Analog IC Design

Lecture 06
Basic Amplifier Stages

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Outline

- Recapping previous key results
- Why amplifiers?
- Basic amplifier operation
- Basic amplifier analysis
- Rin/out shortcuts
  - Looking from drain
  - Looking from source
- GmRout method
- Basic amplifier topologies
  - Common Source (CS)
  - Common Gate (CG)
  - Common Drain (CD) – Source Follower (SF)
- Large signal behavior

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MOSFET in Saturation

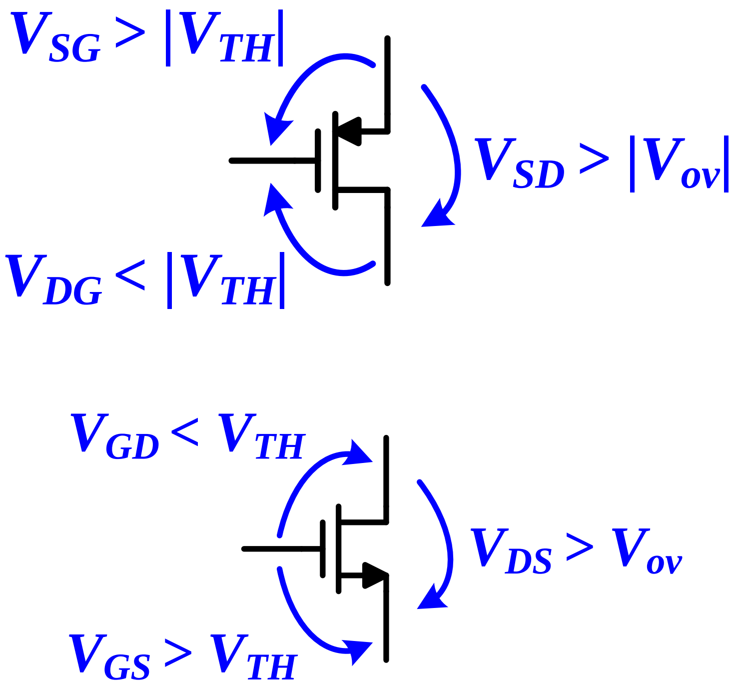
- The channel is pinched off if the difference between the gate and drain voltages is not sufficient to create an inversion layer

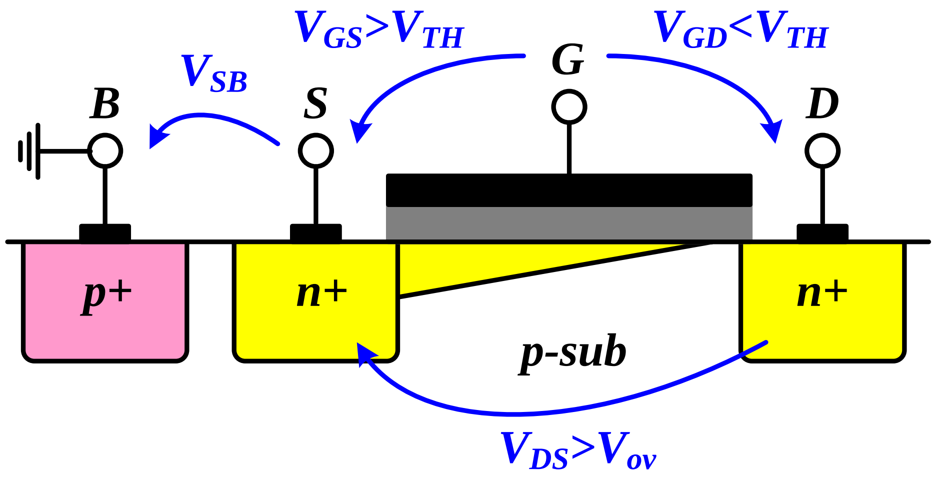
$$V_{GD} \leq V_{TH} \quad OR \quad V_{DS} \geq V_{ov}$$

- Square-law (long channel MOS)

$$I_D = \frac{\mu_n C_{ox}}{2}\frac{W}{L} \cdot V_{ov}^2 (1 + \lambda V_{DS})$$

$$V_{SB} \uparrow \;\Rightarrow\; V_{TH} \uparrow$$





Regions of Operation Summary

OFF (Subthreshold)

$V_{GS} < V_{TH}$

ON

$V_{GS} > V_{TH}$

Triode

$V_{DS} < V_{ov}$

Or

$V_{GD} > V_{TH}$

$$I_D = \mu C_{ox} \frac{W}{L}\left(V_{ov}V_{DS} - \frac{V_{DS}^2}{2}\right)$$

Pinch-Off (Saturation)

$V_{DS} \geq V_{ov}$

Or

$V_{GD} \leq V_{TH}$

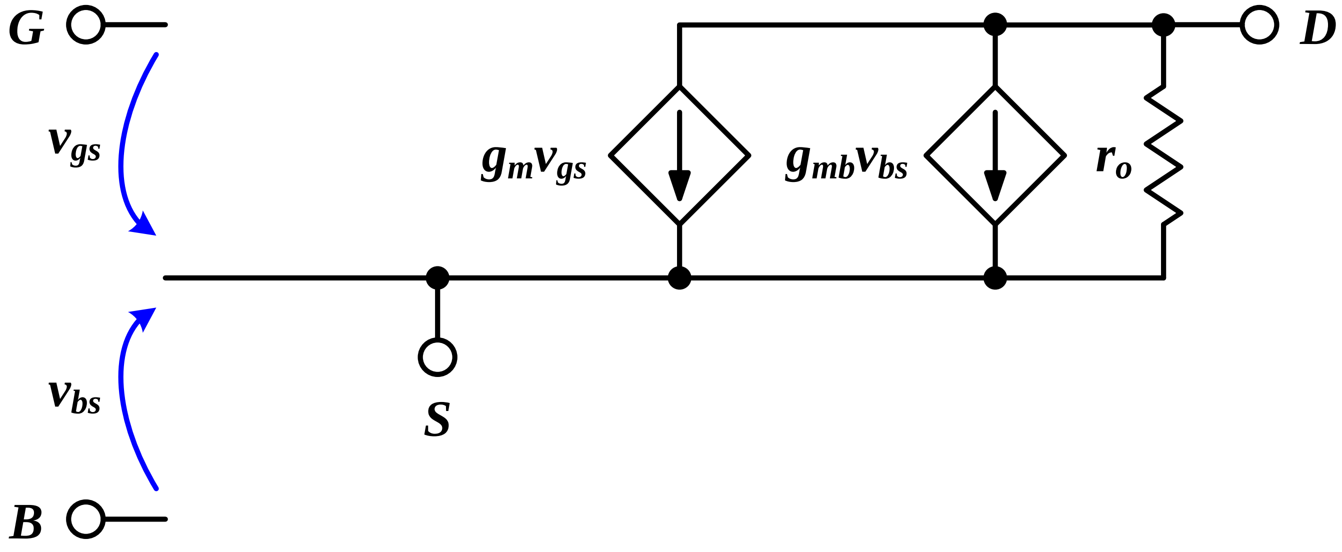
$$I_D = \frac{\mu C_{ox}}{2}\frac{W}{L}V_{ov}^2(1 + \lambda V_{DS})$$

Low-Frequency Small-Signal Model

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \mu C_{ox}\frac{W}{L}V_{ov} = \sqrt{\mu C_{ox}\frac{W}{L}\cdot 2I_D} = \frac{2I_D}{V_{ov}}$$

$$g_{mb} = \eta g_m \qquad \eta \approx 0.1 - 0.25$$

$$r_o = \frac{1}{\partial I_D/\partial V_{DS}} = \frac{V_A}{I_D} = \frac{1}{\lambda I_D} \qquad V_A \propto L \leftrightarrow \lambda \propto \frac{1}{L} \qquad V_{DS}\uparrow V_A \uparrow$$



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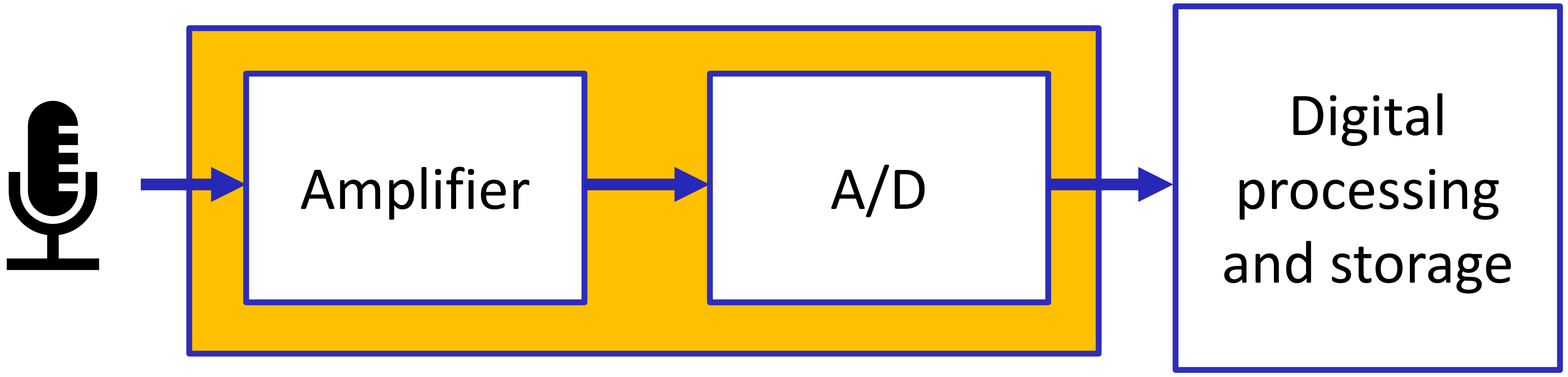
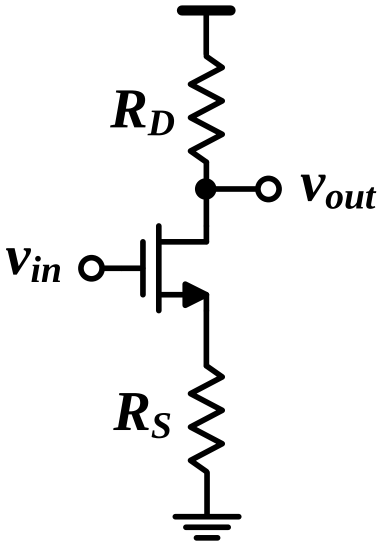
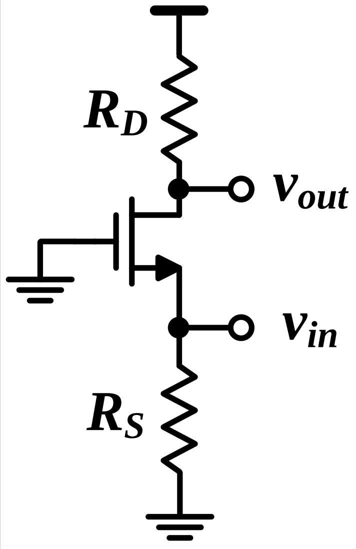
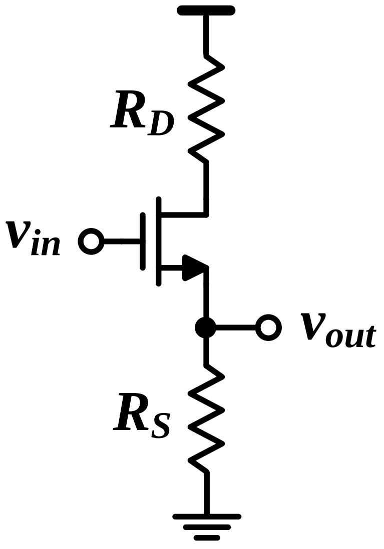
Why Amplifiers?

- All the physical signals in the world around us are analog
  - Voice, light, temperature, pressure, etc.
- We (will) always need an "analog" interface circuit to connect between our physical world and our digital electronics
- The physical signals are usually very weak
  - They must be amplified before any kind of processing
- Amplifiers are also needed in many other applications



Basic MOSFET Amplifier Topologies

	Common Source (CS)	Common Gate (CG)	Common Drain (CD) Source Follower (SF)
			
	Voltage & current amplifier	Voltage amplifier Current buffer	Voltage buffer Current amplifier
In	G	S	G
Out	D	D	S

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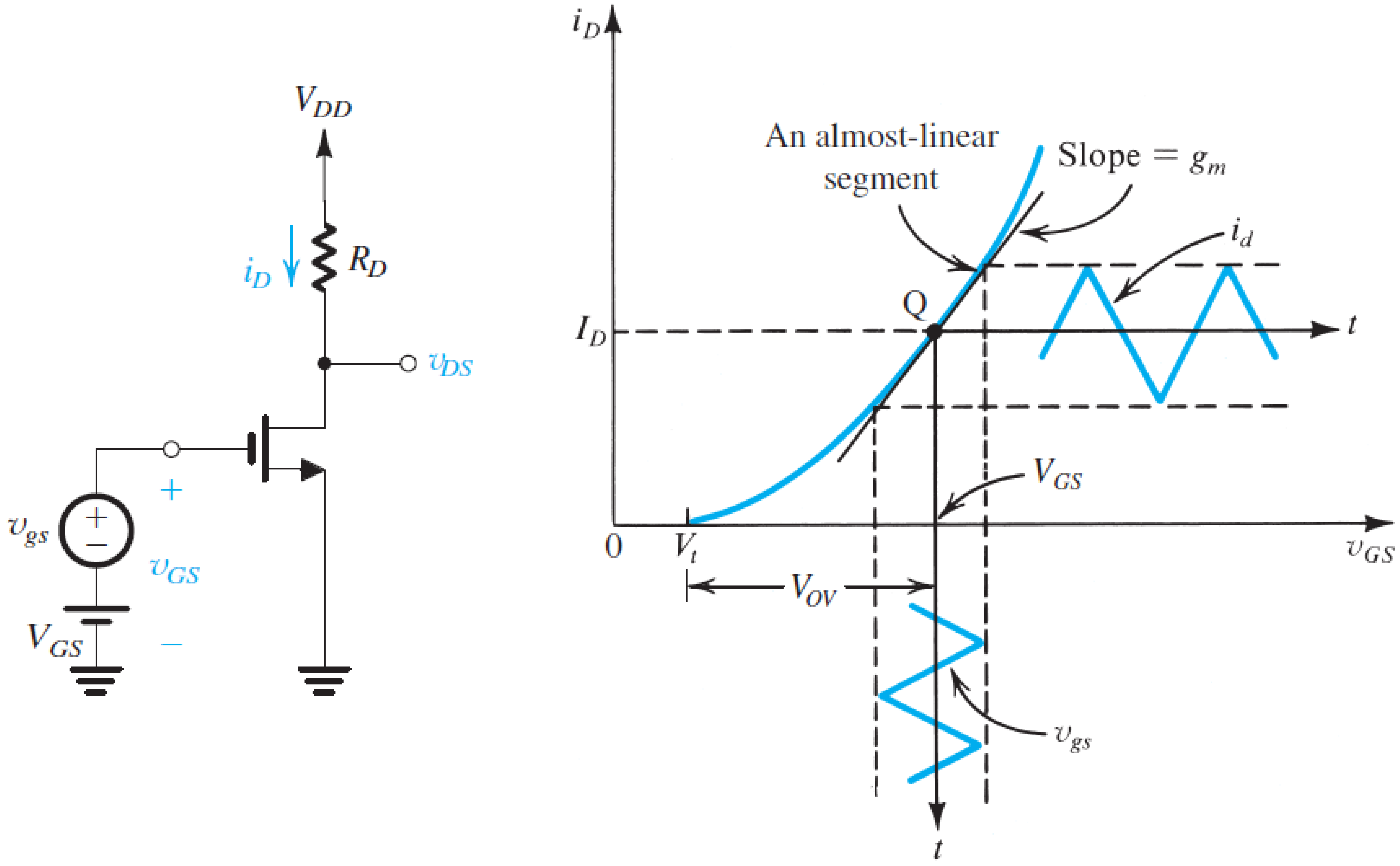
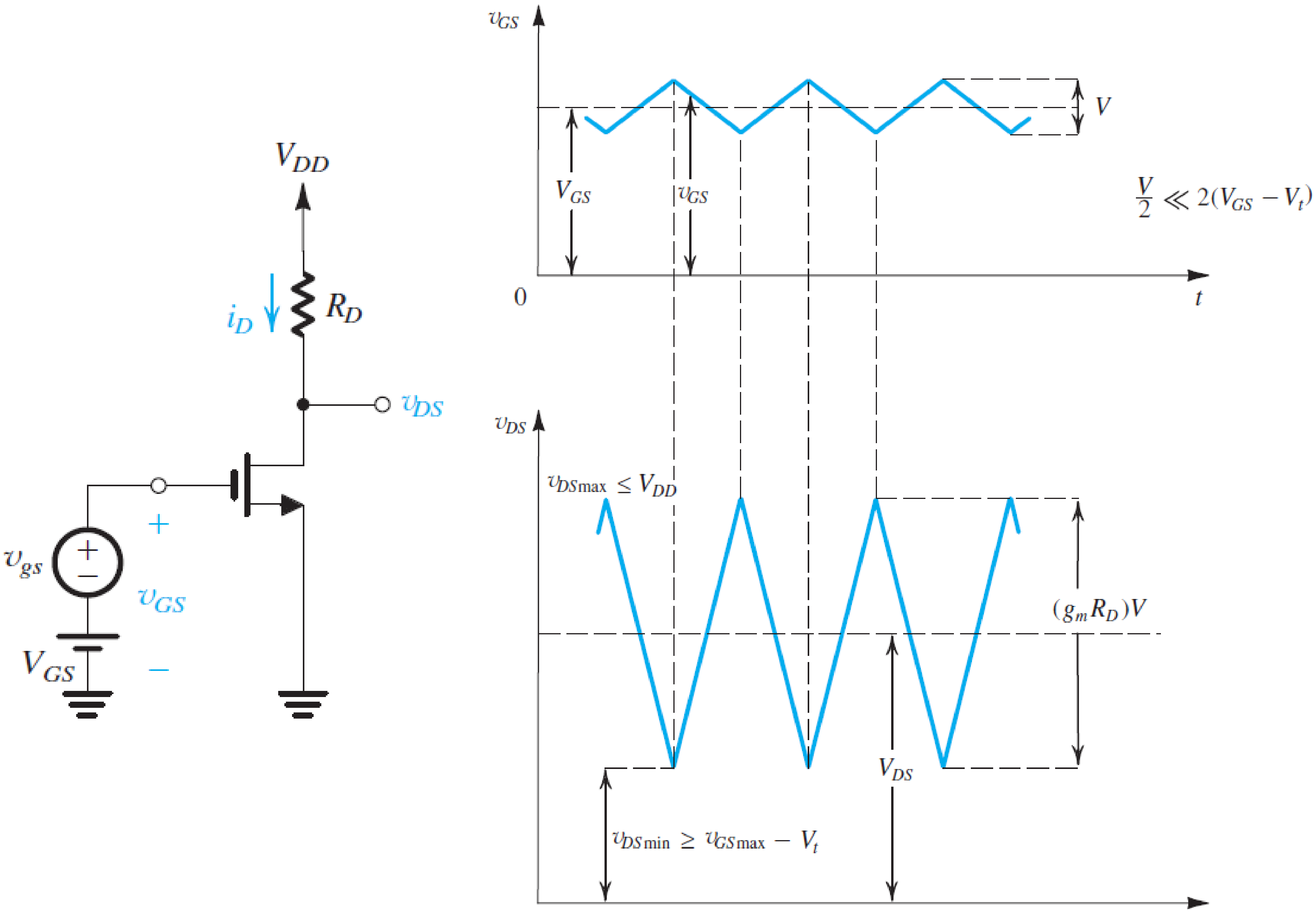
CS Amplifier Example



[Sedra/Smith, 2015]

CS Amplifier Example



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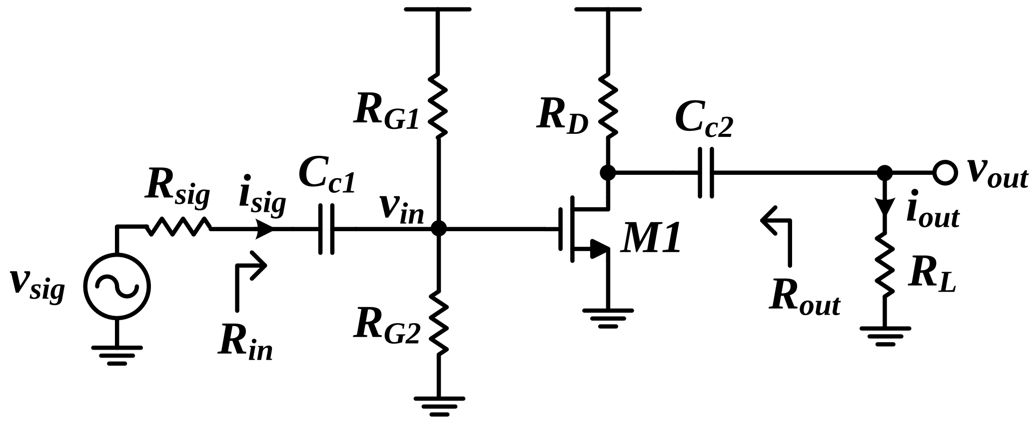
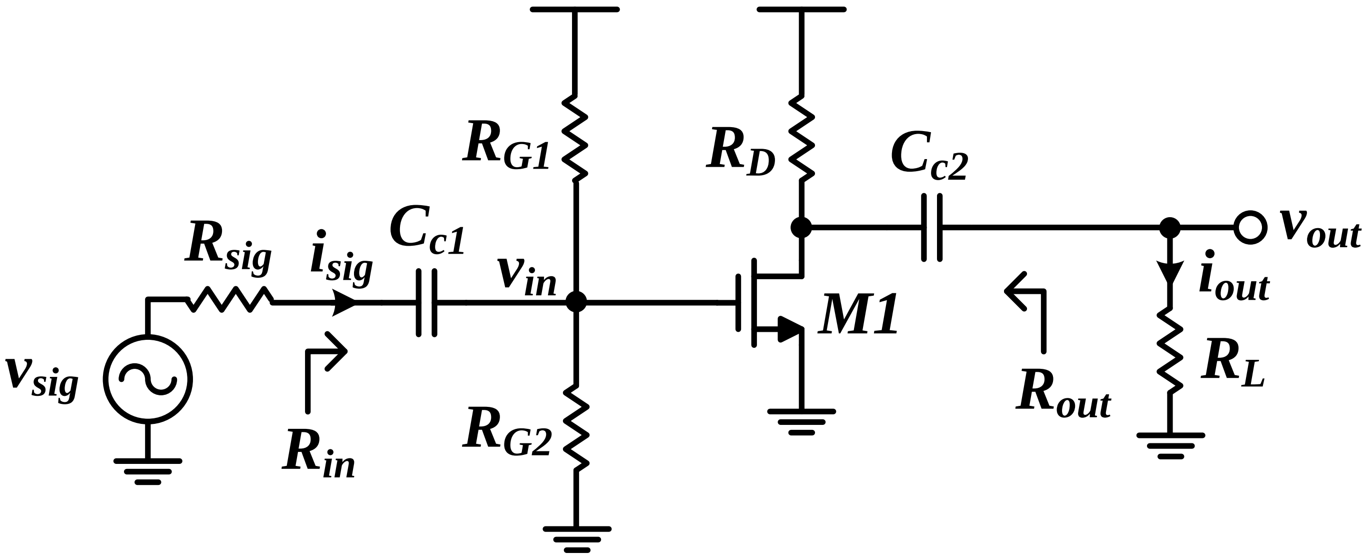
Amplifier Analysis Steps

1. DC analysis
   - Coupling and bypass capacitors → open-circuit (o.c)
   - Calculate Q-point and check operation in saturation $(V_{DS} > V_{ov})$
2. Calculate small signal parameters (g_m, r_o)
3. Draw the small signal equivalent circuit
   - DC voltage source → short-circuit (s.c.)
   - DC current source → open-circuit (o.c.)
   - Coupling and bypass capacitors → short-circuit (s.c)
4. Determine the amplifier parameters
   - Input resistance and output resistance
   - Voltage gain and current gain



Amplifier Parameters

- $R_{in} = \frac{v_{in}}{i_{in}} = \frac{v_{in}}{i_{sig}}$
- $R_{out} = \frac{v_x}{i_x} \ @\ v_{sig} = 0$
- $Voltage\ gain = A_v = \frac{v_{out}}{v_{sig}} = \frac{v_{in}}{v_{sig}} \cdot \frac{v_{out}}{v_{in}} = \frac{R_{in}}{R_{sig}+R_{in}} \cdot \frac{v_{out}}{v_{in}}$
- $Current\ gain = A_i = \frac{i_{out}}{i_{sig}} = \frac{v_{out}/R_L}{v_{in}/R_{in}} = \frac{v_{out}}{v_{in}} \cdot \frac{R_{in}}{R_L}$



Large and Small Signal Analysis

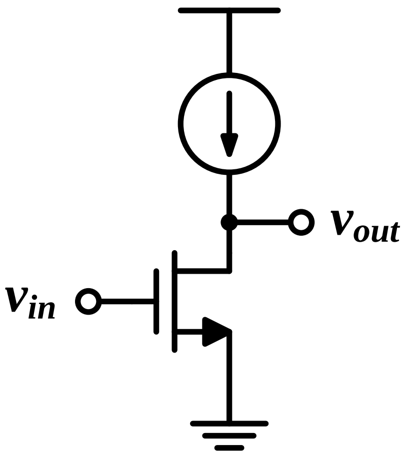
	Large Signal Analysis	Small Signal Analysis
Model	Large signal model	Small signal model
Linearity	Non-linear	Linear
Simulation	DC and transient analysis	AC analysis
Purpose	Calculate bias point, signal swing, distortion, etc.	Calculate $A_v, R_{in}, R_{out}, BW, etc.$
VDC	✓	s.c.
IDC	✓	o.c.
Capacitor	o.c. (in DC)	$1/\omega C$
Inductor	s.c. (in DC)	ωL

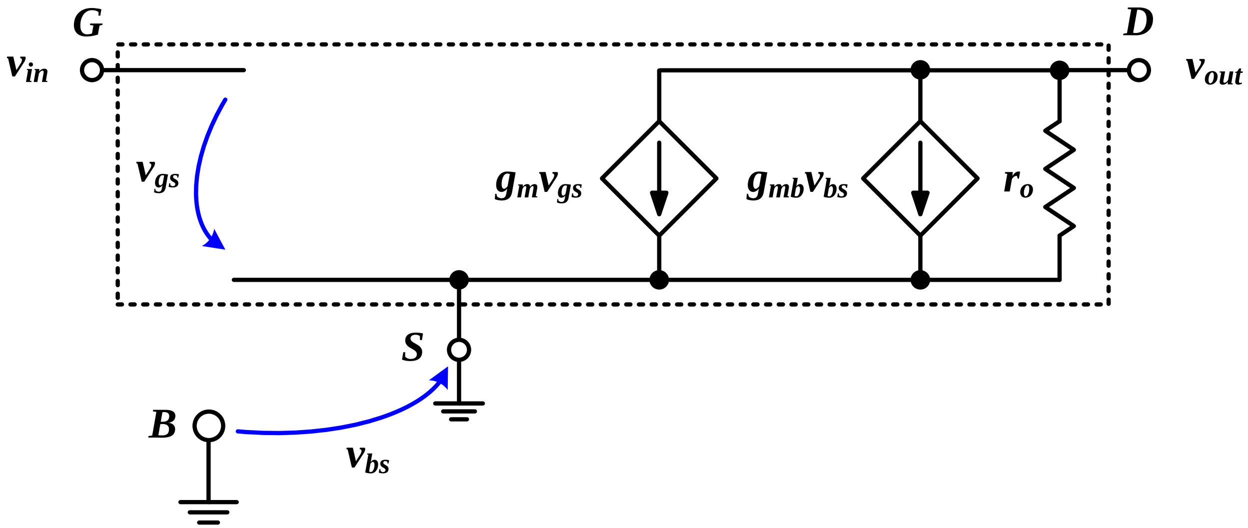
Amplifier Analysis Example

$$v_{out} = -(g_m v_{in}) r_o$$

$$|A_v| = \left|\frac{v_{out}}{v_{in}}\right| = g_m r_o$$

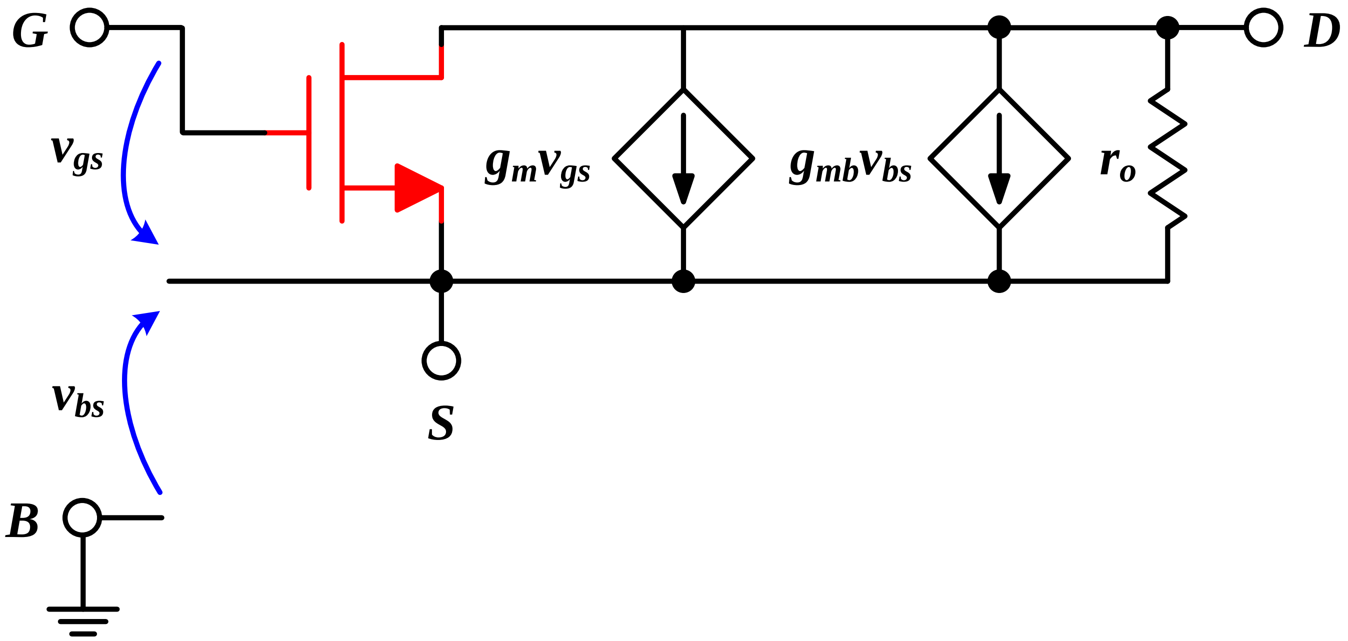
- $g_m r_o$ is the max gain that can be obtained from a single transistor: a.k.a. intrinsic gain





Direct Analysis on Schematics

- No need to draw the small signal model every time
- Just remember we have two VCCSs and r_o between D and S
- If G is ac gnd, then $v_{bs} = v_{gs}$ ➔ g_m and g_{mb} add
  - $g_m \rightarrow g_m + g_{mb}$



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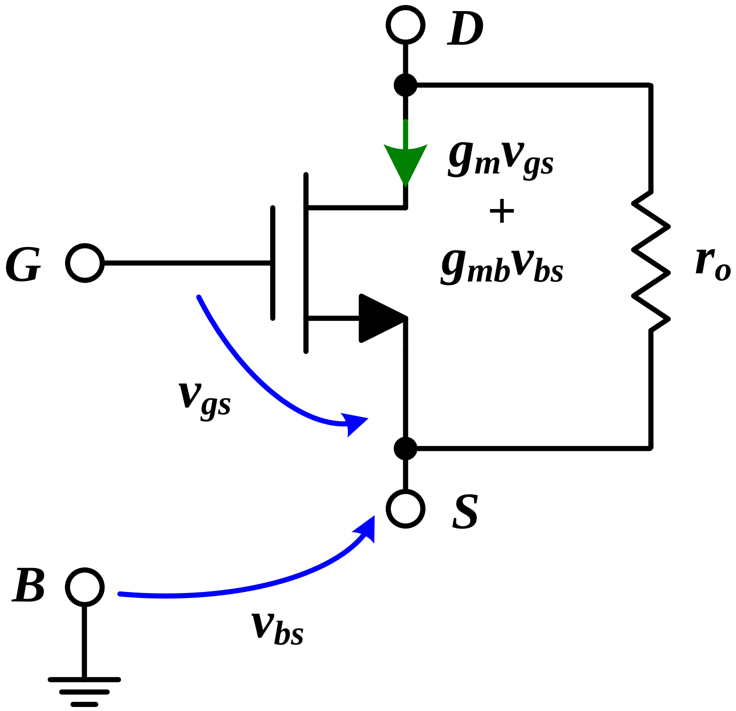


Intrinsic Gain

$$v_{out} = -(g_m v_{in}) r_o$$

$$|A_v| = \left|\frac{v_{out}}{v_{in}}\right| = g_m r_o$$

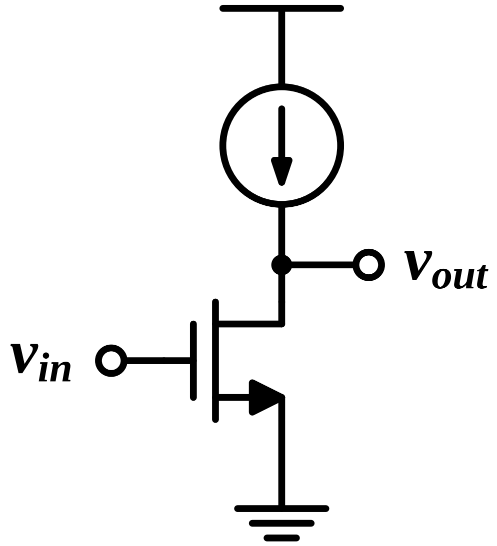
- $g_m r_o$ is the max gain that can be obtained from a single transistor
- Common approximations that we usually use

$$g_m r_o \gg 1$$

$$r_o \gg \frac{1}{g_m}$$

$$g_m + \frac{1}{r_o} \approx g_m$$

$$r_o // \frac{1}{g_m} \approx \frac{1}{g_m}$$



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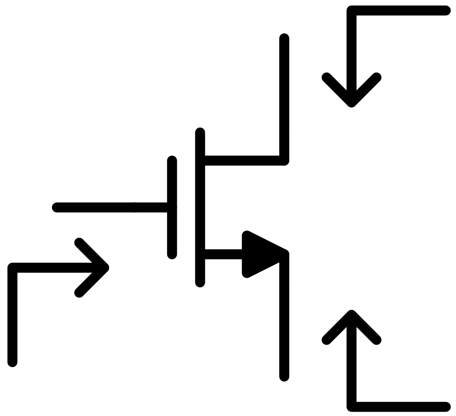
Rin/out Shortcuts

- Find equivalent impedance looking from Gate, Source, and Drain

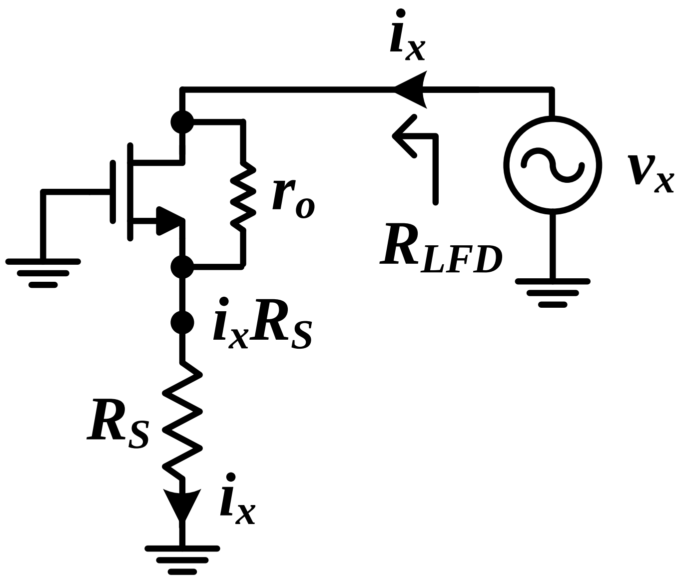


Looking From Drain

- If G is ac gnd, then $v_{bs} = v_{gs}$ ➔ g_m and g_{mb} add
- $v_{gs} = -i_x R_S$ and $g_m r_o \gg 1$
- Apply KCL at D

$$i_x = (g_m + g_{mb})(-i_x R_S) + \frac{v_x - i_x R_S}{r_o}$$

$$R_{LFD} = \frac{v_x}{i_x} = r_o + [(g_m + g_{mb})r_o + 1]R_S \approx r_o[1 + (g_m + g_{mb})R_S]$$



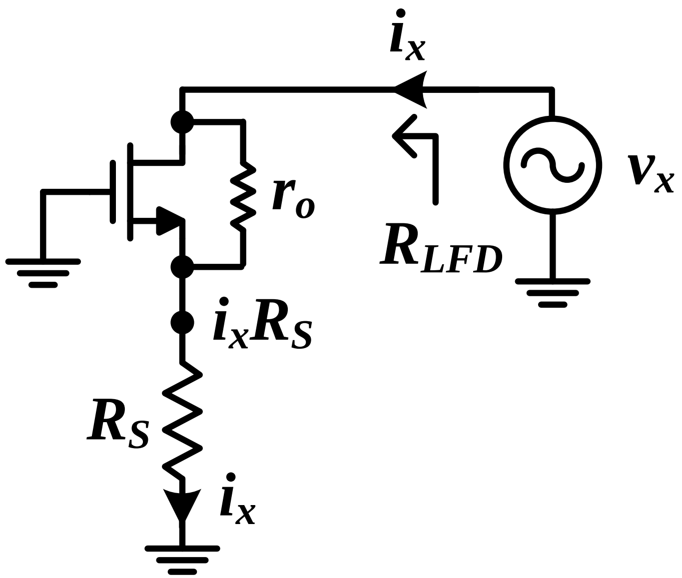
Looking From Drain

$$R_{LFD} \approx r_o[1 + (g_m + g_{mb})R_S]$$

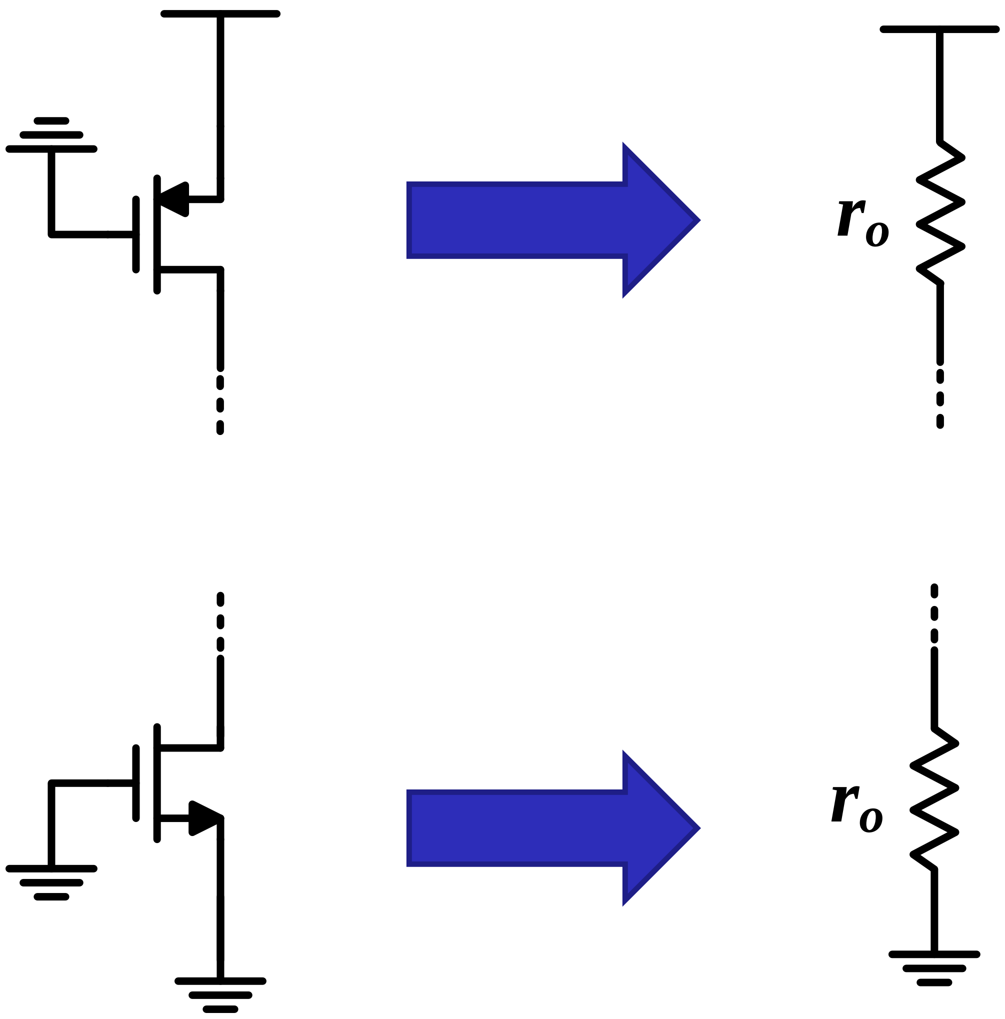
- Special case: $R_S = 0$ (G and S ac s.c.) ➔ **active load**

$$R_{LFD} = r_o$$

- **Drain is a high-impedance node (H.I.N.)**

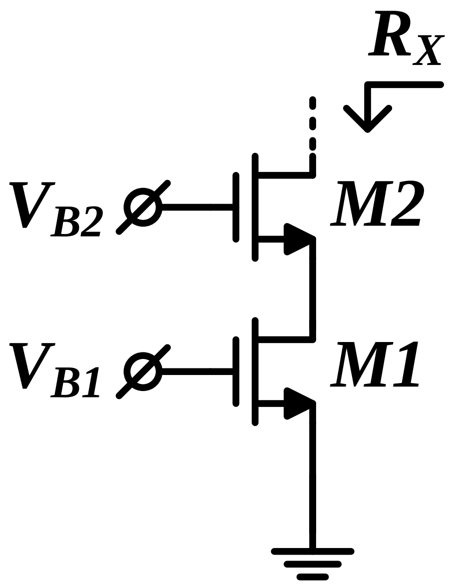


Active Load (Source OFF)



Quiz

- Assume M1 and M2 have the same g_m and r_o, $g_m r_o \gg 1$, and neglect body effect
- Find R_X

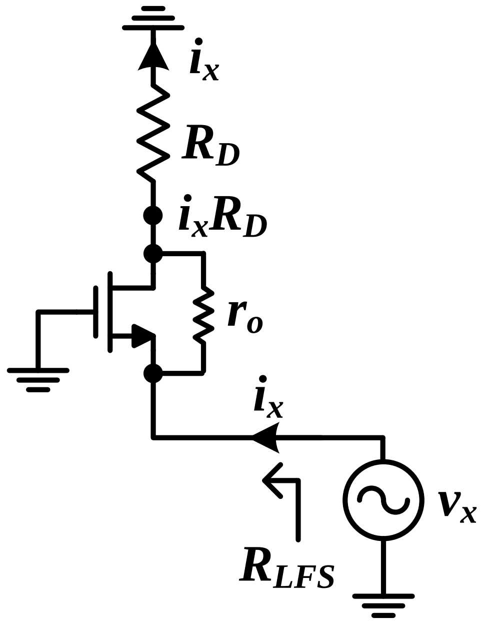


Looking From Source

- $v_{gs} = -v_x$, g_m and g_{mb} add, and $g_m r_o \gg 1$
- Apply KCL at S

$$i_x = (g_m + g_{mb})v_x + \frac{v_x - i_x R_D}{r_o}$$

$$R_{LFS} = \frac{v_x}{i_x} = \frac{1}{g_m + g_{mb} + \frac{1}{r_o}}\left(1 + \frac{R_D}{r_o}\right) \approx \frac{1}{g_m + g_{mb}}\left(1 + \frac{R_D}{r_o}\right)$$



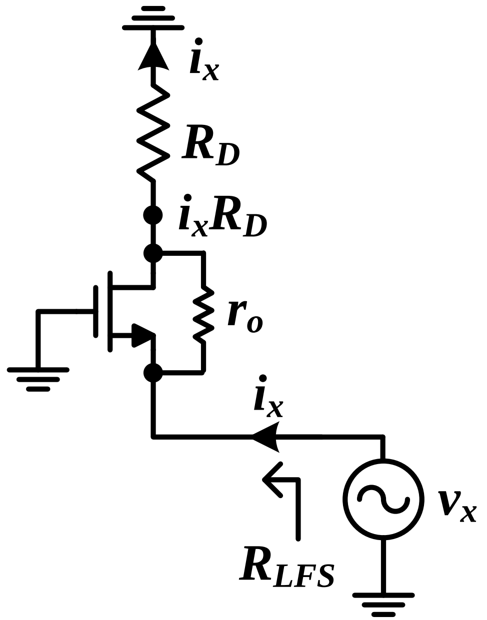
Looking From Source

$$R_{LFS} \approx \frac{1}{g_m + g_{mb}}\left(1 + \frac{R_D}{r_o}\right)$$

- Special case: $R_D = 0$ ➔ **diode connected**

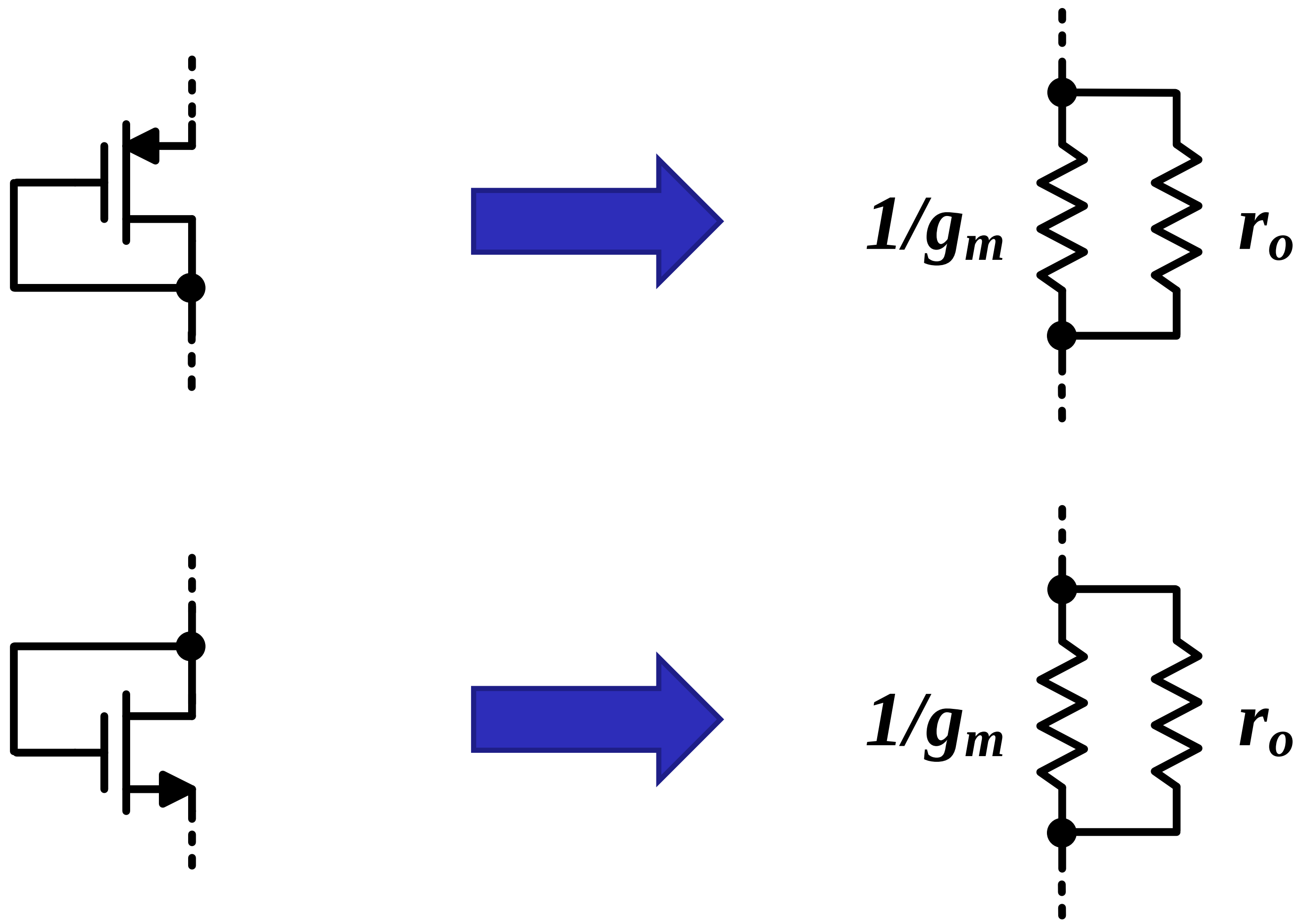
$$R_{LFS} \approx \frac{1}{g_m + g_{mb}}$$

- **Source is a low impedance node (L.I.N.)**



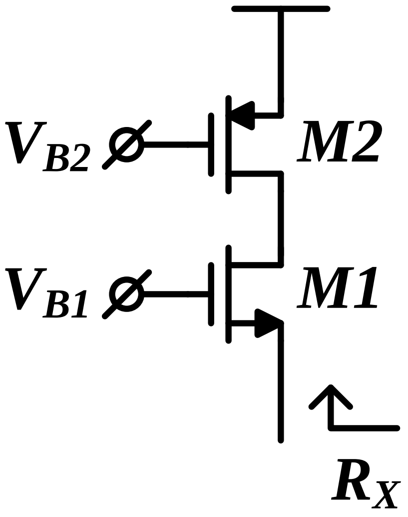
Diode Connected (Source Absorption)

- ❑ Always in saturation ($V_{DS} = V_{GS} > V_{ov}$)
- ❑ Body effect: $g_m \rightarrow g_m + g_{mb}$ (if G is ac gnd)



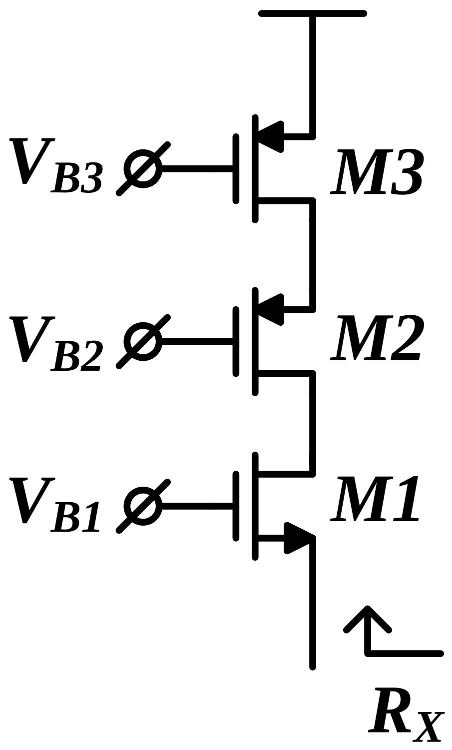
Quiz

- Assume M1 and M2 have the same r_o, $g_m r_o \gg 1$, and neglect body effect
- Find R_X

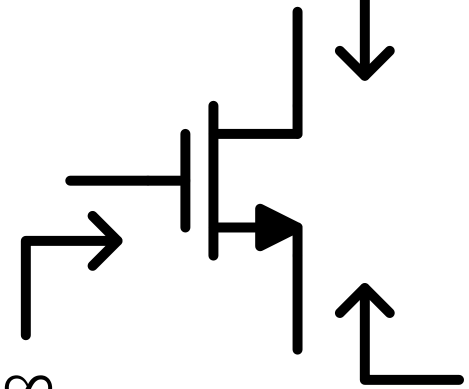


Quiz

- Assume M1, M2, and M3 have the same g_m and r_o, $g_m r_o \gg 1$, and neglect body effect
- Find R_X



Rin/out Shortcuts Summary



$$r_o[1 + (g_m + g_{mb})R_S]$$

H.I.N.

$$\frac{1}{g_m + g_{mb}}\left(1 + \frac{R_D}{r_o}\right)$$

L.I.N.

∞

At low frequencies **ONLY**

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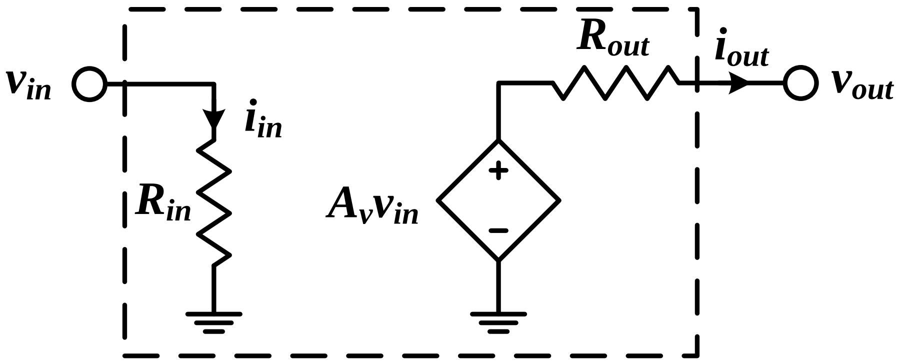
Amplifier Model

- Rin/out

$$R_{in} = \frac{v_{in}}{i_{in}}$$

$$R_{out} = \frac{v_x}{i_x} \text{ @ } v_{in} = 0$$



- O.C. voltage gain (Thevenin model)

$$v_{out,oc} = v_{Thevenin} = A_v v_{in}$$

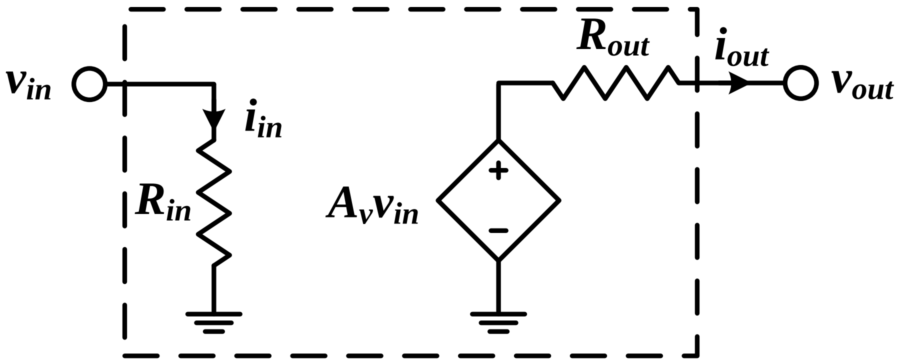
$$A_v = \frac{v_{out,oc}}{v_{in}}$$

Amplifier Model

- Transconductance (Norton model)

$$i_{out,sc} = i_{Norton} = G_m v_{in}$$

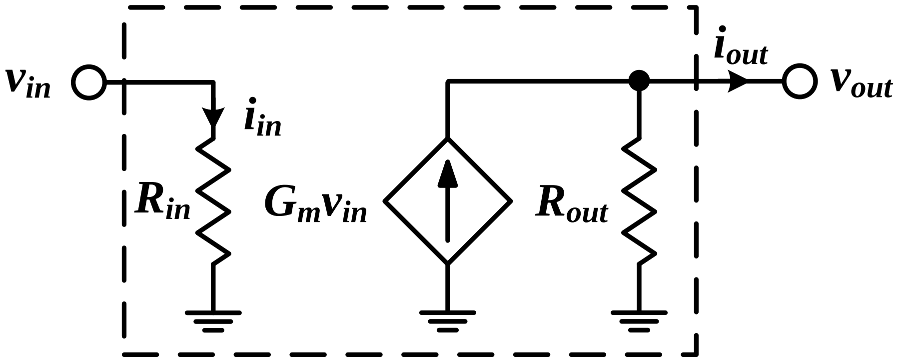
$$G_m = \frac{i_{out,sc}}{v_{in}}$$



- Thevenin ⇔ Norton

$$v_{out,oc} = A_v v_{in} = (G_m v_{in}) R_{out}$$

$$\boxed{A_v = \frac{v_{out,oc}}{v_{in}} = G_m R_{out}}$$

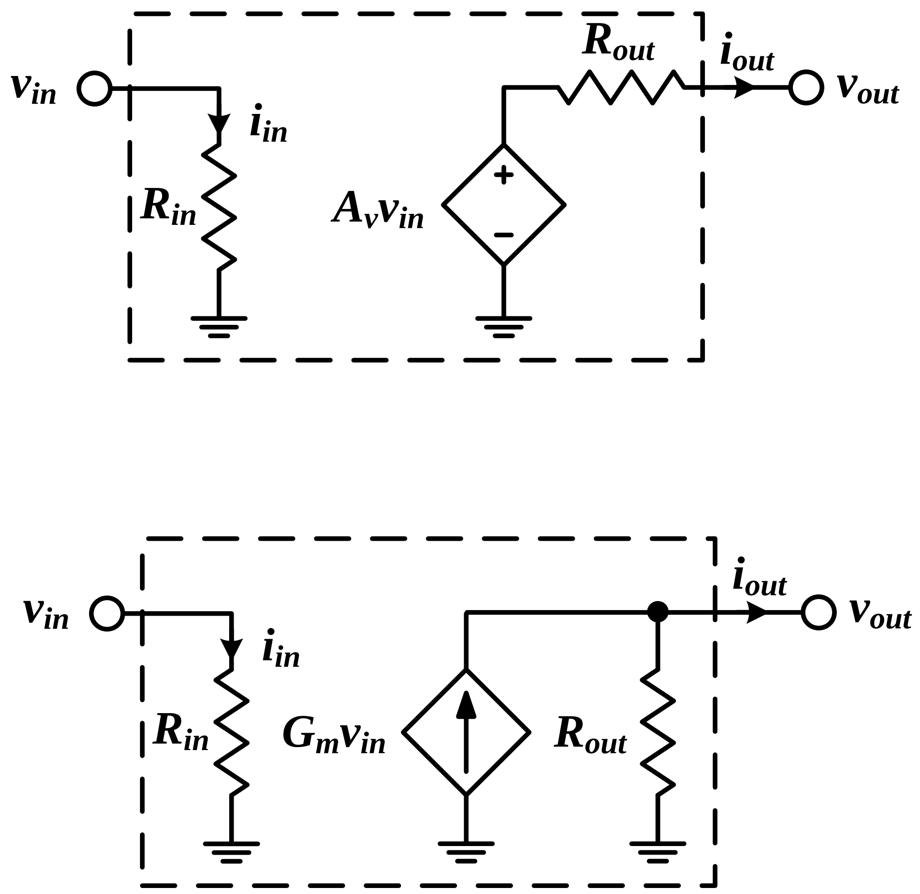


- S.C. Current Gain

$$\boxed{A_i = \frac{i_{out,sc}}{i_{in}} = \frac{i_{out,sc}}{v_{in}/R_{in}} = G_m R_{in}}$$

Why GmRout?

$$R_{in} = v_{in}/i_{in}$$
$$R_{out} = v_x/i_x \ @ \ v_{in} = 0$$
$$G_m = i_{out,sc}/v_{in}$$
$$A_v = G_m R_{out}$$
$$A_i = G_m R_{in}$$



- Divide and conquer
  - Rout simplified: $v_{in} = 0$
  - Gm simplified: $v_{out} = 0$
  - We already need Rin/out and Gm
  - We can quickly and easily get Rin/out from the shortcuts

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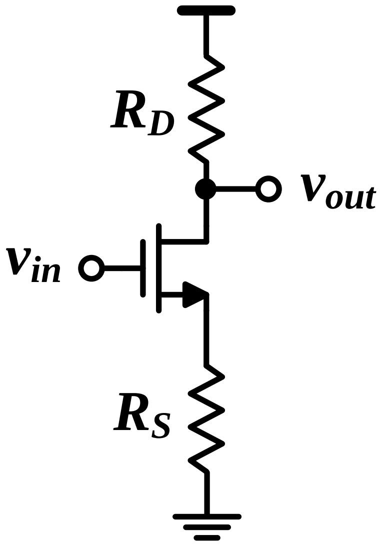
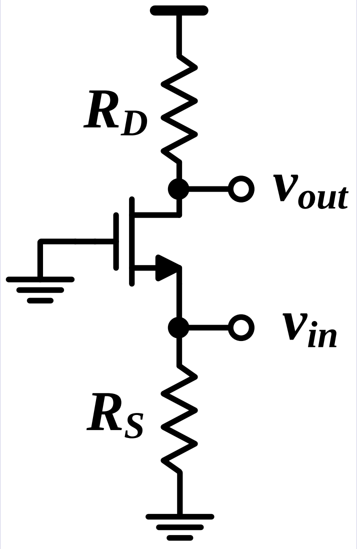
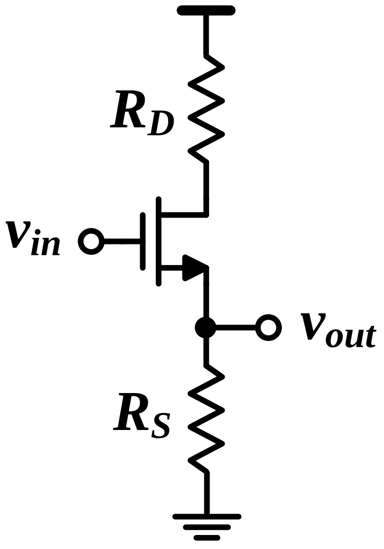
Basic Amplifier Topologies

<table>
<tr><th></th><th>Common Source (CS)</th><th>Common Gate (CG)</th><th>Common Drain (CD)
Source Follower (SF)</th></tr>
<tr><td></td><td>
</td><td>
</td><td>
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Current buffer</td><td>Voltage buffer
Current amplifier</td></tr>
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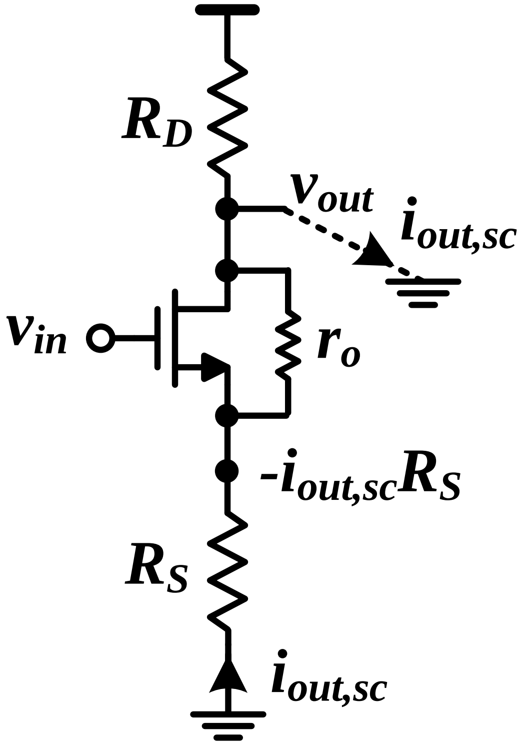
<tr><td>Out</td><td>D</td><td>D</td><td>S</td></tr>
</table>

Common Source (CS)

- Apply KCL at S noting that $v_{gs} = v_{in} - \left(-i_{out,sc} R_S\right)$

$$i_{out,sc} + g_m\left(v_{in} + i_{out,sc} R_S\right) + g_{mb}\left(i_{out,sc} R_S\right) + \frac{i_{out,sc} R_S}{r_o} = 0$$

$$\boxed{G_m = \frac{i_{out,sc}}{v_{in}} \approx \frac{-g_m}{1 + (g_m + g_{mb}) R_S}}$$



$$R_{out} \approx R_D || r_o[1 + (g_m + g_{mb}) R_S]$$

$$A_v = G_m R_{out}$$

- If R_D is ac o.c.: $A_v = -g_m r_o$
- If $R_D \ll R_{LFD}$: $A_v \approx \frac{-g_m R_D}{1+(g_m+g_{mb})R_S}$
- If $R_S = 0$: $A_v = -g_m (R_D // r_o)$

Common Source (CS)

$$\boxed{G_m = \frac{i_{out,sc}}{v_{in}} \approx \frac{-g_m}{1+(g_m+g_{mb})R_S}}$$

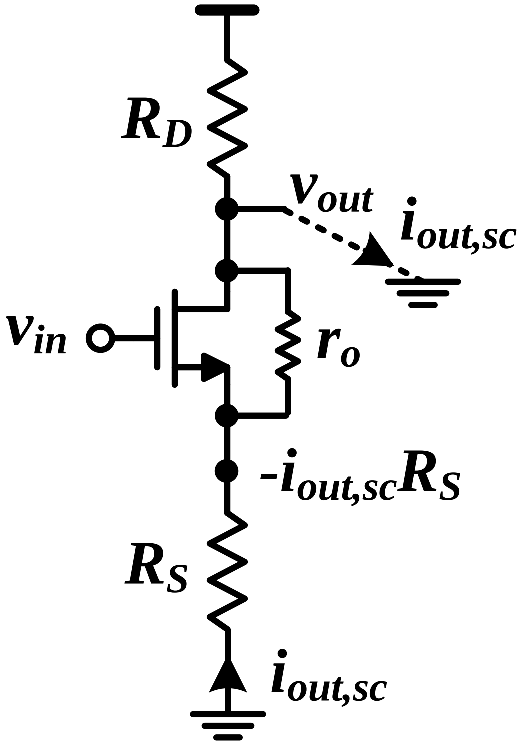
$$R_{out} \approx R_D||r_o[1+(g_m+g_{mb})R_S]$$

$$A_v = G_m R_{out}$$

- ❑ If S and B are ac connected (for PMOS):

$$A_v \approx \frac{-R_D||R_{LFD}}{\frac{1}{g_m}+R_S} = -\frac{Drain\ Res.}{\frac{1}{g_m}+Source\ Res.}$$

- ❑ If $R_S \gg \frac{1}{g_m}$ & $R_D \ll R_{LFD}$: $A_v \approx \frac{-R_D}{R_S} \rightarrow$ Linear gain!
- ❑ R_S reduces G_m → **Source degeneration**
  - ▪ But improves linearity



Common Gate (CG)

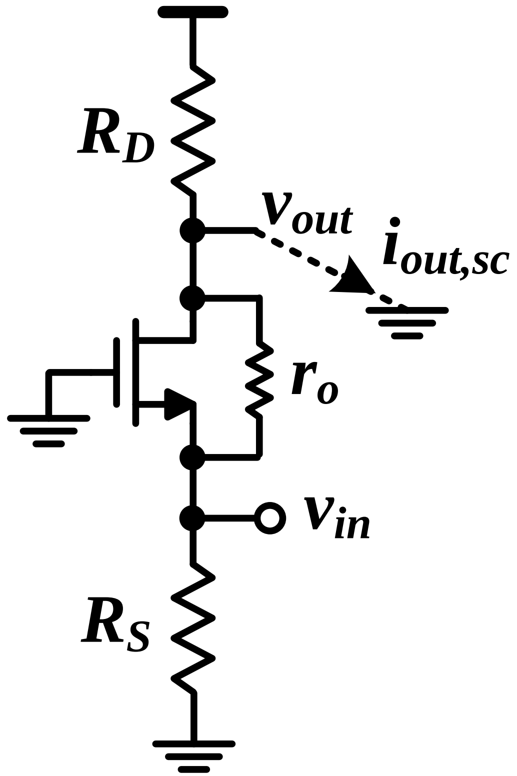
- Apply KCL at D noting that $v_{gs} = v_{bs} = 0 - v_{in}$

$$i_{out,sc} + (g_m + g_{mb})(-v_{in}) - \frac{v_{in}}{r_o} = 0$$

$$\boxed{\boldsymbol{G_m = \frac{i_{out,sc}}{v_{in}} \approx g_m + g_{mb}}}$$

$$R_{out} \approx R_D || r_o \ (\boldsymbol{why?})$$

$$A_v = G_m R_{out}$$



- If R_D is ac o.c.: $A_v = (g_m + g_{mb}) r_o$
- If $R_D \ll r_o$: $A_v \approx (g_m + g_{mb}) \boldsymbol{R_D}$
- If S and B are ac connected: $(g_m + g_{mb}) \rightarrow g_m$
- Note that $A_i = G_m R_{in} <\approx$ **1 (Current Buffer)**

Common Drain (CD) – Source Follower

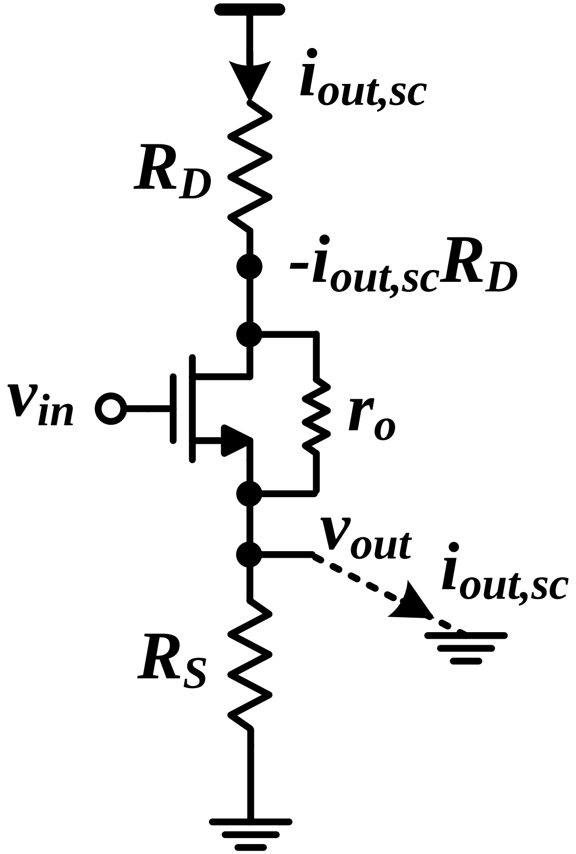
- Apply KCL at D

$$i_{out,sc} - g_m v_{in} + \frac{i_{out,sc} R_D}{r_o} = 0$$

$$\boxed{\boldsymbol{G_m = \frac{i_{out,sc}}{v_{in}} \approx \frac{g_m}{1 + R_D/r_o}}}$$

$$R_{out} \approx R_S || \frac{1}{g_m + g_{mb}}\left(1 + \frac{R_D}{r_o}\right)$$

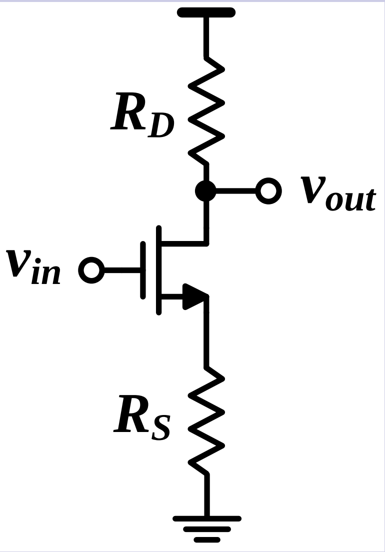
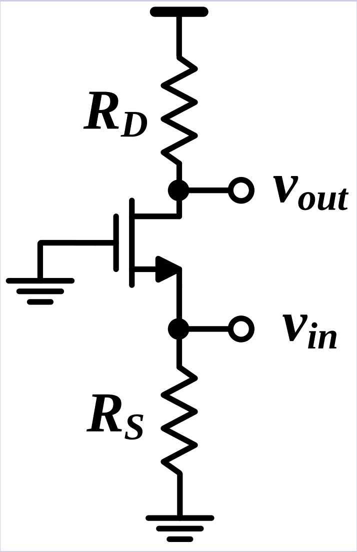
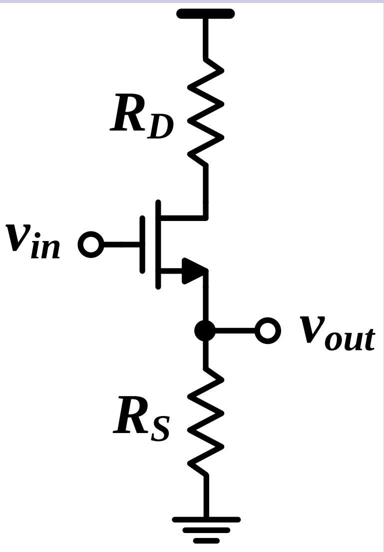
$$A_v = G_m R_{out}$$



- If $R_S \gg R_{LFS}$: $\quad A_v \approx \frac{g_m}{g_m+g_{mb}} < 1$
- If S and B are ac connected (for PMOS):

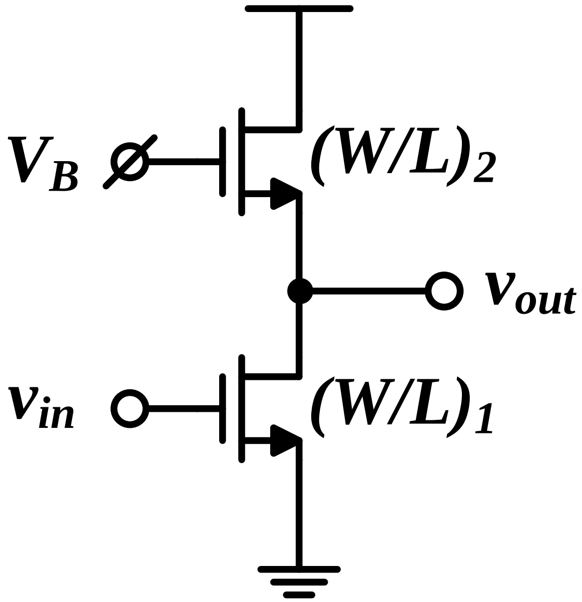
$$A_v \approx \textbf{1 (Voltage buffer)}$$

Summary of Basic Topologies

	CS	CG	CD (SF)
			
	Voltage & current amplifier	Voltage amplifier Current buffer	Voltage buffer Current amplifier
Rin	∞	$R_S \| \| \frac{1}{g_m + g_{mb}}\left(1 + \frac{R_D}{r_o}\right)$	∞
Rout	$R_D \| \| r_o[1 + (g_m + g_{mb})R_S]$	$R_D \| \| r_o$	$R_S \| \| \frac{1}{g_m + g_{mb}}\left(1 + \frac{R_D}{r_o}\right)$
Gm	$\frac{-g_m}{1 + (g_m + g_{mb})R_S}$	$g_m + g_{mb}$	$\frac{g_m}{1 + R_D/r_o}$

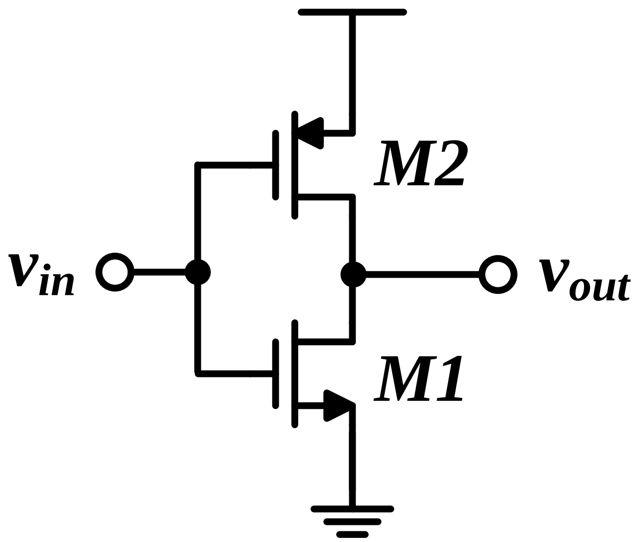
Quiz

- The circuit below shows a CS amplifier With diode-connected load
- Find the gain using GmRout (ignore body effect and CLM).
  - Express the gain in terms of $(W/L)_1$ and $(W/L)_2$.
- This is a “linear” CS amplifier.



Quiz

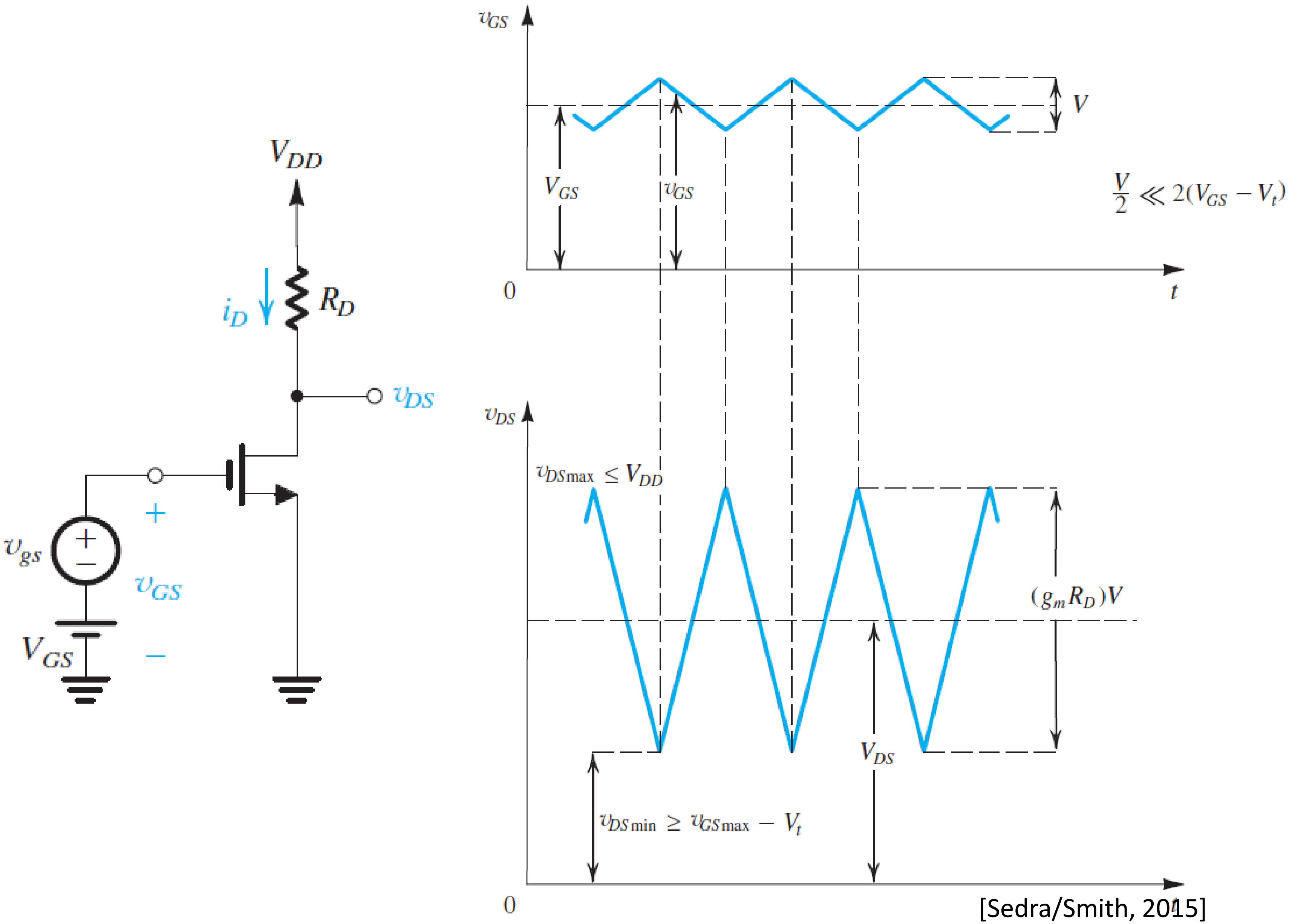
- The circuit below shows a complementary CS amplifier (inverter amp).
- Find the gain using GmRout.



Outline

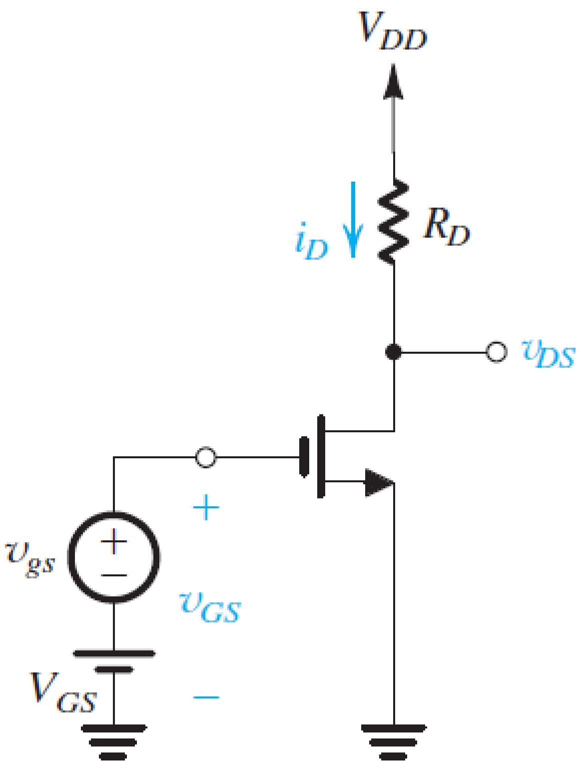
- ❑ Recapping previous key results
- ❑ Why amplifiers?
- ❑ Basic amplifier operation
- ❑ Basic amplifier analysis
- ❑ Rin/out shortcuts
  - Looking from drain
  - Looking from source
- ❑ GmRout method
- ❑ Basic amplifier topologies
  - Common Source (CS)
  - Common Gate (CG)
  - Common Drain (CD) – Source Follower (SF)
- ❑ Large signal behavior

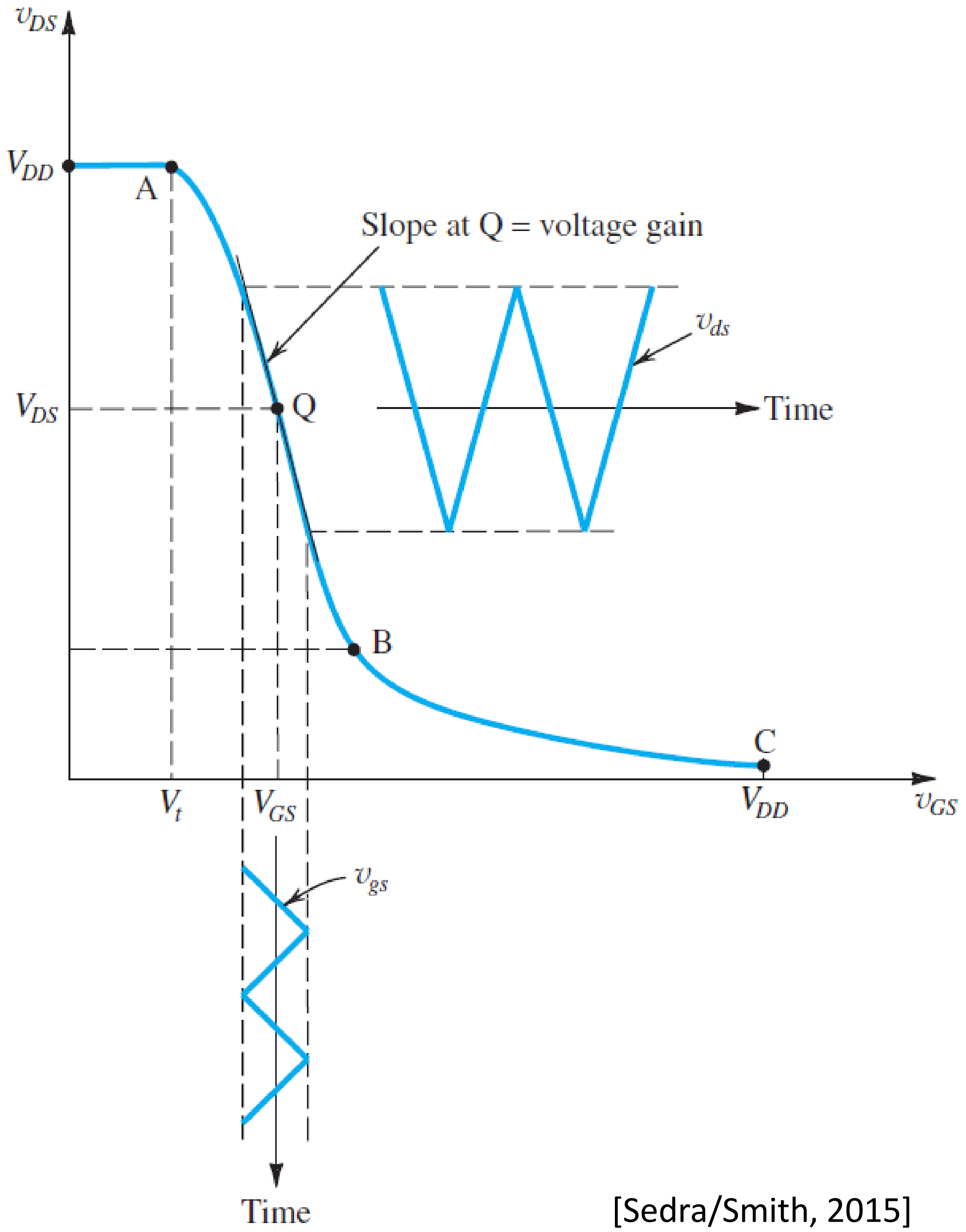
CS Amplifier Example



[Sedra/Smith, 2015]

CS Large Signal Behavior

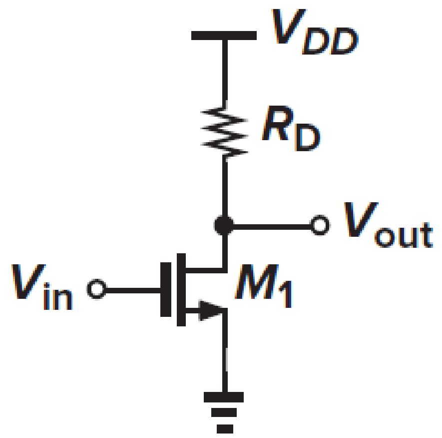


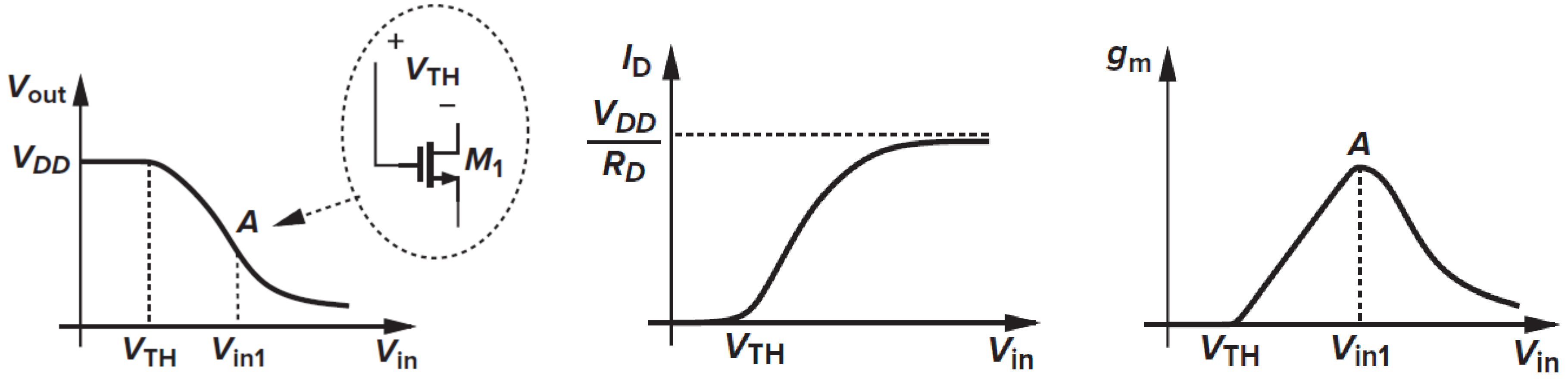


[Sedra/Smith, 2015]

CS Large Signal Behavior

- Gain is non-linear:
  - $A_v = f(V_{in})$
  - $g_m = f(V_{in})$
- For linear gain, A_v should NOT be $f(V_{in})$
- A_v and g_m are max at edge of triode
  - But they are highly non-linear
  - And the available signal swing vanishes





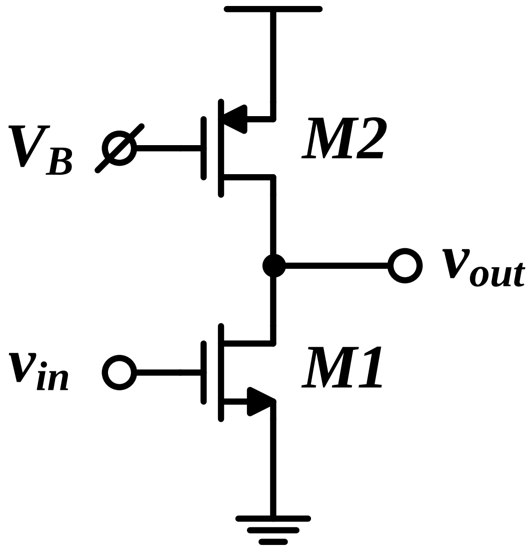
[Razavi, 2017]

Output Signal Swing

$$V_{out,max} = V_{DD} - V_{ov2}$$

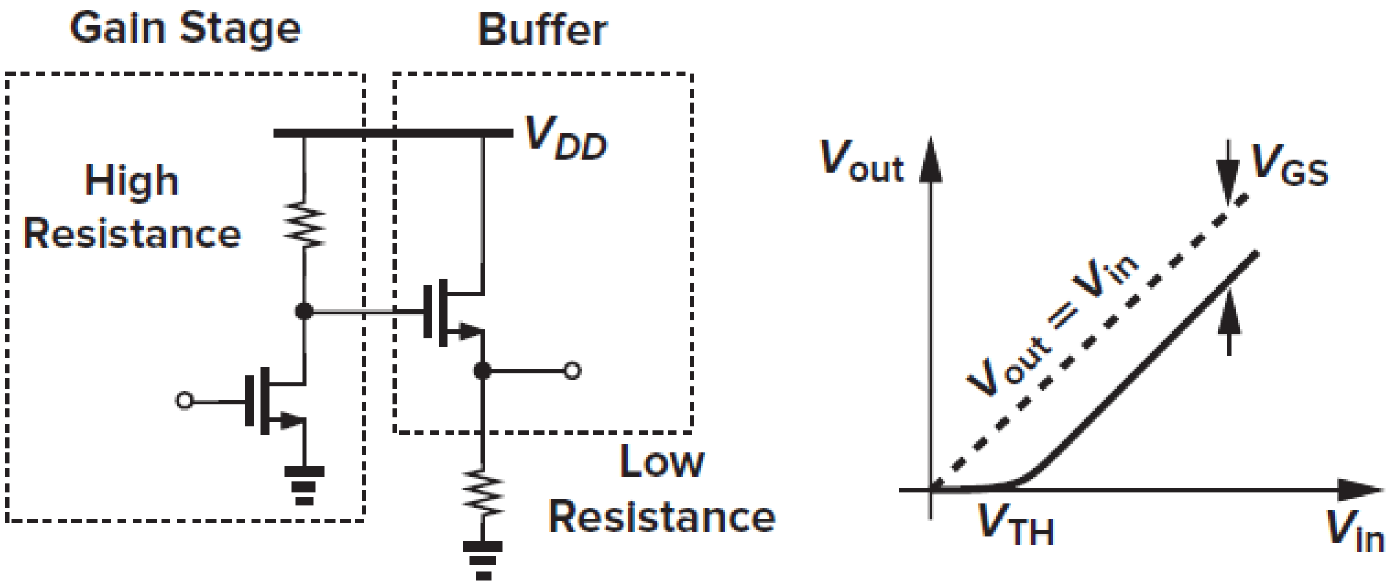
$$V_{out,min} = V_{ov1} = V_{in,max} - V_{TH}$$

- Output swing $\approx V_{DD} - 2V_{ov}$



CD Large Signal Behavior

- Why Source Follower?
  - Buffer
  - Level-shifter



[Razavi, 2017]

Thank you!

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- B. Razavi, “Fundamentals of Microelectronics,” Wiley, 2nd ed., 2014
- B. Razavi, “Design of Analog CMOS Integrated Circuits,” McGraw-Hill, 2nd ed., 2017