

## Nonlinear Finite Element Methods, WiSe 2025/26, Homework 1

Due: Tuesday, November 11, upload on Moodle

Report all work, including any m-files (or Python code) you have written. Please write clearly and be sure to label for which problem each solution is.

Consider the following benchmark: a cantilever beam of length  $L = 5$  m, made of steel ( $E = 200$  GPa,  $G = 80$  GPa) with an IPE 100 profile ( $A = 10.32$  cm<sup>2</sup>,  $A_{web} = 3.63$  cm<sup>2</sup>,  $I = 171$  cm<sup>4</sup>, shear correction factor  $\kappa = A_{web}/A$ , fully fixed on the left and loaded by a single force  $F = 1$  N on the right end.

1. Derive an analytical reference solution according to the Euler-Bernoulli beam theory. Plot the displacement solution.
2. Derive the one-field variational formulation (weak form) using the Timoshenko beam theory. Also draw a Tonti diagram for this case.
3. Discretize the one-field weak form using standard two-node linear finite elements for both displacements and rotations. Conduct a convergence study for the displacement error in the  $L^2$  norm, using  $n_{ele} = 2, 4, 8, 16, 32, 64, 128, 256$  elements and the Euler-Bernoulli analytical solution as a reference.
4. Derive the corresponding two-field variational formulation (weak form), based on the Hellinger-Reissner principle. Also draw a Tonti diagram for this case.
5. Discretize the two-field weak form using standard two-node linear finite elements for all independent fields. Conduct a convergence study for the displacement error in the  $L^2$  norm, using  $n_{ele} = 2, 4, 8, 16, 32, 64, 128, 256$  elements and the Euler-Bernoulli analytical solution as a reference.
6. Compare and discuss the one-field and two-field results. Discuss advantages and disadvantages of the two Timoshenko beam FE formulations and the Euler-Bernoulli beam FE formulation that you know from FEM I.