4 July 2020

وَمَا أُوتِيتُمْ مِنَ الْعِلْمِ إِلَّا قَلِيلًا

13 ذو القعدة 1441

Analog IC Design

Lecture 17
Noise Analysis Fundamentals

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Outline

- Recapping previous key results
- Noise in time and frequency domains
- Resistor thermal noise
- MOSFET thermal and flicker noise
- Signal-to-noise ratio (SNR) and input-referred noise
- Noise analysis example

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MOSFET in Saturation

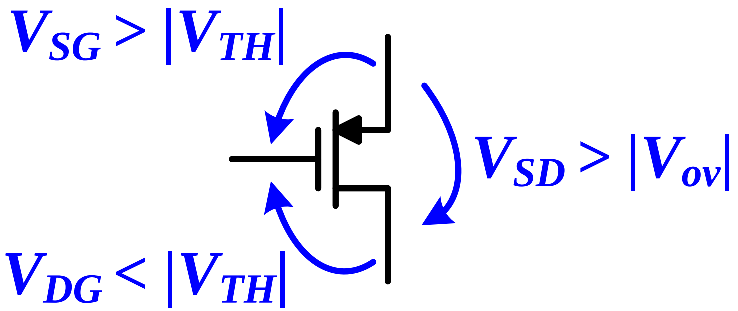
- The channel is pinched off if the difference between the gate and drain voltages is not sufficient to create an inversion layer

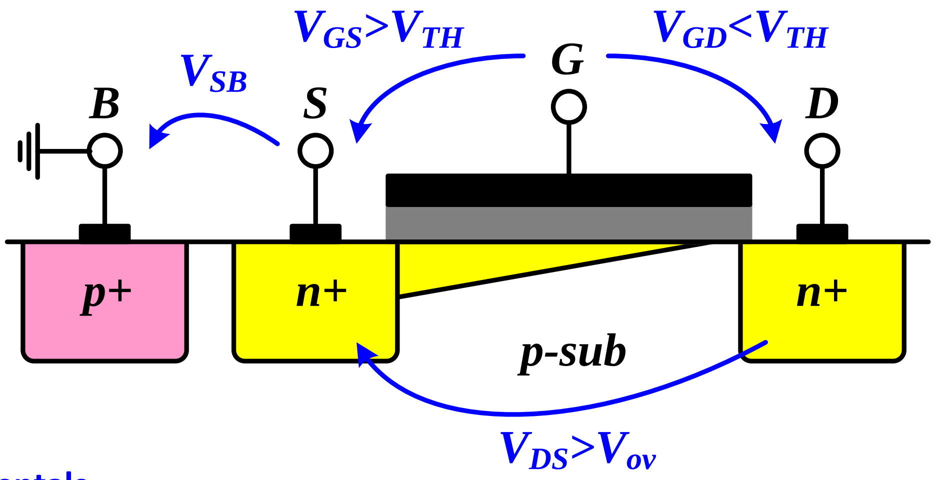
$$V_{GD} \leq V_{TH} \quad or \quad V_{DS} \geq V_{ov}$$

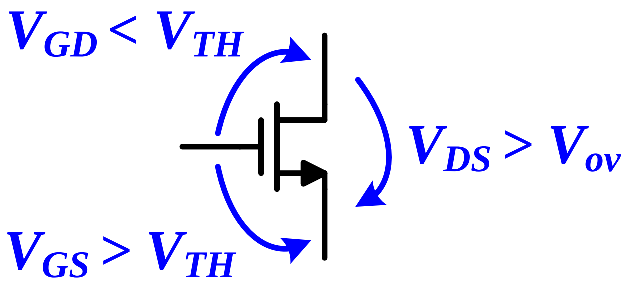
- Square-law (long channel MOS)

$$I_D = \frac{\mu_n C_{ox}}{2}\frac{W}{L} \cdot V_{ov}^2(1 + \lambda V_{DS})$$

$$V_{SB} \uparrow \;\; \Rightarrow \;\; V_{TH} \uparrow$$







Regions of Operation Summary

OFF (Subthreshold)

$V_{GS} < V_{TH}$

ON

$V_{GS} > V_{TH}$

Triode

$V_{DS} < V_{ov}$

Or

$V_{GD} > V_{TH}$

$$I_D = \mu C_{ox} \frac{W}{L}\left(V_{ov}V_{DS} - \frac{V_{DS}^2}{2}\right)$$

Pinch-Off (Saturation)

$V_{DS} \geq V_{ov}$

Or

$V_{GD} \leq V_{TH}$

$$I_D = \frac{\mu C_{ox}}{2}\frac{W}{L}V_{ov}^2(1 + \lambda V_{DS})$$

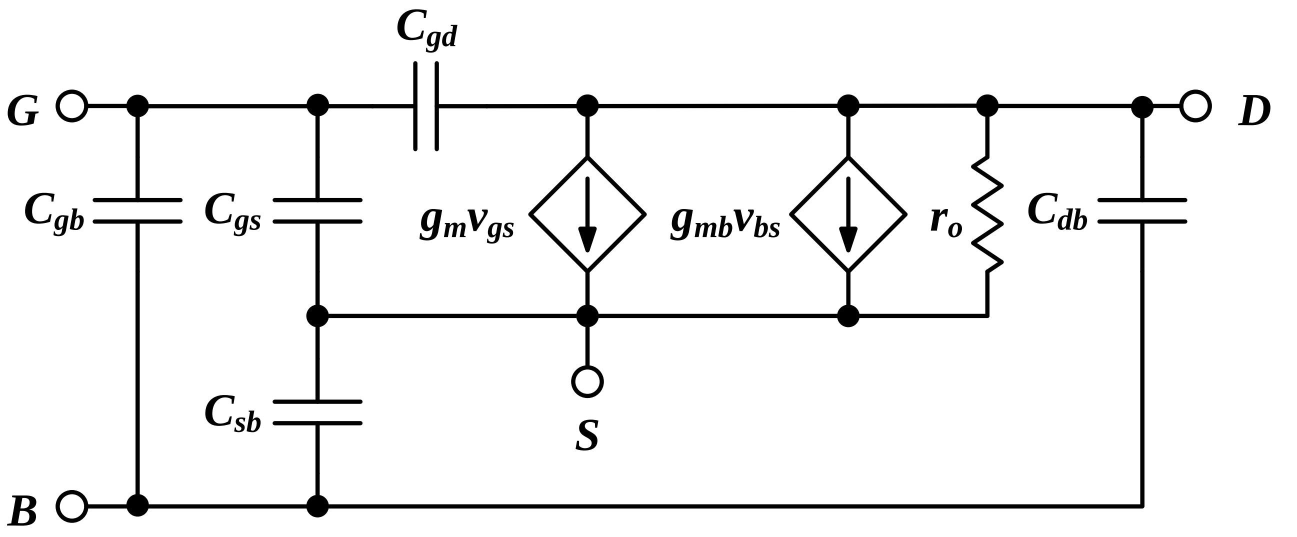
High Frequency Small Signal Model

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \mu C_{ox}\frac{W}{L}V_{ov} = \sqrt{\mu C_{ox}\frac{W}{L}\cdot 2I_D} = \frac{2I_D}{V_{ov}}$$

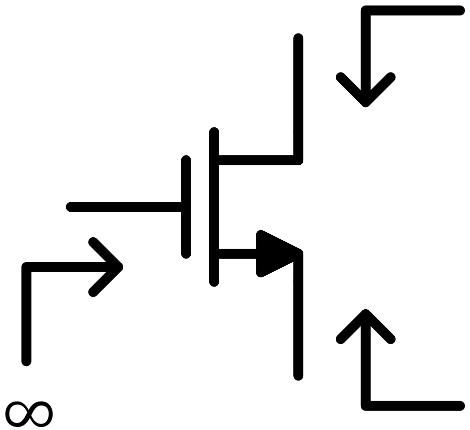
$$g_{mb} = \eta g_m \qquad \eta \approx 0.1 - 0.25$$

$$r_o = \frac{1}{\partial I_D/\partial V_{DS}} = \frac{V_A}{I_D} = \frac{1}{\lambda I_D} \qquad V_A \propto L \leftrightarrow \lambda \propto \frac{1}{L} \qquad V_{DS}\uparrow V_A\uparrow$$

$$C_{gb} \approx 0 \qquad C_{gs} \gg C_{gd} \qquad C_{sb} > C_{db}$$



Rin/out Shortcuts Summary



$$r_o[1 + (g_m + g_{mb})R_S]$$

H.I.N.

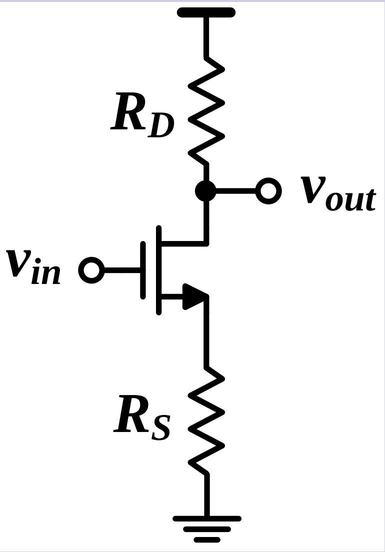
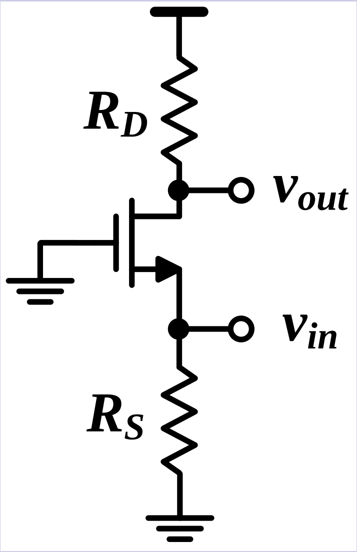
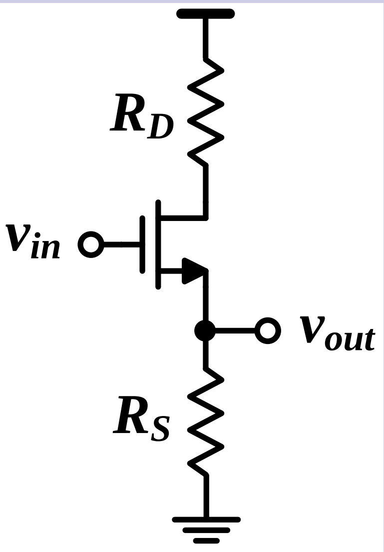
$$\frac{1}{g_m + g_{mb}}\left(1 + \frac{R_D}{r_o}\right)$$

L.I.N.

∞

At low frequencies **ONLY**

Summary of Basic Topologies

	CS	CG	CD (SF)
			
	Voltage & current amplifier	Voltage amplifier Current buffer	Voltage buffer Current amplifier
Rin	∞	$R_S \|\| \frac{1}{g_m + g_{mb}}\left(1 + \frac{R_D}{r_o}\right)$	∞
Rout	$R_D \|\| r_o[1 + (g_m + g_{mb})R_S]$	$R_D \|\| r_o$	$R_S \|\| \frac{1}{g_m + g_{mb}}\left(1 + \frac{R_D}{r_o}\right)$
Gm	$\frac{-g_m}{1 + (g_m + g_{mb})R_S}$	$g_m + g_{mb}$	$\frac{g_m}{1 + R_D/r_o}$

Differential Amplifier

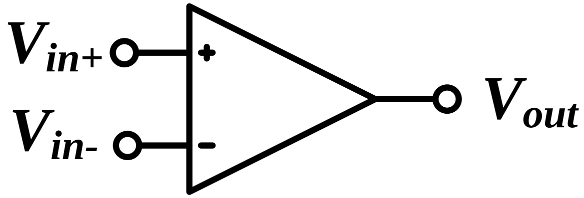
	Pseudo Diff Amp	Diff Pair (w/ ideal CS)	Diff Pair (w/ R_{SS})
A_{vd}	$-g_m R_D$	$-g_m R_D$	$-g_m R_D$
A_{vCM}	$-g_m R_D$	0	$\frac{-g_m R_D}{1+2(g_m+g_{mb})R_{SS}}$
A_{vd}/A_{vCM}	1	∞	$2(g_m+g_{mb})R_{SS} \gg 1$

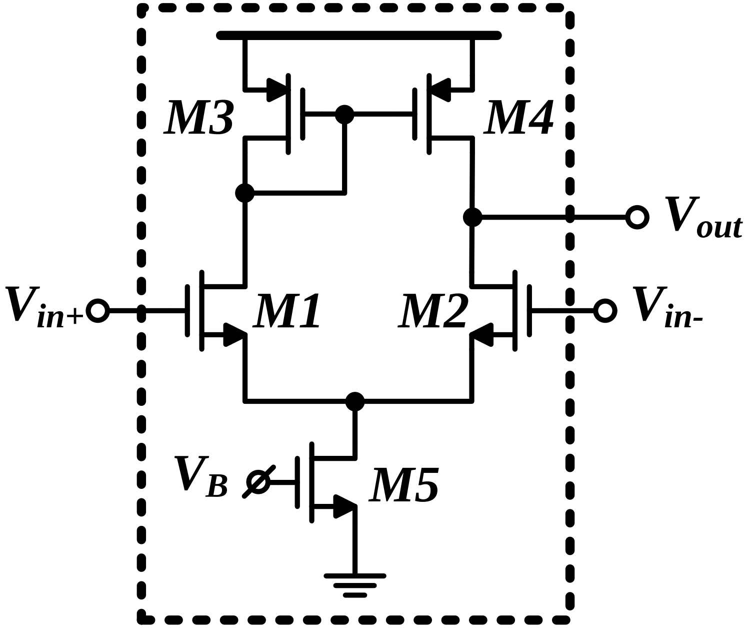
$$A_{vCM2d} = \frac{v_{od}}{v_{iCM}} \approx \frac{\Delta R_D}{2R_{SS}} + \frac{\Delta g_m R_D}{2g_{m1,2}R_{SS}}$$

$$CMRR = \frac{A_{vd}}{A_{vCM2d}}$$

Op-Amp

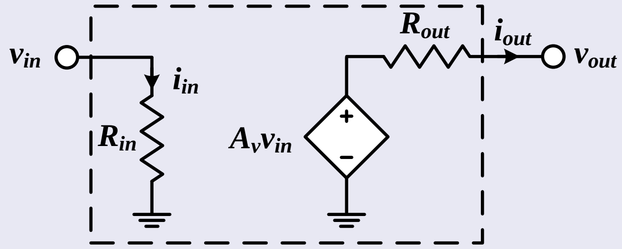
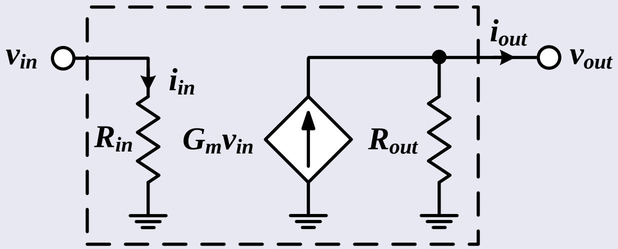
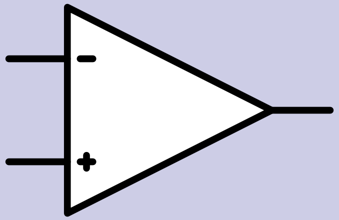
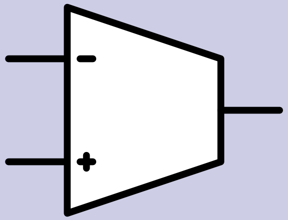
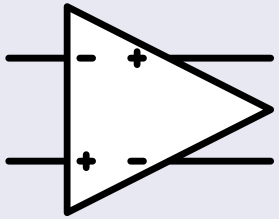
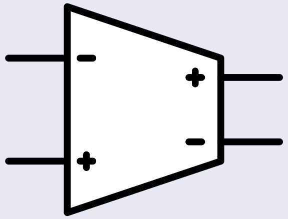
- An op-amp is simply a high gain differential amplifier
  - The gain can be increased by using cascodes and multi-stage amplification
- The diff amp is a key block in many analog and RF circuits
  - DEEP understanding of diff amp is ESSENTIAL





Op-Amp vs OTA

- In short, an OTA is an op-amp without an output stage (buffer)
- Some designers just use op-amp name and symbol for both

	Op-amp	OTA
Rout	LOW	HIGH
Model		
Diff input, SE output		
Fully diff		

V-star ($\boldsymbol{V^*}$)

- V-star (V^*) is inspired by V_{ov} but calculated from actual simulation data

$$g_m = \frac{2I_D}{V^*} \leftrightarrow V^* = \frac{2I_D}{g_m} = \frac{2}{g_m/I_D}$$

- Figures-of-merit in terms of V^*

$$g_m r_o = \frac{2I_D}{V^*} \cdot \frac{1}{\lambda I_D} = \frac{2}{\lambda V^*}$$

$$f_T = \frac{g_m}{2\pi C_{gg}} = \frac{1}{2\pi} \cdot \frac{2I_D}{V^*} \cdot \frac{1}{C_{gg}}$$

$$\frac{g_m}{I_D} = \frac{2}{V^*}$$

- The boundary between weak and strong inversion ($n = 1.2 \rightarrow 1.5$)

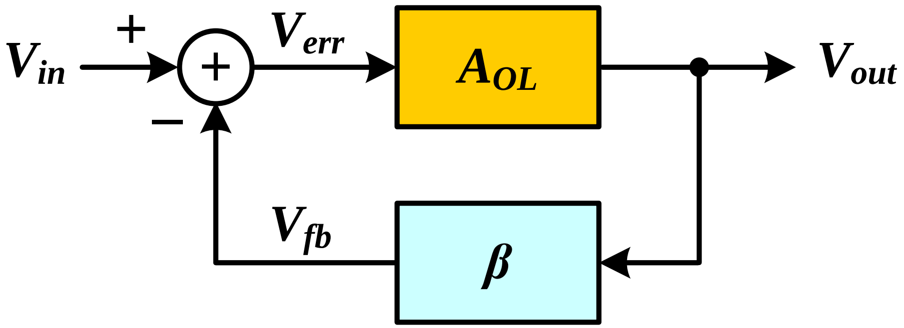
$$V_{ov}(SI) = V^*(WI) = 2nV_T \approx 60 \rightarrow 80mV$$

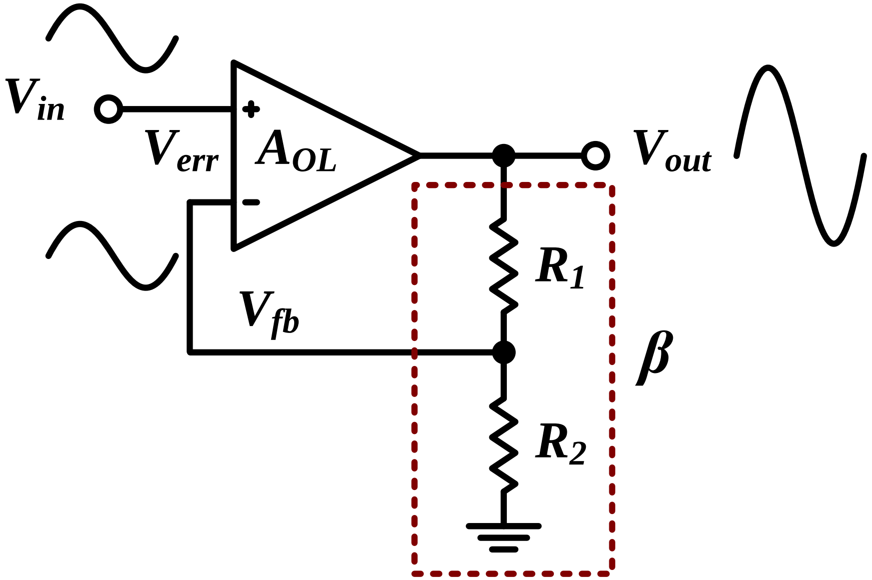
Negative Feedback

$$\beta = \frac{R_2}{(R_1 + R_2)}$$

$$A_{CL} = \frac{V_{out}}{V_{in}} = \frac{A_{OL}}{1 + \beta A_{OL}} = \frac{A_{OL}}{1 + \beta A_{OL}} \approx \frac{1}{\beta} = \frac{R_1 + R_2}{R_2} = 1 + \frac{R_1}{R_2}$$

$$\omega_{p,CL} = (1 + \beta A_{OLo})\omega_{P,OL}$$

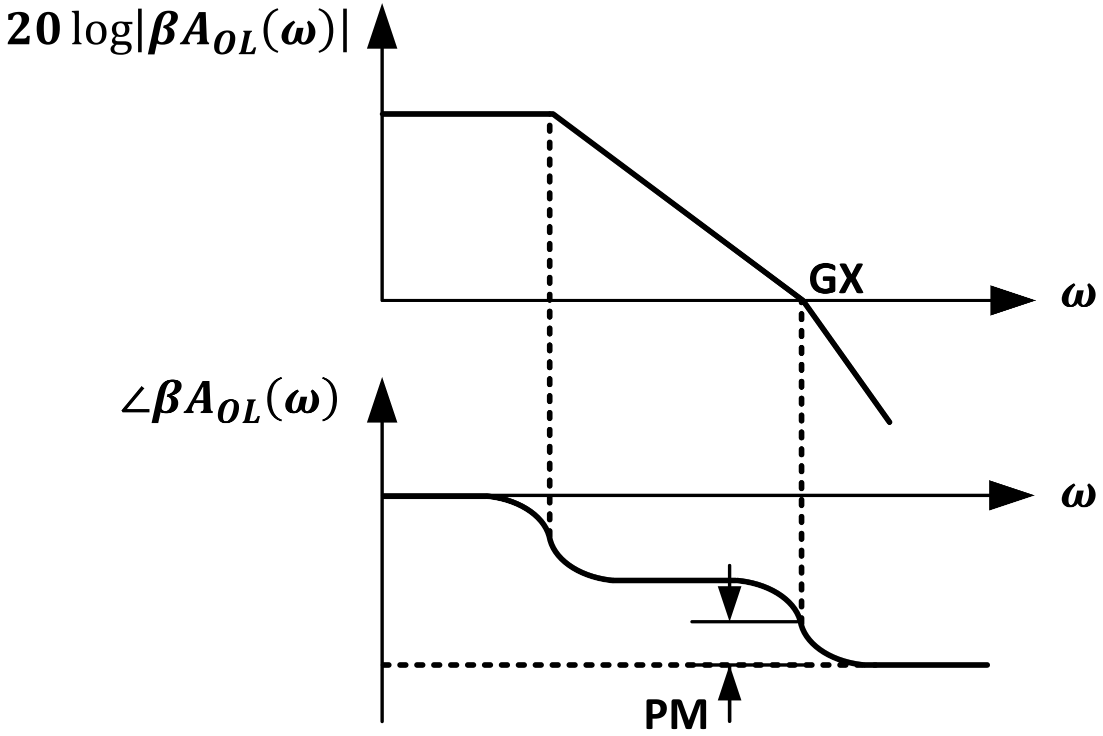




Phase Margin and the Ultimate GBW

- If $\omega_{p2} = \omega_u$: PM = 45°
  - Typically inadequate (peaking/ringing)
- Thus ω_{p2} should be $> \omega_u$ → $\omega_{p1} \ll \omega_u < \omega_{p2}$
  - ω_{p1} defines OL BW and ω_{p2} defines ultimate GBW (max CL BW)

Frequency domain peaking
→ noise amplification
Time domain ringing
→ poor settling time



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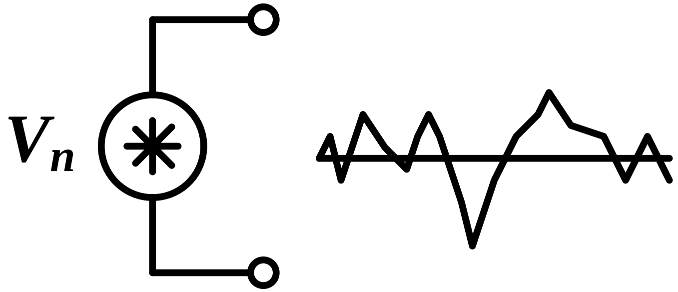
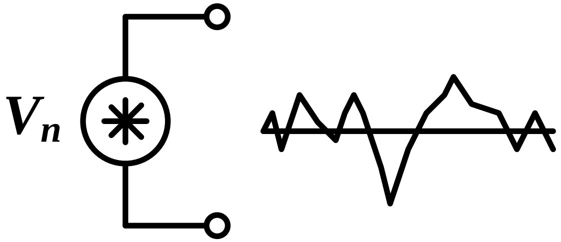
Noise in Time Domain

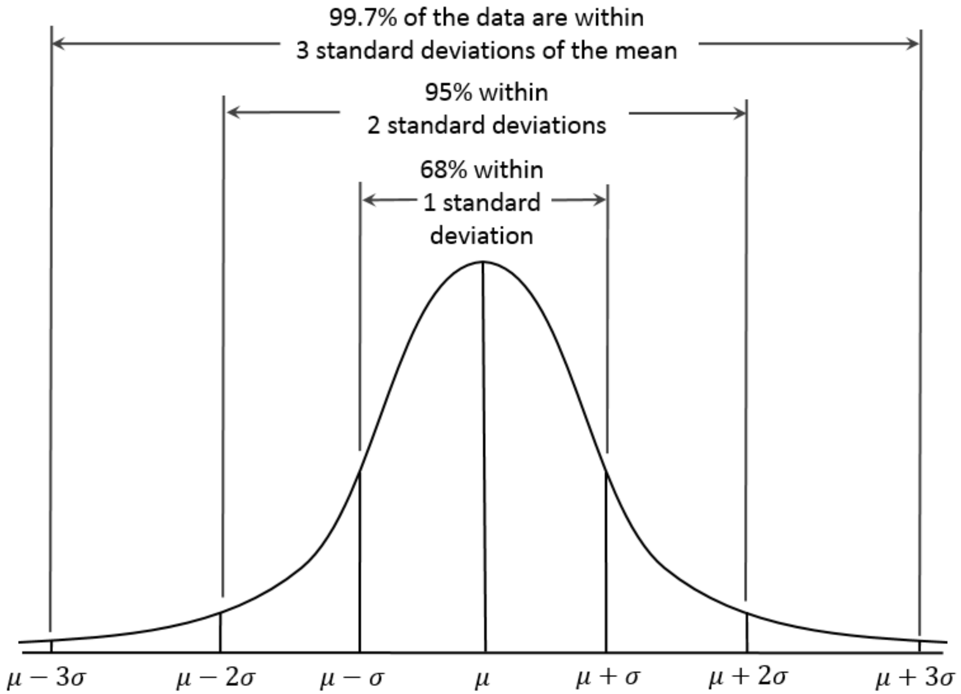
- Noise is an unwanted random signal
- We cannot predict (model) its instantaneous value in advance
- But we can predict (model) noise statistical distribution



Noise Statistical Distribution

- Noise has normal (gaussian) distribution
- The mean is zero (average noise voltage in time domain = 0)
- The variance (σ^2) is the mean-square value
- The standard deviation (σ) is the root-mean-square (rms) value
- Peak-to-peak instantaneous noise voltage is usually within $\pm 3\sigma$





Noise Power and Noise Voltage

- Average power of a periodic signal (in Watts)

$$P_{avg} = \frac{1}{T}\int_{\frac{-T}{2}}^{\frac{T}{2}} P(t)dt = \frac{1}{T}\int_{\frac{-T}{2}}^{\frac{T}{2}} \frac{V^2(t)}{R_L}dt = \frac{1}{R_L}\cdot\frac{1}{T}\int_{\frac{-T}{2}}^{\frac{T}{2}} V^2(t)dt = \frac{\overline{V^2}}{R_L} = \frac{\sigma^2}{R_L}$$

- Average power of a noise signal (non-periodic)

$$P_{avg} = \frac{1}{R_L}\cdot\lim_{T\to\infty}\frac{1}{T}\int_{\frac{-T}{2}}^{\frac{T}{2}} V_n^2(t)dt = \frac{\overline{V_n^2}}{R_L} = \frac{\sigma_n^2}{R_L}$$

- Mean-square noise voltage

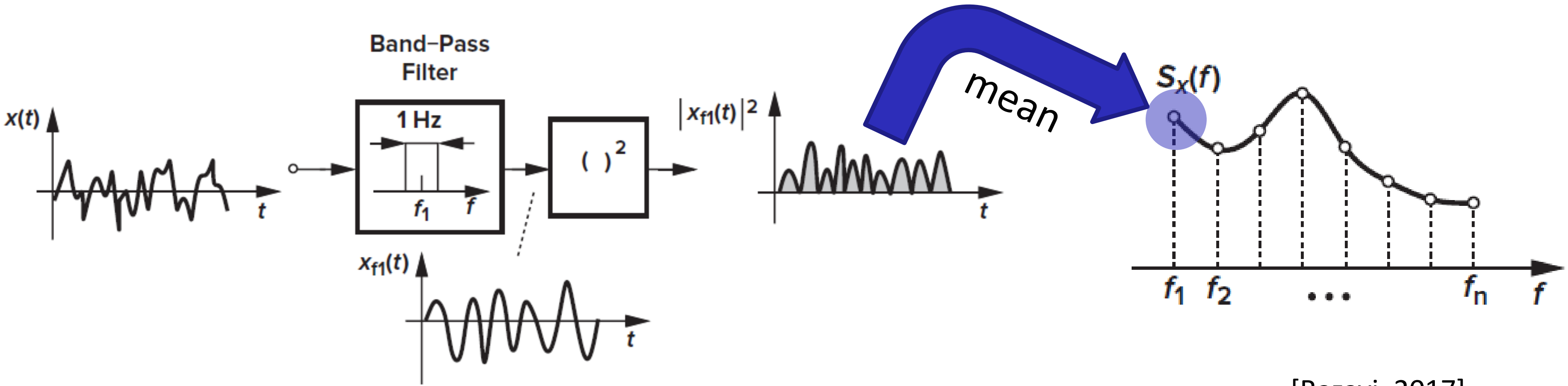
$$\overline{V_n^2} = \lim_{T\to\infty}\frac{1}{T}\int_{\frac{-T}{2}}^{\frac{T}{2}} V_n^2(t)dt = \sigma_n^2 \propto P_{avg}$$

- RMS (root-mean-square) noise voltage

$$V_{nrms} = \sqrt{\overline{V_n^2}} = \sigma_n$$

Noise in Frequency Domain

- A signal $x(t)$ has power spectral density (PSD) $= S_x(f)$
  - How much power is carried around each frequency.
- PSD of a noise signal $x(t)$ at a frequency f_1 is the average power (mean-square value) in a one-hertz bandwidth around f_1
  - Measured in W/Hz or V^2/Hz
  - Sweep f_1 from 0 to ∞
- Voltage noise density: $V_n(f) = \sqrt{S_x(f)} \rightarrow V/\sqrt{Hz}$

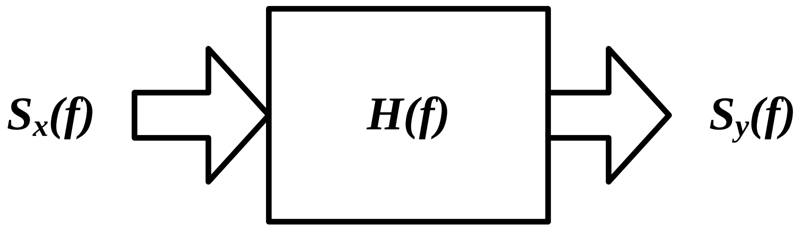


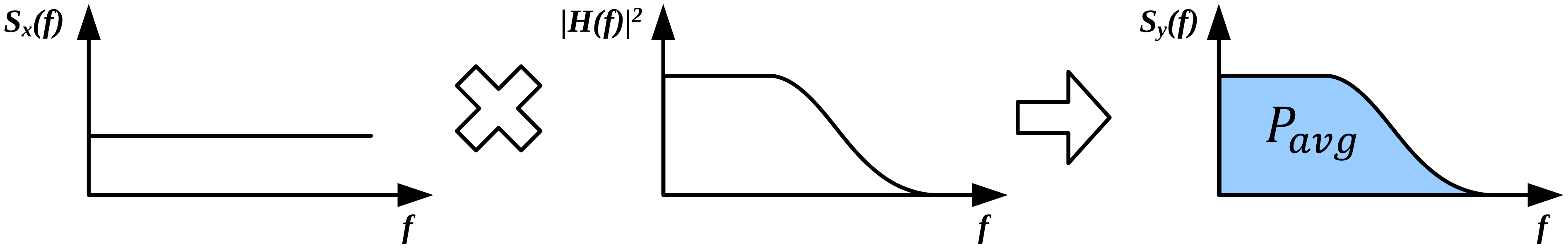
[Razavi, 2017]

White Noise and Noise Shaping

- White noise: noise PSD has the same value at all frequencies.
  - Similar to white light.
- The noise spectrum is shaped by the system transfer function.
- Average output noise power (mean-square value) is the area under the output PSD curve.

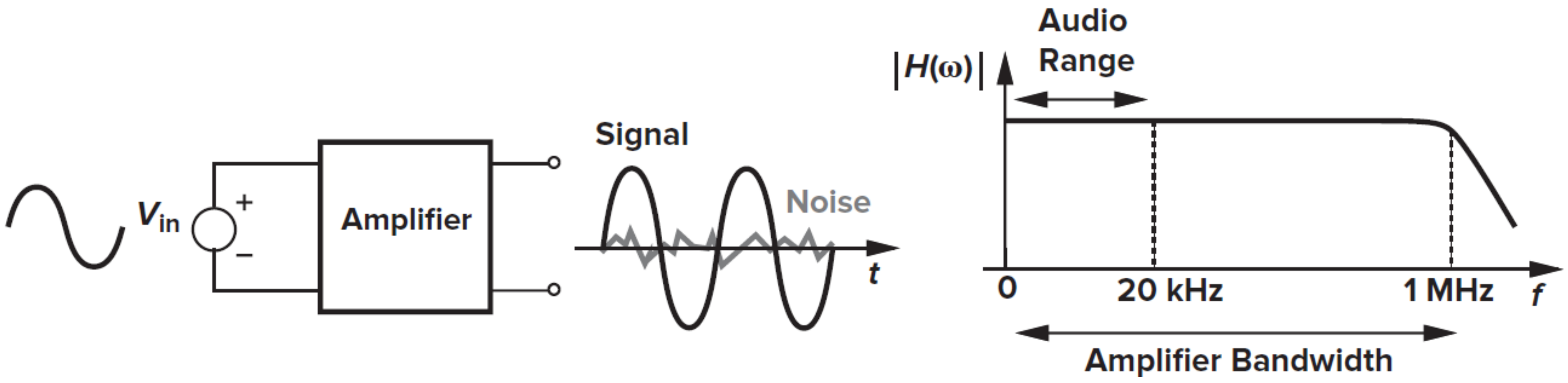
$$\boxed{P_{avg} = \int_{-\infty}^{\infty} S_y(f)\,df}$$





Is Wide BW Good BW?

- Wide BW means more noise power is integrated
- Wide BW is not good BW
  - The amplifier BW should just fit the signal



[Razavi, 2017]

Types of Noise

- External noise
  - A.k.a. interference noise or man-made noise.
  - Unwanted interaction between the outside world and the circuit.
  - Ex: EM interference noise and power supply noise.
  - Can be eliminated by careful design, layout, shielding, etc.
- Internal noise
  - Inherent noise due to the fundamental physical properties of the circuit components.
  - Can be reduced but cannot be eliminated.

Internal Noise Mechanisms

- Thermal noise (a.k.a. Johnson or Nyquist noise):
  - Due to thermal excitation of charge carriers
  - White spectral density
  - Independent of DC current
  - Occurs in all resistive elements (including semiconductors)
- Shot noise:
  - Due to non-smooth DC current (flow of individual carriers)
  - White spectral density
  - Occurs in pn-junctions (and consequently BJT)
- Flicker noise (1/f noise):
  - Due to traps in semiconductors affecting DC current flow
  - Significant noise source in MOSFET

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Resistor Thermal Noise

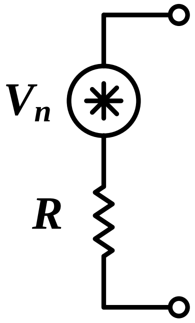
- From thermodynamics, it can be shown that the **spectral density** of resistor thermal noise is given by

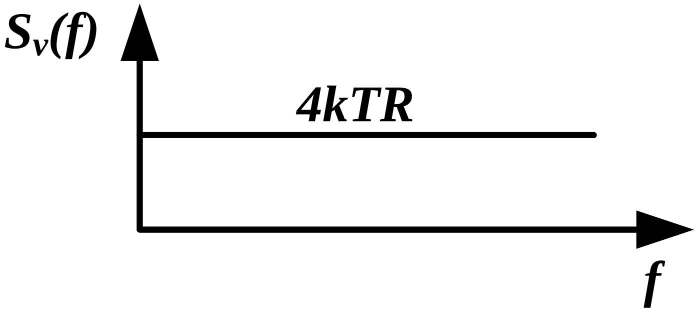
$$V_n^2(f) = S_v(f) = 4kTR$$

$$k = 1.38 \times 10^{-23}\, J/K \text{ and } kT = 4.14 \times 10^{-21}\, J\ @\ T = 300\ K$$

- For $R = 1k\Omega \rightarrow V_n(f) \approx 4\ \frac{nV}{\sqrt{Hz}}$

$$V_n(f) \approx \sqrt{\frac{R}{1\ k}} \times 4\frac{nV}{\sqrt{Hz}}$$





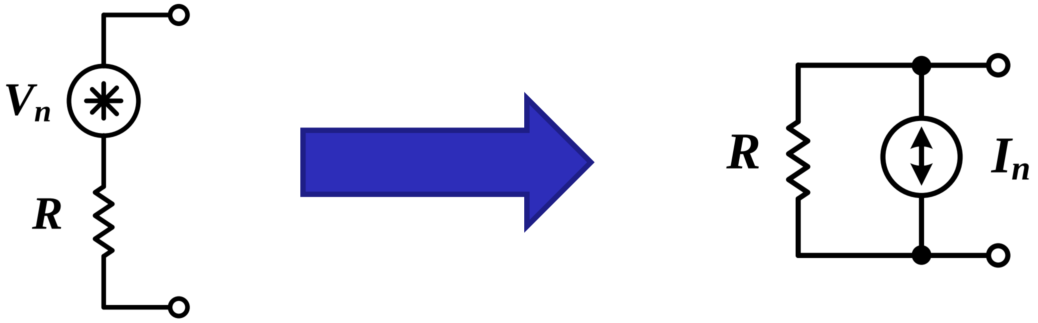
Resistor Thermal Noise

- The Thevenin noise model can be converted to Norton model

$$I_n^2(f) = \frac{V_n^2(f)}{R^2} = \frac{4kT}{R}$$

- For $R = 1k\Omega \rightarrow I_n(f) \approx 4\ \frac{pA}{\sqrt{Hz}}$

$$I_n(f) \approx \sqrt{\frac{1\ k}{R}} \times 4\ \frac{pA}{\sqrt{Hz}}$$



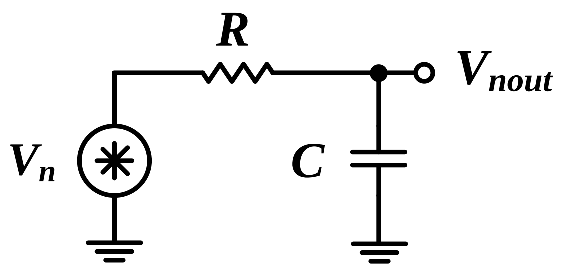
Noise in RC Circuit

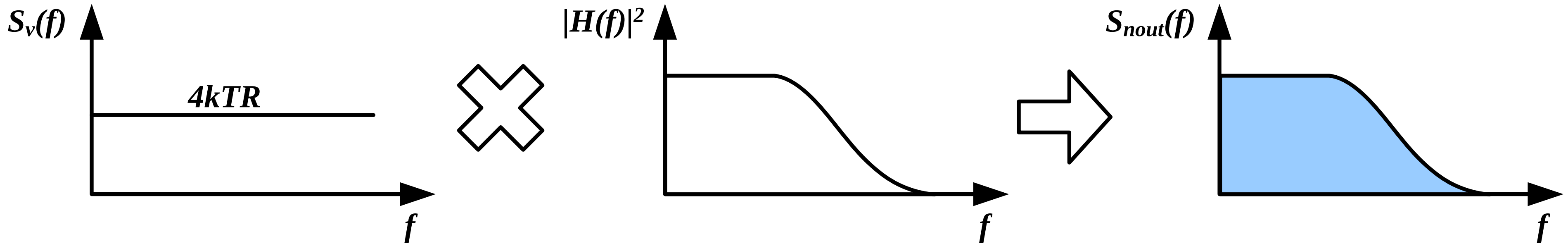
- Resistors never exist alone
  - The BW is always limited by a cap

$$S_{nout}(f) = S_v(f)\left|\frac{V_{nout}(j\omega)}{V_n(j\omega)}\right|^2$$

$$\overline{V_{nout}^2} = V_{noutrms}^2 = \int_{-\infty}^{\infty} S_{nout}(f)\,df$$

$$\boxed{\overline{V_{nout}^2} = \frac{kT}{C}}$$



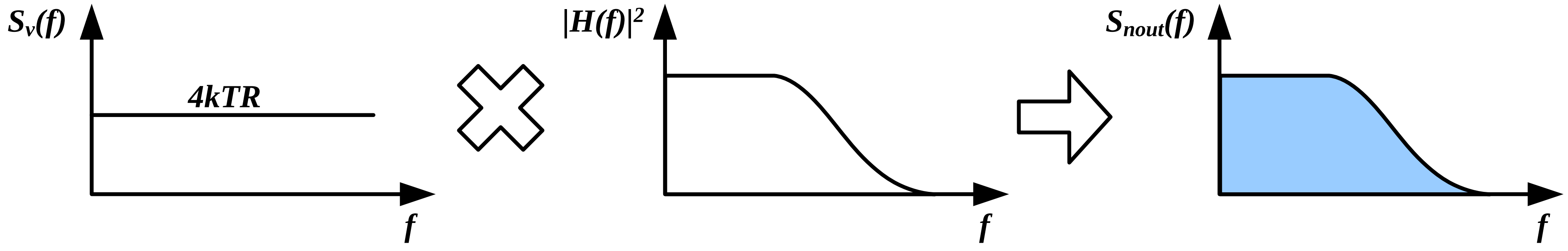


Noise in RC Circuit

$$\overline{V_{nout}^2} = \frac{kT}{C}$$

- RMS noise is independent of R! (why?)
- For $C = 1\,pF$ → $V_{nrms} \approx 64\,\mu Vrms$

$$V_{nrms} \approx \sqrt{\frac{1\,p}{C}} \times 64\,\mu Vrms$$

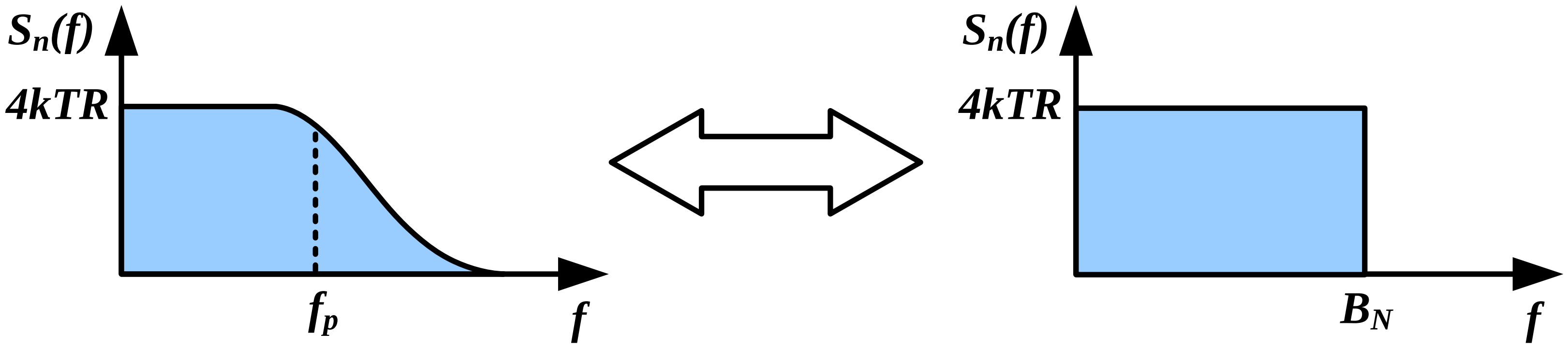


Equivalent Noise Bandwidth

- Define an equivalent noise BW (B_N) such that the area under a brick-wall response is the same area under the actual spectral density curve
- For a first order system

$$V_{nrms}^2 = \int_{-\infty}^{\infty} S_n(f)\,df = 4kTR \times B_N = \frac{kT}{C}$$

$$B_N = \frac{1}{4RC} = \frac{\pi}{2} f_p$$



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tuned circuits

vs

Baseband

Circuits

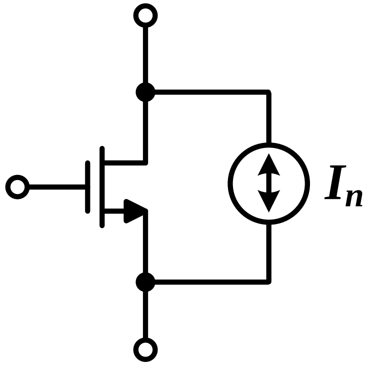
MOSFET Channel Thermal Noise

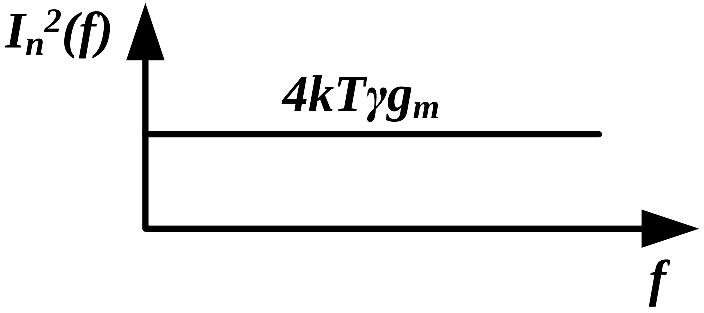
- MOSFET has thermal noise due to the resistive nature of the channel
- It can be shown that noise current **spectral density** is given by

$$I_n^2(f) = 4kT\gamma g_m$$

  - Similar to a resistor with $R = \dfrac{1}{\gamma g_m}$

- γ: MOSFET thermal noise coefficient
  - $\gamma \approx \frac{2}{3}$ for long channel MOSFET, but close to 1 for short channel



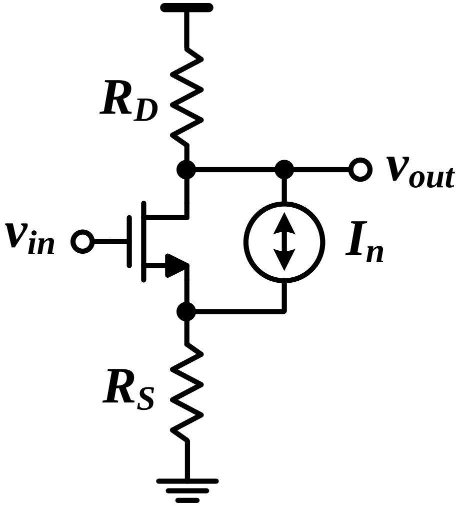


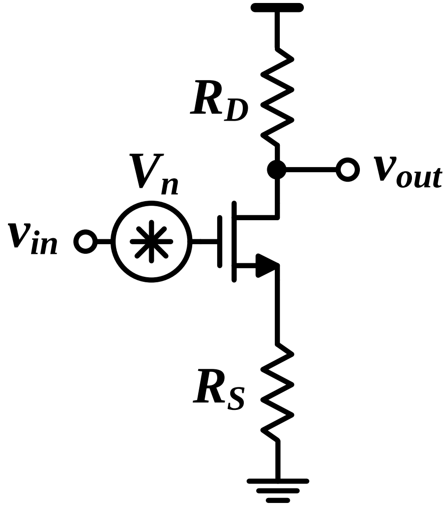
Thermal Noise Referred to Gate

- The noise current can be referred to the gate voltage
- The relation between I_n and V_n is g_m (not G_m)

$$V_n^2(f) = \frac{4kT\gamma}{g_m}$$

  - Can be proven by showing that the $i_{out,sc}$ is the same in both cases (you may ignore body effect and CLM for simplicity)
- <u>**This result is valid at zero gate current**</u> (low/medium frequencies)



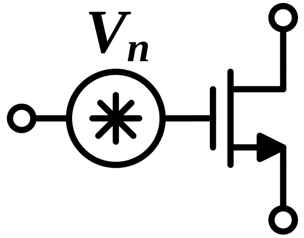


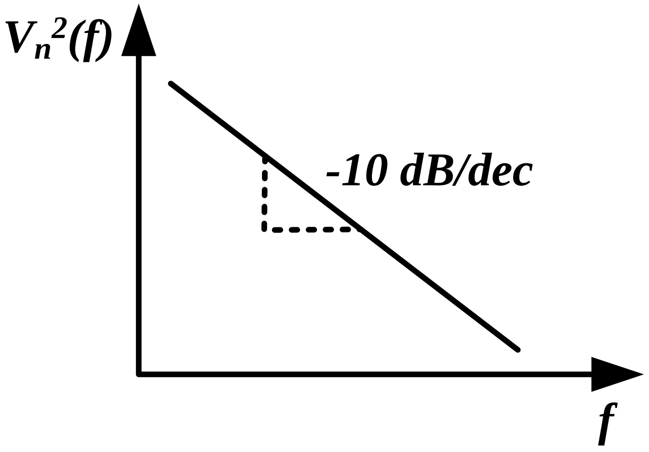
MOSFET Flicker (1/f) Noise

- Mainly due to dangling bonds at the oxide/silicon interface
- It can be shown that noise voltage **spectral density** is given by

$$V_n^2(f) = \frac{K}{C_{ox}WL}\frac{1}{f}$$

- K: Flicker noise coefficient
- Can be reduced by increasing device area
- PMOS has usually much less flicker noise compared to NMOS





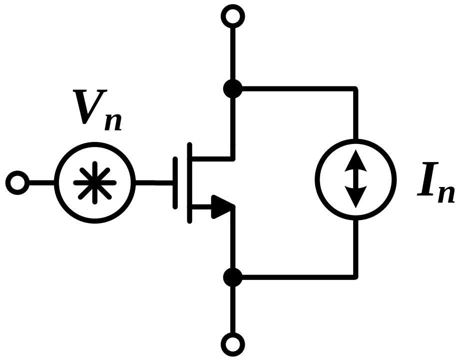
Flicker Noise Corner

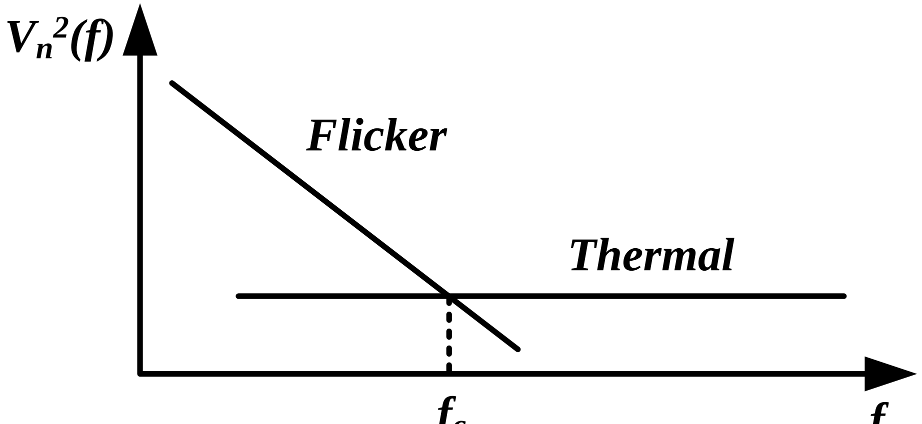
- Tells which type of noise is dominant for a given signal band
- Model both sources as noise current

$$4kT\gamma g_m = \frac{K}{C_{ox}WL}\frac{1}{f_c}\cdot g_m^2$$

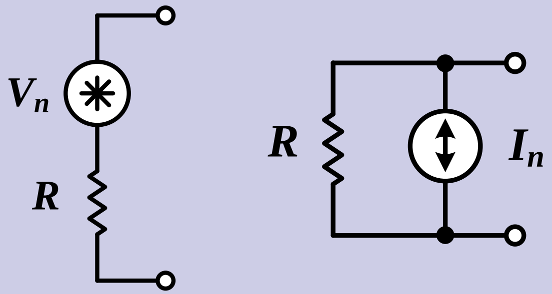
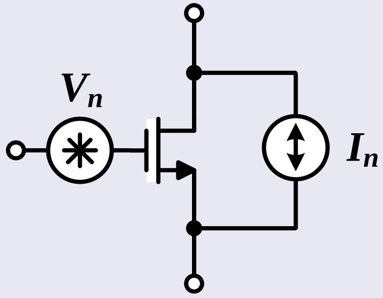
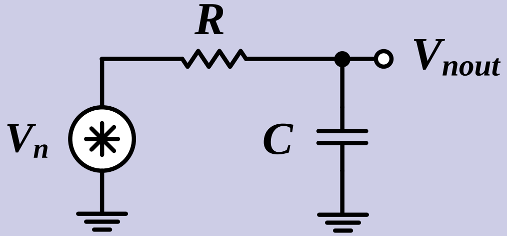
$$f_c = \frac{K}{C_{ox}WL}\cdot g_m \cdot \frac{1}{4kT\gamma}$$

- f_c can be as high as 100s of MHz for DSM nodes.





Noise Models Summary

Resistor thermal noise		$V_n(f) = \sqrt{4kTR} \approx \sqrt{\frac{R}{1\,k}} \times 4\,\frac{nV}{\sqrt{Hz}}$ $I_n(f) = \sqrt{\frac{4kT}{R}} \approx \sqrt{\frac{1\,k}{R}} \times 4\,\frac{pA}{\sqrt{Hz}}$
MOSFET thermal and flicker noise		$I_n^2(f) = 4kT\gamma g_m$ $V_n^2(f) = \frac{K}{C_{ox}WL}\frac{1}{f}$
RMS noise		$V_{noutrms}^2 = 4kTR \times B_N = \frac{kT}{C}$ $B_N = \frac{1}{4RC} = \frac{\pi}{2} f_p$ $V_{noutrms} \approx \sqrt{\frac{1\,p}{C}} \times 64\,\mu Vrms$

Outline

- ❑ Recapping previous key results
- ❑ Noise in time and frequency domains
- ❑ Resistor thermal noise
- ❑ MOSFET thermal and flicker noise
- ❑ Signal-to-noise ratio (SNR) and input-referred noise
- ❑ Noise analysis example

Signal-to-Noise Ratio (SNR)

- Signal-to-Noise Ratio (SNR) is the ratio of signal power to noise power

$$SNR = \frac{P_{signal}}{P_{noise}} = \frac{V_{sigrms}^2}{V_{nrms}^2}$$

- SNR is usually expressed in dB

$$SNR = 10\log\frac{P_{signal}}{P_{noise}} = 20\,log\,\frac{V_{sigrms}}{V_{nrms}}$$

- Example:
  - $V_{sigrms} = 100\ mVrms$
  - $V_{nrms} = 100\ \mu Vrms$
  - SNR $= 60\ dB$

Multiple Noise Sources

- Noise adds in time domain

$$V_{nout}(t) = V_{n1}(t) + V_{n2}(t)$$

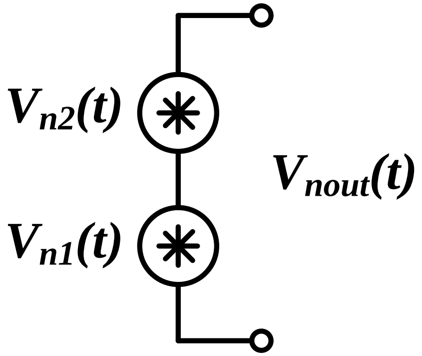
- But remember than $V_n(t)$ is a random variable
  - We cannot add rms values

$$V_{noutrms} \neq V_{n1rms} + V_{n2rms}$$

- If $V_{n1}(t)$ and $V_{n2}(t)$ are uncorrelated (independent random variables)

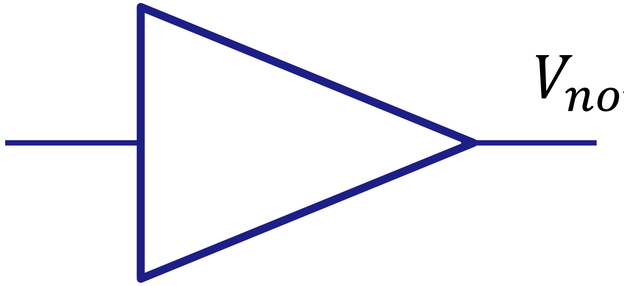
$$V_{noutrms}^2 = V_{n1rms}^2 + V_{n2rms}^2$$

- The largest noise contributor dominates
  - $3^2 + 1^2 \approx 3^2$

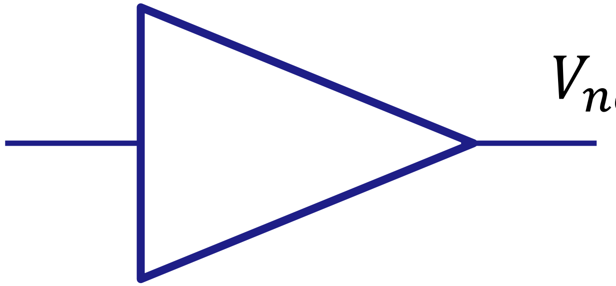


Output-Referred Noise

- Which amplifier has lower noise?



$$V_{noutrms} = 100\ \mu Vrms$$

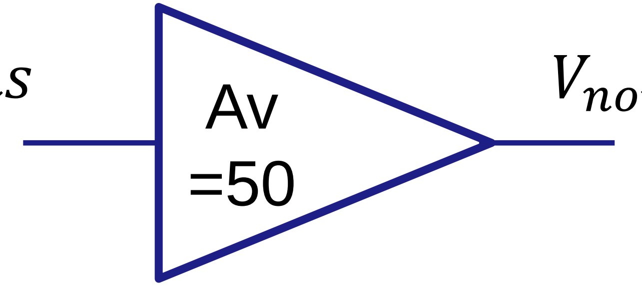


$$V_{noutrms} = 50\ \mu Vrms$$

Input-Referred Noise

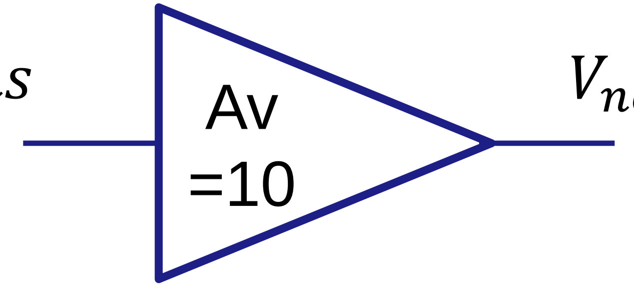
- The output-referred noise does not allow a fair comparison
  - Output-referred noise depends on the amplifier gain
  - But the signal is multiplied by the gain as well
  - For a fair comparison noise should be referred to the input

$V_{ninrms} = 2\ \mu Vrms$



$V_{noutrms} = 100\ \mu Vrms$

$V_{ninrms} = 5\ \mu Vrms$



$V_{noutrms} = 50\ \mu Vrms$

Outline

- ❑ Recapping previous key results
- ❑ Noise in time and frequency domains
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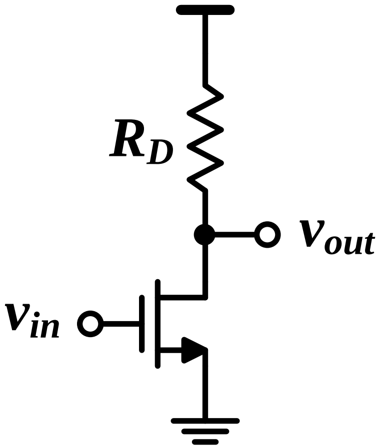
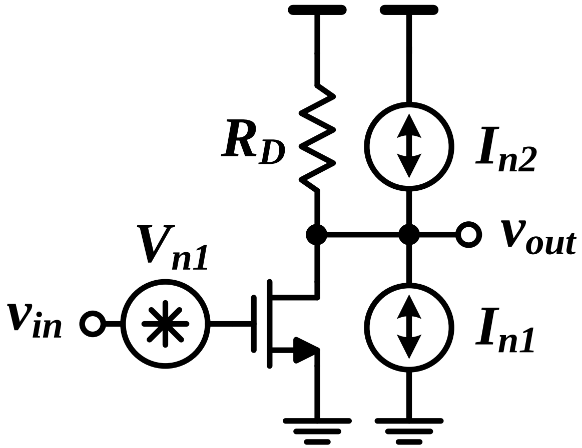
- ❑ Noise analysis example

Ex: CS with Resistive Load

- Deactivate the input signal
- Identify the noise sources
  - Resistor: Thermal
  - MOSFET: Thermal + Flicker
- Find the noise spectral density at output using superposition





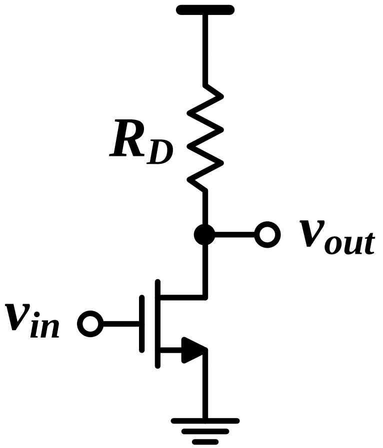
Output Noise Density

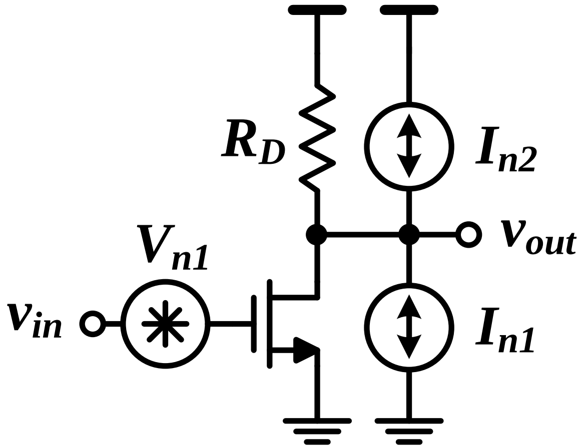
- Apply superposition (noise is small signal)

$$V_{nout}^2(f)' \approx 4kT\gamma g_m R_D^2$$

$$V_{nout}^2(f)'' \approx \frac{K}{C_{ox}WL}\frac{1}{f} \cdot g_m^2 R_D^2$$

$$V_{nout}^2(f)''' \approx \frac{4kT}{R_D} R_D^2$$





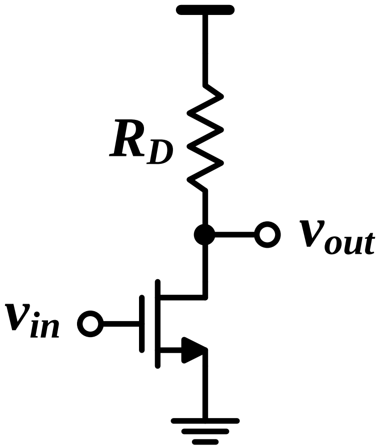
Output Noise Density

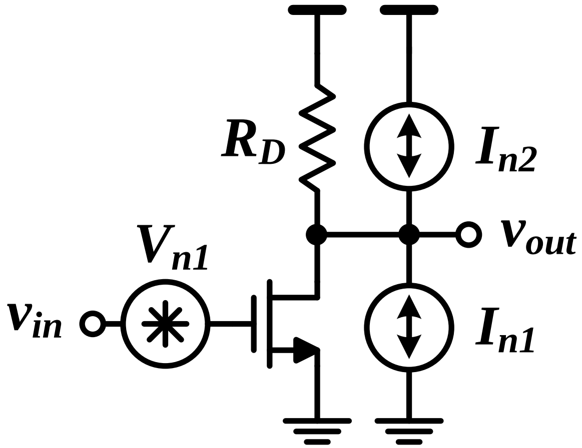
$$V_{nout}^2(f)' \approx 4kT\gamma g_m R_D^2$$

$$V_{nout}^2(f)'' \approx \frac{K}{C_{ox}WL}\frac{1}{f}\cdot g_m^2 R_D^2$$

$$V_{nout}^2(f)''' \approx \frac{4kT}{R_D} R_D^2$$

$$V_{nout}^2(f) \approx \left(4kT\gamma g_m + \frac{K}{C_{ox}WL}\frac{1}{f}\cdot g_m^2 + \frac{4kT}{R_D}\right) R_D^2$$





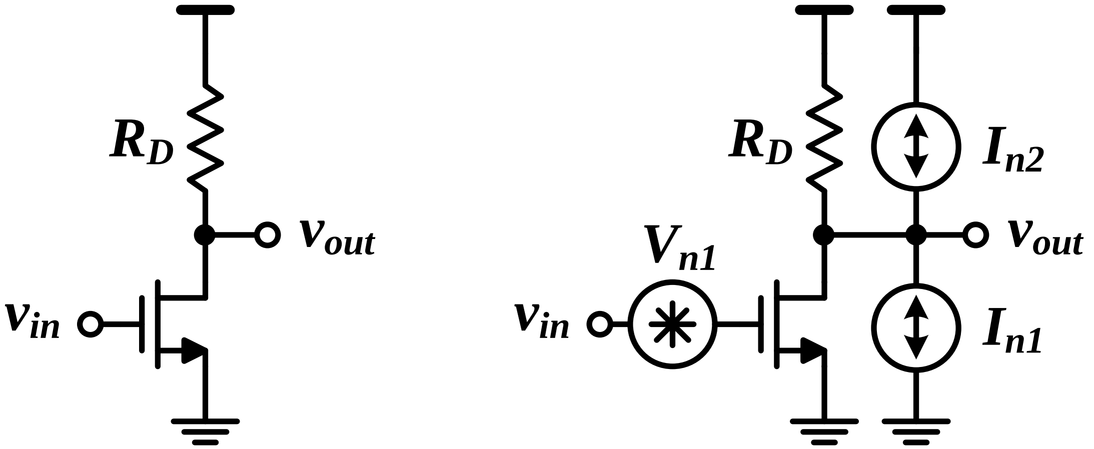
Input-Referred Noise Density

$$V_{nout}^2(f) = \left(4kT\gamma g_m + \frac{K}{C_{ox}WL}\frac{1}{f}\cdot g_m^2 + \frac{4kT}{R_D}\right)R_D^2$$

- For a fair comparison noise should be referred to the input

$$V_{nin}^2(f) = \frac{V_{nout}^2(f)}{A_v^2} = \frac{4kT\gamma}{g_m} + \frac{K}{C_{ox}WL}\frac{1}{f} + \frac{4kT}{g_m^2 R_D}$$

- Fundamental trade-off between power consumption and noise



Noise Analysis Procedure

- Deactivate the input signal
- Identify the **dominant** noise sources → Model as $V_n^2(f)$ or $I_n^2(f)$
- Find the output noise density for each source: $V_{nout,x}^2(f)$
- Calculate the rms output noise of each source

$$V_{noutrms,x}^2 = \overline{V_{nout,x}^2} = V_{nout,x}^2(f) \times B_{N,x}$$

- Calculate total rms noise

$$V_{noutrms,tot}^2 = \overline{V_{nout,tot}^2} = V_{nrms,1}^2 + V_{nrms,2}^2 + \cdots$$

- Calculate the input-referred rms noise voltage

$$V_{ninrms,tot}^2 = V_{noutrms,tot}^2 / A_v^2$$

- For low Z_{in}, input referred noise current must be added

RMS Output Noise

- Assume thermal noise is dominant

$$V_{nout}^2(f) \approx 4kTg_m\left(\gamma + \frac{1}{g_m R_D}\right)R_D^2$$

- Assume BW is limited by a load capacitance C_L

$$V_{noutrms}^2 = \overline{V_{nout}^2} \approx V_{nout}^2(f) \cdot \frac{1}{4R_D C_L} \approx kT(1+\gamma g_m R_D) \cdot \frac{1}{C_L}$$

$$\theta = (1+\gamma g_m R_D) = (1+\gamma|A_v|)$$

$$V_{noutrms}^2 \approx \frac{kT\theta}{C_L}$$

- Amplifier rms output noise can be written in the form $\frac{kT\theta}{C}$, where C the bandwidth-limiting cap, and $\theta > 1$ is topology dependent

SNR

- Assume input signal is a sinusoid with amplitude $= V_p$

$$SNR = \frac{V_{outrms}^2}{V_{noutrms}^2} \approx \left(\frac{V_p}{\sqrt{2}} \cdot g_m R_D\right)^2 \cdot \frac{C_L}{kT\theta}$$
$$= \frac{V_p^2}{2} \frac{g_m^2 R_D^2 C_L}{kT\theta} = \frac{2 V_p^2 V_{R_D}^2 C_L}{{V^*}^2 kT\theta}$$

- Assume $V_{R_D} = \frac{V_{DD}}{2}$ (to maximize output swing)

$$SNR \approx \frac{V_p^2 V_{DD}^2 C_L}{2{V^*}^2 kT\theta}$$

- Using higher V_{DD} improves SNR
  - V_{DD} was scaled from $> 10V$ to sub-$1V$
  - Design at low-supply voltage is quite challenging

SNR

- Assume a maximum rms output amplitude $= \kappa V_{DD}$

$$SNR = \frac{V_{outrms}^2}{V_{noutrms}^2} \approx \frac{(\kappa V_{DD})^2 C_L}{kT\theta} = \frac{(\kappa V_{DD})^2 C_L}{kT(1 + \gamma |A_v|)}$$

- Using higher V_{DD} improves SNR
  - V_{DD} was scaled from $> 10V$ to sub-$1V$
  - Design at low-supply voltage is quite challenging

Noise/Power Tradeoff

- Assume speed spec is fixed

$$SNR \approx \frac{V_p^2 V_{DD}^2 C_L}{2{V^*}^2 kT\theta}$$

$$GBW = \frac{g_m}{C_L}$$

- To improve SNR by 6 dB (equivalent to 1-bit) in a system limited by thermal noise
  - C_L must be quadrupled
  - g_m must be quadrupled to maintain GBW
  - Power dissipation is quadrupled (assuming V^* is constant)

Noise/Speed Tradeoff

- Assume power consumption spec is fixed

$$SNR \approx \frac{V_p^2 V_{DD}^2 C_L}{2{V^*}^2 kT\theta}$$

$$GBW = \frac{g_m}{C_L}$$

- To improve SNR by 6 dB (equivalent to 1-bit) in a system limited by thermal noise
  - C_L must be quadrupled
  - GBW decreases by four-times
- Decreasing V^* may also help
  - But f_T decreases → Lower speed

Thank you!

References

- B. Razavi, "Design of Analog CMOS Integrated Circuits," McGraw-Hill, 2nd ed., 2017.
- T. C. Carusone, D. Johns, and K. W. Martin, "Analog Integrated Circuit Design," 2nd ed., Wiley, 2012.