

## Analog IC Design

## Lecture 09 Frequency Response (2)

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## Outline

- ☐ Recapping previous key results
- Frequency response of CS: Midband, LFR, and HFR (Miller's effect)
- ☐ Frequency response of CG: HFR
- ☐ Frequency response of cascode: HFR
  - Cascode for gain and cascode for BW
- Frequency response of CD: HFR
  - Frequency domain peaking and time domain ringing
  - $Z_{in}$ : negative resistance and  $Z_{out}$ : inductive rise

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#### **MOSFET** in Saturation

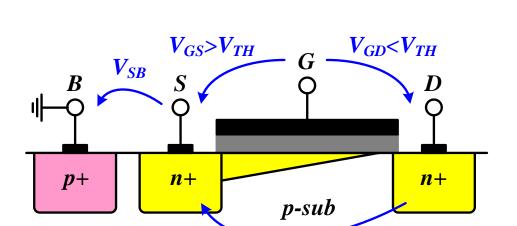
☐ The channel is pinched off if the difference between the gate and drain voltages is not sufficient to create an inversion layer

$$V_{GD} \leq V_{TH} \quad OR \quad V_{DS} \geq V_{ov}$$

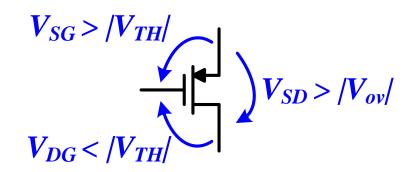
Square-law (long channel MOS)

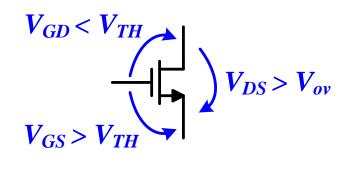
$$I_D = \frac{\mu_n C_{ox}}{2} \frac{W}{L} \cdot V_{ov}^2 (1 + \lambda V_{DS})$$

$$V_{SB} \uparrow \Rightarrow V_{TH} \uparrow$$

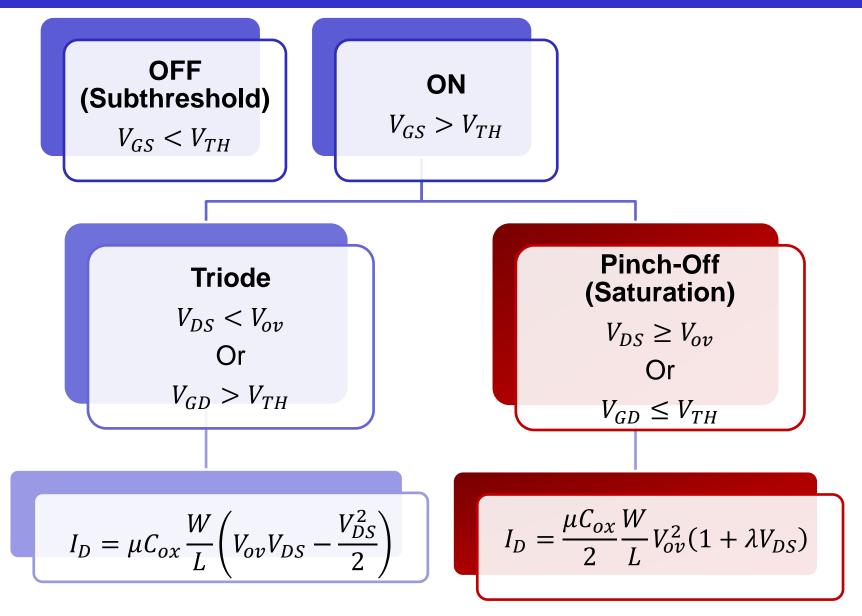


 $V_{DS}>V_{ov}$ 





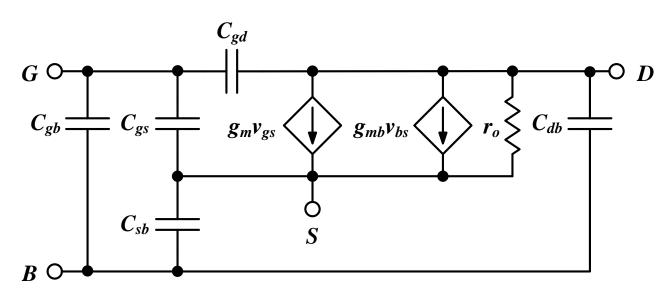
## **Regions of Operation Summary**



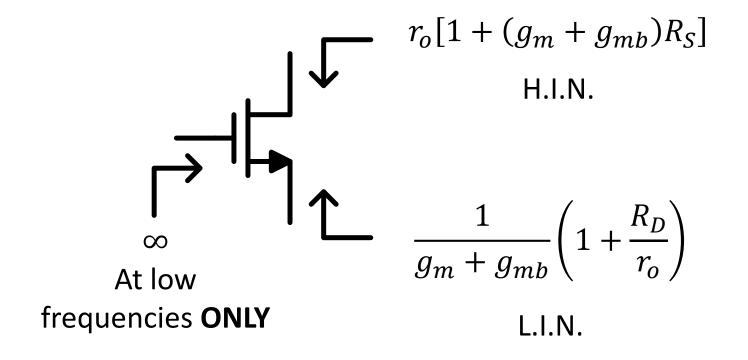
# High Frequency Small Signal Model

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \mu C_{ox} \frac{W}{L} V_{ov} = \sqrt{\mu C_{ox} \frac{W}{L} \cdot 2I_D} = \frac{2I_D}{V_{ov}}$$
$$g_{mb} = \eta g_m \qquad \qquad \eta \approx 0.1 - 0.25$$

$$r_{o} = \frac{1}{\partial I_{D}/\partial V_{DS}} = \frac{V_{A}}{I_{D}} = \frac{1}{\lambda I_{D}}$$
  $V_{A} \propto L \leftrightarrow \lambda \propto \frac{1}{L}$   $V_{DS} \uparrow V_{A} \uparrow$   $C_{gb} \approx 0$   $C_{gs} \gg C_{gd}$   $C_{sb} > C_{db}$ 



# Rin/out Shortcuts Summary



# Summary of Basic Topologies

	CS	CG	CD (SF)
	$R_D$ $v_{in} \circ v_{out}$ $R_S$	$R_D$ $v_{out}$ $R_S$	$R_D$ $v_{in} \circ V_{out}$ $R_S$
	Voltage & current amplifier	Voltage amplifier Current buffer	Voltage buffer Current amplifier
Rin	$\infty$	$R_S  \frac{1}{g_m + g_{mb}} \left(1 + \frac{R_D}{r_o}\right)$	$\infty$
Rout	$R_D    r_o [1 + (g_m + g_{mb}) R_S]$	$R_D  r_o$	$R_S  \frac{1}{g_m + g_{mb}} \left(1 + \frac{R_D}{r_o}\right)$
Gm	$\frac{-g_m}{1+(g_m+g_{mb})R_S}$	$g_m + g_{mb}$	$\frac{g_m}{1+R_D/r_o}$

#### Outline

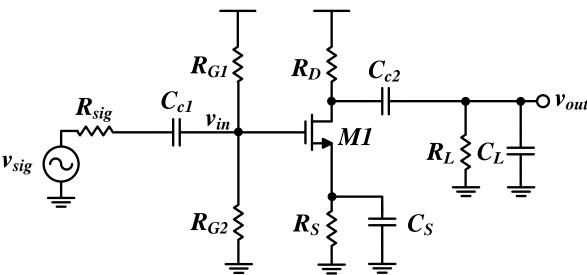
- ☐ Recapping previous key results
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# Frequency Response of CS: Midband

$$A_v = \frac{v_{in}}{v_{sig}} \cdot \frac{v_{out}}{v_{in}}$$

$$\frac{v_{out}}{v_{in}} = G_m R_{out} = -g_m (R_D || R_L || r_o)$$

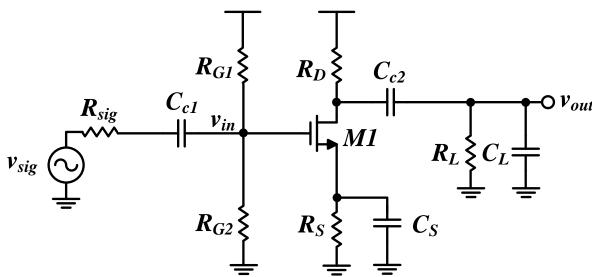
$$\frac{v_{in}}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}}, R_{in} = R_G = R_{G1} || R_{G2}$$



# Frequency Response of CS: LFR

$$\Box C_{C1}: R_{th} = R_{sig} + R_G \rightarrow \omega_{p,C_{C1}} = \frac{1}{(R_{sig} + R_G)C_{C1}} \& \omega_{z,C_{C1}} = 0$$

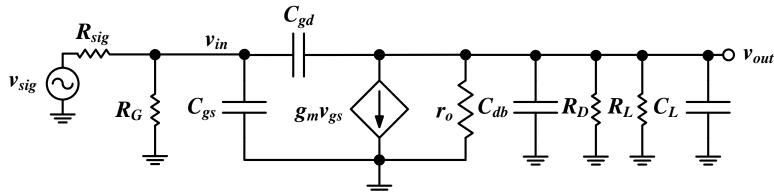
- $\Box \ \ C_{C2}: R_{th} = R_L + R_D || r_o \rightarrow \omega_{p,C_{C2}} = \frac{1}{(R_L + R_D || r_o) C_{C2}} \& \ \omega_{z,C_{C2}} = 0$
- $\Box \ \ C_S: R_{th} = R_S || R_{LFS} \to \omega_{p,C_S} = \frac{1}{(R_S || R_{LFS}) C_S} \& \ \omega_{z,C_S} = \frac{1}{R_S C_S}$
- $\Box$  Usually  $\omega_{p,C_S}$  is dominant:  $\omega_L \approx \omega_{p,C_S}$  (why?)
- ☐ Note that for IC amplifiers we usually use direct coupling (no LFR)



# CS HFR: (1) Miller's Approx + OCTC

- lacktriangle Break the feedback capacitance  $(C_{qd})$  using Miller's approx
- ☐ Each node is associated with a pole
  - $v_{in}$  node  $\rightarrow$  i/p pole  $(\omega_{p,in})$
  - $v_{out}$  node  $\rightarrow$  o/p pole  $(\omega_{p,out})$
- Don't forget the RHP feedforward zero

$$\omega_{z,C_{gd}} = \frac{g_m}{C_{gd}} \rightarrow \text{Usually } \omega_{z,C_{gd}} \text{ is very high (why?)}$$



# CS HFR: (1) Miller's Approx + OCTC

i/p pole: suffers from Miller effect (capacitance multiplication)

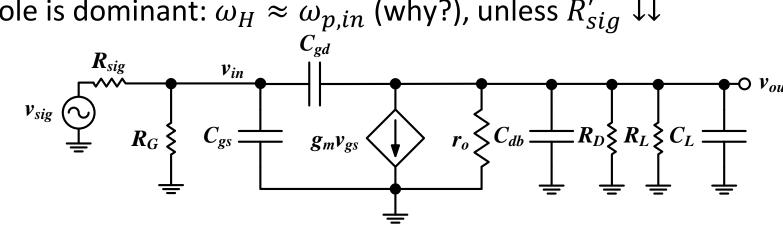
$$R_{th,in} = R_{sig} || R_G = R'_{sig} \rightarrow \omega_{p,in} \approx \frac{1}{R'_{sig} [C_{gs} + C_{gd}(1 + A_o)]}$$

$$A_o = \left| \frac{v_{out}}{v_{in}} \right| = g_m R_{out}$$

 $\Box$  o/p pole:  $R_{th.out} = R_L ||R_D|| r_o$ 

$$\rightarrow \omega_{p,out} \approx \frac{1}{R_{out} \left( C_L + C_{db} + C_{gd} \left( 1 + \frac{1}{A_o} \right) \right)} \approx \frac{1}{R_{out} \left( C_{out} + C_{gd} \right)}$$

Usually i/p pole is dominant:  $\omega_H \approx \omega_{p,in}$  (why?), unless  $R'_{sig} \downarrow \downarrow$ 



# CS HFR: (1) Miller's Approx + OCTC

$$\omega_{H} \approx \frac{1}{\frac{1}{\omega_{p,in}} + \frac{1}{\omega_{p,out}}} = \frac{1}{R'_{sig} \left[ C_{gs} + C_{gd} (1 + g_m R_{out}) \right] + R_{out} \left( C_{out} + C_{gd} \right)}$$

 $\square$  If input pole is dominant (e.g., if  $R'_{sig} \uparrow \uparrow$  or  $C_L \downarrow \downarrow$ )

$$\omega_{H} \approx \frac{1}{R'_{sig} \left[ C_{gs} + C_{gd} (1 + g_{m} R_{out}) \right]} \approx \omega_{p,in}$$

 $\square$  If output pole is dominant (e.g., if  $R'_{sig} \downarrow \downarrow$  or  $C_L \uparrow \uparrow$ )

$$\omega_{H} \approx \frac{1}{R_{out}(C_{out} + C_{gd})} \approx \omega_{p,out}$$

$$GBW = A_v \omega_H = G_m R_{out} \cdot \frac{1}{R_{out}(C_{out} + C_{gd})} = \frac{G_m}{C_{out} + C_{gd}}$$

 $\rightarrow$  independent of  $R_{out}!$ 

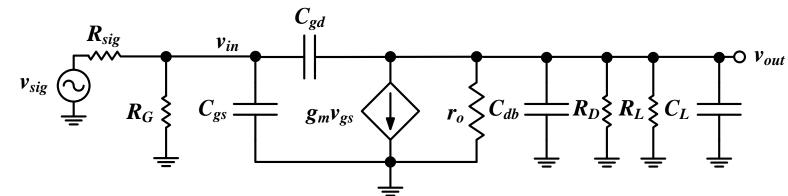
# CS HFR: (2) Just OCTC Technique

$$\square \quad C_{gs}: R_{th,in} = R_{sig} || R_G = R'_{sig} \rightarrow \omega_{p,C_{gs}} = \frac{1}{R'_{sig}C_{gs}}$$

$$\Box \quad C_{out} = C_L + C_{db}: R_{th} = R_L ||R_D|| r_o = R_{out} \rightarrow \omega_{p,C_{out}} = \frac{1}{R_{out}C_{out}}$$

$$\Box C_{gd}: R_{th} = \frac{v_x}{i_x} = R'_{sig}(1 + g_m R_{out}) + R_{out} \Rightarrow \omega_{p,C_{gd}} = \frac{1}{[R'_{sig}(1 + g_m R_{out}) + R_{out}]C_{gd}}$$

$$\omega_H \approx \frac{1}{\frac{1}{\omega_{p,C_{gs}}} + \frac{1}{\omega_{p,C_{out}}} + \frac{1}{\omega_{p,C_{gd}}}} = \frac{R'_{sig}C_{gs} + R_{out}C_{out} + [R'_{sig}(1 + g_m R_{out}) + R_{out}]C_{gd}}$$



# CS HFR: (2) Just OCTC Technique

$$\omega_{H} \approx \frac{1}{R'_{sig} \left[ C_{gs} + C_{gd} (1 + g_{m} R_{out}) \right] + R_{out} \left( C_{out} + C_{gd} \right)}$$

 $\square$  If input pole is dominant (e.g., if  $R'_{sig} \uparrow \uparrow$  or  $C_L \downarrow \downarrow$ )

$$\omega_{H} \approx \frac{1}{R'_{sig} \left[C_{gs} + C_{gd} (1 + g_{m} R_{out})\right]} \approx \omega_{p,in}$$

 $\square$  If output pole is dominant (e.g., if  $R'_{sig} \downarrow \downarrow$  or  $C_L \uparrow \uparrow$ )

$$\omega_{H} \approx \frac{1}{R_{out}(C_{out} + C_{gd})} \approx \omega_{p,out}$$

☐ Same as the expressions obtained from Miller approx

# CS HFR: (3) Exact Analysis + Dominant Pole Approx

- ☐ Surprisingly, exact analysis gives a quite complex expression
  - See [Johns & Martin 2012] or [Razavi 2017]
- ☐ If dominant pole approximation is applied

$$\omega_{pd} \approx \frac{1}{b_1} = \frac{1}{R'_{sig} \left[ C_{gs} + C_{gd} (1 + g_m R_{out}) \right] + R_{out} \left( C_{out} + C_{gd} \right)}$$

- Same result as OCTC (both based on same approximation)
- $oldsymbol{\square}$  Additionally, dominant pole approx gives an expression for  $\omega_{pnd}$

$$\omega_{pnd} \approx \frac{1}{b_2 \omega_{p1}} = \frac{b_1}{b_2} = \frac{g_m C_{gd}}{C_{gd} (C_{gs} + C_{out}) + C_{gs} C_{out}}$$

- If a large cap is connected parallel to  $C_{gd}$ :  $\omega_{pnd} \approx \frac{g_m}{c_{gs} + c_{out}}$ 
  - Can be derived intuitively without analysis (how?)
  - We will need this case when we study two-stage OTA

# Frequency Response of CS: $Z_{in}$

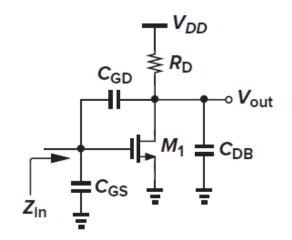
With Miller approx.

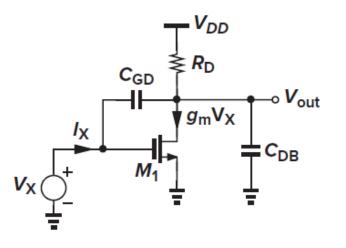
$$Z_{in} = \frac{1}{s[C_{gs} + (1 + g_m R_D)C_{gd}]}$$

lacktriangle Exact Analysis ( $C_{gs}$  adds in parallel)

$$Z_{in} = \frac{V_X}{I_X} = \frac{1 + sR_D(C_{gd} + C_{db})}{sC_{gd}(1 + g_mR_D + sR_DC_{db})}$$

- Extra pole and zero at high frequency ( $\omega_p > \omega_z$ )
- At relatively low frequency the exact solution reduces to Miller approx.





# Frequency Response of CS: $Z_{out}$

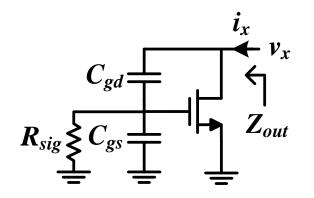
Can we use Miller?

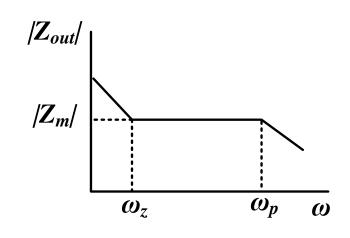
$$i_{x} = f(v_{gs}) = g_{m}v_{gs} + \frac{v_{gs}(1 + sR_{sig}C_{gs})}{R_{sig}}$$

$$v_{x} = f(v_{gs}) = v_{gs} + \frac{v_{gs}(1 + sR_{sig}C_{gs})}{sR_{sig}C_{gd}}$$

$$Z_{out} = \frac{v_{x}}{i_{x}} \approx \frac{1 + sR_{sig}(C_{gs} + C_{gd})}{sC_{gd}g_{m}R_{sig}\left(1 + s\frac{C_{gs}}{g_{m}}\right)}$$

- lacksquare  $r_o$  and  $c_{db}$  add in parallel
- lacktriangle Important special case: If we have a large capacitor parallel to  $C_{ad}$ 
  - $|Z_m| \approx 1/g_m$  → We will need this case when we study Miller OTA



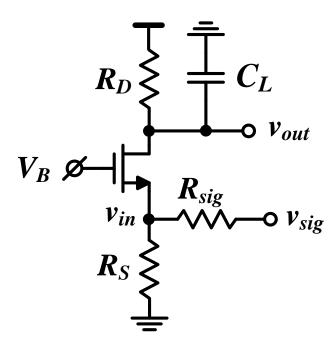


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## Frequency Response of CG: HFR

- $\Box$  i/p pole:  $\omega_{p,in} = \frac{1}{(R_{sig}||R_S||R_{LFS})(C_{gs}+C_{sb})}$
- $\square$  o/p pole:  $\omega_{p,out} = \frac{1}{(R_D||R_{LFD})(C_L + C_{db} + C_{gd})}$
- □ Usually o/p pole is dominant:  $\omega_H \approx \omega_{p,out}$  (why?)
- $\square$  No FB cap  $\rightarrow$  No Miller effect  $\rightarrow BW_{CG} \gg BW_{CS}$



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# Frequency Response of Cascode: HFR

#### Case 1: BW limited by o/p pole ( $R_D \uparrow \uparrow R_{sig} \downarrow \downarrow$ ) (cascode for gain)

$$\Box A_{v} = \frac{v_{out}}{v_{sig}} \approx (g_{m1}r_{o1})(g_{m2}r_{o2}) = A_{v,CS} \cdot (g_{m2}r_{o2})$$

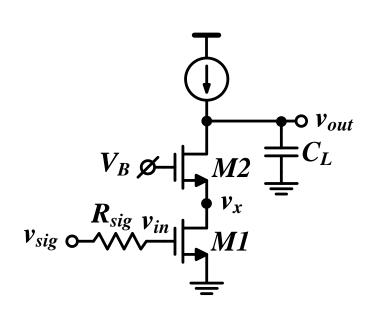
$$\square \quad \omega_{p,out} \approx \frac{1}{r_{o1}(g_{m2}r_{o2})(c_L + c_{db2} + c_{gd2})} = \frac{\omega_{p,out,CS}}{g_{m2}r_{o2}} \rightarrow \text{Dominant}$$

$$\Box GBW = A_v \omega_{p,out} = A_{v,CS} \omega_{p,out,CS}$$
$$= \frac{G_m}{C_{out} + C_{gd2}} \rightarrow \text{Same as CS!}$$

•  $A_{o1} \ll g_{m1}r_{o1}$  Miller effect reduced

$$\square \quad \omega_{p,in} = \frac{1}{R_{sig}(C_{gs1} + C_{gd1}(1 + A_{o1}))} = \omega_{p,in,CS}$$

$$\square \ \omega_{p,x} = \frac{1}{(r_{o1}||R_{LFS2})(C_{gs2} + C_{sb2} + C_{db1} + C_{gd1}(1 + 1/A_{o1}))}$$



# Frequency Response of Cascode: HFR

#### Case 2: BW limited by i/p pole ( $R_D \downarrow \downarrow R_{sig} \uparrow \uparrow$ ) (cascode for BW)

$$\Box A_v = \frac{v_{out}}{v_{sig}} \approx g_{m1} R_D \approx A_{v,CS} \rightarrow \text{Similar to CS!}$$

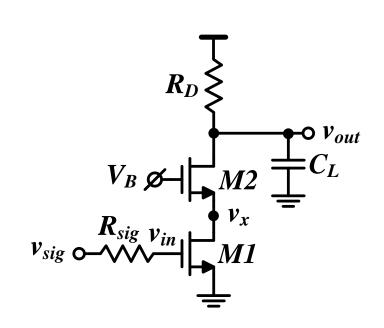
$$\square \ \omega_{p,in} \approx \frac{1}{R_{sig}(C_{gs1} + 2C_{gd1})} > \omega_{p,in,CS}$$

- Miller effect significantly reduced
- → BW extension!

$$\square \ \omega_{p,x} \approx \frac{1}{(r_{o1}||\frac{1}{g_{m2}})(C_{gs2} + C_{sb2} + C_{db1} + 2C_{gd1})}$$

$$\square \ \omega_{p,out} \approx \frac{1}{R_D(C_L + C_{db2} + C_{gd2})}$$

$$\square$$
  $GBW = A_v \omega_{v,in} > GBW$  of CS

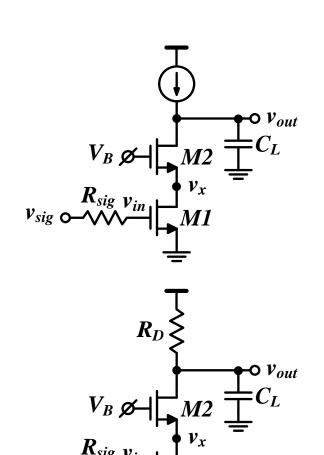


# Cascode HFR: Recapping

☐ If BW is limited by o/p pole

• 
$$GBW = A_v \omega_{p,out} = G_m R_{out} \cdot \frac{1}{R_{out} C_{out}} = \frac{G_m}{C_{out}}$$

- Cascode can be used **to trade gain for bandwidth** by modifying  $R_{out}$
- But  $GBW = A_v \omega_{p,out}$  remain unchanged
- If BW is limited by i/p pole
  - Cascode can provide higher BW (Miller ↓)
  - The gain may be higher as well
  - $GBW = A_v \omega_{p,in}$  increases
  - Also improves reverse isolation (RF LNAs)
- ☐ See Example 10.10 in Sedra/Smith 7<sup>th</sup> ed.

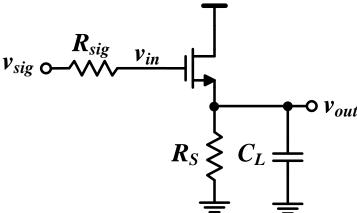


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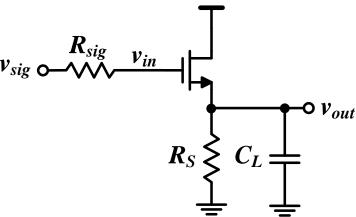
# Frequency Response of CD: HFR

- ☐ Assume real and widely spaced poles (revisited next slide)
- lacktriangleq Apply Miller: Ideally  $A_o = v_{out}/v_{in} \approx 1 \Rightarrow C_{gs}$  is **bootstrapped**
- $\Box$  i/p pole:  $\omega_{p,in} = \frac{1}{R_{sig} \left( C_{gd} + C_{gs} (1 A_o) \right)}$
- Both poles are at high frequency (why?) → Large BW
- lacksquare Don't forget the LHP feedforward zero:  $\omega_Z=rac{g_m}{c_{gs}}\uparrow\uparrow$



# CD HFR: Why Approximations Fail?

- ☐ The two poles are nearby and possibly complex conjugate
- ☐ OCTC technique and Miller approx cannot be used ⊗



## CD HFR: Exact Analysis

Simple circuit, but exact analysis gives a complex expression!

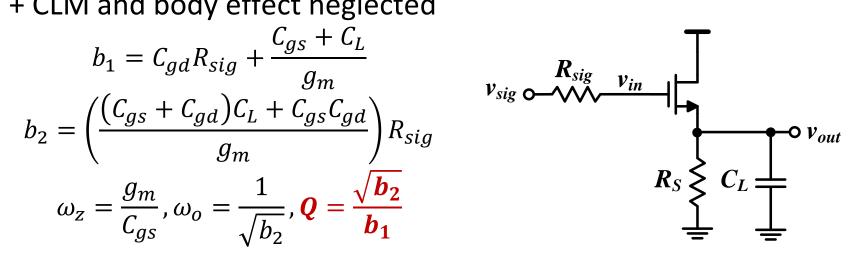
$$\begin{split} \frac{v_{out}}{v_{sig}} &= A_M \frac{1 + s/\omega_z}{\left(1 + \frac{s}{\omega_{p1}}\right) \left(1 + \frac{s}{\omega_{p2}}\right)} = A_M \frac{1 + s/\omega_z}{1 + \left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right) s + \frac{s^2}{\omega_{p1}\omega_{p2}}} \\ &= A_M \frac{1 + s/\omega_z}{1 + b_1 s + b_2 s^2} = A_M \frac{1 + s/\omega_z}{1 + \frac{1}{Q} \frac{s}{\omega_o} + \frac{s^2}{\omega_o^2}} = A_M \frac{1 + s/\omega_z}{1 + 2\zeta \frac{s}{\omega_o} + \frac{s^2}{\omega_o^2}} \end{split}$$

Special case:  $R_S \uparrow \uparrow$  (IDC) + CLM and body effect neglected

$$b_{1} = C_{gd}R_{sig} + \frac{C_{gs} + C_{L}}{g_{m}}$$

$$b_{2} = \left(\frac{\left(C_{gs} + C_{gd}\right)C_{L} + C_{gs}C_{gd}}{g_{m}}\right)R_{sig}$$

$$\omega_{z} = \frac{g_{m}}{C_{gs}}, \omega_{o} = \frac{1}{\sqrt{b_{2}}}, \mathbf{Q} = \frac{\sqrt{b_{2}}}{b_{1}}$$

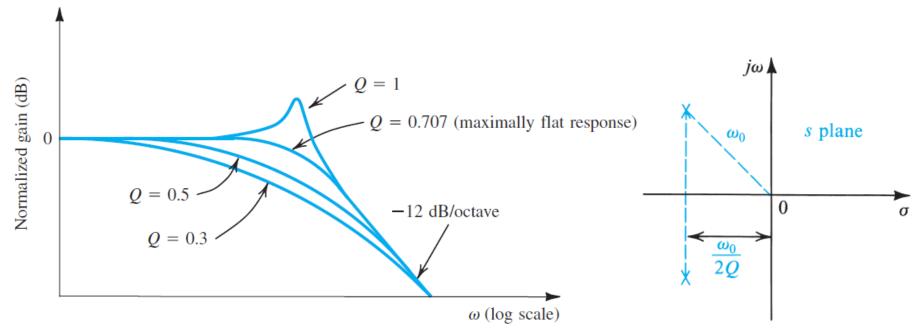


# Peaking and Ringing

- $\square$   $\omega_z \uparrow \uparrow$  is ignored
- $\square$  Q > 0.5 ( $\zeta < 1$ ): Underdamped system (complex conj. poles)
  - Ringing (overshoot) in step response (time domain)

% overshoot= 
$$100 e^{\frac{-\pi}{\sqrt{4Q^2-1}}}$$

 $Q > \frac{1}{\sqrt{2}} = 0.707$  ( $\zeta < 0.707$ ): Peaking in frequency response



# **Driving Large Capacitive Load**

 $\square$  Special case:  $R_S \uparrow \uparrow$  (IDC) + CLM and body effect neglected +  $C_L \uparrow \uparrow$ 

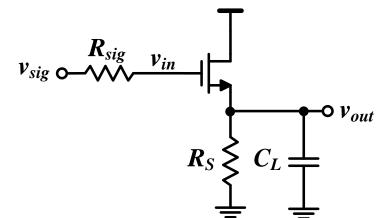
$$b_1 = C_{gd}R_{sig} + \frac{C_{gs} + C_L}{g_m} \approx \frac{C_L}{g_m}$$

$$b_2 = \left(\frac{\left(C_{gs} + C_{gd}\right)C_L + C_{gs}C_{gd}}{g_m}\right)R_{sig} \approx \frac{\left(C_{gs} + C_{gd}\right)C_LR_{sig}}{g_m}$$

$$\omega_z = \frac{g_m}{C_{gs}}$$

$$\omega_o = \frac{1}{\sqrt{b_2}} \approx \sqrt{\frac{g_m}{(C_{gs} + C_{gd})C_L R_{sig}}} \qquad R_{sig} \qquad V_{sig} \sim V$$

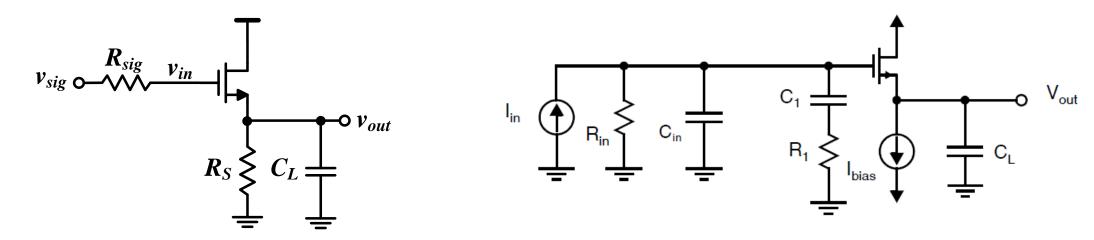
$$Q = \frac{\sqrt{b_2}}{b_1} \approx \sqrt{\frac{g_m(C_{gs} + C_{gd})R_{sig}}{C_L}}$$



 $\square$  Increasing  $C_L$  eventually decreases  $Q \rightarrow \omega_{p,out}$  becomes dominant

## Suppressing the Overshoot

- $\Box$  Space the two poles far apart  $\rightarrow$  single dominant pole
  - Increase  $C_L$  (till Q < 0.5)
  - Or increase  $C_{in}$  (adds to  $C_{gd}$ )  $\rightarrow$  but buffer becomes less useful!
- More clever solution
  - A compensation network ( $R_1$  and  $C_1$ ) can be used to compensate for the negative input impedance and prevent overshoots
  - See [Johns and Martin, 2012] Section 4.4 for more details



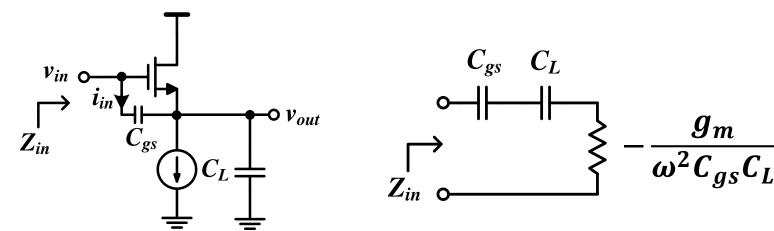
09: Frequency Response (2) [Johns and Martin, 2012]

## $oldsymbol{Z_{in}}$ of CD

- $\square v_{gs} = i_{in}/sC_{gs}$
- $\Box v_{in} = i_{in}/sC_{gs} + (i_{in} + g_m i_{in}/sC_{gs})(r_o||1/g_{mb}||1/sC_L)$   $Z_{in} = \frac{v_{in}}{i_{in}} = 1/sC_{gs} + (1 + g_m/sC_{gs})(r_o||1/g_{mb}||1/sC_L)$
- $\square$  If  $1/sC_L$  is dominant (e.g., driving large cap load, or @ high freq)

$$Z_{in} \approx \frac{1}{sC_{gs}} + \frac{1}{sC_L} + \frac{g_m}{s^2C_{gs}C_L} = \frac{1}{j\omega C_{gs}} + \frac{1}{j\omega C_L} - \frac{g_m}{\omega^2C_{gs}C_L} \rightarrow \text{-ve res!}$$

- ☐ Can be used in oscillators, and may make amplifiers unstable!
- lacktriangle Note that  $C_{qd}$  shunts  $Z_{in}$  at high frequency

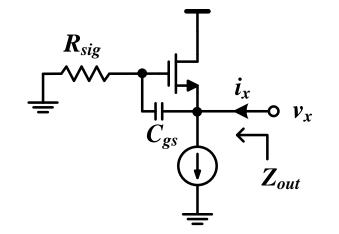


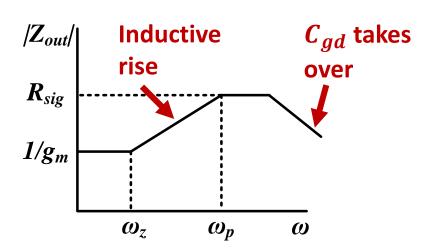
# $Z_{out}$ of CD

$$\Box i_{\chi} = \frac{v_{\chi}}{1/sC_{gs} + R_{sig}} + g_m \cdot \frac{v_{\chi}}{1/sC_{gs} + R_{sig}} \cdot \frac{1}{sC_{gs}}$$

$$Z_{out} = \frac{v_x}{i_x} = \frac{1}{g_m} \left( \frac{1 + sR_{sig}C_{gs}}{1 + s\frac{C_{gs}}{g_m}} \right)$$

- $\square$  By intuition:  $\omega \downarrow \downarrow$ :  $Z_{out} \approx 1/g_m$  and  $\omega \uparrow \uparrow$ :  $Z_{out} \approx R_{sig}$
- $\Box$  Usually  $R_{sig} > 1/g_m$  (buffer)  $\rightarrow$  inductive rise
- lacktriangle Note that  $C_{gd}$  shunts  $R_{sig}$  at high frequency ( $pprox 1/R_{sig}C_{gd}$ )
- $\square$  Body resistance  $(1/g_{mb})$  and  $r_o$  add to  $Z_{out}$  in parallel





# Thank you!

#### References

- ☐ A. Sedra and K. Smith, "Microelectronic Circuits," Oxford University Press, 7<sup>th</sup> ed., 2015
- ☐ B. Razavi, "Design of Analog CMOS Integrated Circuits," McGraw-Hill, 2<sup>nd</sup> ed., 2017
- T. C. Carusone, D. Johns, and K. W. Martin. "Analog Integrated Circuit Design," Wiley, 2<sup>nd</sup> ed., 2012

## Quiz

- $\Box$  LD = 100 nH, Cgd = 10 fF, gm = 10 mS, w = 10 Gr/s
- ☐ Ignore VA and other caps
- $\Box$  Assume Av >> 1
- $\square$  Rin = ?

