

d-separation

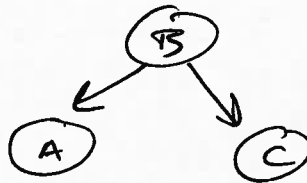
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We've been looking at examples (types) of conditional independence.

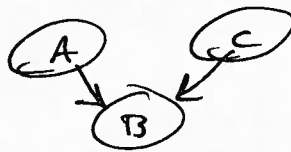
Causal chain



Common cause



Common effect



The conditional independence, independence, and dependence between nodes, or sets of nodes in a network are formalized with concept of d-separation. Extends to sets of nodes as well.

Start w/ definitions

Path: A path between two sets of nodes

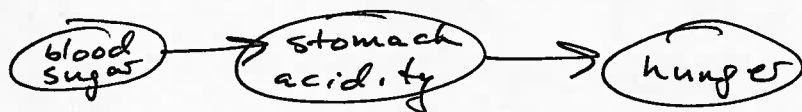
X and Y is any sequence of nodes between a member of X and a member of Y s.t. every adjacent pair of nodes is ~~to be~~ connected by an arc, and no node appears twice.

Blocked Path : A path is blocked, (2)
given a set of nodes E , if there is a node Z on the path for which at least one of the three conditions holds:

1. Z is in E and Z has one arc on the path leading in and one arc leading out



ex:



there is a dependency between these nodes, low blood sugar causes (triggers) generating stomach acid, which triggers hunger sensation.

Assume stomach acid not measured, you only have blood sugar measurement and hunger awareness.

Active path: Between blood sugar and hunger. w/o stomach acid knowledge. Path carries information, there is a dependence between blood sugar and hunger.

Measure stomach acid. Now, path is now blocked by stomach acidity. Our belief about hunger requires

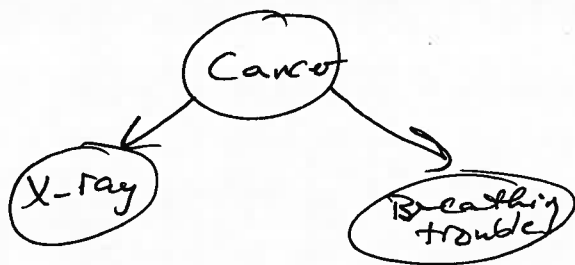
only knowledge of middle Node, knowledge of blood sugar will not change our belief about hunger.

Inactive Path : Path is blocked

d-separation is dependence separation
blood sugar and hunger are d-separated because they are conditional independent on stomach acidity. All paths between them need to be inactive.

Nodes X and Y are d-separated if all paths that connect them are inactive and $X \perp\!\!\!\perp Y \mid E$.

2. Z is in E and Z has both path arcs leading out (common cause)



There is dependence between x-ray and breathing w/o knowledge of cancer. It's not causal, we don't say that positive x-ray causes breathing difficulty, but knowing that patient has

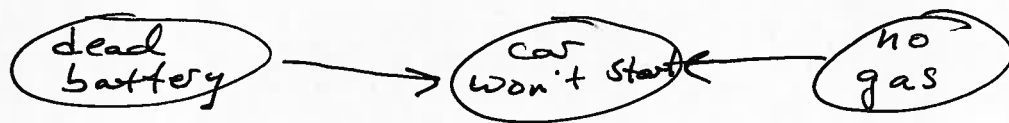
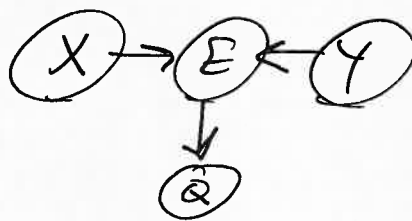
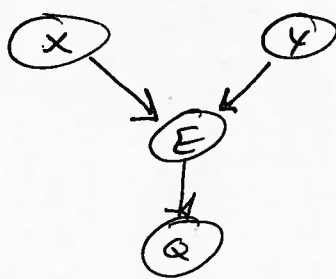
had positive X-ray will influence your (4) belief about their breathing trouble.

Observe that patient has cancer, this blocks the dependence between X-ray and breathing.

Once you observe cancer, observing X-ray adds no new information about breathing trouble.

3. Neither Z nor any of its descendants of Z is in E , and both paths lead in to Z . (common effect)

This one is different than the first two



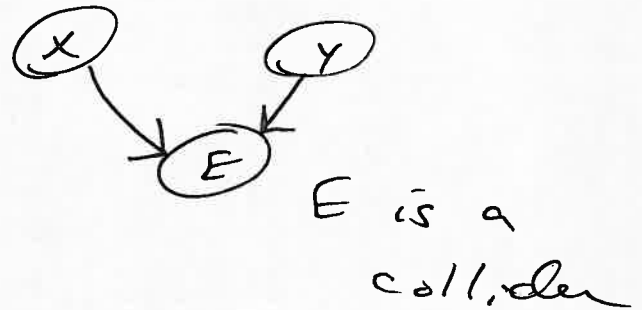
Initially, there is no dependence relation between dead battery and no gas.

Car won't start is our evidence.

The path between dead battery and no gas is inactive due to collider node.

(5)

Collider: arcs from variables that influence it collide, blocking association between the variables.



What happens is relation between dead battery and no gas when we observe car won't start.

Knowing battery is charged tells you nothing about no gas.

Knowing battery is charged and observing car won't start tells you car has no gas.

Observing E (car won't start) activates the dependence between X and Y (dead battery and no gas)

Also true for descendants of E

Observe g also activates dependence between X and Y.

