Review of Deep self-consistent learning of local volatility.

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1 Summary

The reviewed paper is proposing a joint calibration of call/put prices and local volatility. The novelty of the study consists in two separate networks for learning different quantities of interest :

1. A network for interpolating price with respect to coordinates strike and maturity.

2. A network for learning local volatility with the same coordinates.

The first network is regularized with shape constraints for embedding terminal conditions / static arbitrages and Dupire equation with a neural local volatility. The second network is learning local volatility from Dupire formula applied to the adjoint derivatives of the first network.

These two networks are calibrated sequentially with a Bayesian procedure recalling Expectation-Maximization algorithms : the penalty term L_{dup} (of the second network, if I understand well, but this should be clarified as required below) is regularising neural prices with a prior on local volatility.

The numerical results demonstrate the relevance of Dupire regularisation with a smoother adjoint local volatility and with a better generalization error thanks to information provided beyond market knots.

Furthermore, with an appropriate change of variable and transformation of neural price outputs, the pricing RMSEs and repricing errors for local volatility outperform the ones of [CCD20] and [CCC$^+$21].

2 Comments

- How is noise filtered in market data (cf beginning of page 4) ? Did you mean that using a separate neural network for local volatility provides by construction a continuous surface, in constrast with dupire volatility obtained by neural price AAD as in [CCD20]?

- The statement that no (calendar or butterfly) arbitrage can be synthetized as the single condition (10) is not proved and it is intuitively wrong. Please prove or amend this statement and draw the consequences of the changes in the rest of the paper (including the numerical implementation and results), detailing them in the response letter to the revision of the paper.

- No github link available for reproducing results even if data are supplied with github from [CCD20] and [CCC$^+$21]. Which (pre-)training schemes for the neural networks? (which look over-parametrized).

- Delete the first lines of the Introduction (before 'An option is a financial contract...'), which are very badly written (or rewrite them completely).

- Justify the weight design on page 9 in Section 3.2.2. Why not the standard vega weighting ?

- Shape constraints are imposed with penalization which is sometimes referred as a soft-constraint approach in the literature leading to violations at some locations. Do you observe any violations of L_{arb} at some collocation points (in the paper's terminology)? Some measure of theses violations would be welcomed.

- Only the penalization coefficient λ_{dup} is estimated in your analysis. Do you have any insight for setting the other coefficients (λ_{arb}, λ_{ini}) equal to 1?

- An RMSE on local volatility calibrated on discrete data does not mean much because of non-uniqueness of the solution to such calibration problem. What counts more is the pricing RMSEs and repricing errors (which, having said this, are duly documented in the paper).

- In section 4.1, page 13, "recovered from Eqs. (11) and (17)" should be "recovered from Eqs. (11) and (13)".

3 Recommendation

The numerical results look good but the above points need to be addressed. Major revision.

References

[CCC+21] Marc Chataigner, Areski Cousin, Stéphane Crépey, Matthew Dixon, and Djibril Gueye. Beyond surrogate modeling: Learning the local volatility via shape constraints. *SIAM Journal on Financial Mathematics*, 12(3):SC58–SC69, 2021.

[CCD20] Marc Chataigner, Stéphane Crépey, and Matthew Dixon. Deep local volatility. *Risks*, 8(3):82, 2020.