Analog IC Design

Lecture 18
Noise in Amplifier Circuits

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Outline

- ❑ Recapping previous key results
- ❑ Noise in Amplifiers

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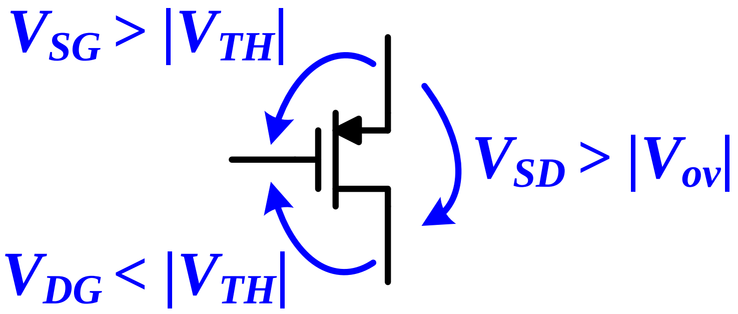
- Noise in Amplifiers

MOSFET in Saturation

- The channel is pinched off if the difference between the gate and drain voltages is not sufficient to create an inversion layer

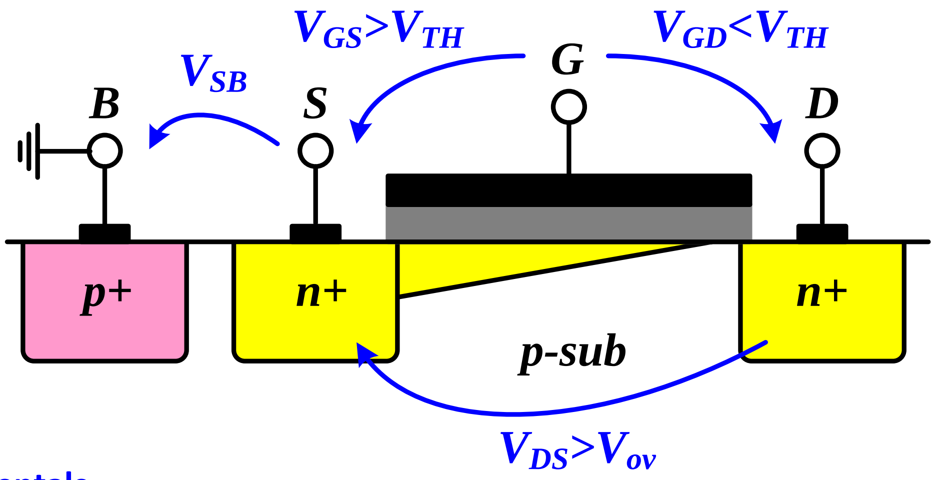
$$V_{GD} \leq V_{TH} \quad or \quad V_{DS} \geq V_{ov}$$

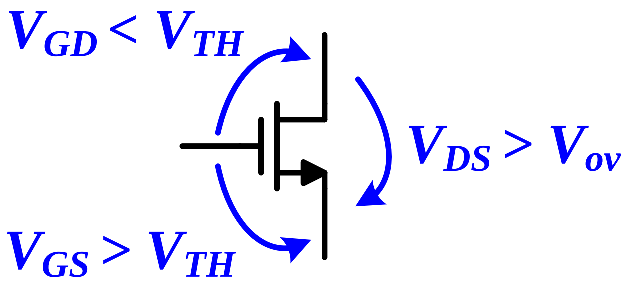


- Square-law (long channel MOS)

$$I_D = \frac{\mu_n C_{ox}}{2} \frac{W}{L} \cdot V_{ov}^2 (1 + \lambda V_{DS})$$

$$V_{SB} \uparrow \ \Rightarrow \ V_{TH} \uparrow$$





Regions of Operation Summary

OFF (Subthreshold)

$V_{GS} < V_{TH}$

ON

$V_{GS} > V_{TH}$

Triode

$V_{DS} < V_{ov}$

Or

$V_{GD} > V_{TH}$

$$I_D = \mu C_{ox} \frac{W}{L}\left(V_{ov}V_{DS} - \frac{V_{DS}^2}{2}\right)$$

Pinch-Off (Saturation)

$V_{DS} \geq V_{ov}$

Or

$V_{GD} \leq V_{TH}$

$$I_D = \frac{\mu C_{ox}}{2}\frac{W}{L}V_{ov}^2(1 + \lambda V_{DS})$$

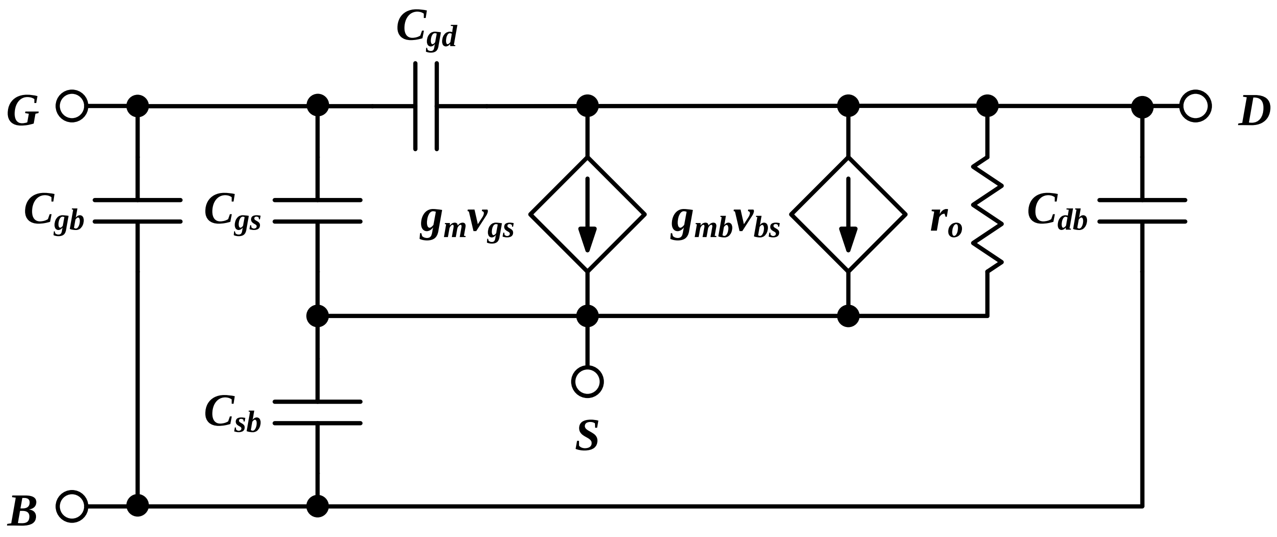
High Frequency Small Signal Model

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \mu C_{ox} \frac{W}{L} V_{ov} = \sqrt{\mu C_{ox} \frac{W}{L} \cdot 2 I_D} = \frac{2 I_D}{V_{ov}}$$

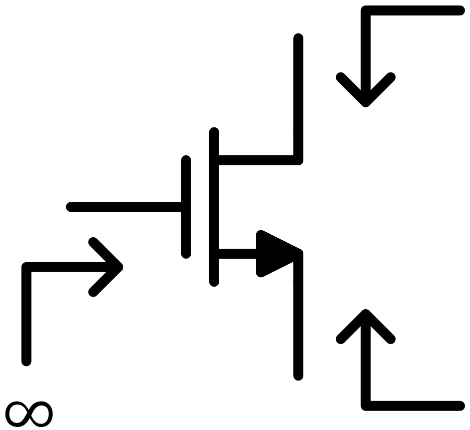
$$g_{mb} = \eta g_m \qquad \eta \approx 0.1 - 0.25$$

$$r_o = \frac{1}{\partial I_D / \partial V_{DS}} = \frac{V_A}{I_D} = \frac{1}{\lambda I_D} \qquad V_A \propto L \leftrightarrow \lambda \propto \frac{1}{L} \qquad V_{DS} \uparrow V_A \uparrow$$

$$C_{gb} \approx 0 \qquad C_{gs} \gg C_{gd} \qquad C_{sb} > C_{db}$$



Rin/out Shortcuts Summary



$$r_o[1 + (g_m + g_{mb})R_S]$$

H.I.N.

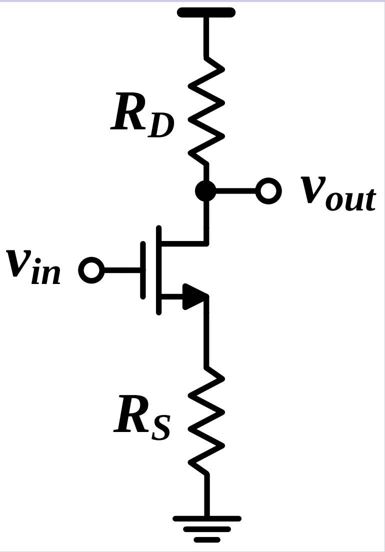
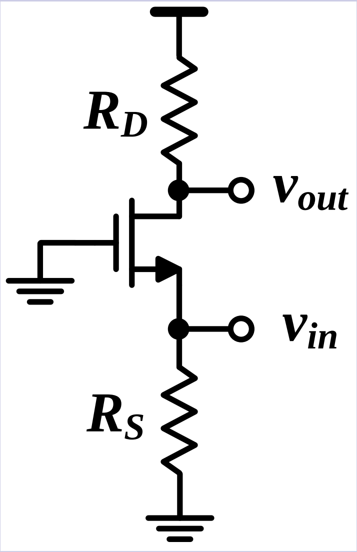
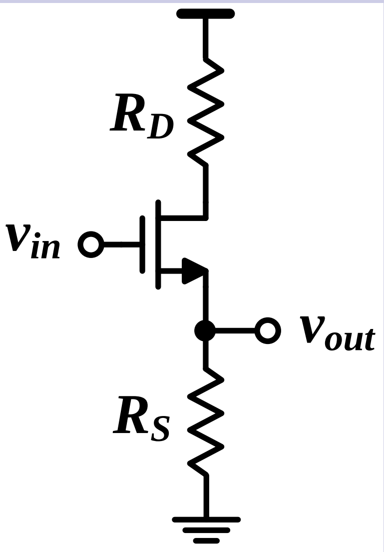
$$\frac{1}{g_m + g_{mb}}\left(1 + \frac{R_D}{r_o}\right)$$

L.I.N.

∞

At low frequencies **ONLY**

Summary of Basic Topologies

	CS	CG	CD (SF)
			
	Voltage & current amplifier	Voltage amplifier Current buffer	Voltage buffer Current amplifier
Rin	∞	$R_S \| \| \frac{1}{g_m + g_{mb}}\left(1 + \frac{R_D}{r_o}\right)$	∞
Rout	$R_D \| \| r_o[1 + (g_m + g_{mb})R_S]$	$R_D \| \| r_o$	$R_S \| \| \frac{1}{g_m + g_{mb}}\left(1 + \frac{R_D}{r_o}\right)$
Gm	$\frac{-g_m}{1 + (g_m + g_{mb})R_S}$	$g_m + g_{mb}$	$\frac{g_m}{1 + R_D/r_o}$

Differential Amplifier

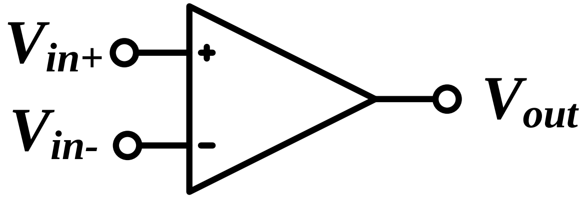
	Pseudo Diff Amp	Diff Pair (w/ ideal CS)	Diff Pair (w/ R_{SS})
A_{vd}	$-g_m R_D$	$-g_m R_D$	$-g_m R_D$
A_{vCM}	$-g_m R_D$	0	$\dfrac{-g_m R_D}{1+2(g_m+g_{mb})R_{SS}}$
A_{vd}/A_{vCM}	1	∞	$2(g_m+g_{mb})R_{SS} \gg 1$

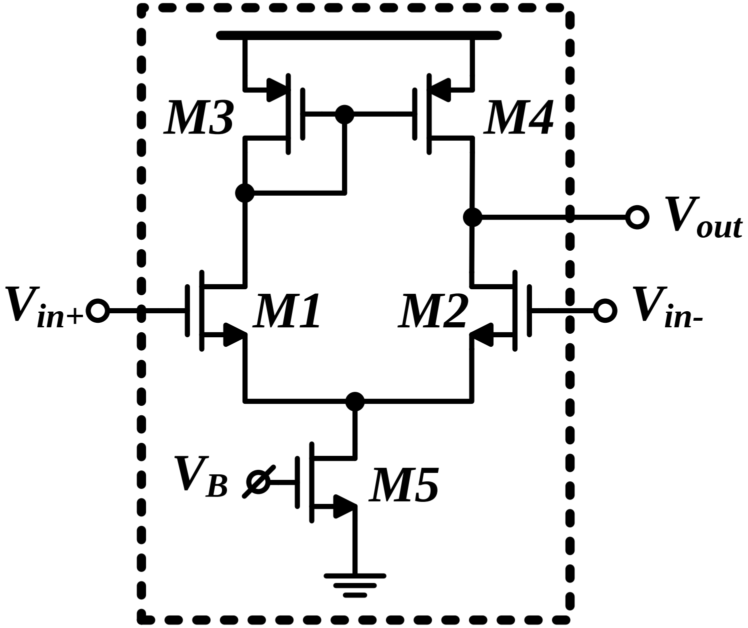
$$A_{vCM2d} = \frac{v_{od}}{v_{iCM}} \approx \frac{\Delta R_D}{2R_{SS}} + \frac{\Delta g_m R_D}{2g_{m1,2}R_{SS}}$$

$$CMRR = \frac{A_{vd}}{A_{vCM2d}}$$

Op-Amp

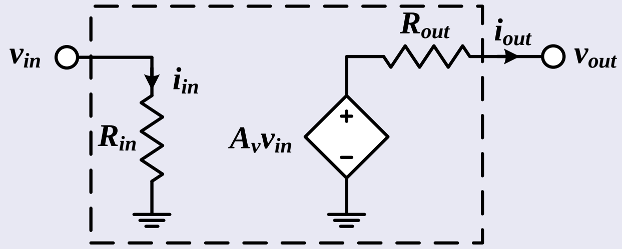
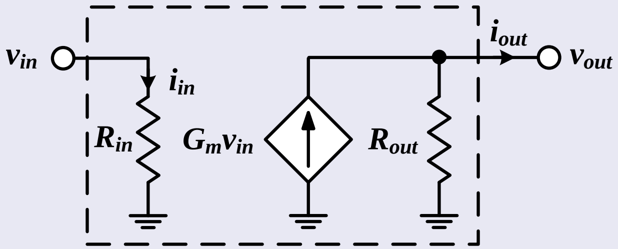
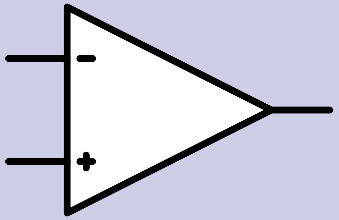
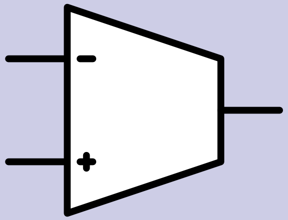
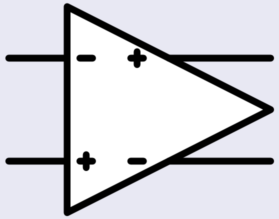
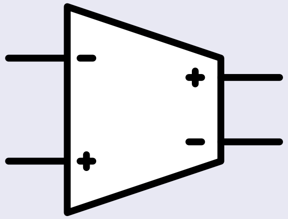
- An op-amp is simply a high gain differential amplifier
  - The gain can be increased by using cascodes and multi-stage amplification
- The diff amp is a key block in many analog and RF circuits
  - DEEP understanding of diff amp is ESSENTIAL





Op-Amp vs OTA

- ❑ In short, an OTA is an op-amp without an output stage (buffer)
- ❑ Some designers just use op-amp name and symbol for both

	Op-amp	OTA
Rout	LOW	HIGH
Model		
Diff input, SE output		
Fully diff		

V-star ($\boldsymbol{V^*}$)

- V-star (V^*) is inspired by V_{ov} but calculated from actual simulation data

$$g_m = \frac{2I_D}{V^*} \leftrightarrow V^* = \frac{2I_D}{g_m} = \frac{2}{g_m/I_D}$$

- Figures-of-merit in terms of V^*

$$g_m r_o = \frac{2I_D}{V^*} \cdot \frac{1}{\lambda I_D} = \frac{2}{\lambda V^*}$$

$$f_T = \frac{g_m}{2\pi C_{gg}} = \frac{1}{2\pi} \cdot \frac{2I_D}{V^*} \cdot \frac{1}{C_{gg}}$$

$$\frac{g_m}{I_D} = \frac{2}{V^*}$$

- The boundary between weak and strong inversion ($n = 1.2 \rightarrow 1.5$)

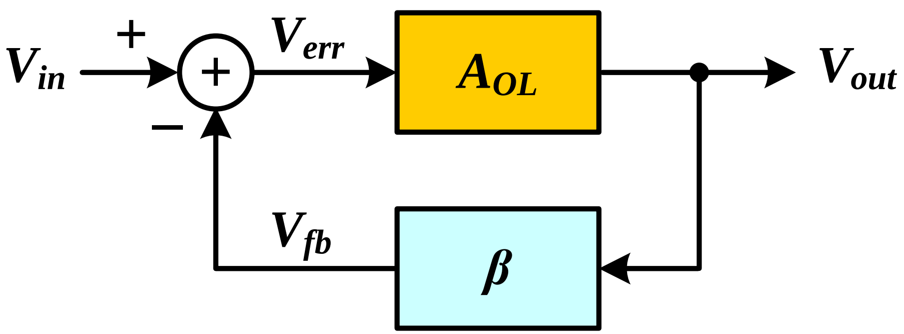
$$V_{ov}(SI) = V^*(WI) = 2nV_T \approx 60 \rightarrow 80mV$$

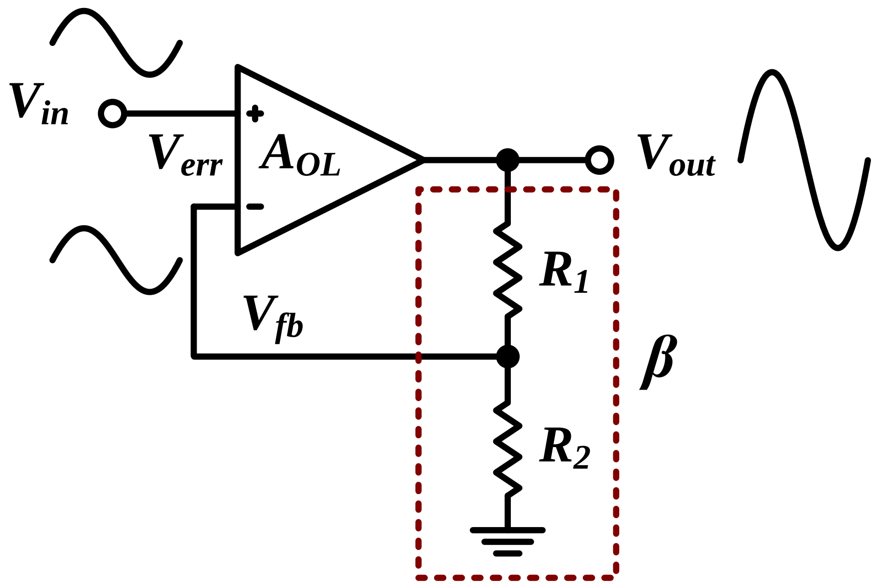
Negative Feedback

$$\beta = \frac{R_2}{(R_1 + R_2)}$$

$$A_{CL} = \frac{V_{out}}{V_{in}} = \frac{A_{OL}}{1+\beta A_{OL}} = \frac{A_{OL}}{1+\beta A_{OL}} \approx \frac{1}{\beta} = \frac{R_1 + R_2}{R_2} = 1 + \frac{R_1}{R_2}$$

$$\omega_{p,CL} = (1 + \beta A_{OLo})\omega_{P,OL}$$

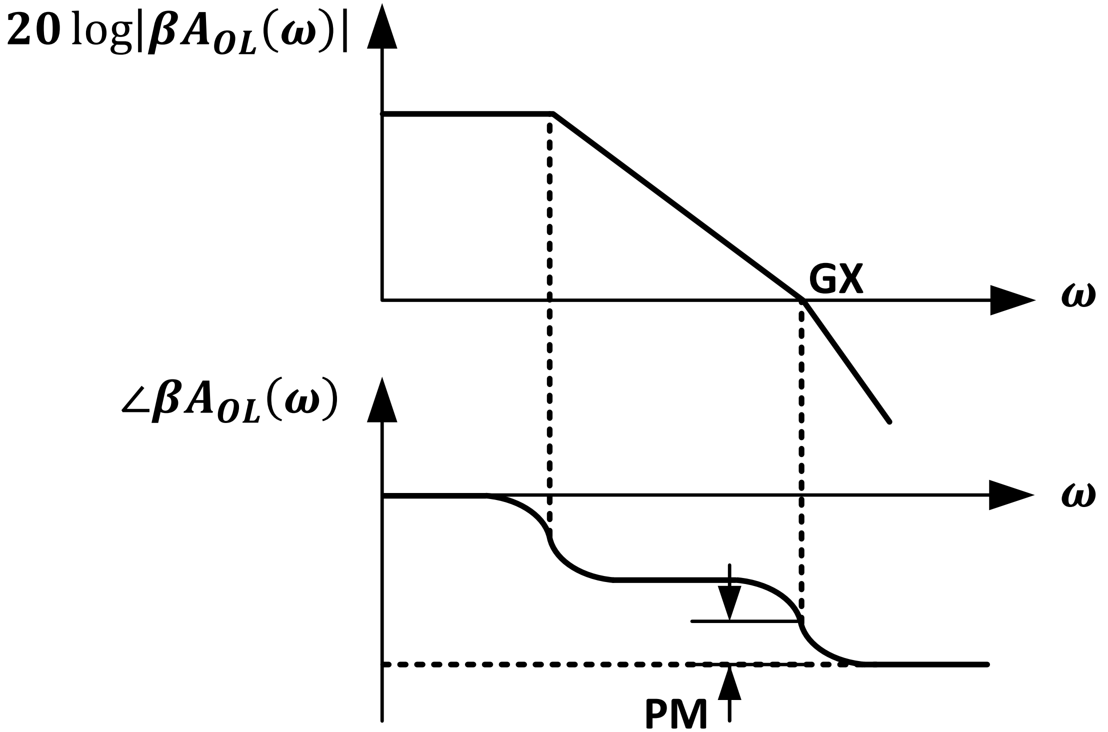




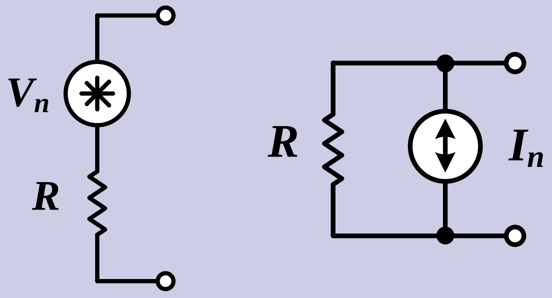
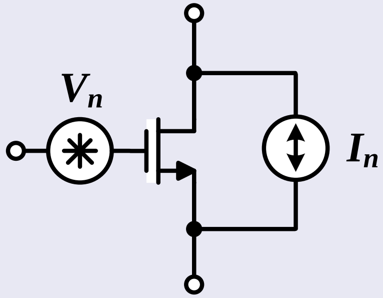
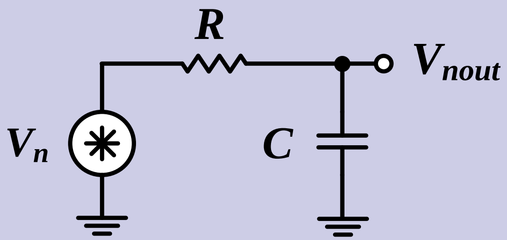
Phase Margin and the Ultimate GBW

- If $\omega_{p2} = \omega_u$: PM = 45°
  - Typically inadequate (peaking/ringing)
- Thus ω_{p2} should be $> \omega_u$ → $\omega_{p1} \ll \omega_u < \omega_{p2}$
  - ω_{p1} defines OL BW and ω_{p2} defines ultimate GBW (max CL BW)

Frequency domain peaking
→ noise amplification
Time domain ringing
→ poor settling time



Noise Models

Resistor thermal noise		$V_n(f)=\sqrt{4kTR}\approx\sqrt{\frac{R}{1\,k}}\times 4\,\frac{nV}{\sqrt{Hz}}$ $I_n(f)=\sqrt{\frac{4kT}{R}}\approx\sqrt{\frac{1\,k}{R}}\times 4\,\frac{pA}{\sqrt{Hz}}$
MOSFET thermal and flicker noise		$I_n^2(f)=4kT\gamma g_m$ $V_n^2(f)=\frac{K}{C_{ox}WL}\frac{1}{f}$
RMS noise		$V_{noutrms}^2=4kTR\times B_N=\frac{kT}{C}$ $B_N=\frac{1}{4RC}=\frac{\pi}{2}f_p$ $V_{noutrms}\approx\sqrt{\frac{1\,p}{C}}\times 64\,\mu Vrms$

Noise Analysis Procedure

- Deactivate the input signal
- Identify the **dominant** noise sources → Model as $V_n^2(f)$ or $I_n^2(f)$
- Find the output noise density for each source: $V_{nout,x}^2(f)$
- Calculate the rms output noise of each source

$$V_{noutrms,x}^2 = V_{nout,x}^2(f) \times B_{N,x}$$

- Calculate total rms noise

$$V_{noutrms,tot}^2 = V_{nrms,1}^2 + V_{nrms,2}^2 + \cdots$$

- Calculate the input-referred rms noise voltage

$$V_{ninrms,tot}^2 = V_{noutrms,tot}^2 / A_v^2$$

- For low Z_{in}, input referred noise current must be added

Outline

- Recapping previous key results
- Noise in Amplifiers

Noise in Amplifiers

- Common source amplifier
- Common gate amplifier
- Common drain amplifier
- Cascode amplifier
- Differential amplifier
- Common OTA topologies
  - 5T OTA
  - Telescopic cascode OTA
  - Folded cascode OTA
  - Two-stage OTA

CS Amplifier with Resistive Load

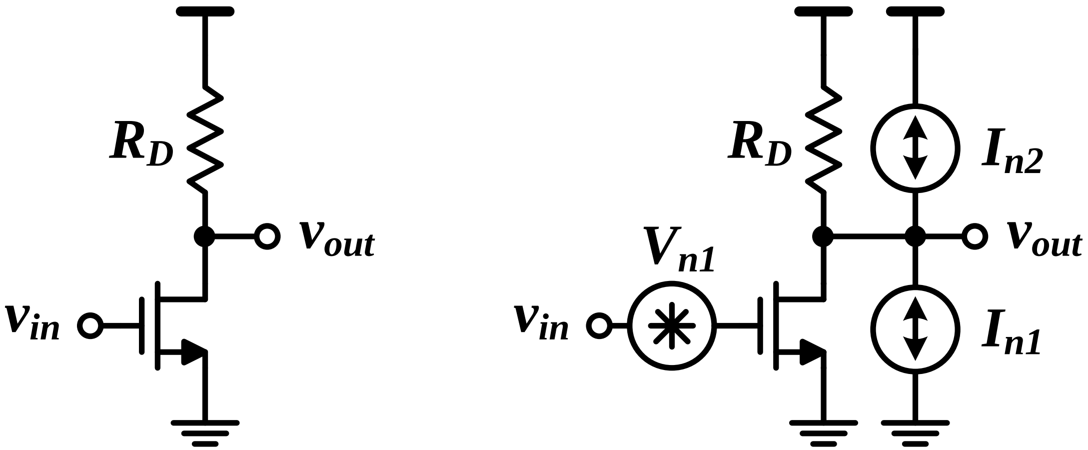
- Deactivate the input and find the noise spectral density at output

$$V_{nout}^2(f) = \left(4kT\gamma g_m + \frac{4kT}{R_D}\right) R_D^2 + \frac{K}{C_{ox}WL}\frac{1}{f}\cdot (g_m R_D)^2$$

- Divide by gain to get input-referred noise

$$V_{nin}^2(f) = \frac{V_{nout}^2(f)}{(g_m R_D)^2} = \frac{4kT}{g_m}\left(\gamma + \frac{1}{g_m R_D}\right) + \frac{K}{C_{ox}WL}\frac{1}{f}$$

- Maximize g_m → noise-power trade-off



CS Amplifier with Active Load

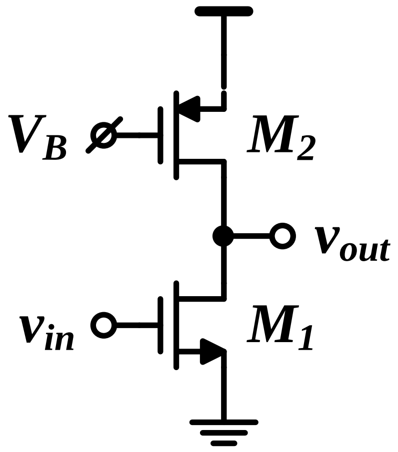
- Find the noise spectral density at output

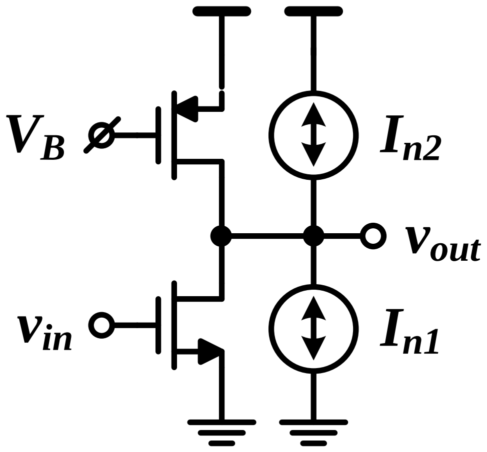
$$V_{nout}^2(f) = (4kT\gamma g_{m1} + 4kT\gamma g_{m2})R_{out}^2$$

- Divide by gain to get input-referred noise

$$V_{nin}^2(f) = \frac{V_{nout}^2(f)}{(g_{m1}R_{out})^2} = \frac{4kT\gamma}{g_{m1}}\left(1 + \frac{g_{m2}}{g_{m1}}\right) = \frac{4kT\gamma\alpha}{g_{m1}}$$

- Maximize g_{m1} (transconductor) and minimize g_{m2} (current source)

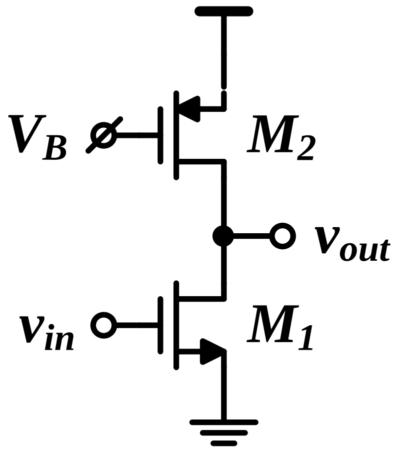


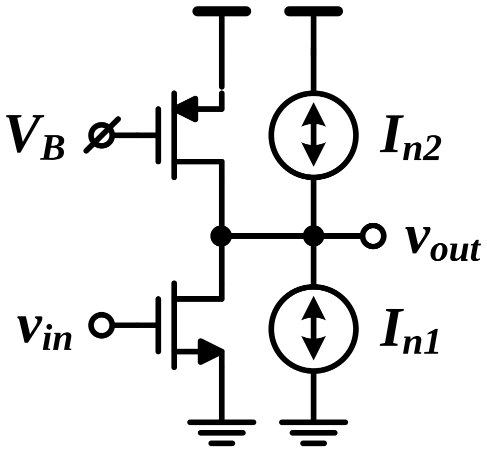


Small or Large g_m?

$$V_{nin}^2(f) = \frac{4kT\gamma}{g_{m1}}\left(1 + \frac{g_{m2}}{g_{m1}}\right) = \frac{4kT\gamma\alpha}{g_{m1}}$$

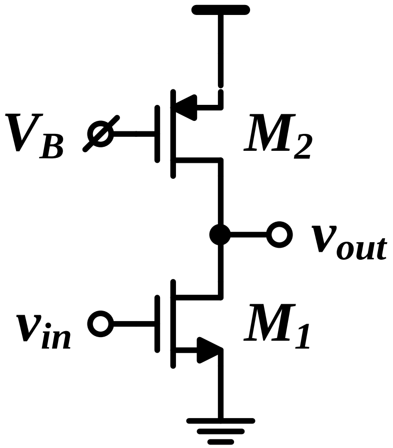
- From noise perspective, do we want small or large g_m?
- Large g_m for amplifier/transconductor (g_{m1}) → Large gm/ID
  - Noise power $\propto g_{m1}$ but signal power $\propto g_{m1}^2$
- Small g_m for constant current source (g_{m2}) → Small gm/ID
  - Noise power $\propto g_{m2}$ but signal power independent of g_{m2}





Design for Low Noise: Thermal Noise

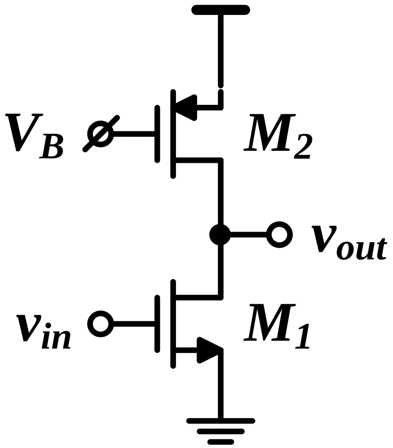
$$V_{nin}^2(f) = \frac{4kT\gamma}{g_{m1}}\left(1 + \frac{g_{m2}}{g_{m1}}\right)$$



- Maximize g_{m1}
  - $I_D \uparrow \rightarrow$ power consumption $\uparrow$
  - Or $(g_m/I_D)_1 \uparrow \rightarrow V_1^* \downarrow \rightarrow f_T \downarrow$ (area and capacitance $\uparrow$)
- Minimize g_{m2}
  - $(g_m/I_D)_2 \downarrow \rightarrow V_2^* \uparrow \rightarrow$ headroom $\downarrow$

Design for Low Noise: Flicker Noise

$$V_{nin}^2(f) = \frac{K_N}{C_{ox}(WL)_1}\frac{1}{f} + \frac{K_P}{C_{ox}(WL)_2}\frac{1}{f}\left(\frac{g_{m2}}{g_{m1}}\right)^2$$



- If $L_1 = L_2$: NMOS dominates
  - Because $K_N > K_P$ and $\mu_N > \mu_P$
- Increase W_1 → $I_D \uparrow$ (if V^* is constant) → power consumption ↑
- Increase L_1 → $f_T \downarrow$ → area and capacitance ↑

CS Amplifier SNR

- Assume BW is limited by a load capacitance C_L

$$V_{noutrms}^2 = V_{nout}^2(f) \cdot \frac{1}{4R_{out}C_L} \approx \frac{kT\gamma\alpha g_{m1}R_{out}}{C_L}$$

- Assume input signal is a sinusoid with amplitude $= V_p$

$$SNR = \frac{V_{outrms}^2}{V_{noutrms}^2} \approx \left(\frac{V_p}{\sqrt{2}} \cdot g_{m1}R_{out}\right)^2 \cdot \frac{C_L}{kT\gamma\alpha g_{m1}R_{out}}$$
$$= \frac{V_p^2}{2}\frac{g_{m1}R_{out}C_L}{kT\gamma\alpha} = \frac{V_p^2}{2}\frac{g_{m1}R_{out}C_L}{kT\gamma\left(1+\frac{g_{m2}}{g_{m1}}\right)}$$

- Maximize g_{m1} (transconductor) and minimize g_{m2} (current source)
  - Large gm/ID for transconductor and small gm/ID for load

CS Amplifier SNR

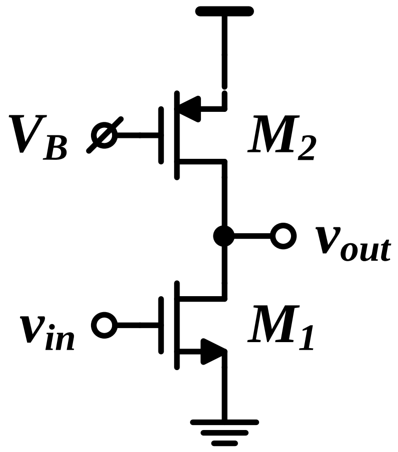
- Assume a maximum rms output amplitude $= \kappa V_{DD}$

$$SNR = \frac{V_{outrms}^2}{V_{noutrms}^2} \approx (\kappa V_{DD})^2 \cdot \frac{C_L}{kT\gamma\alpha g_{m1} R_{out}}$$
$$= \frac{(\kappa V_{DD})^2 C_L}{kT\gamma\left(1 + \frac{g_{m2}}{g_{m1}}\right)|A_v|}$$

- Maximize g_{m1} (transconductor) and minimize g_{m2} (current source)
  - Large gm/ID for transconductor and small gm/ID for load

Dominant Noise Contributors By Inspection

- At least two devices contribute to the amplifier noise
  - One input transistors and one load transistors
- Determine dominant noise contributors by inspection
  - Estimate the gain from the gate of the transistor to the output
    - Is it comparable to the gain from the input to output?

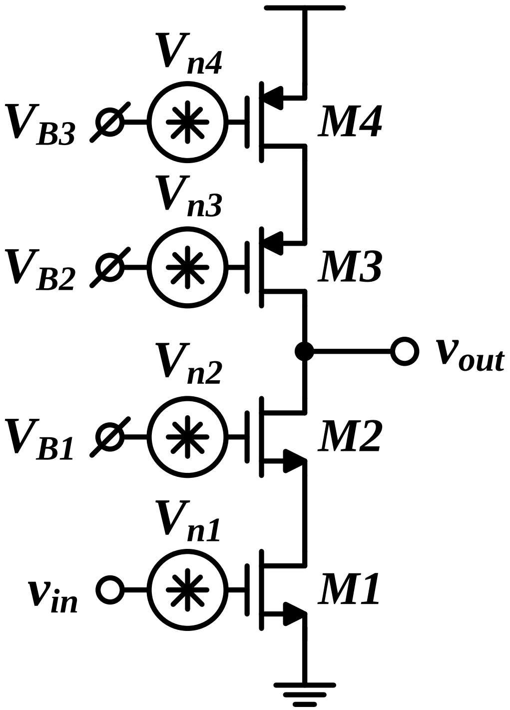


Cascode Amplifier

- Identify dominant noise contributors
  - Noise signals with high gain paths → V_{n1} and V_{n4} only (why?)
  - But $V_{n2,3}$ contribution may be large at high frequencies (why?)

$$V_{nout}^2(f) \approx (4kT\gamma g_{m1} + 4kT\gamma g_{m4})R_{out}^2$$

$$V_{nin}^2(f) \approx \frac{V_{nout}^2(f)}{(g_{m1}R_{out})^2} = \frac{4kT\gamma}{g_{m1}}\left(1 + \frac{g_{m4}}{g_{m1}}\right) = \frac{4kT\gamma\alpha}{g_{m1}}$$

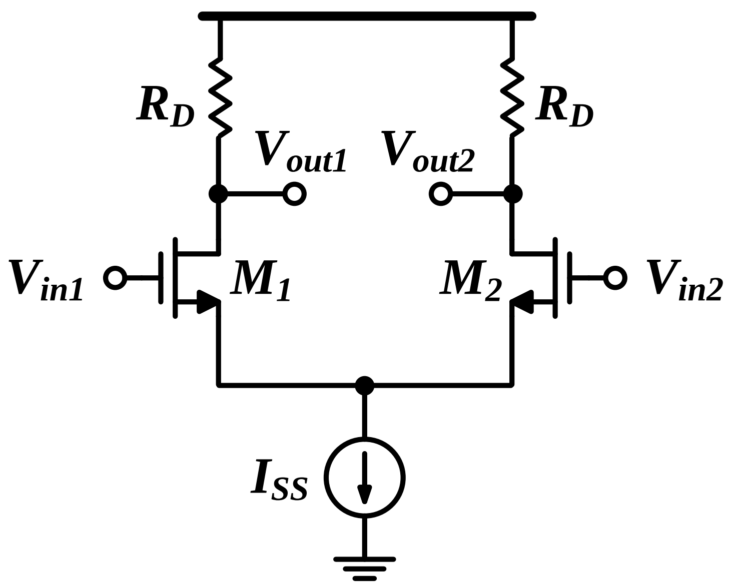


Differential Pair with Resistive Load

- Use half-circuit principle
- Twice the noise of CS amplifier (variances add)

$$V_{nin}^2(f) = 2 \times \frac{4kT}{g_m}\left(\gamma + \frac{1}{g_m R_D}\right) + 2 \times \frac{K}{C_{ox}WL}\frac{1}{f}$$

- But SNR is improved by a factor of two = 3 dB compared to CS (why?)

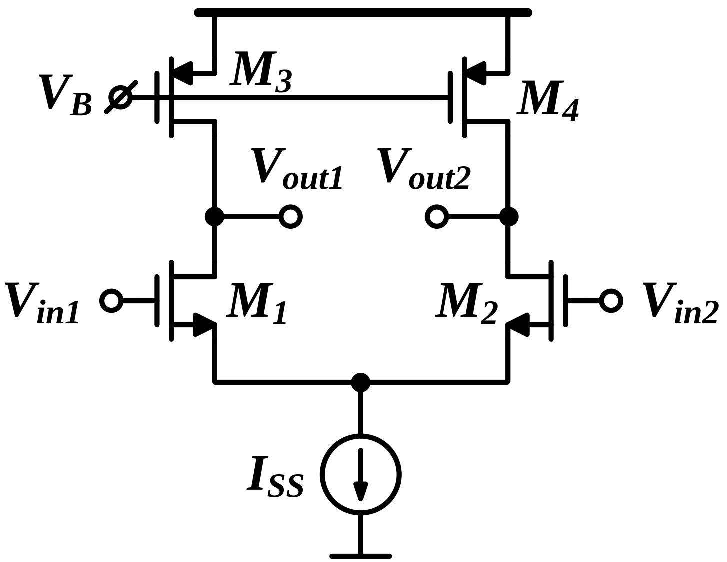


Differential Pair with Active Load

- Use half-circuit principle
- Twice the noise of CS amplifier (variances add)

$$V_{nin}^2(f) = \frac{8kT\gamma}{g_{m1,2}}\left(1 + \frac{g_{m3,4}}{g_{m1,2}}\right) + \frac{2K_N}{C_{ox}(WL)_{1,2}}\frac{1}{f} + \frac{2K_P}{C_{ox}(WL)_{3,4}}\frac{1}{f}\left(\frac{g_{m3,4}}{g_{m1,2}}\right)^2$$

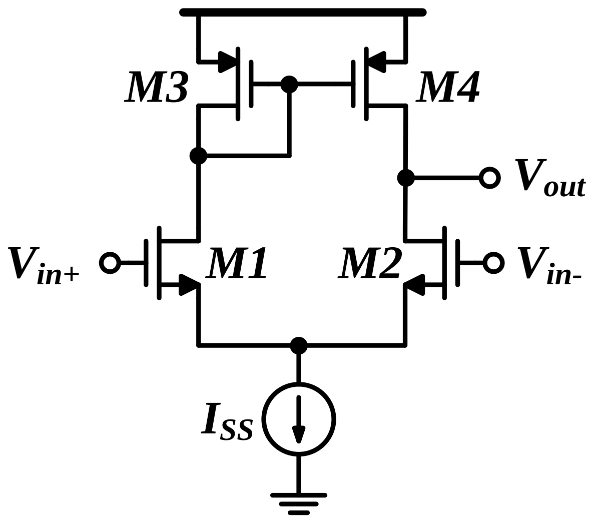
- But SNR is improved by a factor of two = 3 dB compared to CS (why?)

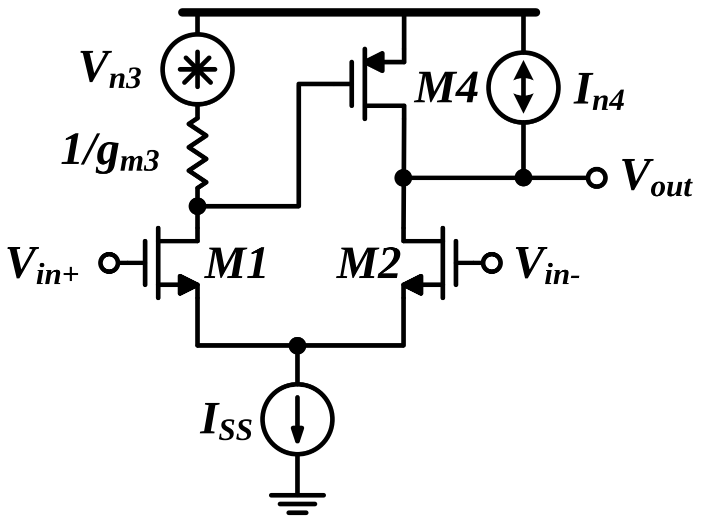


Differential Pair with CM Load (5T OTA)

- Noise sources of M1 and M2 are already input referred
- M3 is diode connected and drain of M1 is H.I.N.: $V_{gs4} \approx V_{n3}$
- Same as differential pair noise:

$$V_{nin}^2(f) = \frac{8kT\gamma}{g_{m1,2}}\left(1 + \frac{g_{m3,4}}{g_{m1,2}}\right) + \frac{2K_N}{C_{ox}(WL)_{1,2}}\frac{1}{f} + \frac{2K_P}{C_{ox}(WL)_{3,4}}\frac{1}{f}\left(\frac{g_{m3,4}}{g_{m1,2}}\right)^2$$

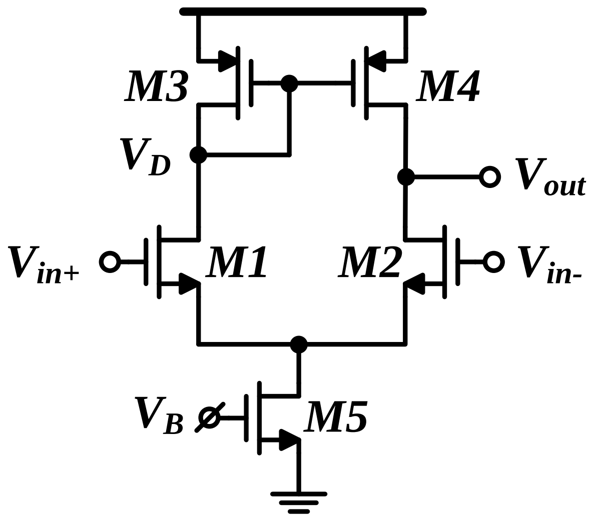




Differential Pair with CM Load (5T OTA)

- Noise of tail current source appears at output, even if we assume perfect matching
  - Noise of tail current source split equally between M1 and M2
  - V_{out} follow V_D → $R_{out} = 1/g_{m3}$ → negligible contribution

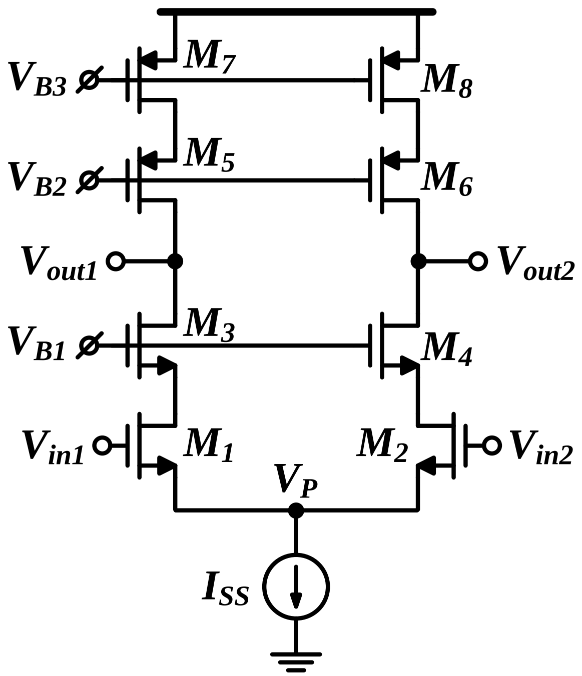
$$V_{nin,M5}^2(f) = \left(\frac{I_{n5}}{2}\right)^2 \cdot \frac{1}{g_{m3}^2} \cdot \frac{1}{g_{m1}^2 (r_{o2}||r_{o4})^2}$$



Telescopic Cascode

- The noise of the cascode devices is negligible at low frequencies
- M1,2 and M7,8 are the primary noise sources

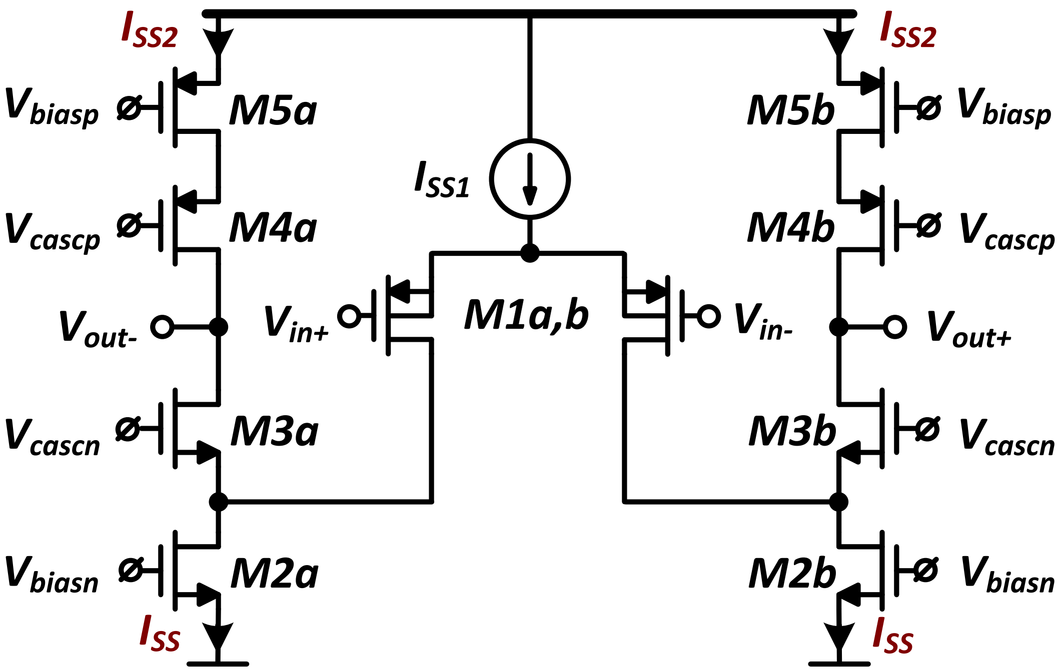
$$V_{nin}^2(f) = \frac{8kT\gamma}{g_{m1,2}}\left(1+\frac{g_{m7,8}}{g_{m1,2}}\right) + \frac{2K_N}{C_{ox}(WL)_{1,2}}\frac{1}{f} + \frac{2K_P}{C_{ox}(WL)_{7,8}}\frac{1}{f}\left(\frac{g_{m7,8}}{g_{m1,2}}\right)^2$$



Folded Cascode

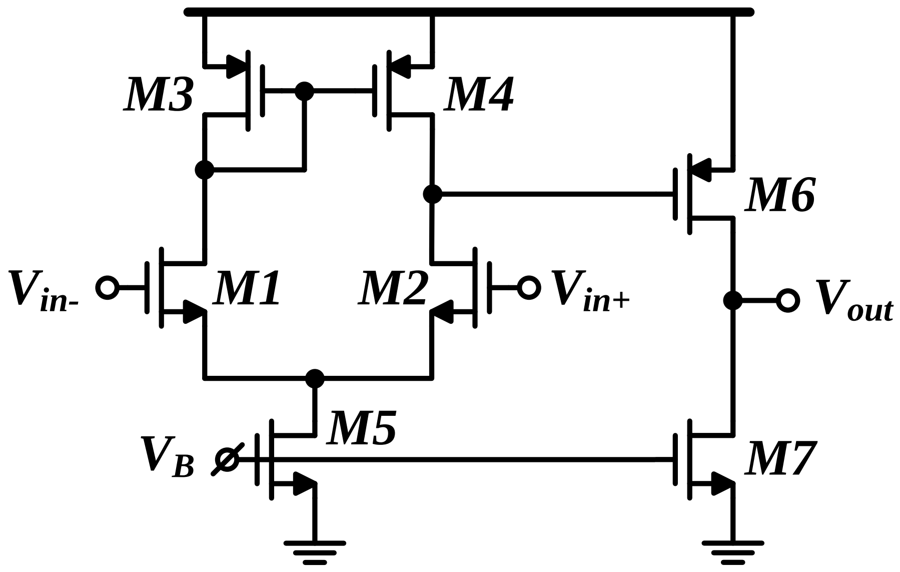
- The noise of the cascode devices is negligible at low frequencies
- M1, M2, and M5 are the primary noise sources

$$V_{nin}^2(f) = \frac{8kT\gamma}{g_{m1}}\left(1 + \frac{g_{m2}}{g_{m1}} + \frac{g_{m5}}{g_{m1}}\right) = \frac{8kT\gamma\alpha}{g_{m1}}$$



Two-Stage OTA

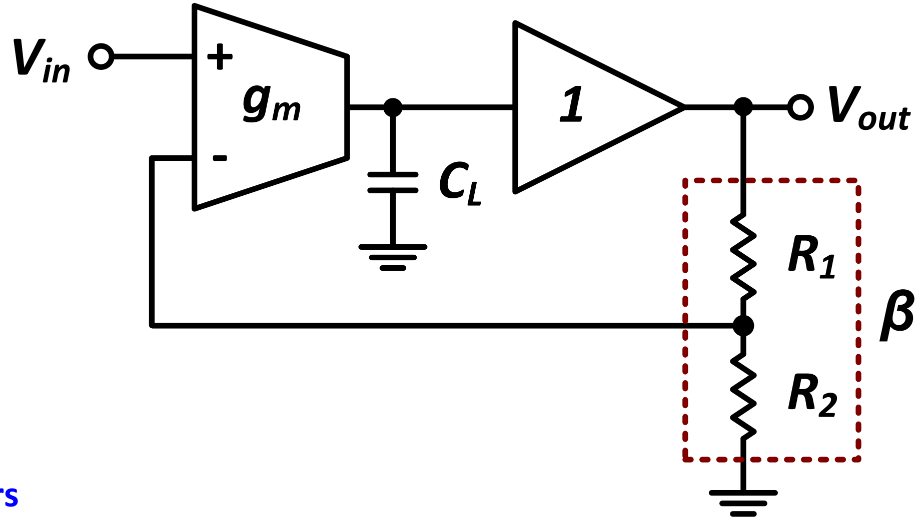
- The noise of the second stage is divided by the gain of the first stage when referred to the input
  - Usually noise of first stage is dominant
  - A general result for multi-stage amplifiers
- More accurate result for Miller compensated OTA shortly

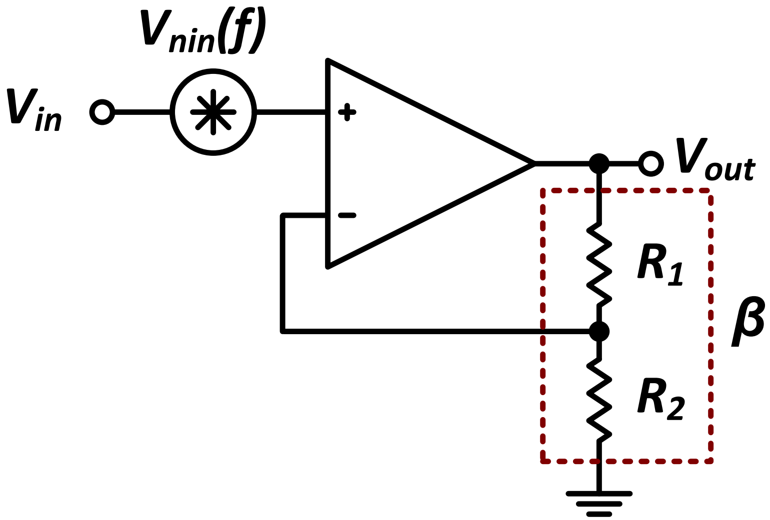


Single-Stage Noise in Closed-loop

$$\omega_u = \frac{g_m}{C_L} \qquad A_{CL} \approx \frac{1}{\beta} \qquad BW_{CL} \approx \frac{1}{2\pi}\frac{\omega_u}{A_{CL}} = \frac{1}{2\pi}\beta\omega_u \qquad V_{nin}^2(f) = \frac{8kT\gamma\alpha}{g_m}$$

$$V_{noutrms}^2 = \overline{V_{nout}^2} = V_{nin}^2(f) \cdot A_{CL}^2 \cdot \frac{\pi}{2} BW_{CL} = \frac{8kT\gamma\alpha}{g_m} \cdot \frac{1}{\beta^2} \cdot \frac{\beta g_m}{4C_L}$$
$$= \frac{2kT\gamma\alpha}{\beta C_L}$$



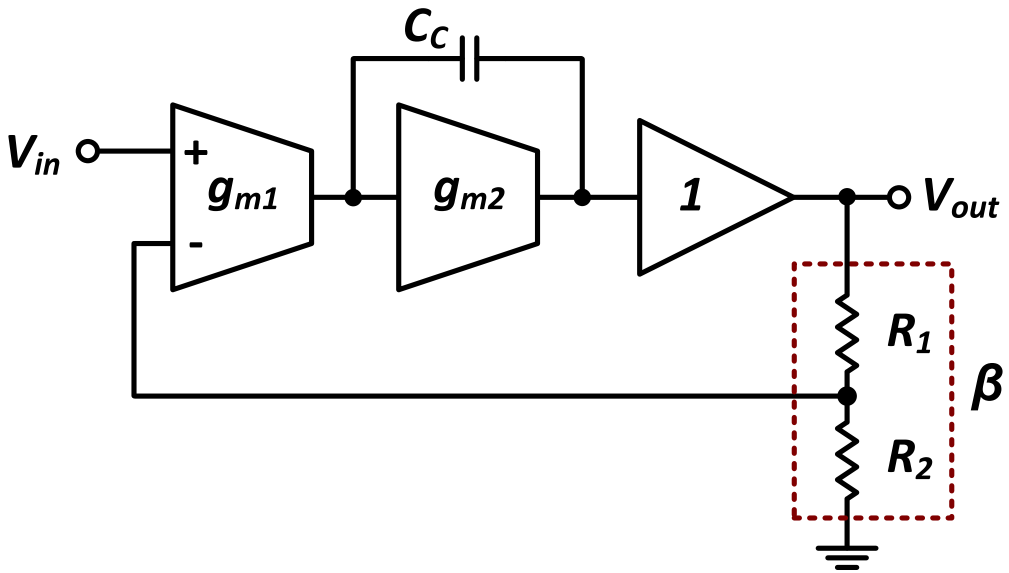


Two-Stage Noise in Closed-loop

$$\overline{V_{nout1}^2} \approx V_{nin1}^2(f) \cdot A_{CL}^2 \cdot \frac{\pi}{2} BW_{CL} = \frac{2kT\gamma\alpha_1}{\beta C_C}$$

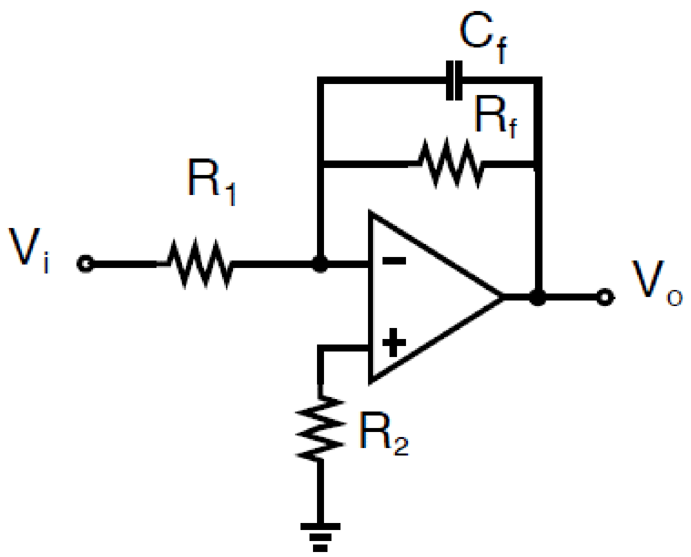
$$\overline{V_{nout2}^2} \approx 4kT\gamma g_{m2}\alpha_2 \cdot \frac{1}{g_{m2}^2} \cdot \frac{g_{m2}}{4C_2} = \frac{kT\gamma\alpha_2}{C_2}$$

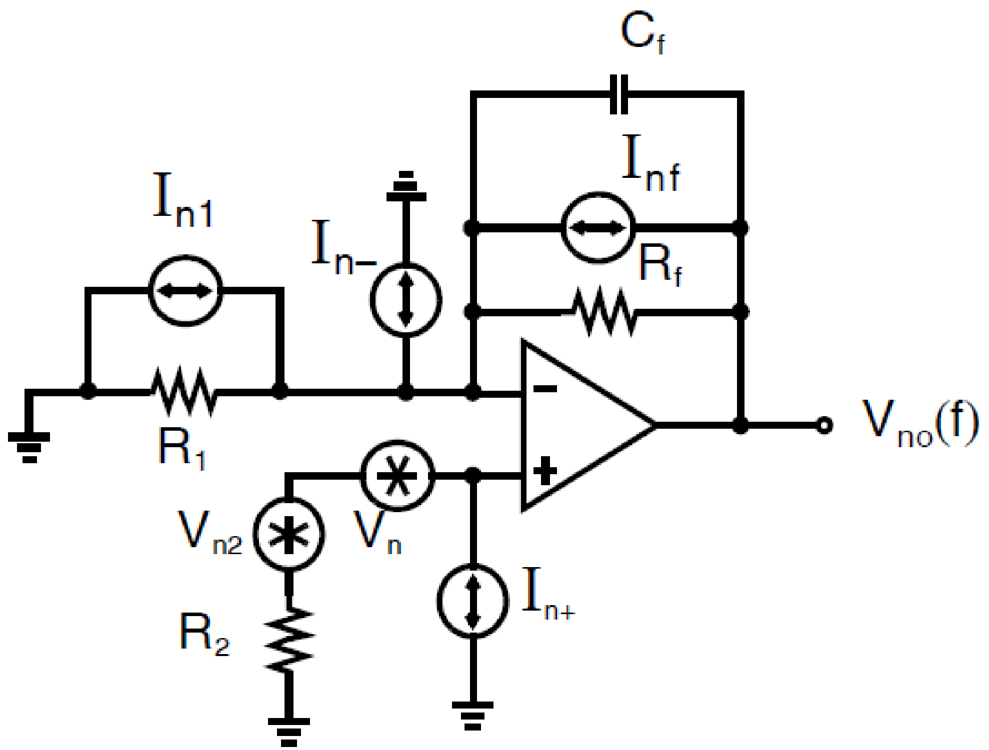
$$\overline{V_{nout}^2} = \overline{V_{nout1}^2} + \overline{V_{nout2}^2} \approx \frac{2kT\gamma\alpha_1}{\beta C_C}\left(1 + \frac{\beta\alpha_2 C_C}{2\alpha_1 C_2}\right)$$



Noise Analysis Example

- HW: Read Section 9.4.1 and solve Example 9.10 in [Johns and Martin, 2015]
- Inverting amplifier (R_f and R_1) and LPF (R_f and C_f)
- $V_n, I_{n-}, and\ I_{n+}$ model the op-amp equivalent input noise





[Johns and Martin, 2015]

References

- B. Razavi, "Design of Analog CMOS Integrated Circuits," McGraw-Hill, 2nd ed., 2017.
- T. C. Carusone, D. Johns, and K. W. Martin, "Analog Integrated Circuit Design," 2nd ed., Wiley, 2012.
- B. Murmann, EE214 Course Reader, Stanford University.

Thank you!