وَمَا أُوتِيتُم مِّنَ الْعِلْمِ إِلَّا قَلِيلًا

Analog IC Design

Lecture 12
The Five-Transistor (5T) OTA

Dr. Hesham A. Omran

Integrated Circuits Laboratory (ICL)
Electronics and Communications Eng. Dept.
Faculty of Engineering
Ain Shams University

Outline

- Recapping previous key results
- Op-amp vs OTA
- Differential to single-ended conversion
- Five-transistor OTA Analysis
    A. Small signal analysis
        1. Diff small signal analysis
        2. CM small signal analysis
    B. Large signal analysis
        1. Diff large signal analysis
        2. CM large signal analysis
- Effect of mismatch
- 5T OTA frequency response

Outline

- ❑ Recapping previous key results
- ❑ Op-amp vs OTA
- ❑ Differential to single-ended conversion
- ❑ Five-transistor OTA Analysis
  - A. Small signal analysis
    1. Diff small signal analysis
    2. CM small signal analysis
  - B. Large signal analysis
    1. Diff large signal analysis
    2. CM large signal analysis
- ❑ Effect of mismatch
- ❑ 5T OTA frequency response

MOSFET in Saturation

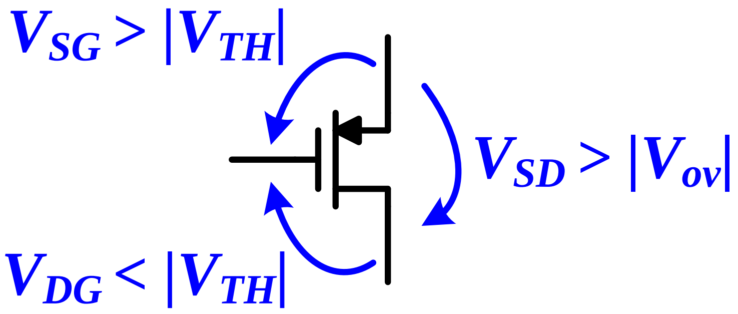
- The channel is pinched off if the difference between the gate and drain voltages is not sufficient to create an inversion layer

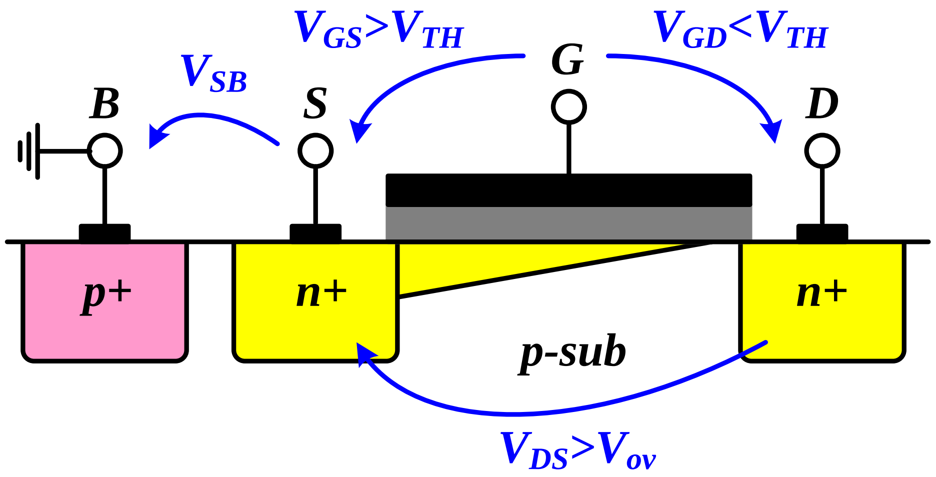
$$V_{GD} \leq V_{TH} \quad OR \quad V_{DS} \geq V_{ov}$$

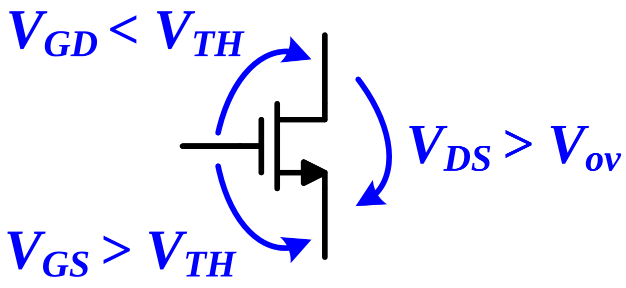
- Square-law (long channel MOS)

$$I_D = \frac{\mu_n C_{ox}}{2}\frac{W}{L} \cdot V_{ov}^2(1 + \lambda V_{DS})$$

$$V_{SB} \uparrow \quad \Rightarrow \quad V_{TH} \uparrow$$







Regions of Operation Summary

OFF (Subthreshold)

$V_{GS} < V_{TH}$

ON

$V_{GS} > V_{TH}$

Triode

$V_{DS} < V_{ov}$

Or

$V_{GD} > V_{TH}$

$$I_D = \mu C_{ox} \frac{W}{L} \left(V_{ov} V_{DS} - \frac{V_{DS}^2}{2} \right)$$

Pinch-Off (Saturation)

$V_{DS} \geq V_{ov}$

Or

$V_{GD} \leq V_{TH}$

$$I_D = \frac{\mu C_{ox}}{2} \frac{W}{L} V_{ov}^2 (1 + \lambda V_{DS})$$

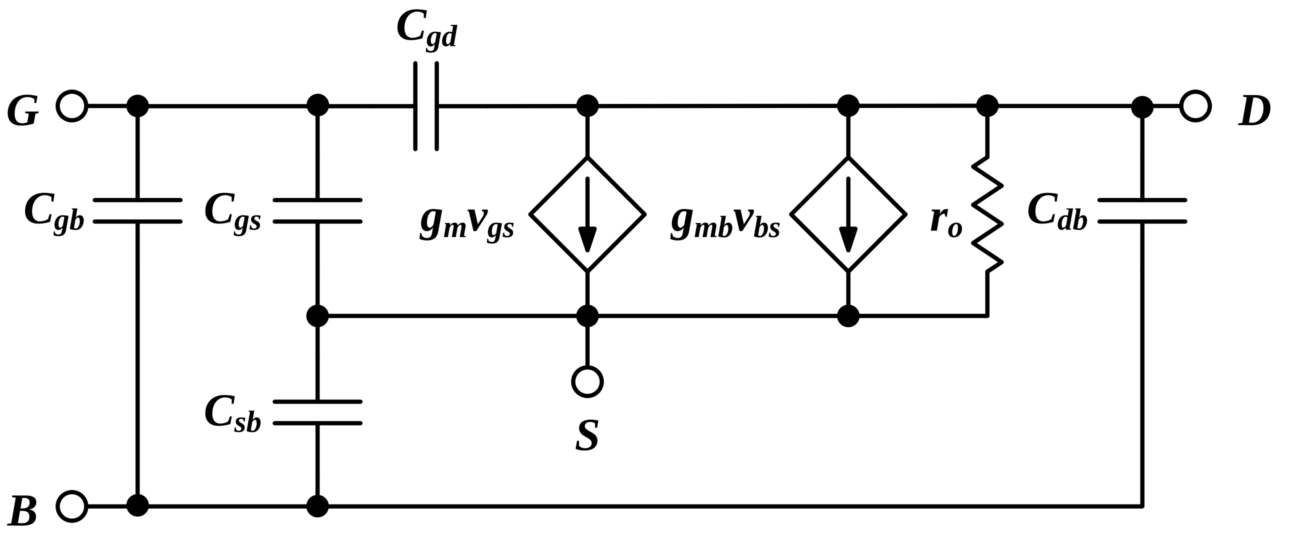
High Frequency Small Signal Model

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \mu C_{ox}\frac{W}{L}V_{ov} = \sqrt{\mu C_{ox}\frac{W}{L}\cdot 2I_D} = \frac{2I_D}{V_{ov}}$$

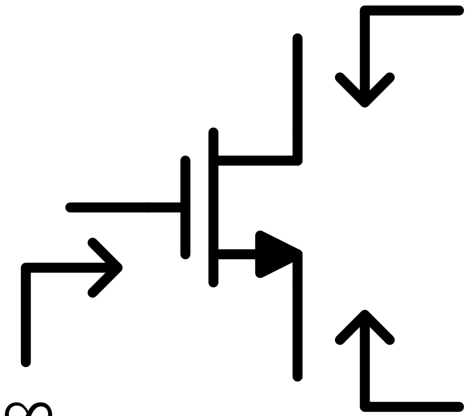
$$g_{mb} = \eta g_m \qquad \eta \approx 0.1 - 0.25$$

$$r_o = \frac{1}{\partial I_D/\partial V_{DS}} = \frac{V_A}{I_D} = \frac{1}{\lambda I_D} \qquad V_A \propto L \leftrightarrow \lambda \propto \frac{1}{L} \qquad V_{DS}\uparrow V_A \uparrow$$

$$C_{gb} \approx 0 \qquad C_{gs} \gg C_{gd} \qquad C_{sb} > C_{db}$$



Rin/out Shortcuts Summary



$$r_o[1 + (g_m + g_{mb})R_S]$$

H.I.N.

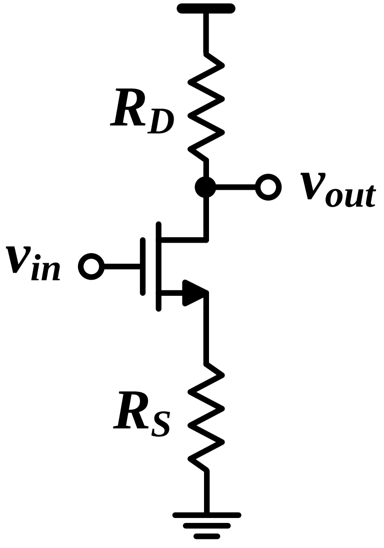
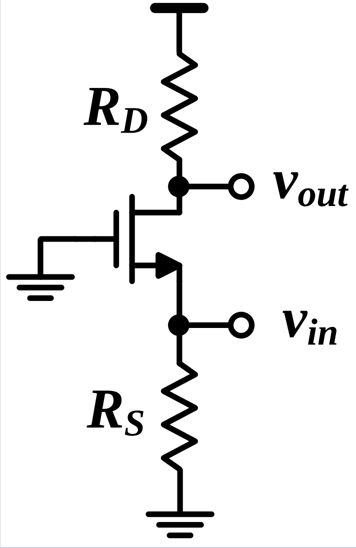
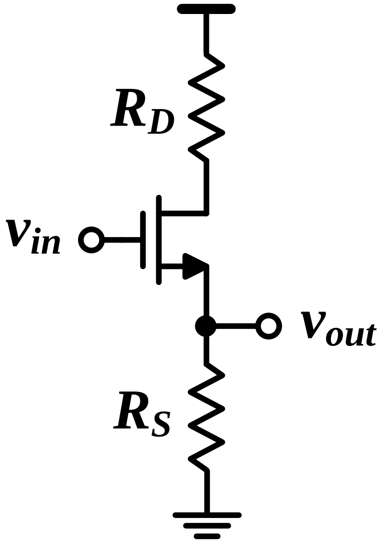
∞

At low frequencies **ONLY**

$$\frac{1}{g_m + g_{mb}}\left(1 + \frac{R_D}{r_o}\right)$$

L.I.N.

Summary of Basic Topologies

	CS	CG	CD (SF)
			
	Voltage & current amplifier	Voltage amplifier Current buffer	Voltage buffer Current amplifier
Rin	∞	$R_S \vert\vert \dfrac{1}{g_m + g_{mb}}\left(1 + \dfrac{R_D}{r_o}\right)$	∞
Rout	$R_D \vert\vert r_o[1 + (g_m + g_{mb})R_S]$	$R_D \vert\vert r_o$	$R_S \vert\vert \dfrac{1}{g_m + g_{mb}}\left(1 + \dfrac{R_D}{r_o}\right)$
Gm	$\dfrac{-g_m}{1 + (g_m + g_{mb})R_S}$	$g_m + g_{mb}$	$\dfrac{g_m}{1 + R_D/r_o}$

Differential Amplifier

	Pseudo Diff Amp	Diff Pair (w/ ideal CS)	Diff Pair (w/ R_{SS})
A_{vd}	$-g_m R_D$	$-g_m R_D$	$-g_m R_D$
A_{vCM}	$-g_m R_D$	0	$\frac{-g_m R_D}{1 + 2(g_m + g_{mb})R_{SS}}$
A_{vd}/A_{vCM}	1	∞	$2(g_m + g_{mb})R_{SS} \gg 1$

$$A_{vCM2d} = \frac{v_{od}}{v_{iCM}} \approx \frac{\Delta R_D}{2R_{SS}} + \frac{\Delta g_m R_D}{2g_{m1,2}R_{SS}}$$

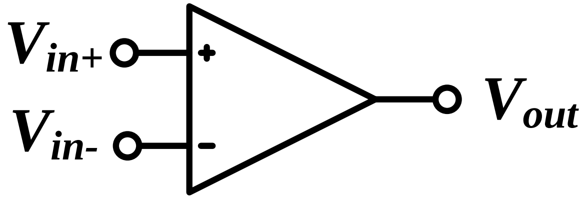
$$CMRR = \frac{A_{vd}}{A_{vCM2d}}$$

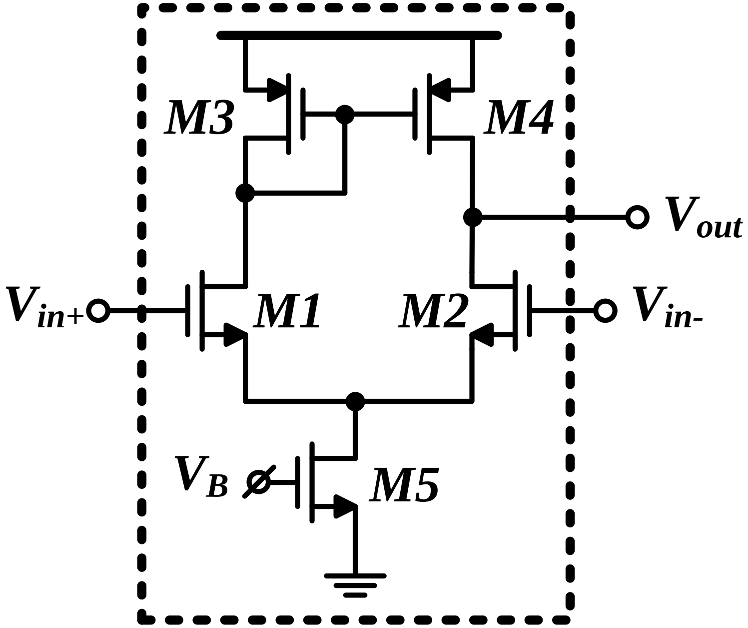
Outline

- Recapping previous key results
- Op-amp vs OTA
- Differential to single-ended conversion
- Five-transistor OTA Analysis
  - A. Small signal analysis
    1. Diff small signal analysis
    2. CM small signal analysis
  - B. Large signal analysis
    1. Diff large signal analysis
    2. CM large signal analysis
- Effect of mismatch
- 5T OTA frequency response

Op-Amp

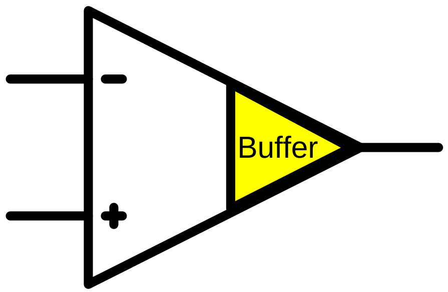
- An op-amp is simply a high gain differential amplifier
  - The gain can be increased by using cascodes and multi-stage amplification
- The diff amp is a key block in many analog and RF circuits
  - DEEP understanding of diff amp is ESSENTIAL





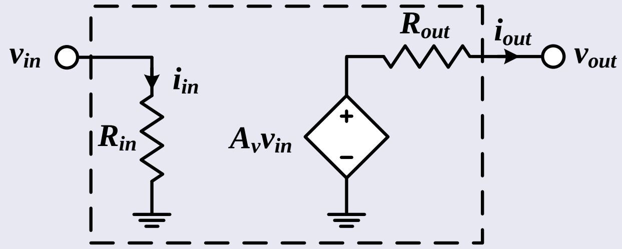
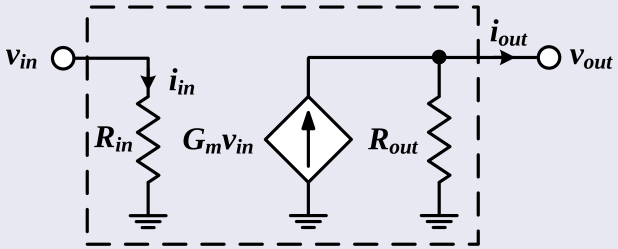
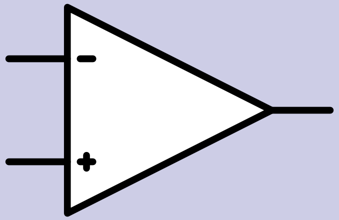
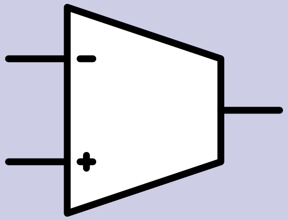
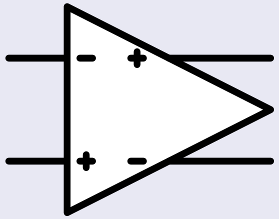
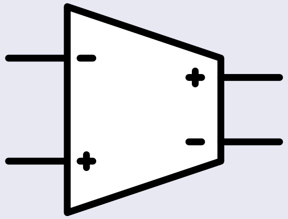
Op-Amp vs OTA

- Ideal op-amp has infinite R_{in}, infinite gain, and zero R_{out}
- Practical op-amp has HIGH R_{in}, HIGH gain, and LOW R_{out}
  - LOW R_{out} required to avoid loading when driving resistive loads
  - The op-amp is usually implemented as a multistage amplifier
  - The last stage (output stage) is a buffer to provide LOW R_{out}
- IC CMOS op-amps usually drive capacitive loads
  - Usually, there is no need for LOW R_{out}
  - The output stage (buffer) is not required
  - These modern integrated op-amps are usually called Operational Transconductance Amplifiers (OTAs)



Op-Amp vs OTA

- In short, an OTA is an op-amp without an output stage (buffer)
- Some designers just use op-amp name and symbol for both

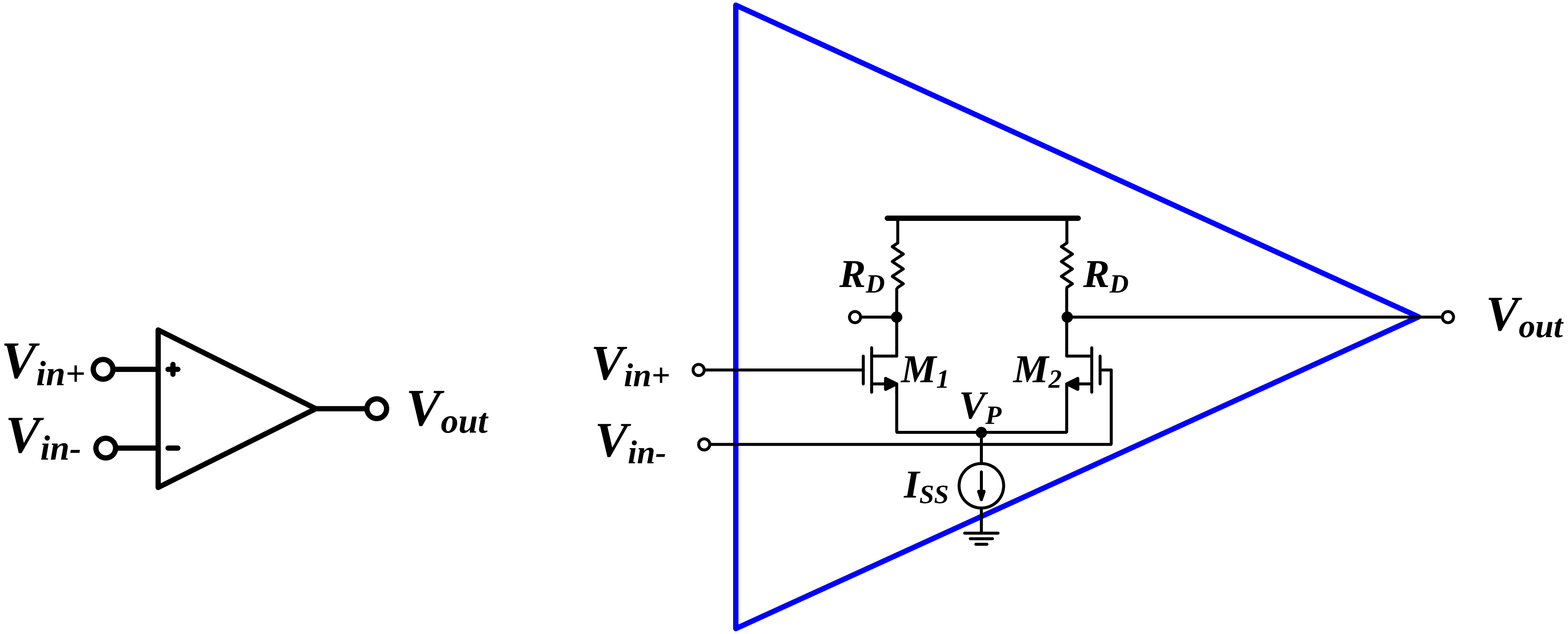
	Op-amp	OTA
Rout	LOW	HIGH
Model		
Diff input, SE output		
Fully diff		

Outline

- ❑ Recapping previous key results
- ❑ Op-amp vs OTA
- ❑ Differential to single-ended conversion
- ❑ Five-transistor OTA Analysis
  - A. Small signal analysis
    1. Diff small signal analysis
    2. CM small signal analysis
  - B. Large signal analysis
    1. Diff large signal analysis
    2. CM large signal analysis
- ❑ Effect of mismatch
- ❑ 5T OTA frequency response

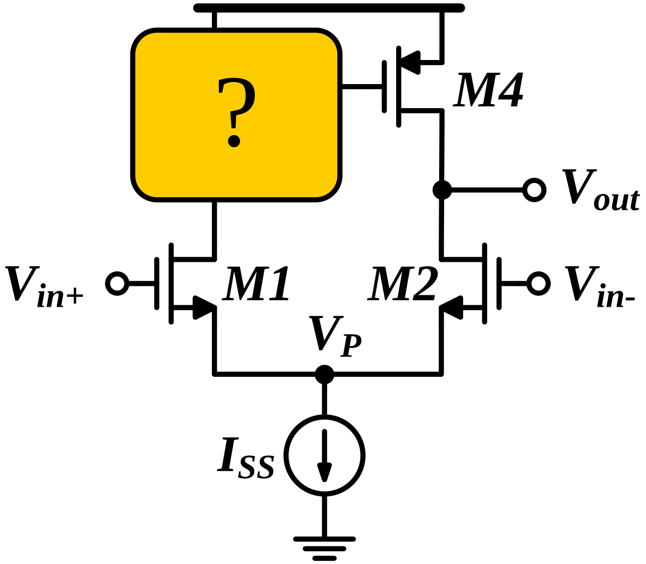
How to Get SE Output?

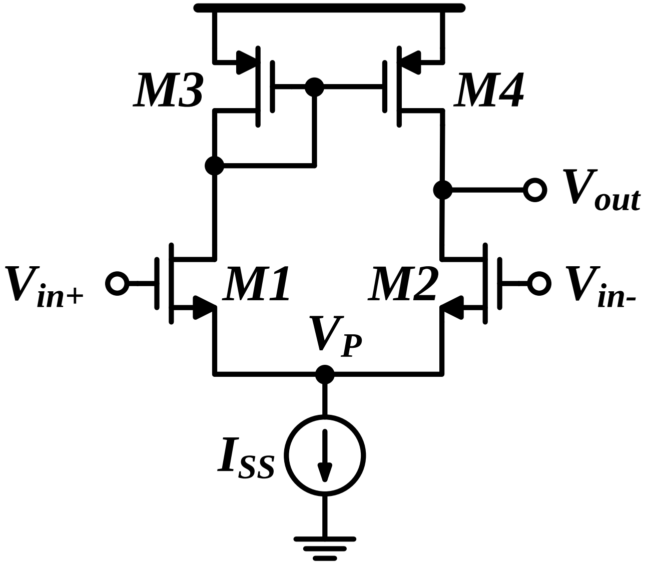
- Trivial solution: discard one output!
  - But the gain is halved (and the CMRR is poor $\sim g_m R_{SS}$)
- Better solution: use a diff to single-ended (SE) converter (but how?!)



5T OTA

- A.k.a. diff pair with active load, diff pair with CM load, unbalanced diff pair.
- Can be viewed in two ways
  1. We use a current mirror to transfer the small signal current from the left side to the right side.
  2. M3 is a diode-connected load and M4 is a CS amplifier that transfers the small signal voltage from the left side to the right side.





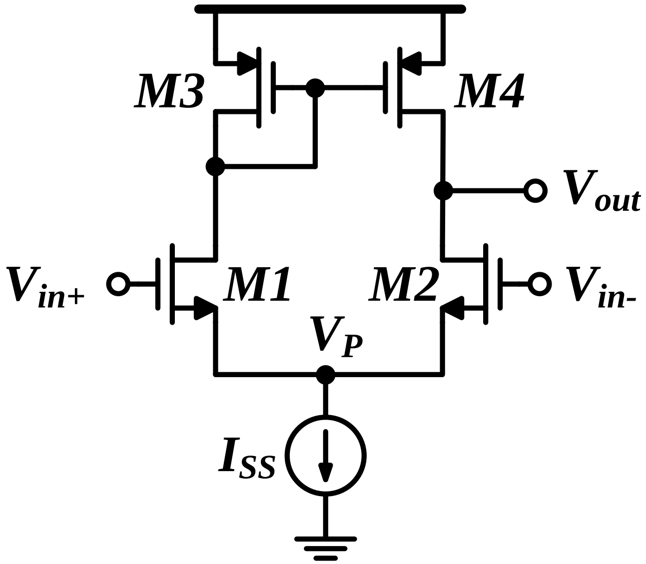
Outline

- ❑ Recapping previous key results
- ❑ Op-amp vs OTA
- ❑ Differential to single-ended conversion
- ❑ Five-transistor OTA Analysis
  - A. Small signal analysis
    1. Diff small signal analysis
    2. CM small signal analysis
  - B. Large signal analysis
    1. Diff large signal analysis
    2. CM large signal analysis
- ❑ Effect of mismatch
- ❑ 5T OTA frequency response

5T OTA Analysis

- In general, half-circuit principle cannot be used due to asymmetry

- Similar to diff amp, we carry four types of analysis:
  A. Small signal analysis
     1. Diff small signal analysis
     2. CM small signal analysis
  B. Large signal analysis
     1. Diff large signal analysis
     2. CM large signal analysis



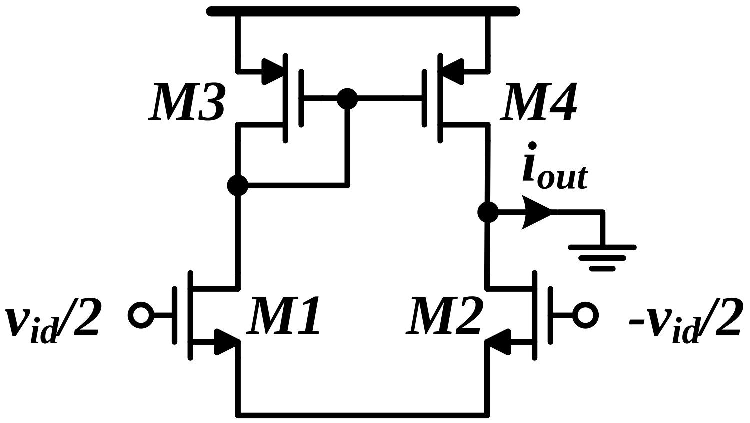
A1. Diff Small Signal Analysis

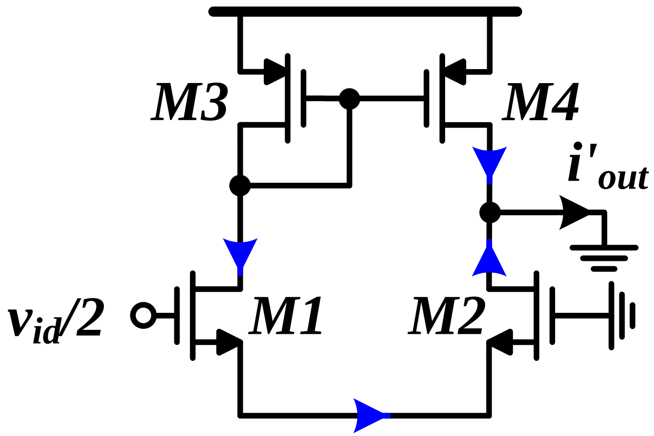
- Solve it using superposition as a whole using GmRout (Can we use half-circuit? Why?)
- Each side contributes to i_{out} twice: directly and through the mirror

$$i_{out} = i'_{out} + i''_{out}$$

$$i_{out} \approx \frac{g_{m1}}{1 + g_{m1}(1/g_{m2})} \cdot \mathbf{2} \cdot \frac{v_{id}}{2} + \frac{g_{m2}}{1 + g_{m2}(1/g_{m1})} \cdot \mathbf{2} \cdot \frac{v_{id}}{2}$$

$$G_m = \frac{i_{out}}{v_{id}} \approx g_{m1,2}$$





A1. Diff Small Signal Analysis

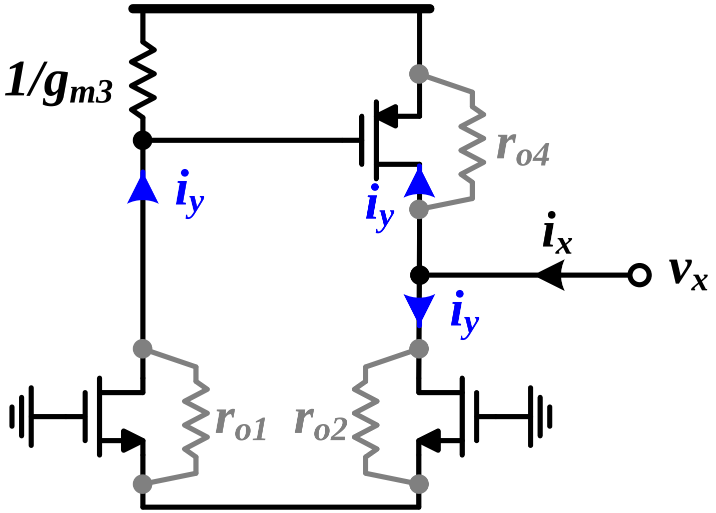
- Can we use half-circuit? Why?

$$R_{out} = \frac{v_x}{i_x} = \frac{v_x}{2i_y}||r_{o4}$$

$$\frac{v_x}{i_y} = R_{LFD,M2} \approx r_{o2}\left(1 + g_{m2}R_{LFS,M1}\right) \approx 2r_{o2}$$

$$R_{out} \approx r_{o2}||r_{o4}$$

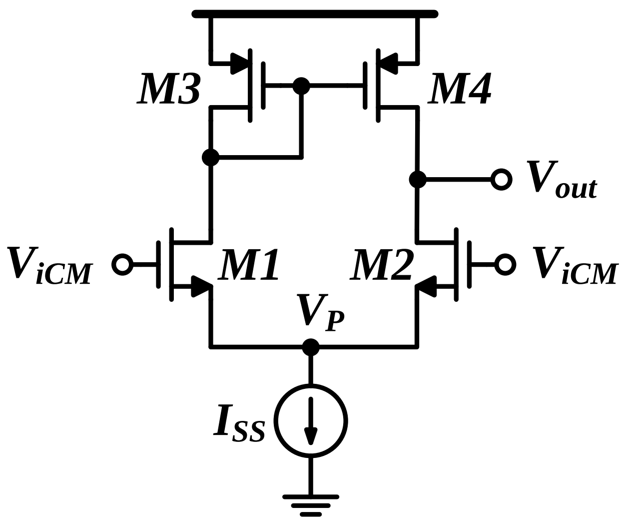
$$A_{vd} = \frac{v_{out}}{v_{id}} \approx g_{m1,2}(r_{o2}||r_{o4})$$

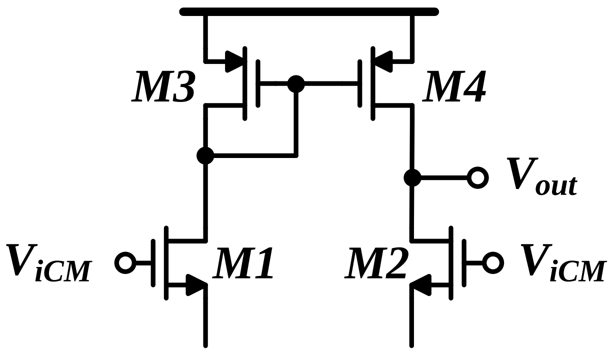


A2. CM Small Signal Analysis

- If the tail CS is ideal
  - The two sides will generate current in the same direction
  - Thus both currents must be zero

$$v_{out} = 0 \;\Rightarrow\; A_{vCM} = \frac{v_{out}}{v_{iCM}} = 0 \;\Rightarrow\; CMRR = \frac{A_{vd}}{A_{vCM}} \to \infty$$





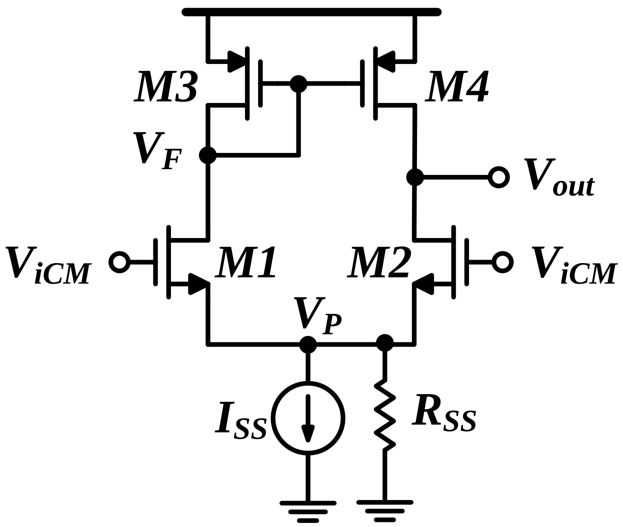
A2. CM Small Signal Analysis

- For non-ideal tail CS: $A_{vCM} \neq 0$ and $CMRR \neq \infty$
- The presence of R_{SS} complicates the analysis. Using SLiCAP yields:

$$H_s = -\frac{1.0\, g_{\mathrm{m12}}\, r_{\mathrm{o12}}\, r_{\mathrm{o34}}}{2.0\, R_{\mathrm{SS}} + r_{\mathrm{o12}} + r_{\mathrm{o34}} + 2.0\, R_{\mathrm{SS}}\, g_{\mathrm{m12}}\, r_{\mathrm{o12}} + 2.0\, R_{\mathrm{SS}}\, g_{\mathrm{m34}}\, r_{\mathrm{o34}} + g_{\mathrm{m34}}\, r_{\mathrm{o12}}\, r_{\mathrm{o34}} + 2.0\, R_{\mathrm{SS}}\, g_{\mathrm{m12}}\, g_{\mathrm{m34}}\, r_{\mathrm{o12}}\, r_{\mathrm{o34}}}$$

$$A_{vCM} \approx -\frac{1}{2 g_{m3,4} R_{SS}}$$

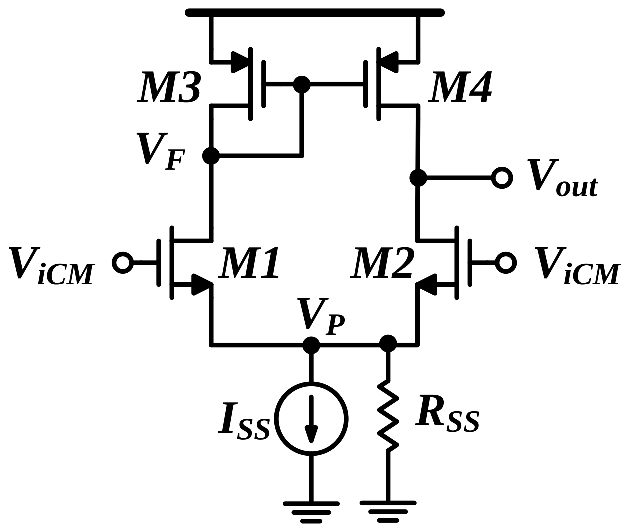
- Can we derive the same result intuitively?

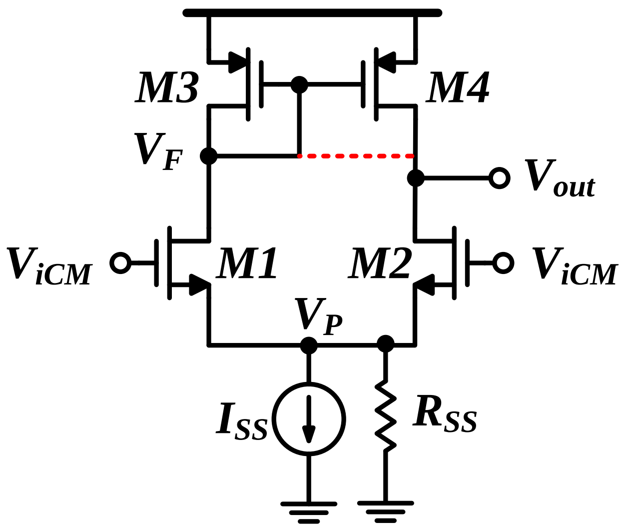


[www.analog-electronics.eu/slicap/slicap.html]

A2. CM Small Signal Analysis

- M1 and M2 have the same V_{GS}: Difference in I_D depends on V_{DS}
- M3 and M4 have the same V_{GS}: Difference in I_D depends on V_{DS}
- Assume $V_F < V_{out}$
  - $I_{D1} < I_{D2}$ and $I_{D3} > I_{D4}$ → Not a valid assumption
- Assume $V_F > V_{out}$
  - $I_{D1} > I_{D2}$ and $I_{D3} < I_{D4}$ → Not a valid assumption
- The only valid assumption is $V_F = V_{out}$ → As if there is a s.c.!



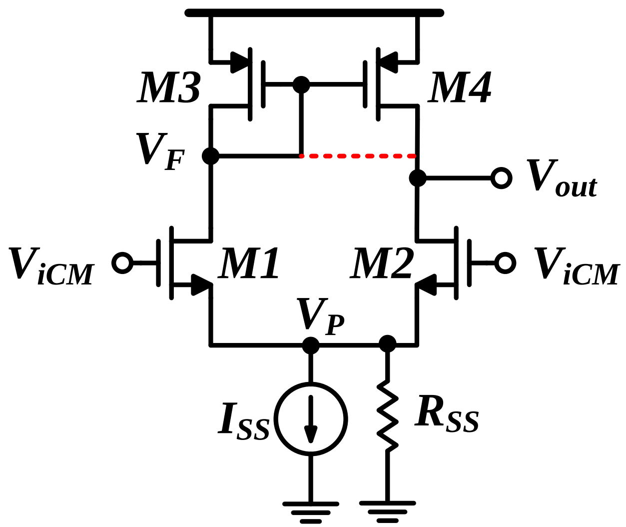


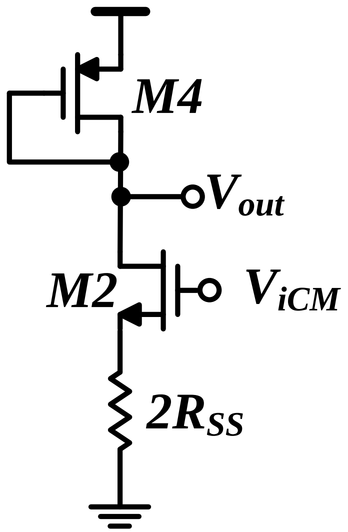
A2. CM Small Signal Analysis

- For CM input, V_{out} follows V_F → As if there is a s.c. between them
- The circuit becomes symmetric → Apply half circuit principle

$$A_{vCM} = \frac{V_{out}}{V_{iCM}} = G_m R_{out} \approx \frac{-g_{m1,2}}{1 + 2g_{m1,2}R_{SS}} \frac{1}{g_{m3,4}} \approx -\frac{1}{2g_{m3,4}R_{SS}}$$

- Same as SLiCAP's result ☺

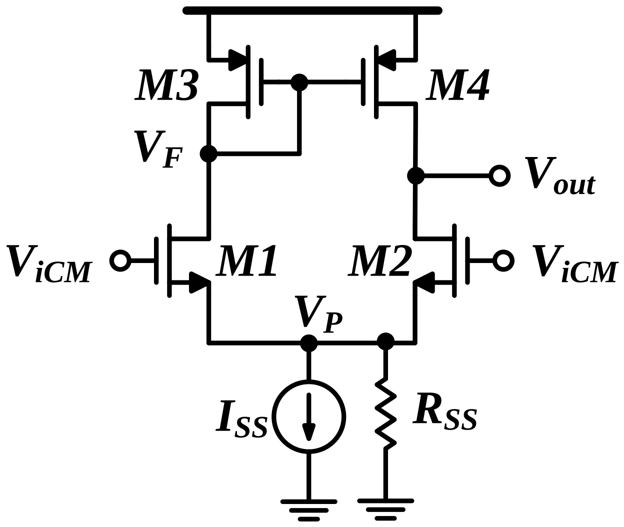


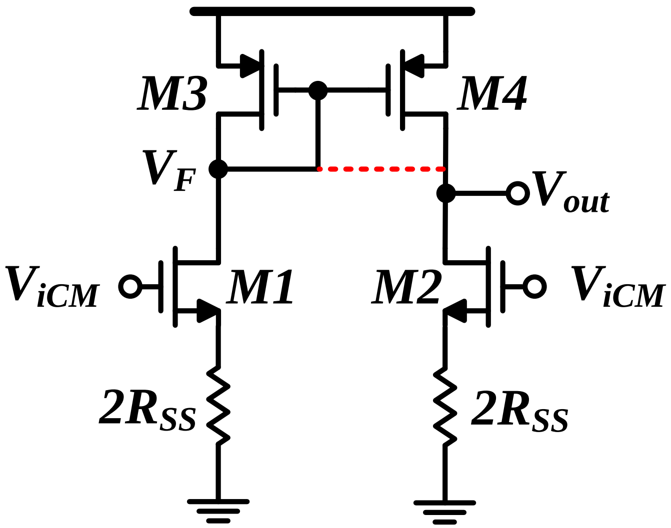


A2. CM Small Signal Analysis

- **Method #2:** Ignore $r_{o1,2}$ for simplicity (CLM of M1,2 ignored) → apply half-circuit at the sources
- M1 and M2 generate the same current (Δi_d) → $\therefore$ M3 and M4 have the same current → $\therefore$ Not just their V_{GS} is equal, but their V_{DS} must be equal → $\therefore$ $V_{out} = V_F$
- s.c. between V_{out} and V_F → apply half-circuit at the drains

$$A_{vCM} = \frac{V_{out}}{V_{iCM}} = G_m R_{out} \approx \frac{-g_{m1,2}}{1 + 2g_{m1,2}R_{SS}} \frac{1}{g_{m3,4}} \approx -\frac{1}{2g_{m3,4}R_{SS}}$$





A2. CM Small Signal Analysis

- Strictly speaking, R_{out} for diff and cm must be the same!
  - R_{out} is independent of input type and location (we already deactivate the input when we calculate Rout)

$$R_{out} \approx r_{o2}||r_{o4}$$

- It can be shown that G_m is actually given by

$$G_m \approx \frac{-g_{m1,2}}{1+2g_{m1,2}R_{SS}} \frac{1}{g_{m3,4}(r_{o2}//r_{o4})} \approx -\frac{1}{2g_{m3,4}R_{SS}(r_{o2}//r_{o4})}$$

  - An intuitive derivation is not very easy.

$$A_{vCM} = \frac{V_{out}}{V_{iCM}} = G_m R_{out} \approx -\frac{1}{2g_{m3,4}R_{SS}}$$

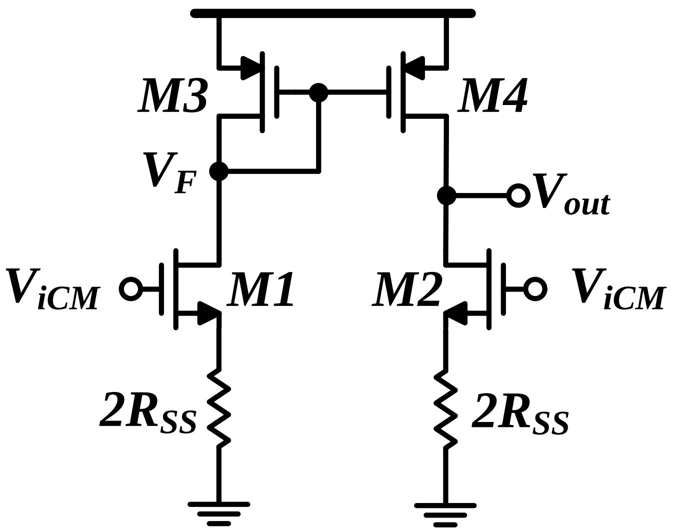
A2. CM Small Signal Analysis

- **Method #3:** Use GmRout (Neglect $r_{o1,2}$ BUT do NOT neglect r_{o3}, otherwise $A_{vCM} \to 0$)

$$G_m = \frac{i_{o,sc}}{v_{iCM}} = \frac{g_{m1}}{1+2g_{m1}R_{SS}} \cdot \left(\frac{1}{g_{m3}}||r_{o3}\right) \cdot g_{m4} - \frac{g_{m2}}{1+2g_{m2}R_{SS}}$$

$$= \frac{g_{m1,2}}{1+2g_{m1,2}R_{SS}} \left(\frac{g_{m3,4}r_{o3,4}}{1+g_{m3,4}r_{o3,4}} - 1\right) \approx \frac{-1}{2g_{m3,4}r_{o3,4}R_{SS}}$$

$$A_{vCM} = G_m R_{out} \approx \frac{-1}{2g_{m3,4}r_{o3,4}R_{SS}} \cdot r_{o4} = -\frac{1}{2g_{m3,4}R_{SS}}$$



CMRR

$$A_{vd} = \frac{V_{out}}{V_{id}} \approx g_{m1,2}(r_{o2}//r_{o4})$$

$$A_{vCM} = \frac{V_{out}}{V_{iCM}} \approx -\frac{1}{2g_{m3,4}R_{SS}}$$

$$CMRR \approx g_{m1,2}(r_{o2}//r_{o4}) \cdot 2g_{m3,4}R_{SS} \sim (g_m r_o)^2$$

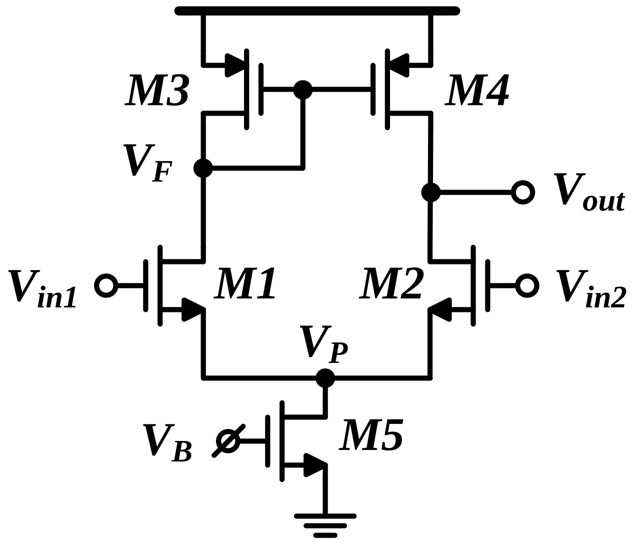
- The CMRR is much better than the trivial case of getting SE output by dropping one differential output
  - For the trivial case $CMRR \approx g_m R_{SS} \sim g_m r_o$
- CM noise will affect the SE output even in the case of perfect symmetry
  - A clear disadvantage compared to fully diff OTA
- The effective R_{SS} degrades at high frequency

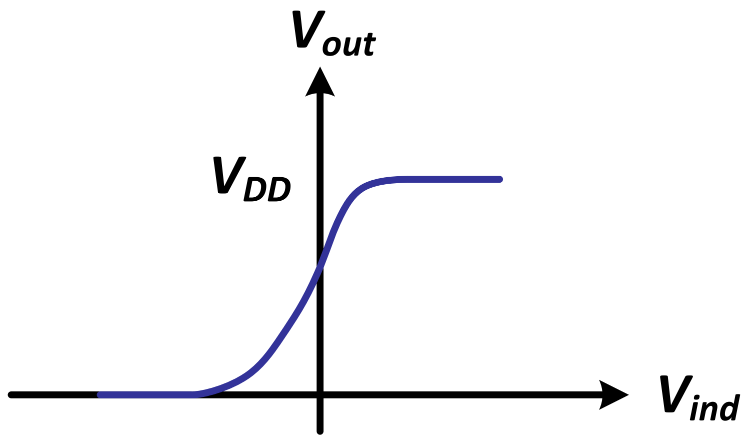
Outline

- Recapping previous key results
- Op-amp vs OTA
- Differential to single-ended conversion
- Five-transistor OTA Analysis
  A. Small signal analysis
     1. Diff small signal analysis
     2. CM small signal analysis
  B. Large signal analysis
     1. Diff large signal analysis
     2. CM large signal analysis
- Effect of mismatch
- 5T OTA frequency response

B1. Diff Large Signal Analysis

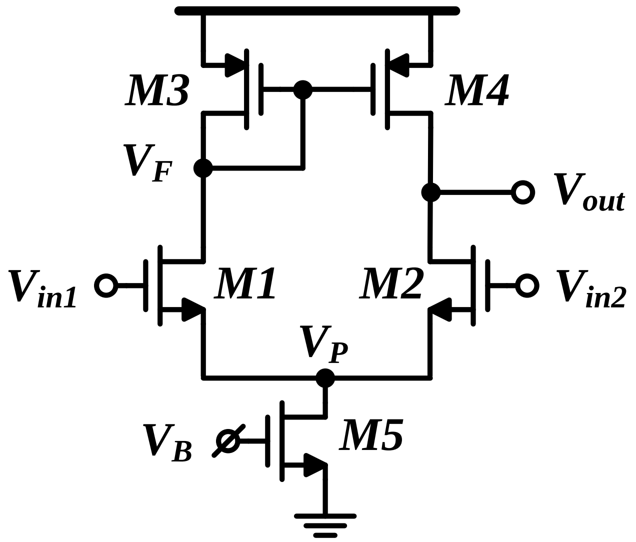
- $V_{id} = (V_{in1} - V_{in2}) \gg 0$
  - M1 and M3 ON → M4 ON (triode) → small resistance
  - M2 OFF → large resistance
  - $V_{out} \approx V_{DD}$

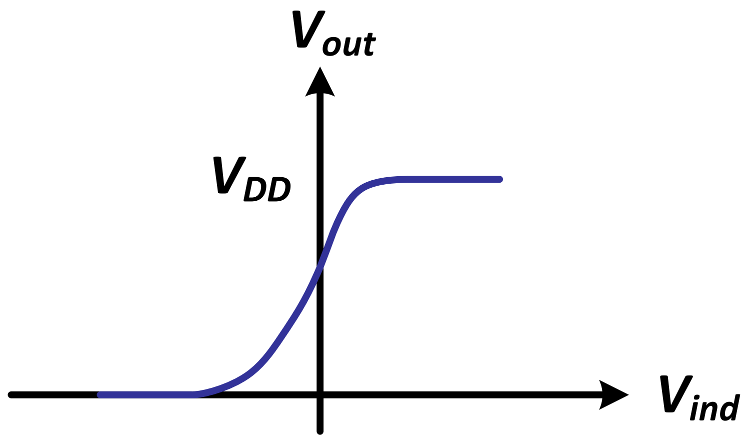




B1. Diff Large Signal Analysis

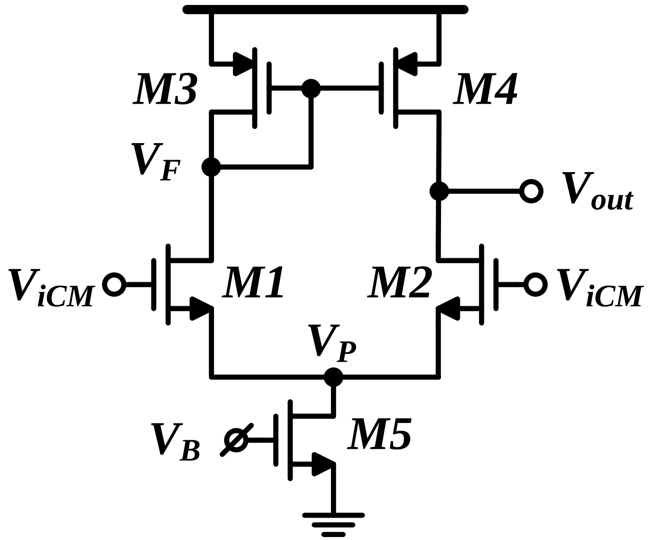
- $V_{id} = (V_{in1} - V_{in2}) \ll 0$
  - M1 and M3 OFF → M4 OFF → large resistance
  - M2 and M5 ON (triode) → small resistance
  - $V_{out} \approx 0$





B2. CM Large Signal Analysis

- All transistors must be in saturation



- Tail CS in sat

$$V_{iCM} \geq V_{THN} + V_{ov1} + V_{ov5}$$

- Input pair in sat

$$V_{iCM} \leq V_{DD} - |\boldsymbol{V_{THP}}| - |V_{ov3}| + V_{THN}$$

Outline

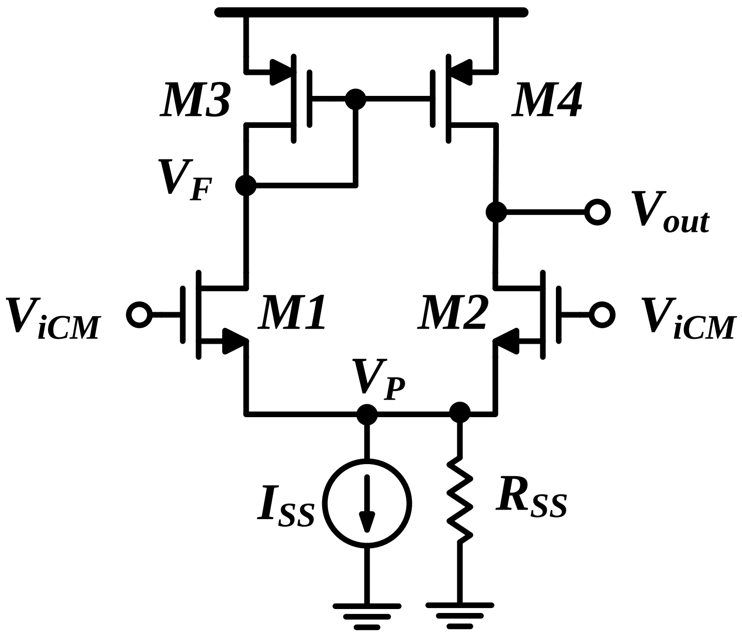
- Recapping previous key results
- Op-amp vs OTA
- Differential to single-ended conversion
- Five-transistor OTA Analysis
  - A. Small signal analysis
    1. Diff small signal analysis
    2. CM small signal analysis
  - B. Large signal analysis
    1. Diff large signal analysis
    2. CM large signal analysis
- Effect of mismatch
- 5T OTA frequency response

Effect of Mismatch (in Input Pair)

- Use superposition (left + right)

$$A_{vCM} = \frac{v'_{out}}{v_{iCM}} + \frac{v''_{out}}{v_{iCM}} \approx \left(\frac{\mathbf{2} g_{m1}}{1 + g_{m1}\left(\frac{1}{g_{m2}} || R_{SS}\right)} - \frac{\mathbf{2} g_{m2}}{1 + g_{m2}\left(\frac{1}{g_{m1}} || R_{SS}\right)} \right) R_{out}$$

$$= \frac{2\Delta g_m R_{out}}{1 + (g_{m1} + g_{m2}) R_{SS}} \approx \frac{\Delta g_m R_{out}}{g_{m1,2} R_{SS}}$$



CMRR with Mismatch

- Overall CM small signal response (matched + mismatch)

$$A_{vCM} \approx \frac{-\frac{g_{m1,2}}{g_{m3,4}}}{1+2g_{m1,2}R_{SS}} + \frac{2\Delta g_m R_{out}}{1+2g_{m1,2}R_{SS}} \approx \frac{-\frac{g_{m1,2}}{g_{m3,4}} + 2\Delta g_m R_{out}}{2g_{m1,2}R_{SS}}$$

- Common-mode rejection ratio (CMRR) (@low frequency!)

$$A_{vd} = g_{m1,2}R_{out}$$

$$CMRR = \frac{A_{vd}}{A_{vCM}} \approx \frac{2g_{m1,2}R_{SS}}{-\frac{1}{g_{m3,4}R_{out}} + 2\frac{\Delta g_m}{g_{m1,2}}}$$

Outline

- Recapping previous key results
- Op-amp vs OTA
- Differential to single-ended conversion
- Five-transistor OTA Analysis
  - A. Small signal analysis
    1. Diff small signal analysis
    2. CM small signal analysis
  - B. Large signal analysis
    1. Diff large signal analysis
    2. CM large signal analysis
- Effect of mismatch
- 5T OTA frequency response

Differential Frequency Response: Poles

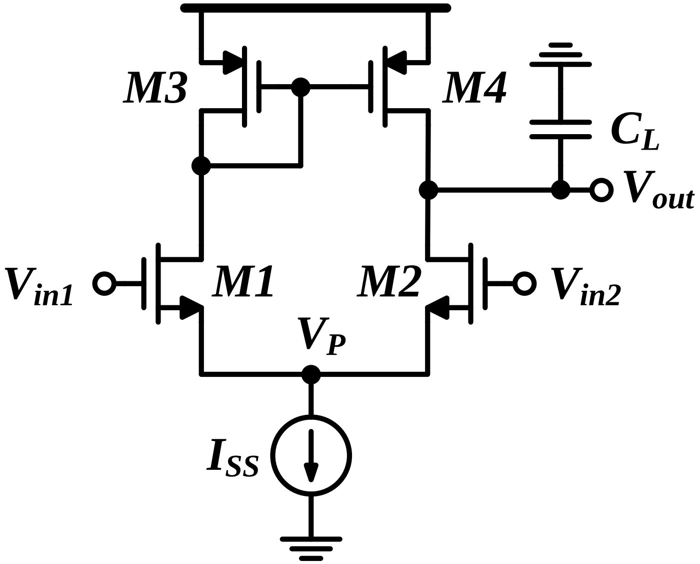
- Output pole: $C_{out} \approx C_{db4} + C_{db2} + f\left(C_{gd4}\right) + C_{gd2} + C_L$

$$\omega_{p1} \approx \frac{1}{R_{out} C_{out}}$$

- Mirror pole: $C_{mirr} \approx C_{gs3} + C_{gs4} + C_{db3} + C_{db1} + C_{gd1} + f\left(C_{gd4}\right)$

$$\omega_{p2} \approx \frac{g_{m3}}{C_{mirr}}$$

v_p acts as virtual ground at high frequency (why?)



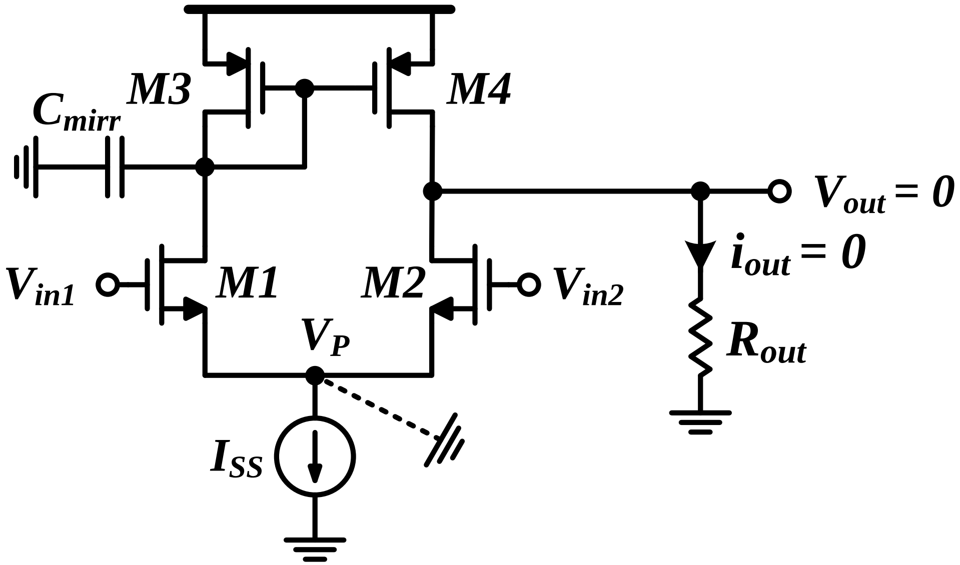
Differential Frequency Response: LHP Zero

- At s_z: $v_{out} = 0 \Rightarrow i_{out} = 0 \Rightarrow g_{m4}v_{sg4} = g_{m2}v_{gs2}$

$$g_{m1}\left(\frac{1}{g_{m3}}||\frac{1}{s_z C_{mirr}}\right)\cdot g_{m4}\cdot v_{in1} = g_{m2}\cdot v_{in2}$$

$$s_z = -\frac{2g_{m3,4}}{C_{mirr}} \Rightarrow \ \omega_z = 2\omega_{p2}$$

- Can we get same result by inspection?
  - What happens to $i_{out,sc}$?



v_p acts as virtual ground at high frequency (why?)

Thank you!

References

- A. Sedra and K. Smith, “Microelectronic Circuits,” 7th ed., Oxford University Press, 2015
- B. Razavi, “Fundamentals of Microelectronics,” 2nd ed., Wiley, 2014
- B. Razavi, “Design of Analog CMOS Integrated Circuits,” McGraw-Hill, 2nd ed., 2017
- T. C. Carusone, D. Johns, and K. W. Martin, “Analog Integrated Circuit Design,” 2nd ed., Wiley, 2012