وَمَا أُوتِيتُمْ مِنَ الْعِلْمِ إِلَّا قَلِيلًا

Analog IC Design

Lecture 02
Review on Circuits and Systems Basics

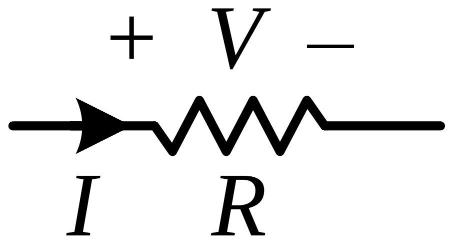
Dr. Hesham A. Omran

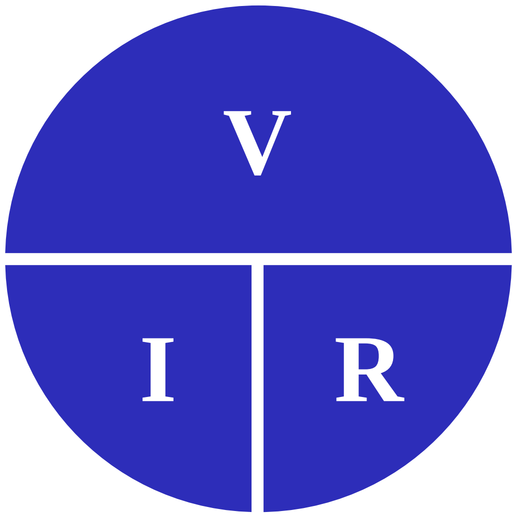
Integrated Circuits Lab (ICL)
Electronics and Communications Eng. Dept.
Faculty of Engineering
Ain Shams University

Outline

- Circuits review
  - Ohm’s law, KCL, KVL
  - Thevenin and Norton equivalents
  - Superposition
  - Capacitance
- Systems review
  - Laplace transform
  - Poles and zeros
  - Frequency response
  - First-order system
  - Second-order system

Ohm's Law





$$V = IR$$

$$I = \frac{V}{R}$$

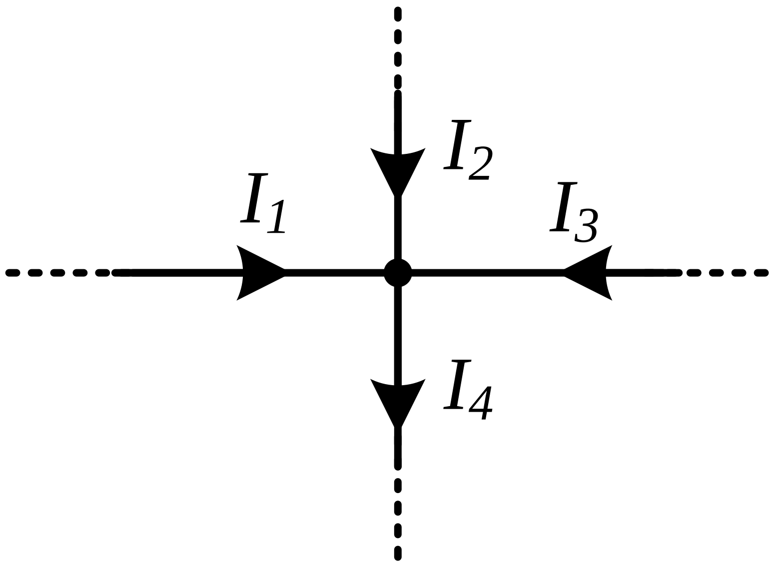
$$R = \frac{V}{I}$$

Kirchhoff's Current Law (KCL)

- The sum of all currents flowing into a node is zero.

$$\Sigma I = 0$$

$$I_1 + I_2 + I_3 - I_4 = 0$$



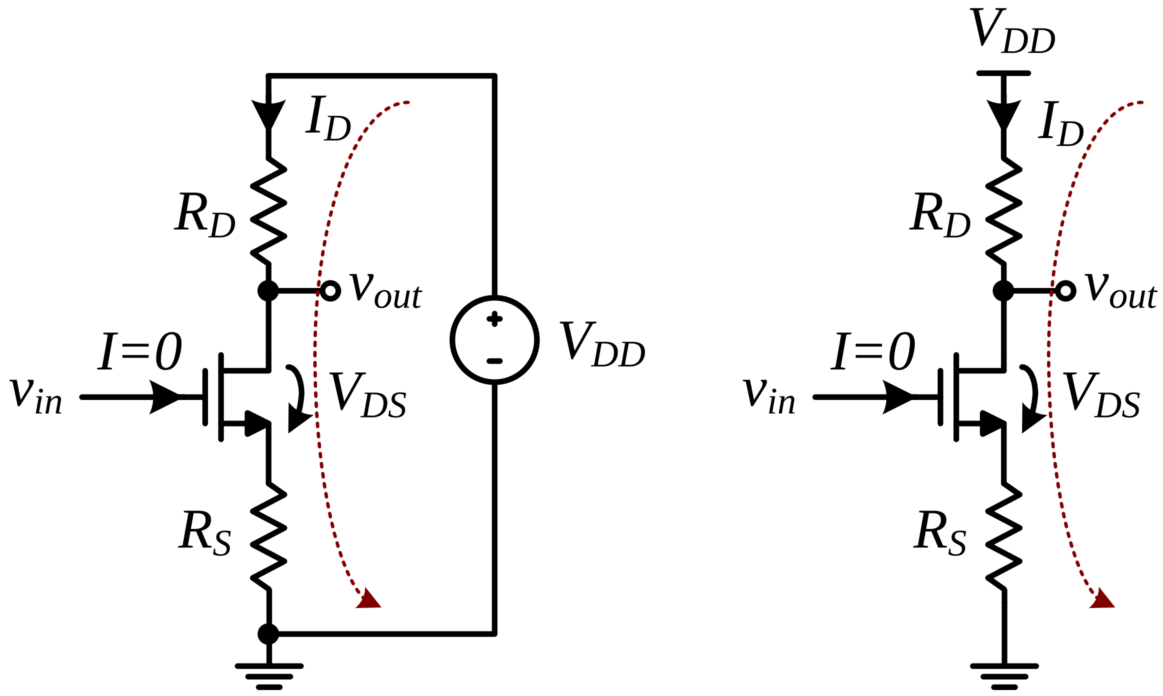
Kirchhoff's Voltage Law (KVL)

- The sum of all voltage drops around any closed loop is zero

$$\Sigma V = 0$$

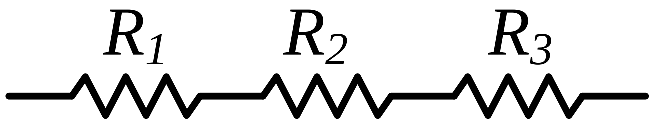
$$-V_{DD} + I_D R_D + V_{DS} + I_D R_S = 0$$

$$V_{DD} = I_D (R_D + R_S) + V_{DS}$$



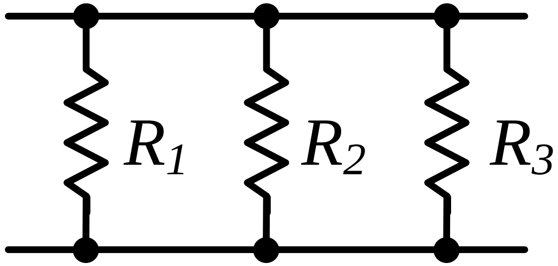
Resistor Combinations

- Resistors in series: Largest resistor dominates



$$R_{eq} = R_1 + R_2 + R_3$$

- Resistors in parallel: Smallest resistor dominates



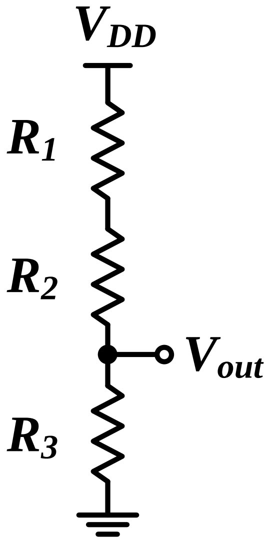
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

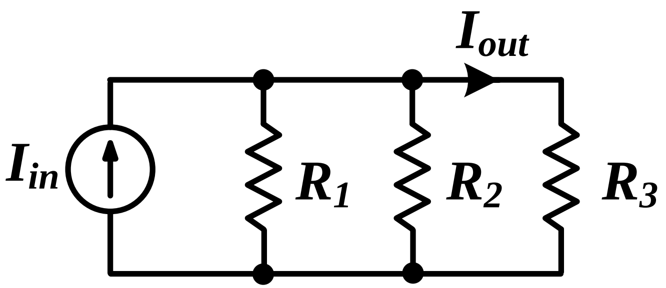
Voltage and Current Dividers

- Voltage divider → the largest resistor takes most of the voltage
- Current divider → the smallest resistor (largest conductance) takes most of the current
  - Remember that current flows in the least resistance path

$$V_{out} = V_{DD} \cdot \frac{R_3}{R_1 + R_2 + R_3}$$

$$I_{out} = I_{in} \cdot \frac{G_3}{G_1 + G_2 + G_3}$$



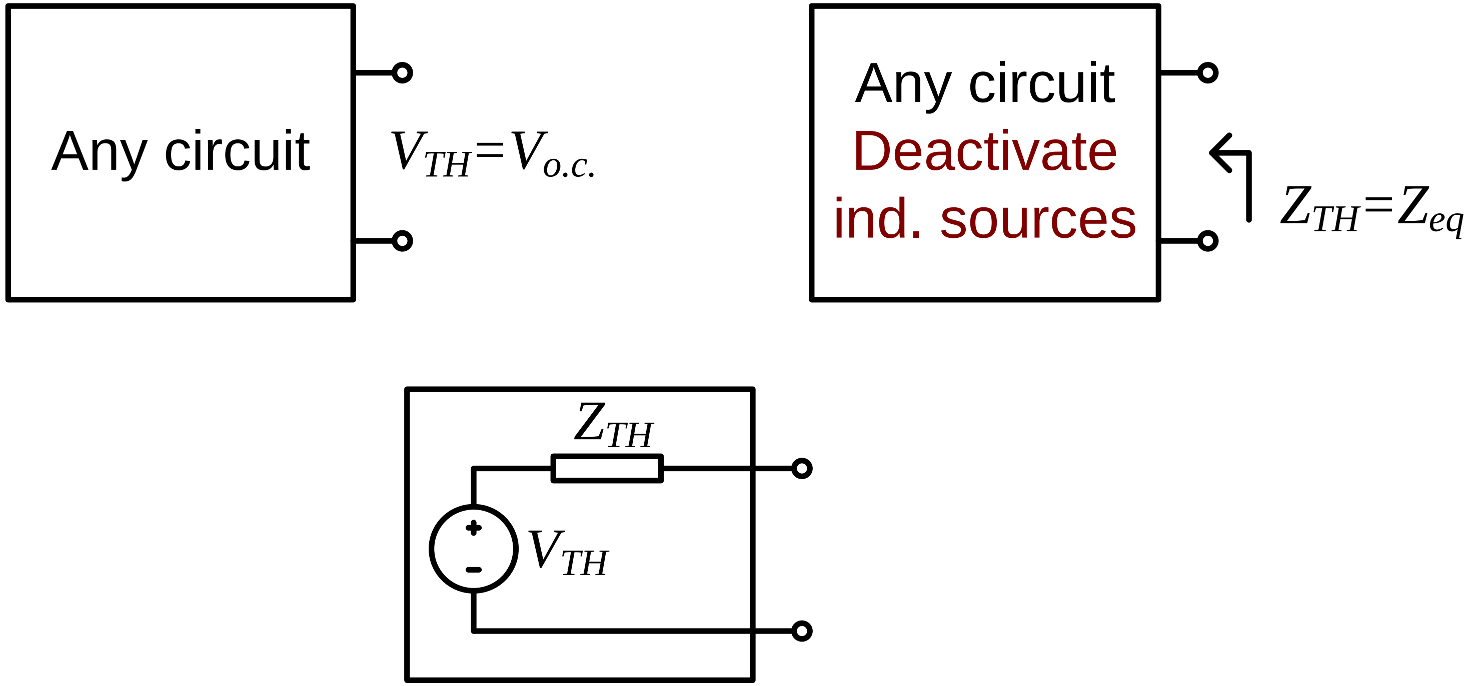


Thevenin Equivalent Circuit

- Any one port circuit can be replaced by a voltage source and a series impedance

$$V_{TH} = V_{o.c.}$$

$$Z_{TH} = Z_{eq} \text{ (turn OFF all independent sources)}$$



Norton Equivalent Circuit

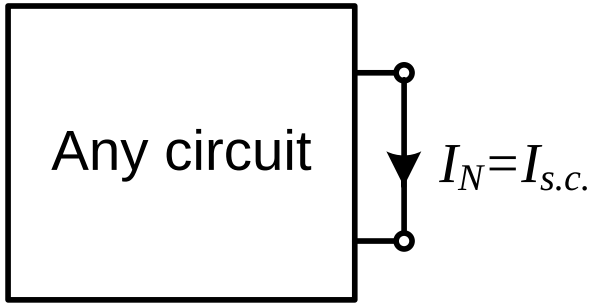
- Any one port circuit can be replaced by a current source and a parallel impedance

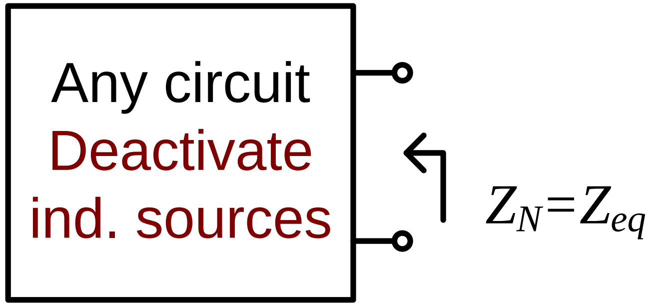
$$I_N = I_{s.c.}$$

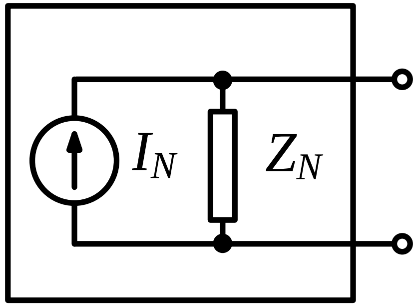
$$Z_N = Z_{eq} \text{ (turn OFF all independent sources)}$$

$$\boldsymbol{Z_N = Z_{TH}}$$

$$\boldsymbol{V_{TH} = V_{o.c.} = I_N \times Z_N}$$







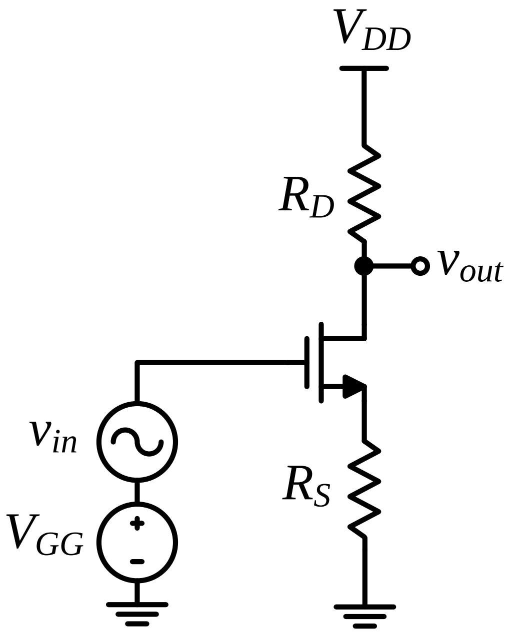
Superposition Theorem

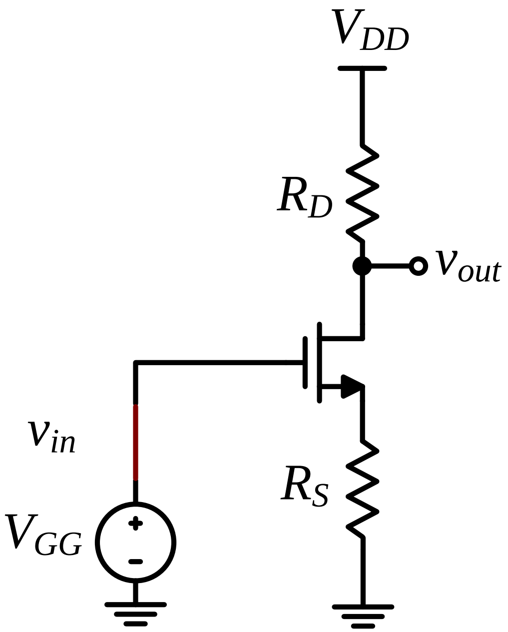
- Deactivate all independent sources except one
  - Independent voltage source → short circuit (s.c.)
  - Independent current source → open circuit (o.c.)
  - Do NOT deactivate dependent sources
- Solve the circuit
- Repeat the previous two steps for every source
- Algebraically add all the results

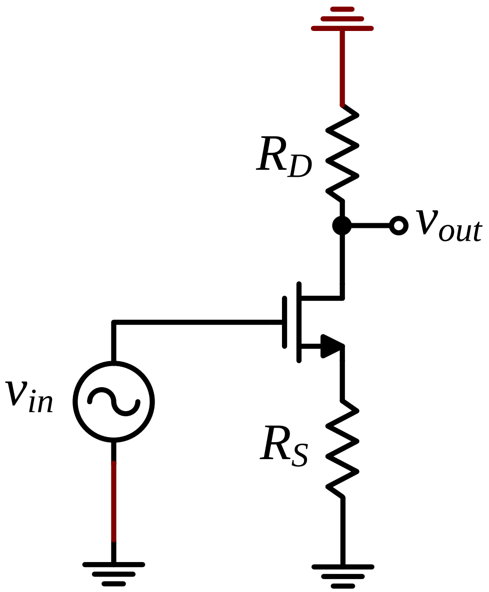
We use this frequently to separate DC and AC solutions

Superposition Theorem

DC + AC = DC + AC







Capacitance

$$Q = CV$$

$$i = \frac{dQ}{dt} = C\frac{dV}{dt}$$

$$V = V_o \cos\omega t = V_o \cdot Re\left\{e^{j\omega t}\right\} \Rightarrow V_o e^{j\omega t}$$

$$i = C\frac{dV}{dt} = j\omega C\left(V_o e^{j\omega t}\right) = j\omega C \cdot V$$

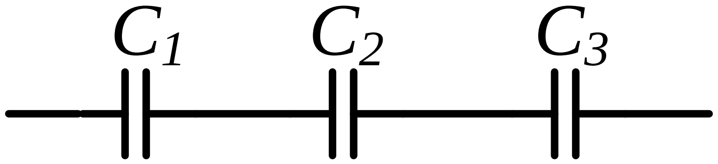
$$Z_C = \frac{V}{i} = \frac{1}{j\omega C} = \frac{1}{sC} \Rightarrow X_C = \frac{1}{\omega C}$$

$$\omega \uparrow\uparrow \Rightarrow X_C \to 0 \Rightarrow s.c.$$

$$\omega \downarrow\downarrow \Rightarrow X_C \to \infty \Rightarrow o.c.$$

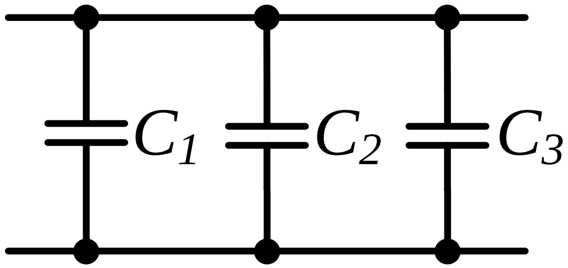
Capacitance Combinations

- Capacitors in series: Smallest capacitor dominates



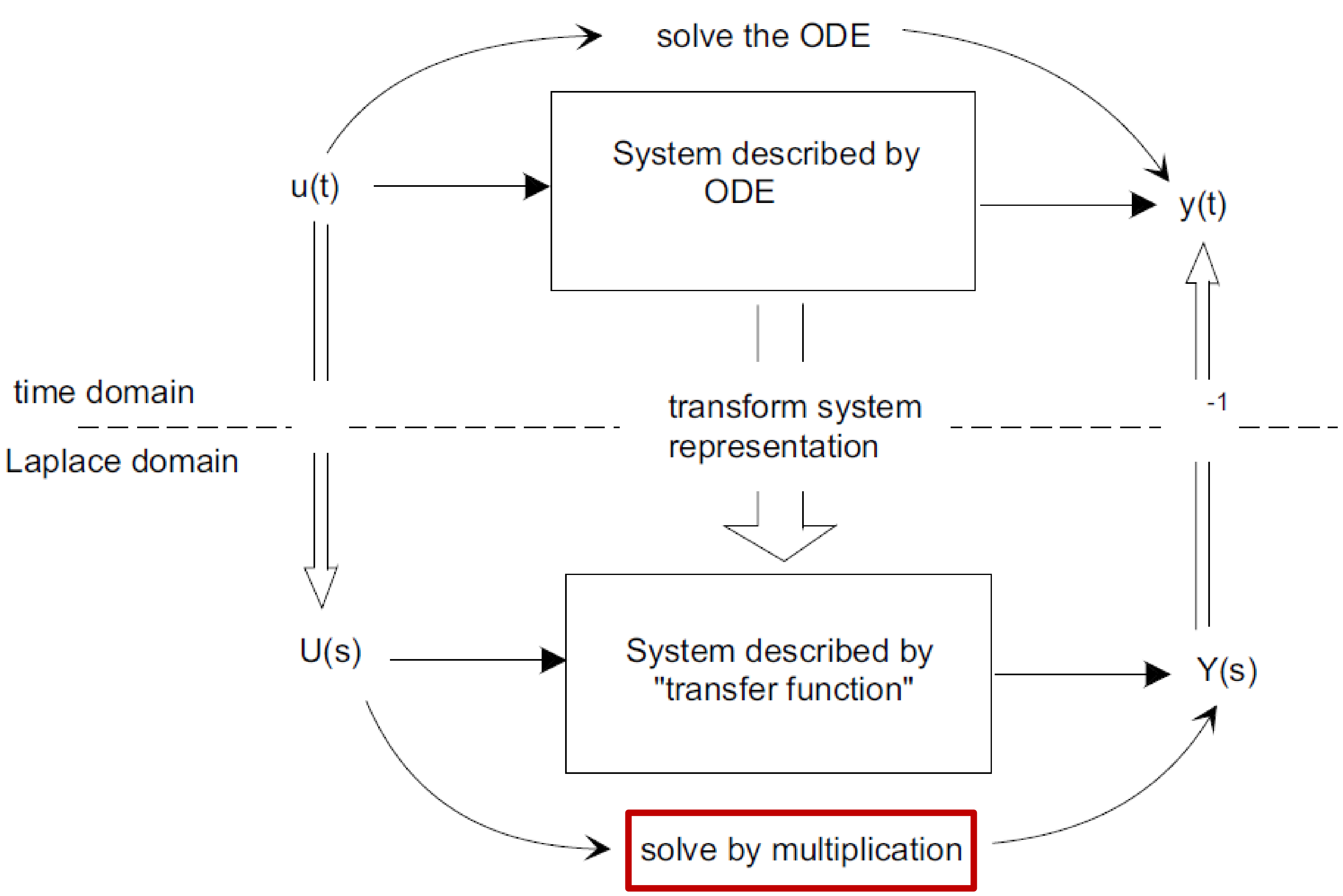
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

- Capacitors in parallel: Largest capacitor dominates



$$C_{eq} = C_1 + C_2 + C_3$$

Laplace Transform (LT)



[D. Rowell, MIT 2.004, 2008]

Laplace Transform (LT)

Time domain	Laplace domain
e^{at}	$\dfrac{1}{s-a}$
$\int_0^t f(t)dt$	$\dfrac{1}{s}F(s)$
$\dfrac{df(t)}{dt}$	$sF(s)$
$\delta(t)$	1
$u(t)$	$\dfrac{1}{s}$

Impulse Response and Step Response

Time domain	Laplace domain
e^{at}	$\dfrac{1}{s-a}$
$\int_0^t f(t)dt$	$\dfrac{1}{s}F(s)$
$\dfrac{df(t)}{dt}$	$sF(s)$
$\delta(t)$	1
$u(t)$	$\dfrac{1}{s}$

System input	System response (output) in Laplace domain
Unit impulse: $\delta(t)$	$H(s)$
Unit step: $u(t)$	$\dfrac{1}{s}H(s)$

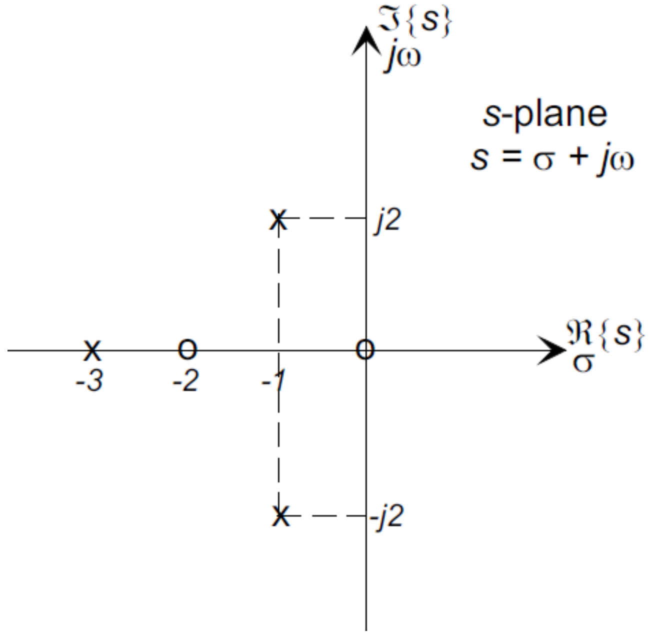
Poles and Zeros

- Transfer function

$$H(s) = \frac{N(s)}{D(s)}$$

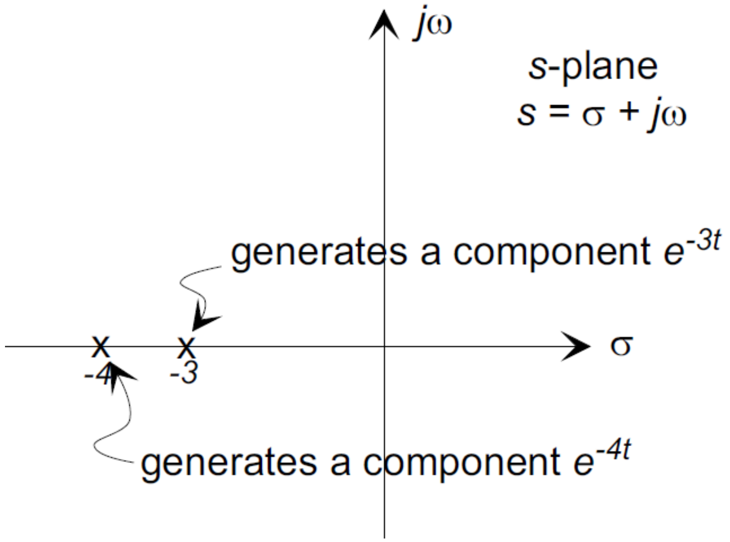
- Zeros: roots of the numerator ➔ $N(s) = 0$
- Poles: roots of the denominator (characteristic eq.) ➔ $D(s) = 0$
- For physical systems, poles & zeros are real or complex conjugate
- Example:

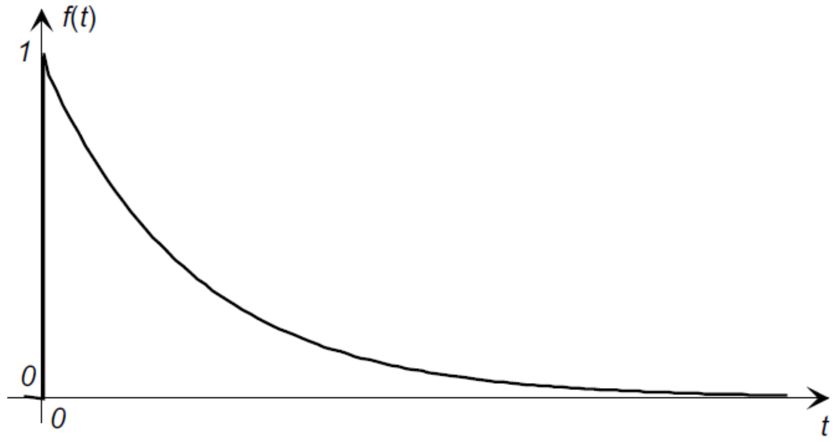
$$\begin{aligned} G(s) &= \frac{5s^2 + 10s}{s^3 + 5s^2 + 11s + 15} \\ &= \frac{5s(s+2)}{(s+3)(s^2+2s+5)} \\ &= \frac{5s(s+2)}{(s+3)(s+(1+j2))(s+(1-j2))} \end{aligned}$$

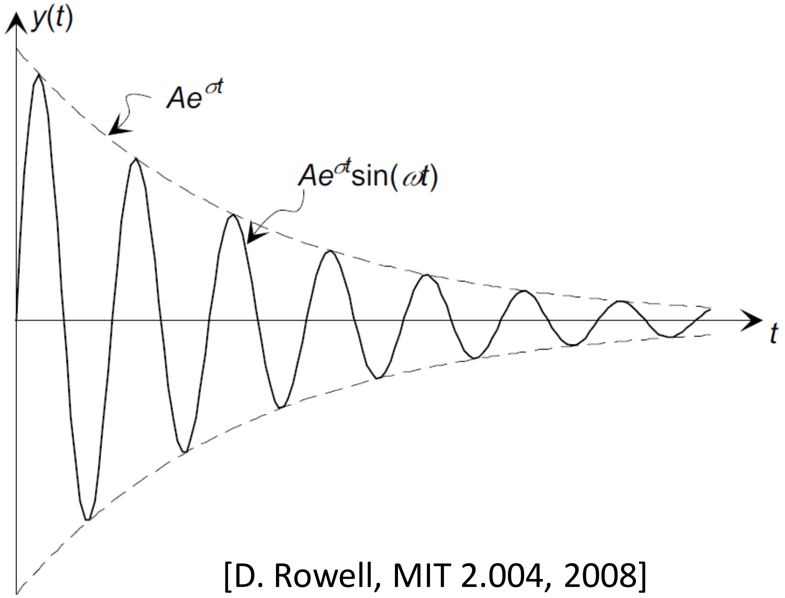


[D. Rowell, MIT 2.004, 2008]

Real and Complex Poles



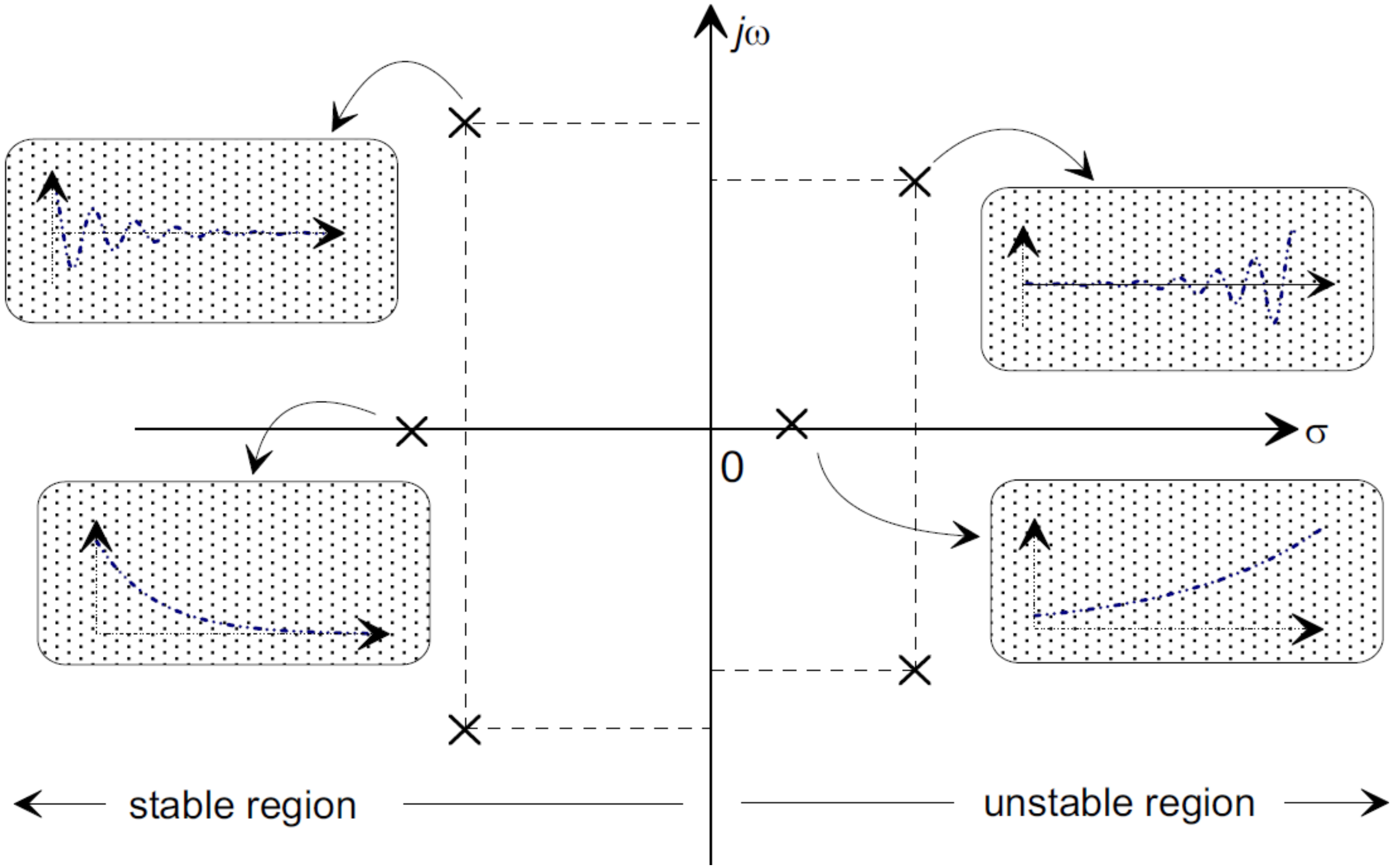




[D. Rowell, MIT 2.004, 2008]

LHP and RHP Poles

- Poles in LHP: Decaying exponential ➔ Stable system
  - BIBO: Bounded input bounded output
- Poles in RHP: Growing exponential ➔ Unstable system



[D. Rowell, MIT 2.004, 2008]

Frequency Response

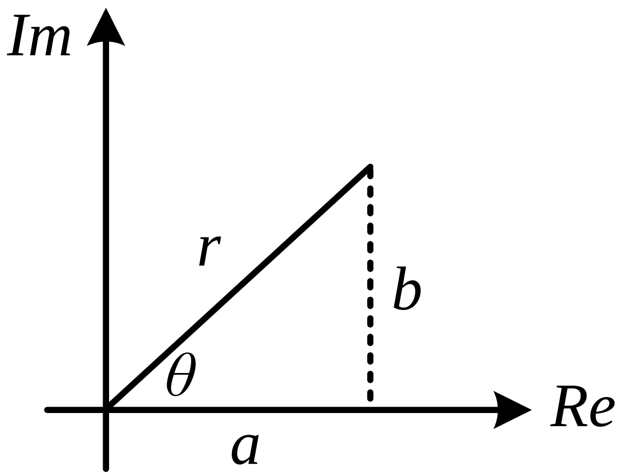
- Transfer function

$$H(s) = \frac{N(s)}{D(s)}$$

- Fourier Transform is a special case of Laplace Transform: $s \Rightarrow j\omega$
  - $\sigma = 0$ ➔ Steady state response for sinusoidal input
- Transfer function ➔ Frequency response: $s \Rightarrow j\omega$

$$H(j\omega) = \frac{V_{out}(j\omega)}{V_{in}(j\omega)} = |H(j\omega)|e^{j\phi}$$

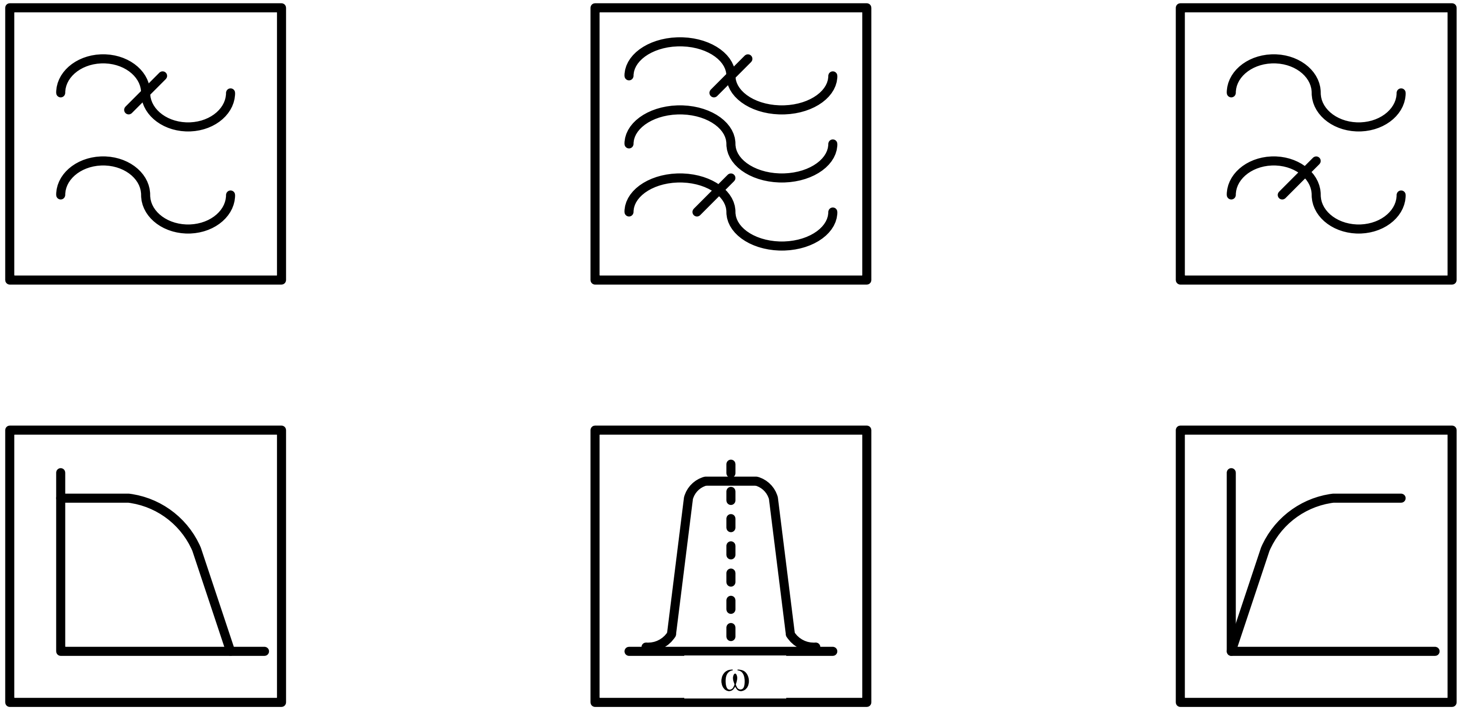
- $a + jb = re^{j\theta}$
- $r =$ Magnitude$(a + jb) = \sqrt{a^2 + b^2}$
- $\theta =$ Phase$(a + jb) = \tan^{-1}\frac{b}{a}$



Frequency Response

- Y-axis: magnitude of frequency response, x-axis: frequency

LPF | BPF | HPF

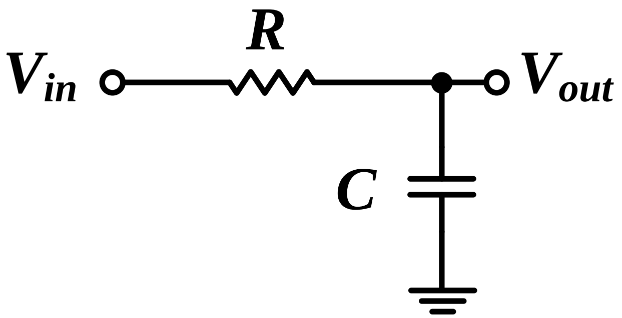


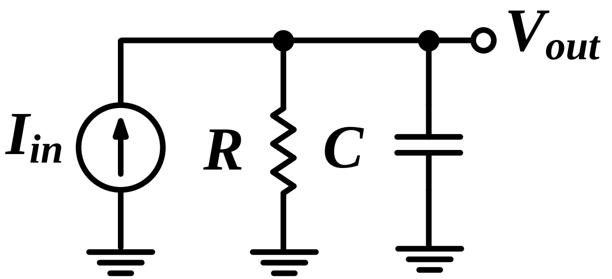
First-Order LPF

$$H(s) = \frac{v_{out}(s)}{v_{in}(s)} = \frac{1/sC}{R + 1/sC} = \frac{1}{1 + sRC} = \frac{1}{1 + s\tau}$$

$$H(j\omega) = \frac{v_{out}(j\omega)}{v_{in}(j\omega)} = \frac{1/j\omega C}{R + 1/j\omega C} = \frac{1}{1 + j\omega RC} = \frac{1}{1 + \frac{j\omega}{\omega_c}}$$

- $\tau = RC$: time constant
- $\omega_c = \frac{1}{\tau} = \frac{1}{RC}$: cutoff/corner frequency
- Poles: $s_p = -\frac{1}{\tau} = -\omega_c$
- Zeros: ?
- $|H(j\omega)| = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_c}\right)^2}}$
- $P\big(H(j\omega)\big) = -\tan^{-1}\frac{\omega}{\omega_c}$





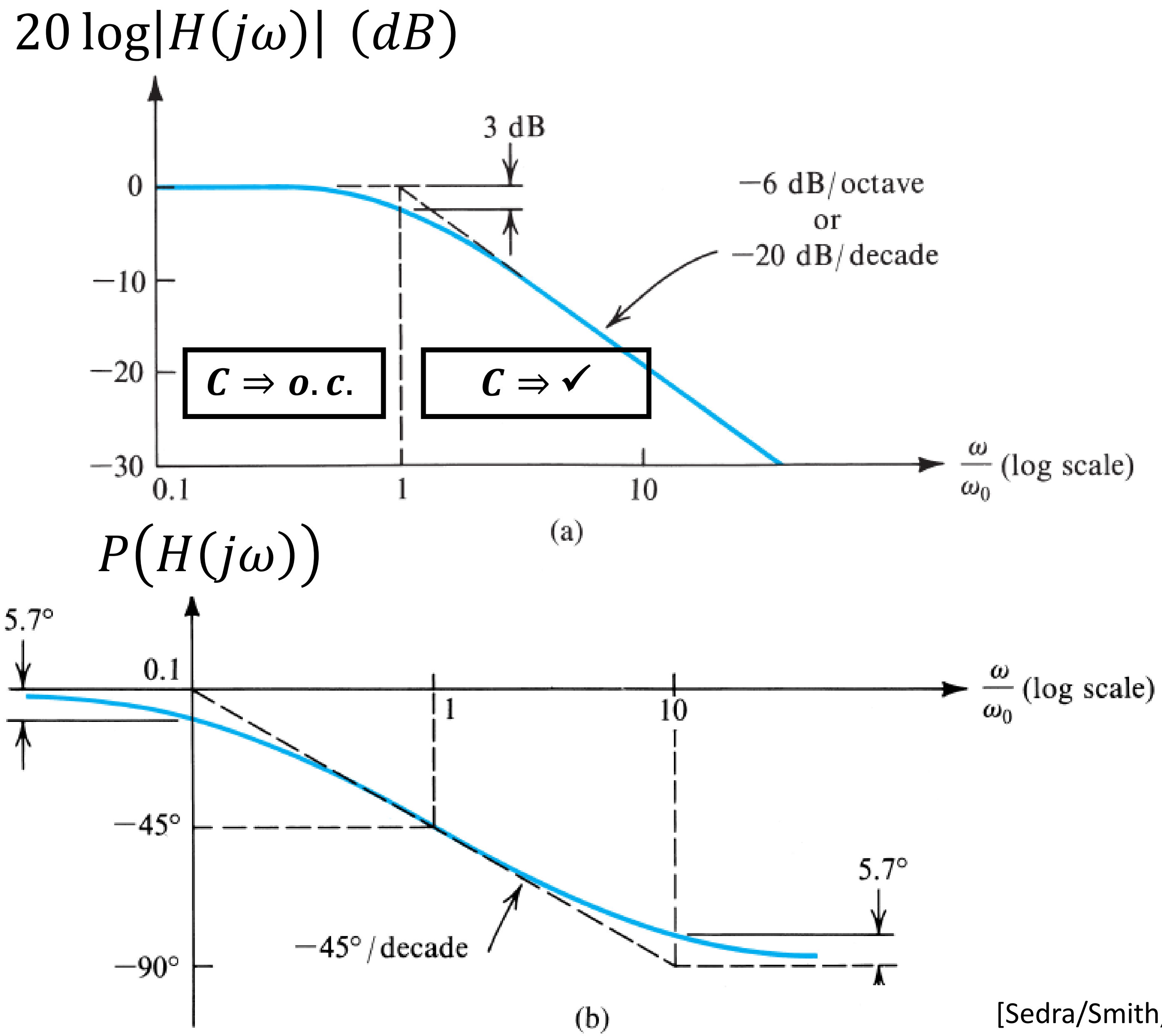
Bode Plot Rules

	Pole	Zero
Magnitude	-20 dB/decade Actual Mag @ pole: -3 dB	+20 dB/decade Actual Mag @ zero: +3 dB
Phase	-90^{o} Actual Phase @ pole: -45^{o}	LHP zero: $+90^{o}$ Actual Phase @ zero: $+45^{o}$
		RHP zero: -90^{o} Actual Phase @ zero: -45^{o}

→ RHP: Right-half plane ($Re\{s\} > 0$)

→ LHP: Left-half plane ($Re\{s\} < 0$)

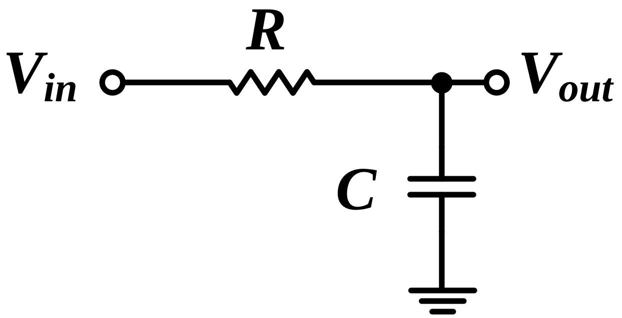
First-Order LPF Bode Plot

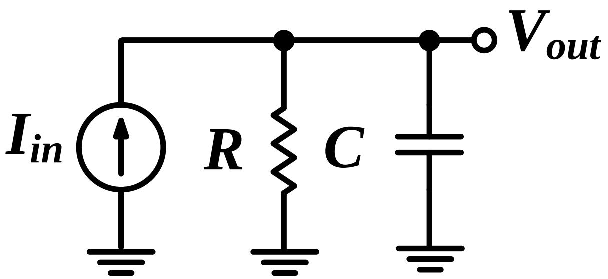


[Sedra/Smith, 2015]

First-Order LPF Impulse and Step Response

$$H(s) = \frac{v_{out}(s)}{v_{in}(s)} = \frac{1/sC}{R + 1/sC} = \frac{1}{1 + sRC} = \frac{1}{1 + s\tau}$$



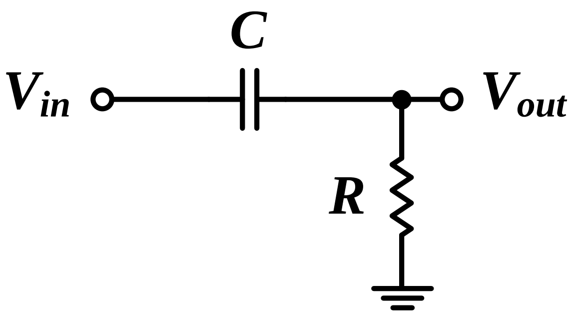


First-Order HPF

$$H(s) = \frac{v_{out}(s)}{v_{in}(s)} = \frac{R}{R + 1/sC} = \frac{sRC}{1 + sRC} = \frac{s\tau}{1 + s\tau}$$

$$H(j\omega) = \frac{v_{out}(j\omega)}{v_{in}(j\omega)} = \frac{R}{R + 1/j\omega C} = \frac{j\omega RC}{1 + j\omega RC} = \frac{\frac{j\omega}{\omega_c}}{1 + \frac{j\omega}{\omega_c}}$$

- Poles: $s_p = -\frac{1}{\tau} = -\omega_c$
- Zeros: $s_z = 0$
- $|H(j\omega)| = \frac{\frac{\omega}{\omega_c}}{\sqrt{1+\left(\frac{\omega}{\omega_c}\right)^2}}$
- $P\big(H(j\omega)\big) = 90^o - \tan^{-1}\frac{\omega}{\omega_c}$



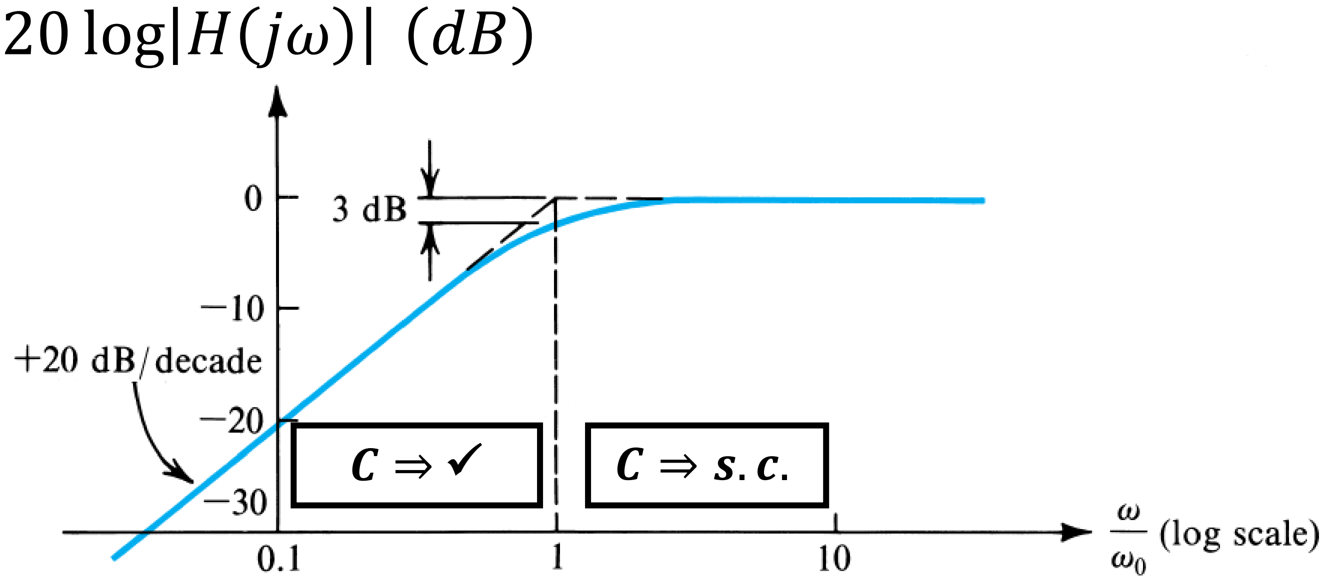
Bode Plot Rules

<table>
<tr><th></th><th>Pole</th><th>Zero</th></tr>
<tr><td>Magnitude</td><td>-20 dB/decade
Actual Mag @ pole: -3 dB</td><td>+20 dB/decade
Actual Mag @ zero: +3 dB</td></tr>
<tr><td rowspan="2">Phase</td><td rowspan="2">-90º
Actual Phase @ pole: -45º</td><td>LHP zero:
+90º
Actual Phase @ zero: +45º</td></tr>
<tr><td>RHP zero:
-90º
Actual Phase @ zero: -45º</td></tr>
</table>

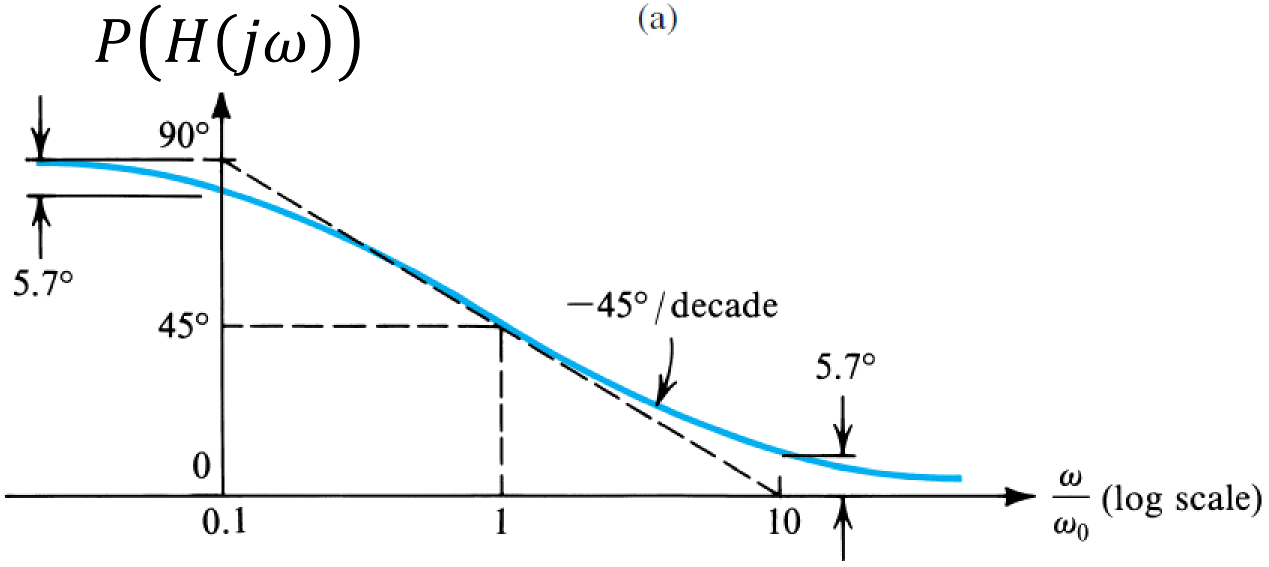
→ RHP: Right-half plane ($Re\{s\} > 0$)

→ LHP: Left-half plane ($Re\{s\} < 0$)

First-Order HPF Bode Plot



(a)



(b)

[Sedra/Smith, 2015]

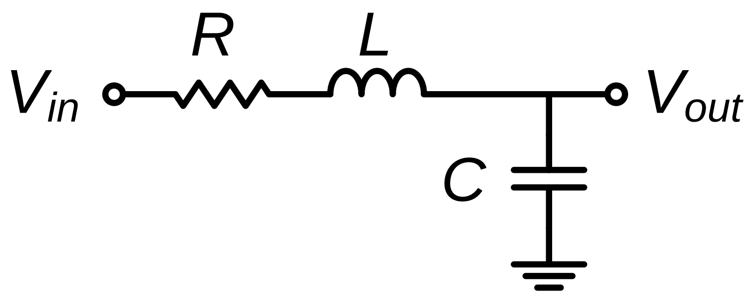
Second-Order System: LC LPF

$$H(s) = \frac{Z_C}{R + Z_L + Z_C} = \frac{1}{LCs^2 + RCs + 1}$$

$$\omega_o^2 = \frac{1}{LC} \quad and \quad Q = \frac{\omega_o L}{R} = \frac{1}{\omega_o RC} = \frac{1}{2\zeta}$$

- Higher R means higher damping (ζ) and lower quality factor (Q)

$$H(s) = \frac{1}{\left(\frac{s}{\omega_o}\right)^2 + \frac{s}{\omega_o Q} + 1} = \frac{\omega_o^2}{s^2 + \left(\frac{\omega_o}{Q}\right)s + \omega_o^2} = \frac{\omega_o^2}{s^2 + (2\zeta\omega_o)s + \omega_o^2}$$

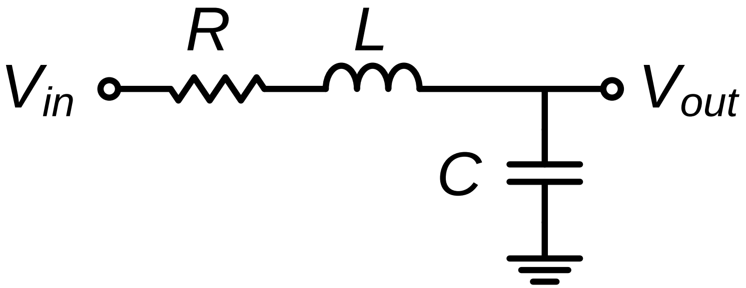


Second-Order Passive LC LPF

$$H(s) = \frac{1}{\left(\frac{s}{\omega_o}\right)^2 + \frac{s}{\omega_o Q} + 1} = \frac{\omega_o^2}{s^2 + \left(\frac{\omega_o}{Q}\right)s + \omega_o^2}$$

- The poles occur at $s^2 + \left(\frac{\omega_o}{Q}\right)s + \omega_o^2 = 0$

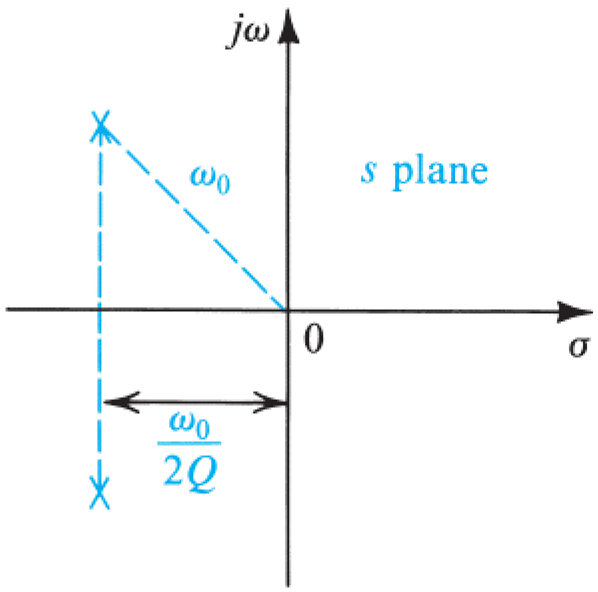
$$s_{p1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = -\frac{\omega_o}{2Q} \pm \frac{\omega_o}{2}\sqrt{\frac{1}{Q^2} - 4}$$



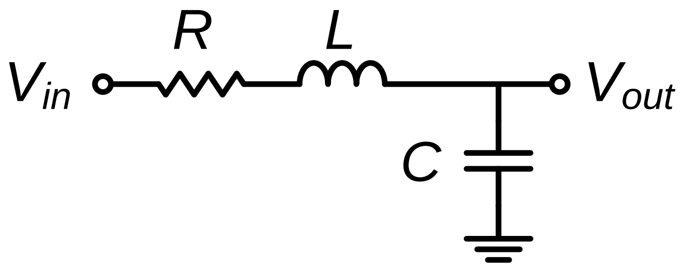
Second-Order System Poles

$$H(s) = \frac{1}{\left(\frac{s}{\omega_o}\right)^2 + \frac{s}{\omega_o Q} + 1} = \frac{1}{\left(\frac{s}{\omega_o}\right)^2 + \frac{s}{\omega_o/2\zeta} + 1}$$

$$s_{p1,2} = -\frac{\omega_o}{2Q} \pm \frac{\omega_o}{2}\sqrt{\frac{1}{Q^2} - 4}$$



- If $Q < 0.5$ $(\zeta > 1)$: overdamped system, roots are real, negative, and distinct, like two first-order RC filters in cascade
- If $Q = 0.5$ $(\zeta = 1)$: critical damped system, roots are real, negative, and equal
- If $Q > 0.5$ $(\zeta < 1)$: underdamped system, roots are complex conjugate



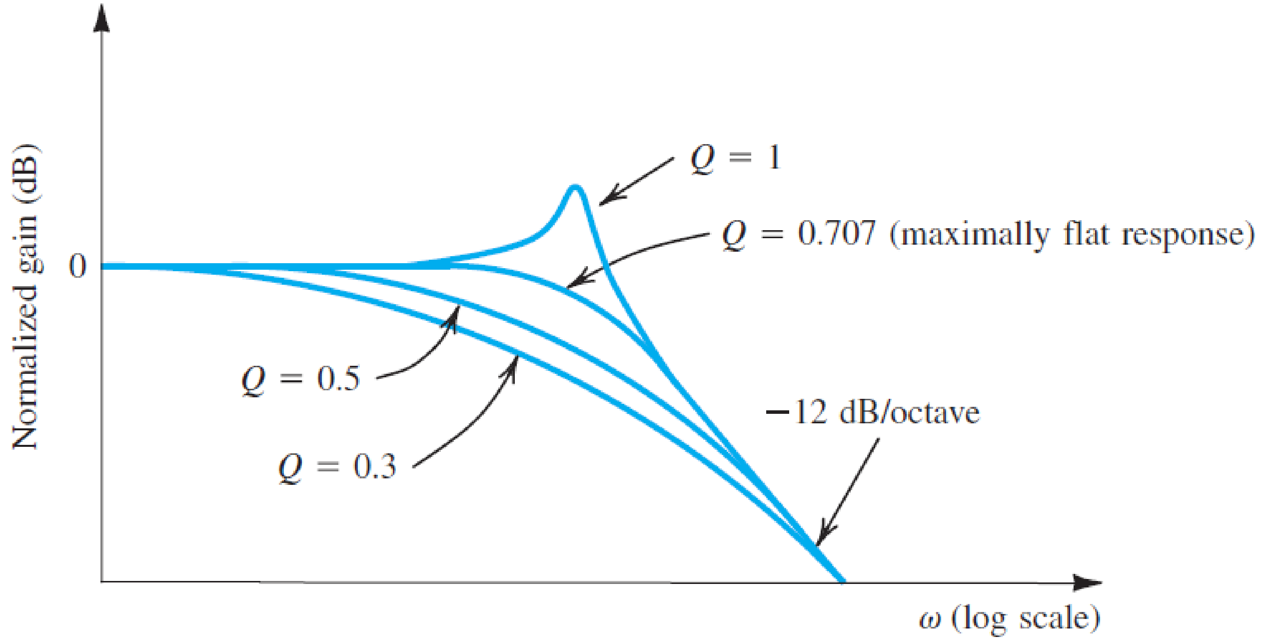
[Sedra/Smith, 2015] & [Johns and Martin, 2012]

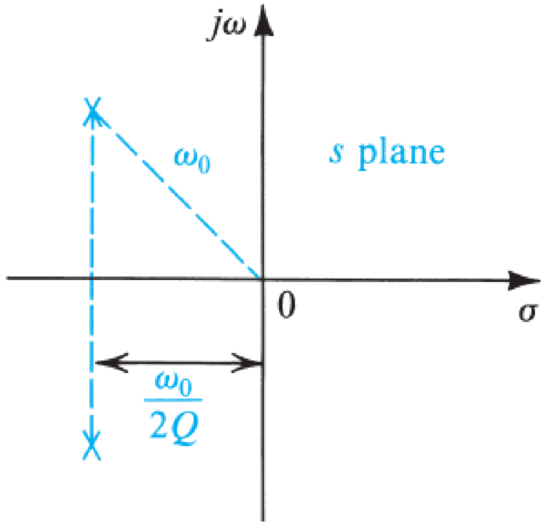
Ringing and Peaking

- $Q > 0.5$ ($\zeta < 1$): Underdamped system (complex conjugate poles)
  - Ringing (overshoot) in step response (time domain)

$$\%\ \text{overshoot} = 100\, e^{\frac{-\pi}{\sqrt{4Q^2-1}}}$$

- $Q > \frac{1}{\sqrt{2}} = 0.707$ ($\zeta < 0.707$): Peaking in frequency response





[Sedra/Smith, 2015] & [Johns and Martin, 2012]

Thank you!

References

- T. Floyd and D. Buchla, “Electronics Fundamentals, Circuits, Devices, and Applications,” 8th ed., Pearson, 2014.
- A. Sedra and K. Smith, “Microelectronic circuits,” Oxford University Press, 7th ed., 2015.
- B. Razavi, “Fundamentals of microelectronics,” 2nd ed., Wiley, 2014.

Order of a Circuit

- The order of a circuit is the order of the differential equation describing the circuit
- Viewed in s-domain, it is the order of the denominator of the transfer function (the characteristic equation)
  - The number of poles of the system
- The order is also the number of state variables in the circuit (the variables that control the behavior of the circuit)
  - For C: state variable is voltage
  - For L: state variable is current
- A practical rule of thumb:
  - Find the number of independent inductor currents and capacitor voltages
  - Or the number of independent initial conditions that can be assigned to state variables
- Beware of pole-zero cancellation (e.g., two capacitors forming a voltage divider)