Stopping Criteria for PINNs

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Contents

1 Introduction

Moving averages are a crucial statistical tool used in various fields, including economics, finance, and engineering. They are particularly useful in smoothing data series and identifying underlying trends. This document provides an overview of moving averages and discusses their application in the training of Physics-Informed Neural Networks (PINNs).

2 Types of Moving Averages

There are several types of moving averages, each with unique characteristics and applications:

2.1 Simple Moving Average (SMA)

The Simple Moving Average (SMA) is the arithmetic mean of a set number of data points over a specific period. It is calculated by adding up a set of values and then dividing by the count of those values.

Formula:

$$SMA = \frac{1}{N} \sum_{i=1}^{N} P_i \tag{1}$$

where N is the number of time periods and P_i are the data points.

Characteristics:

- Easy to calculate and interpret.
- Useful for identifying overall trends.
- Treats all data points equally.

2.2 Exponential Moving Average (EMA)

The Exponential Moving Average (EMA) places greater weight on more recent data points, making it more responsive to new information.

Formula:

$$EMA_t = \alpha \cdot P_t + (1 - \alpha) \cdot EMA_{t-1} \tag{2}$$

where α is the smoothing factor, P_t is the current data point, and EMA_{t-1} is the previous EMA value.

Characteristics:

- More responsive to recent changes in data.
- Complex calculation.
- Useful for dynamic and rapidly changing datasets.

2.3 Weighted Moving Average (WMA)

The Weighted Moving Average (WMA) is similar to the EMA but allows for customizable weighting, giving more control over how recent and past data are valued.

Formula:

$$WMA = \frac{\sum_{i=1}^{N} w_i \cdot P_i}{\sum_{i=1}^{N} w_i} \tag{3}$$

where w_i are the weights and P_i are the data points.

Characteristics:

- Balances between recent and past data.
- Customizable weighting.
- Requires more effort to set up and optimize.

3 Application in PINN Training

When training Physics-Informed Neural Networks (PINNs), selecting the right moving average as a stopping criterion is crucial. The choice depends on the nature of the training process and the characteristics of the data.

3.1 Choosing the Right Moving Average

- **SMA** is suitable for stable training processes with less fluctuation.
- **EMA** is preferred for dynamic processes where recent changes are more indicative of future performance.
- **WMA** offers a balanced approach, suitable for a mix of recent and historical performance metrics.

4 Conclusion

The choice of moving average in PINN training should be based on the specific requirements of the training process and the nature of the dataset. It is often beneficial to experiment with different types and parameters to find the most suitable one for your specific case.