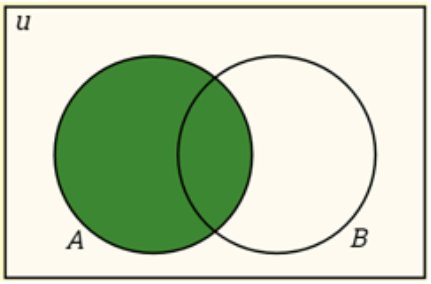
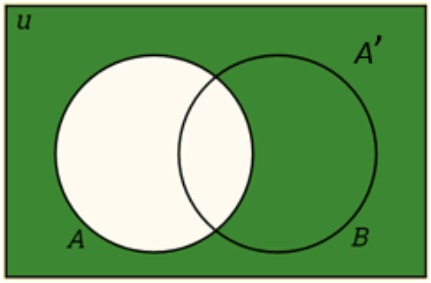
Set Theory

Cantor's defintion of a set

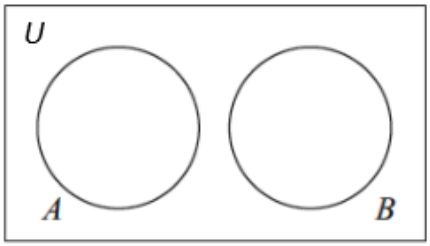
A set is a collection of definite and seperate objects. These objects are called elements of the set.



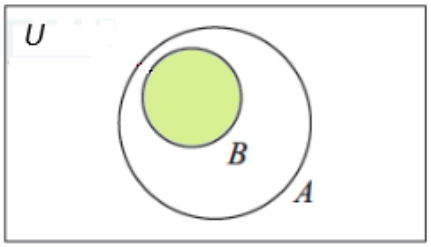
Set A



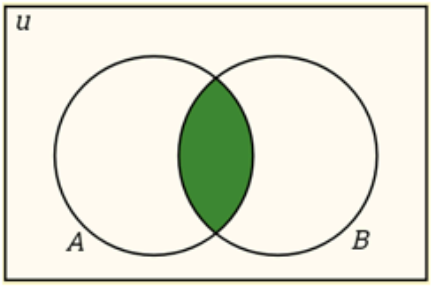
A' the complement of A



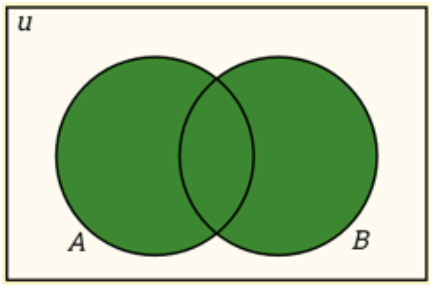
A and B are disjoint sets



B is proper subset of A $\quad B \subset A$



Both A and B
A intersect B $\quad A \cap B$



Either A or B
A union B $\quad A \cup B$

An empty set is denoted by { }, it is an unique set with no elements.

Ordered tuples

An ordered n-tuple is defined by $(x_1, x_2, ..., x_n)$

An ordered pair is defined by (x_1, x_2)

An ordered triple is defined by (x_1, x_2, x_3)

Rule: $(a, b) = (c, d)$ iff (in and only if) $(a = c), (b = d)$

Cartesian products

The Cartesian product $A_1 \times A_2 \times ... \times A_n$ of ordered n-tuples $(a_1, a_2, ..., a_n) | (a_1 \in A_1) \wedge (a_2 \in A_2) \wedge ... \wedge (a_n \in A_n$

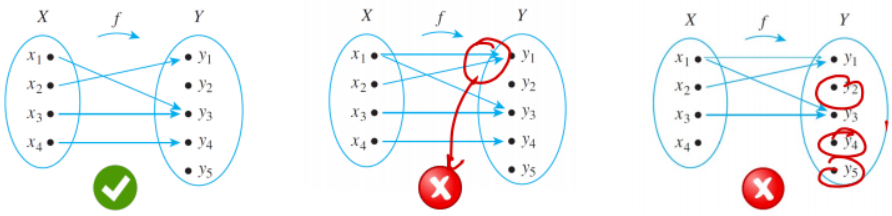
Functions

A function is built from two sets, X and Y. What the function does is build a relation between these two sets following these two rules:

- Every element of X is related to some elements in Y
- No element of X is related to more than one element in Y

Following cases are also allowed:

- There can be elements of the co-domain Y that are not function of any input element.
- Different inputs can mapped to the same element of Y.



Notable functions

- The identity function: $I : X \to X$, where each element is mapped to itself.
- The exponential function
- The log-function
- The Hamming-Distance Function
- The image function

Exponential rules

Laws of Exponents

If b and c are any positive real numbers and u and v are any real numbers, the following laws of exponents hold true:

$$b^u b^v = b^{u+v} \quad 7.2.1$$

$$(b^u)^v = b^{uv} \quad 7.2.2$$

$$\frac{b^u}{b^v} = b^{u-v} \quad 7.2.3$$

$$(bc)^u = b^u c^u \quad 7.2.4$$

Logarithmic rules

Theorem 7.2.1 Properties of Logarithms

For any positive real numbers b, c and x with $b \neq 1$ and $c \neq 1$:

a. $\log_b(xy) = \log_b x + \log_b y$

b. $\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$

c. $\log_b(x^a) = a \log_b x$

d. $\log_c x = \frac{\log_b x}{\log_b c}$

Convolution

- Check slides for explaination
- See this link for a demo