

Nonlinear Finite Element Methods, WiSe 2025/26, Homework 1

Due: Tuesday, November 11, upload on Moodle

Report all work, including any m-files (or Python code) you have written. Please write clearly and be sure to label for which problem each solution is.

Consider the following benchmark: a cantilever beam of length $L = 5$ m, made of steel ($E = 200$ GPa, $G = 80$ GPa) with an IPE 100 profile ($A = 10.32\text{ cm}^2$, $A_{web} = 3.63\text{ cm}^2$, $I = 171\text{ cm}^4$, shear correction factor $\varkappa = A_{web}/A$, fully fixed on the left and loaded by a single force $F = 1$ N on the right end.

1. Derive an analytical reference solution according to the Euler-Bernoulli beam theory. Plot the displacement solution.
2. Derive the one-field variational formulation (weak form) using the Timoshenko beam theory. Also draw a Tonti diagram for this case.
3. Discretize the one-field weak form using standard two-node linear finite elements for both displacements and rotations. Conduct a convergence study for the displacement error in the L^2 norm, using $n_{ele} = 2, 4, 8, 16, 32, 64, 128, 256$ elements and the Euler-Bernoulli analytical solution as a reference.
4. Derive the corresponding two-field variational formulation (weak form), based on the Hellinger-Reissner principle. Also draw a Tonti diagram for this case.
5. Discretize the two-field weak form using standard two-node linear finite elements for all independent fields. Conduct a convergence study for the displacement error in the L^2 norm, using $n_{ele} = 2, 4, 8, 16, 32, 64, 128, 256$ elements and the Euler-Bernoulli analytical solution as a reference.
6. Compare and discuss the one-field and two-field results. Discuss advantages and disadvantages of the two Timoshenko beam FE formulations and the Euler-Bernoulli beam FE formulation that you know from FEM I.