Calculate $P(a|\neg)$: interred

$$P(a) = P(a|b,e)P(b)P(e) + P(a|b,\neg e)P(b)P(\neg e)$$
$$+ P(a|\neg b,e)P(\neg b)P(e) + P(a|\neg b,\neg e)P(\neg b)P(\neg e)$$

Same as $P(a, B, E)$

Sum out over B, E

$$= (.95 \times .001 \times .002) + (.94 \times .001 \times .998) + (.29 \times .999 \times .002) + (.001 \times .999 \times .998)$$

$$P(a) = .0025$$

$$P(a|b) = \frac{P(a,b,E)}{P(b,E)}$$

a alarm given burglary

B, E indep.

$$= \frac{P(a|b,e)P(b)P(e) + P(a|b,\neg e)P(b)P(\neg e)}{P(b)P(e) + P(b)P(\neg e)}$$

$$= \frac{(.95 \times .001 \times .002) + (.94 \times .001 \times .998)}{(.001 \times .002) + (.001 \times .998)}$$

$$\approx .94 \quad \text{or} \quad .94002$$

$$P(b|a) = \frac{P(a|b)P(b)}{P(a)} = \frac{.94 \times .001}{.0025} = .376$$

$$P(a|j) = \frac{P(j|a)P(a)}{P(j)}$$

$$= \frac{.90 \times .0025}{P(j)}$$

$P(j)$ same as $P(j, A)$ sum out over A

$$= P(j|a)P(a) + P(j|\neg a)P(\neg a)$$

$$= (.90 \times .0025) + (.05 \times .9975)$$

$$= .0521$$

$$P(a|j) = \frac{.90 \times .0025}{.0521} \approx .0432$$

A more complicated problem:

$P(b|j)$: Probability of a burglary given that John has called.

$P(b|j)$ calculated using:

$$\frac{P(j|b)P(b)}{P(j)}$$

Notice that Mary not included $P(j)$ and $P(m)$ are independent.

$\Rightarrow$

All entries are:

1. $P(j|a)P(a|b,e)P(b)P(e)$
2. $P(j|a)P(a|b,\neg e)P(b)P(\neg e)$
3. $P(j|a)P(a|\neg b,e)P(\neg b)P(e)$
4. $P(j|a)P(a|\neg b,\neg e)P(\neg b)P(\neg e)$
5. $P(j|\neg a)P(\neg a|b,e)P(b)P(e)$
6. $P(j|\neg a)P(\neg a|b,\neg e)P(b)P(\neg e)$
7. $P(j|\neg a)P(\neg a|\neg b,e)P(\neg b)P(e)$
8. $P(j|\neg a)P(\neg a|\neg b,\neg e)P(\neg b)P(\neg e)$

Entries where b = true
1, 2, 5, 6

$$P(j|b) = \frac{P(j,b,A,E)}{P(b,A,E)}$$

alarm is true or false
needs to also include
Earthquake, t or f.

$$= \frac{\begin{array}{l}P(j|a)P(a|b,e)P(b)P(e) + \\ P(j|a)P(a|b,\neg e)P(b)P(\neg e) + \\ P(j|\neg a)P(\neg a|b,e)P(b)P(e) + \\ P(j|\neg a)P(\neg a|b,\neg e)P(b)P(\neg e)\end{array}}{\begin{array}{l}P(a|b,e)P(b)P(e) + P(a|b,\neg e)P(b)P(\neg e) + \\ P(\neg a|b,e)P(b)P(e) + P(\neg a|b,\neg e)P(b)P(\neg e)\end{array}}$$

$$= \frac{\begin{array}{l}(.90 \times .95 \times .001 \times .002) + \\ (.90 \times .94 \times .001 \times .998) + \\ (.05 \times .05 \times .001 \times .002) + \\ (.05 \times .06 \times .001 \times .998)\end{array}}{\begin{array}{l}(.95 \times .001 \times .002) + (.94 \times .001 \times .998) + \\ (.05 \times .001 \times .002) + (.06 \times .001 \times .998)\end{array}}$$

$$= .849 = P(j \mid b)$$

$$P(b \mid j) = \frac{P(j \mid b)\, P(b)}{P(j)}$$

$$= \frac{.849 \times .001}{.0521} = .0163$$