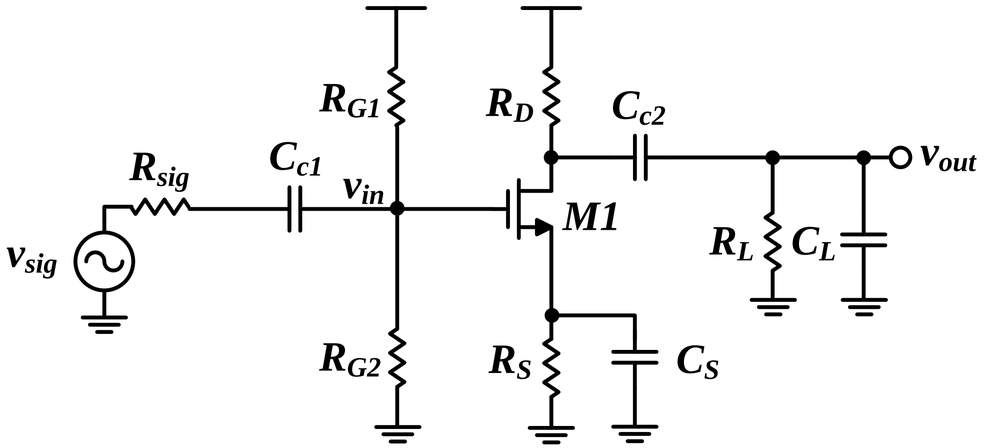
- ❑ Recapping previous key results
- ❑ Bode plot review
- ❑ Where are the capacitors?
- ❑ Approximate analysis techniques
  - Short-circuit time constant (SCTC) and open-circuit time constant (OCTC) techniques
  - Dominant pole approximation
- ❑ IC amplifier frequency response
  - Unity gain frequency (UGF) and gain-bandwidth product (GBW)
- ❑ Calculating zeros and poles by inspection
  - Associating poles with nodes
- ❑ Miller's theorem

Calculating Zeros by Inspection

1. Find the value $s = s_z$ that makes $H(s) = 0 \Rightarrow v_{out} = 0$

- Ex1: C: $v_o = 0\ if\ Z_C = \infty$
  - $Z_C = \frac{1}{sC}$
  - $\Rightarrow s_z = 0$
- Ex2: C_{c1}: $v_o = 0\ if\ Z_{C_1} = \infty$
  - $Z_{C_1} = \frac{1}{sC_1}$
  - $\Rightarrow s_{z1} = 0$
- Ex3: C_S: $v_o = 0\ if\ Z_S = \infty$
  - $Z_S = \frac{R_S}{1+sR_SC_S}$
  - $\Rightarrow s_{z2} = -\frac{1}{R_SC_S}$

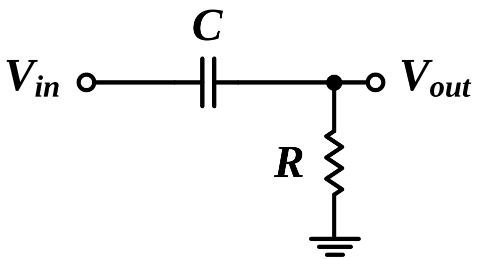


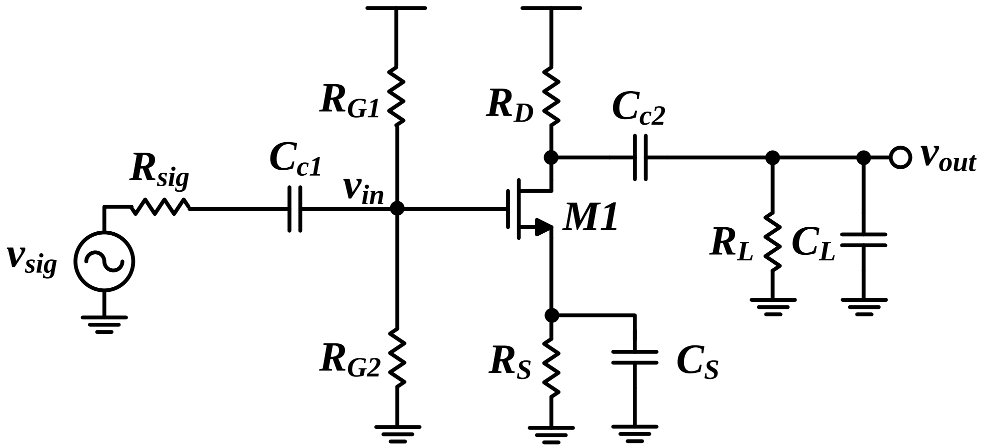


Calculating Poles by Inspection

1. Set $v_{sig} = 0$ (deactivate independent sources)
2. Calculate Thevenin resistance $\left(R_{th,i}\right)$ seen by each cap $\left(C_i\right)$
3. $s_{p,i} = -\frac{1}{R_{th,i}C_i}$

- Ex1: $C: R_{th} = R$
  - $\Rightarrow s_p = -\frac{1}{RC}$
- Ex2: $C_{c1}: R_{th} = R_{sig} + R_G$
  - $\Rightarrow s_{p1} = -\frac{1}{\left(R_{sig}+R_G\right)C_{c1}}$
- Ex3: $C_S: R_{th} \approx R_S // \frac{1}{g_m}$
  - $\Rightarrow s_{p2} = -\frac{1}{\left(R_S//\frac{1}{g_m}\right)C_S}$



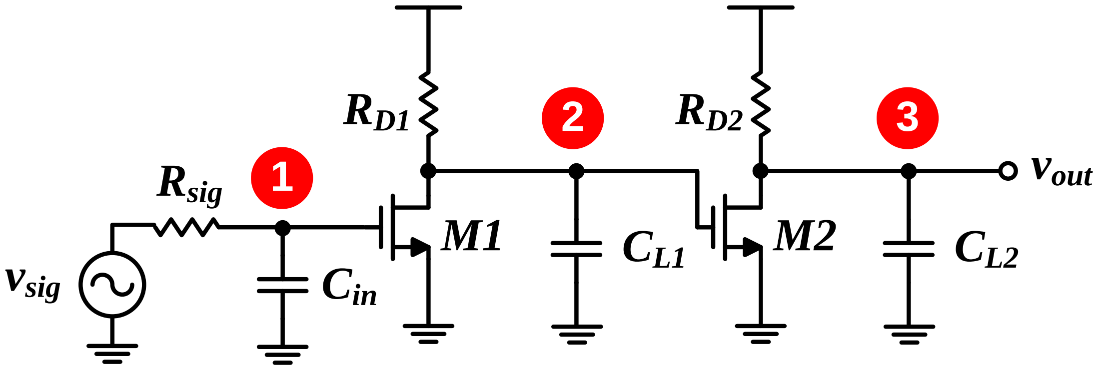


Associating Poles with Nodes

1. Set $v_{sig} = 0$ (deactivate independent sources)
2. Calculate Thevenin resistance $\left(R_{th,i}\right)$ seen by each cap $\left(C_i\right)$
3. $s_{p,i} = -\frac{1}{R_{th,i}C_i}$

- Example: Ignore MOSFET r_o and capacitance
  - **Each node is associated with a pole**
  - **H.I.N. dominates**

$$H(s) = \frac{(g_{m1}R_{D1})(g_{m2}R_{D2})}{\left(1 + sR_{sig}C_{in}\right)(1 + sR_{D1}C_{L1})(1 + sR_{D2}C_{L2})}$$



Outline

- Recapping previous key results
- Bode plot review
- Where are the capacitors?
- Approximate analysis techniques
  - Short-circuit time constant (SCTC) and open-circuit time constant (OCTC) techniques
  - Dominant pole approximation
- IC amplifier frequency response
  - Unity gain frequency (UGF) and gain-bandwidth product (GBW)
- Calculating zeros and poles by inspection
  - Associating poles with nodes
- Miller's theorem

Miller's Theorem

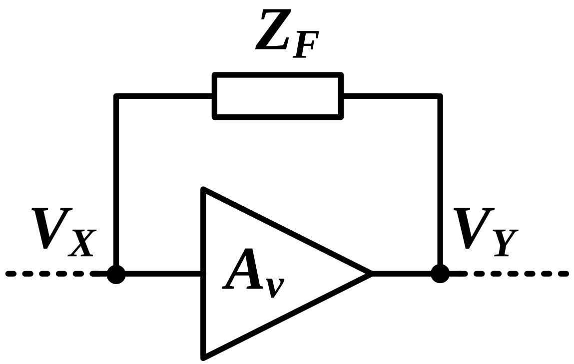
$$A_v = \frac{V_Y}{V_X}$$

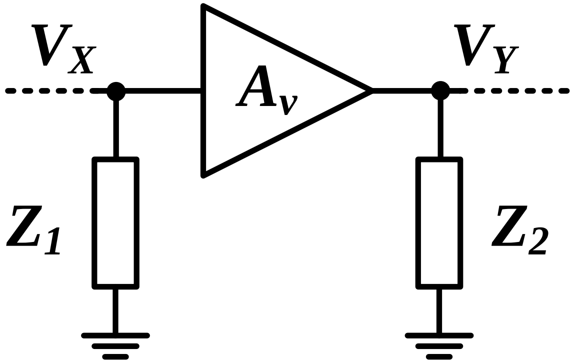
$$\frac{V_X - V_Y}{Z_F} = \frac{V_X}{Z_1}$$

$$Z_1 = \frac{Z_F}{1 - \frac{V_Y}{V_X}} = \frac{Z_F}{1 - A_v}$$

$$\frac{V_Y - V_X}{Z_F} = \frac{V_Y}{Z_2}$$

$$Z_2 = \frac{Z_F}{1 - \frac{V_X}{V_Y}} = \frac{Z_F}{1 - \frac{1}{A_v}}$$





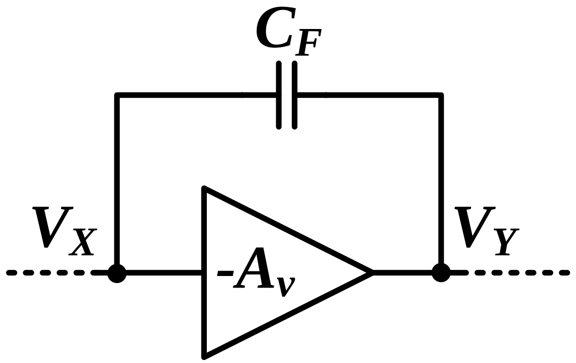
- ❑ Note: Miller's Theorem cannot be used for Z_{out} calculation (why?)

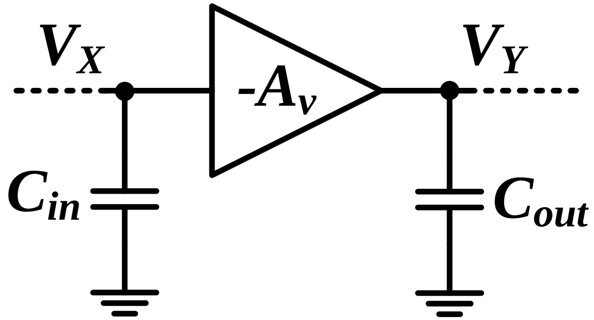
Miller Effect

- Miller Effect: Capacitance multiplication if $Z_F = 1/sC_F$

$$Z_1 = \frac{Z_F}{1+A_v} \approx \frac{Z_F}{A_v} = \frac{1}{sA_vC_F} \Rightarrow \boldsymbol{C_{in} = A_vC_F}$$

$$Z_2 = \frac{Z_F}{1+\frac{1}{A_v}} \approx Z_F = \frac{1}{sC_F} \Rightarrow C_{out} \approx C_F$$

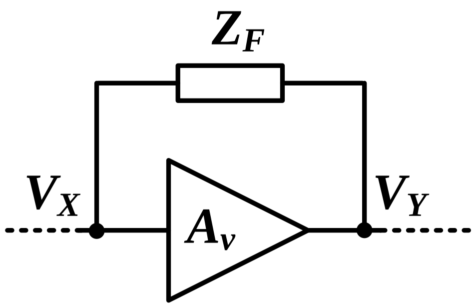


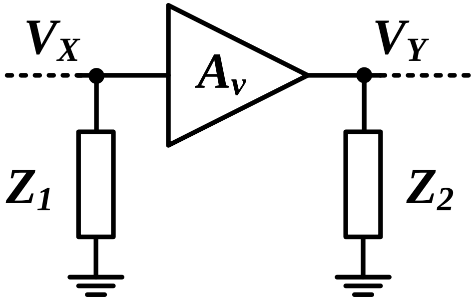


Miller's Approximation

$$Z_1 = \frac{Z_F}{1 - A_v} \ \& \ Z_2 = \frac{Z_F}{1 - \frac{1}{A_v}}$$

- ❑ But A_v is a function of frequency!
- ❑ Miller's Approximation: Substitute with the low frequency gain
  - $A_v(s) \approx A_o$
  - **Gives good approx for the dominant pole ONLY (why?)**
  - It does not tell about the feedforward zero (next slide)

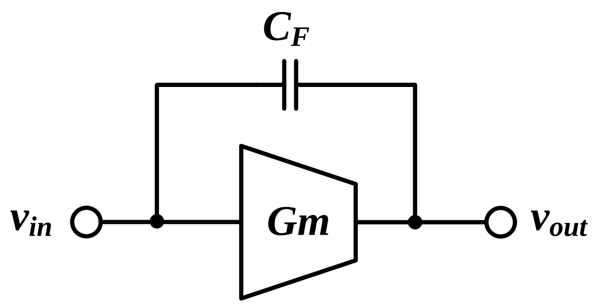


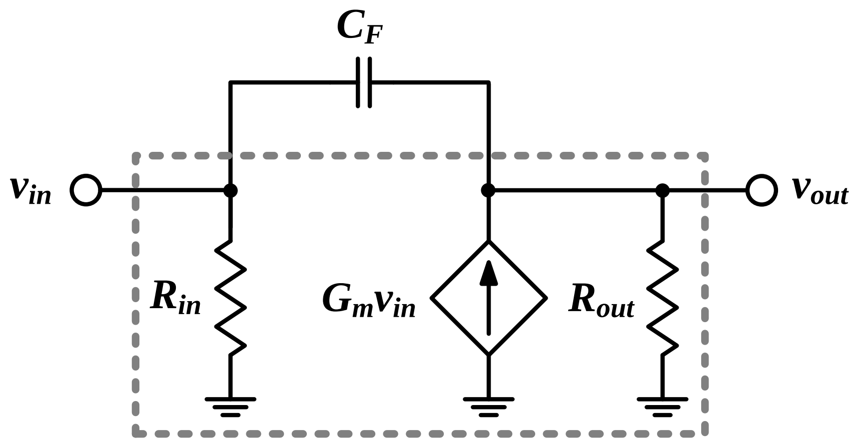


The Feedforward Zero

- $v_{out} = 0 \ \rightarrow i_{out} = 0$
- $v_{in} sC_F = -G_m v_{in}$

$$\boxed{s_z = -\frac{G_m}{C_F}}$$





- LHP zero if G_m is +ve (e.g. CD)
- **RHP zero if G_m is –ve (e.g. CS)**
  - Mag inc and phase drops
  - Very bad for FB loop stability
  - More on this when we study op-amp design

Thank you!

References

- A. Sedra and K. Smith, "Microelectronic Circuits," 7th ed., Oxford University Press, 2015
- B. Razavi, "Fundamentals of Microelectronics," 2nd ed., Wiley, 2014
- B. Razavi, "Design of Analog CMOS Integrated Circuits," McGraw-Hill, 2nd ed., 2017
- N. Weste and D. Harris, "CMOS VLSI Design," 4th ed., Pearson, 2010
- T. C. Carusone, D. Johns, and K. W. Martin, "Analog Integrated Circuit Design," 2nd ed., Wiley, 2012
- R. J. Baker, "CMOS circuit design," 3rd ed., Wiley, 2010
- B. Murmann, EE214 Course Reader, Stanford University