Solid Mechanics 2 Tutorial Sheets

Tutorial Sheets and Answers for DE2's Enjoyment

Tutorial Sheet 1: Planar Kinematics Answers

Topics covered are:

- Types of motion
- Rotation around a fixed axis
- Relative velocity
- Instantaneous centres

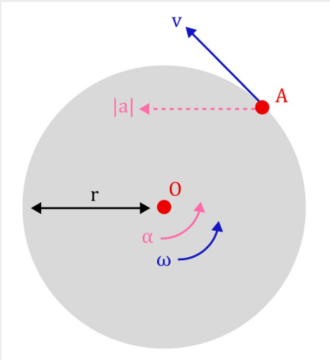
Tips

- Always draw the situation!
- The order of the cross product matters, $\omega \times r \neq r \times \omega$

Question 1

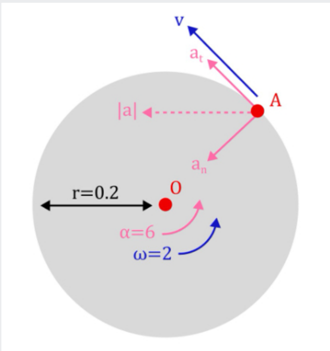
At the instant shown, the disk has angular velocity is 2 rad/s counter clockwise and angular acceleration 6 rad/s ^{2}. Its radius is 0.2 m.

What are the magnitudes of the velocity and acceleration of point A?



Answer

It is useful to sketch the situation



Using the basic equations of motion and subbing in values, velocity can be calculated

$$\begin{aligned} v &= r\omega = 0.2 \times 2 \\ &= 0.4m/s \end{aligned}$$

The magnitude of acceleration can be calculated using the normal and tangential components, then using basic pythagoras to find the magnitude

$$\begin{aligned} a_n &= r\omega^2 = 0.2 \times 2^2 \\ &= 0.8m/s^2 \end{aligned}$$

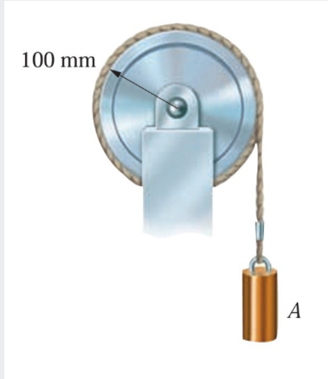
$$\begin{aligned} a_t &= r\alpha = 0.2 \times 6 \\ &= 1.2m/s^2 \end{aligned}$$

$$\begin{aligned} |a| &= \sqrt{a_n^2 + a_t^2} \\ &= \sqrt{0.8^2 + 1.2^2} \\ &= 1.44m/s^2 \end{aligned}$$

Question 2

The mass A starts from rest at t=0 and falls with a constant acceleration of 8 m/s ^{2}. When the mass has fallen one meter, determine the magnitudes of:

(a) The angular velocity of the pulley.
(b) The tangential and normal components of acceleration of a point at the outer edge of the pulley.



Answer

(a) a=8, u=0 (starting from rest), s=1, calculate v in order to work out everything else. Using SUVAT

$$\begin{aligned} v^2 &= a^2 + 2as \\ v &= \sqrt{0^2 + 2 \times 8 \times 1} \\ &= 4m/s \end{aligned}$$

Use this with the given equations

$$\begin{aligned} v &= r\omega \\ 4 &= 0.1 \times \omega \\ \omega &= 40rad/s \end{aligned}$$

(b) Using the given equations and v from before

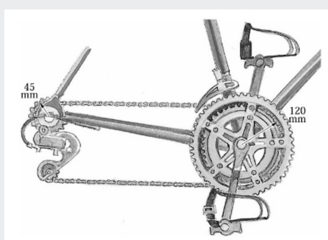
$$\begin{aligned} a_n &= \frac{v^2}{r} = \frac{4^2}{0.1} \\ &= 160m/s^2 \end{aligned}$$

We need alpha to find tangential accel

$$\begin{aligned} \alpha &= \frac{a}{r} = \frac{8}{0.1} = 80rad/s^2 \\ a_t &= r\alpha = 0.1 \times 80 \\ &= 8m/s^2 \end{aligned}$$

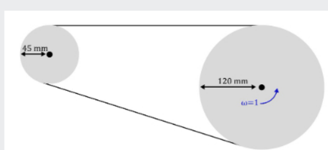
Question 3

(a) If the bicycle's 120 mm radius sprocket wheel rotates through one revolution, through how many revolutions does the 45 mm gear turn?
(b) If the angular velocity of the sprocket wheel is 1 rad/s, what is the angular velocity of the gear?



Answer

Sketch the situation so it's a bit more comprehensible.



(a) As the gears are attached by the chain, when the large sprocket wheel rotates by a certain length, the small one must also pass that length through. Hence we can simply use ratios.

Circumference of wheel

$$\begin{aligned} l &= \pi d = \pi \times 0.12 \times 2 \\ &= 0.754 \text{ m} \end{aligned}$$

'Length' passed through by gear

$$\begin{aligned} 0.754 &= x \times \pi \times 2 \times 0.045 \\ x &= 2.67 \text{ revs} \end{aligned}$$

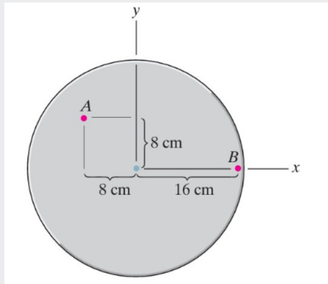
(b) The chain must be moving at a constant rate (velocity), therefore

$$\begin{aligned} v_s &= v_g \\ r_s\omega_s &= r_g\omega_g \end{aligned}$$

$$0.12 \times 1 = 0.045 \times \omega_g$$
$$\omega_g = 2.67rad/s$$

Question 4

The disk is rotating about the origin with a constant clockwise angular velocity of 100 rpm. Determine the x and y components of velocity of points A and B (in cm/s).



Answer

First, convert rpm to rad/s, or all the calculations will be messed up. You can do this manually - or just plug it into your calculator!

$$\frac{100}{60}.2\pi = 10.47rad/s = \omega$$

Then using the basic $v = \omega r$ equation

Point A:

$$v_x = 10.47 \times 8 = 83.77$$
$$v_y = 10.47 \times 8 = 83.77$$

In vector form

$v_A = 83.77i + 83.77j$ cm/s

Point B:

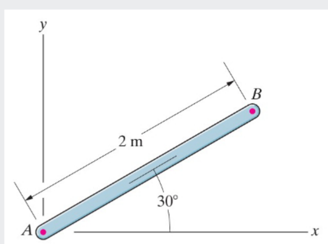
$$v_x = 10.47 \times 16 = 167.5$$
$$v_y = 10.47 \times 0 = 0$$

In vector form $v_B = 167.55j$ cm/s

Question 5

The bar is moving in the x–y plane and is rotating in the counterclockwise direction. The magnitude of the velocity of point A relative to point B is 8 m/s. Relative to a nonrotating referenece frame with origin A, what is the

(a) Angular velocity of the bar.
(b) Velocity of B relative to the reference frame in vector form.

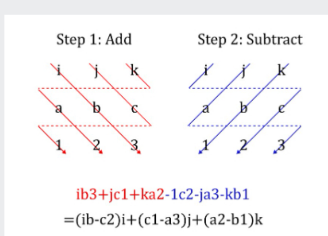


Answer

(a)

$$\omega = \frac{v}{r} = \frac{8}{2}$$
$$= 4rad/s$$

(b) As we are dealing with vectors, we need to use the cross product. This is really important from now on so make sure you are comfortable with calculating this. I recommend the method below, but whatever works for you!



To calculate velocity of B

$$v_B = \omega \times r$$
$$= 4k \times 2(cos(30)i + sin(30)j) = \begin{vmatrix} i & j & k \\ 0 & 0 & 4 \end{vmatrix}$$

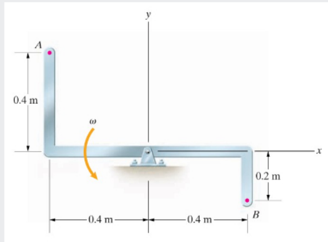
$$\left| 2cos(30) \quad 2sin(30) \quad 0 \right|$$

Using my cross product method

$$0 \times 0i - 4 \times 2sin(30)i + 4 \times 2cos(30)j - 0 \times 0j + 0 \times 2sin(30)i - 0 \times 2cos(30k)$$
$$= -4i + 6.93j$$

Question 6

The bar is rotating in the counterclockwise direction with angular velocity ω. The magnitude of the velocity of point A relative to point B is 6 m/s. Determine the velocity of point B (relative to the origin).



Answer

The distance B from A can be simply found

$$r_{A/B} = \sqrt{(0.4+0.4)^2 + (0.4+0.2)^2}$$

The angular velocity of the bar is constant through the whole bar

$$v_{A/B} = \omega . r_{A/B}$$
$$\omega = \frac{6}{1} = 6rad/s$$

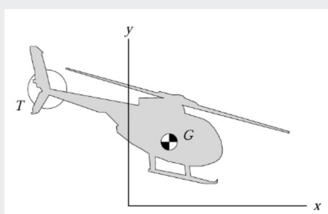
From there, velocity of B can be calculated as normal

$$v_B = \omega \times r_B = 6k \times (0.4i - 0.2j)$$
$$v_B = 1.2i + 2.4j \text{ m/s}$$

Remember to use the cross product!

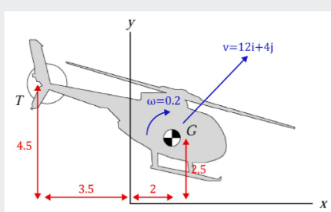
Question 7

The helicopter is in planar motion in the x–y plane. At the instant shown, the position of its center of mass, G, is x=2m, y=2.5m, and its velocity is $v_G = 12i + 4j$ (m/s). The position of point T, where the tail rotor is mounted, is x= −3.5m, y=4.5m. The helicopter's angular velocity is 0.2 rad/s clockwise. What is the velocity of point T?



Answer

Draw the situation



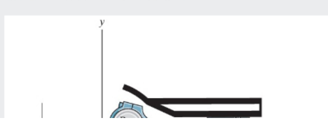
We currently only know the velocity of G. To find the velocity of T, we need to find the velocity of T from G, and add the velocity of G. See it as velocity 'origin to G + G to T'.

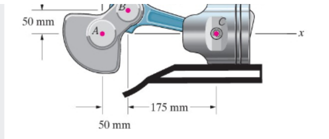
$$r_{T/G} = (-3.5-2)i + (4.5-2.5)j = -5.5i + 2j \text{ m}$$

$$v_T = v_G + \omega \times r_{T/G}$$
$$= 12i + 4j + \begin{vmatrix} i & j & k \\ 0 & 0 & -0.2 \\ -5.5 & 2 & 0 \end{vmatrix}$$
$$= 12.4i + 5.1j \text{ m/s}$$

Question 8

At the instant shown, the piston's velocity is $v_C = -14i$ m/s. What is the angular velocity of the crank AB, which rotates around A?





Answer

Looking at the mechanism, we can see that C is limited to only i. Also knowing that A is static, we can set up some equations.

$$\begin{gathered} v_B = v_A + \omega_{BA} \times r_{BA} \\ 0 + \omega_{BA} k \times (0.05i + 0.05j) \\ = -0.05\omega_{BA} i + 0.05\omega_{BA} j \end{gathered}$$

$$\begin{gathered} v_B = v_C + \omega_{BC} \times r_{BC} \\ -14i + \omega_{BC} k \times (0.175i - 0.05j) \\ = -14i - 0.05\omega_{BC} i - 0.175\omega_{BC} j \end{gathered}$$

Then analyse looking at each component of velocity

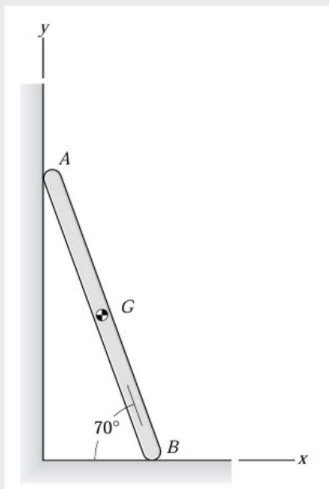
$$\begin{gathered} (i) - 0.05\omega_{BA} + 0.05\omega_{BC} = -14 \\ (j) 0.05\omega_{BA} = -0.175\omega_{BC} \end{gathered}$$

And using simultaneous equations

$$\omega_{BA} = 218 \text{ rad/s}$$

Question 9

Points A and B of the 2 m bar slide on the plane surfaces. Point B is moving to the right at 3 m/s. What is the velocity of the midpoint G of the bar?



Answer

Take advantage of the constraints from the floor and wall to solve this; the velocity of A has only j component, the velocity of B has only i.

$$v_A = v_B + \omega \times r_{A/B}$$

$$v_A j = 3i + \begin{vmatrix} i & j & k \\ 0 & 0 & \omega \\ -2\cos(70) & 2\sin(70) & 0 \end{vmatrix}$$

$$v_A = -1.88\omega i - 0.7\omega j + 3i$$

Now equate i and j components

$$\begin{gathered} (i) 0 = -1.88\omega + 3 \rightarrow \omega = 1.6 \\ (j) v_A = -0.7\omega \end{gathered}$$

Now work out the velocity of G. You can do this from point B or point A (using the above relation) but given point B's velocity is given in the question, I'd advise you to go from point B just in case you make a mistake.

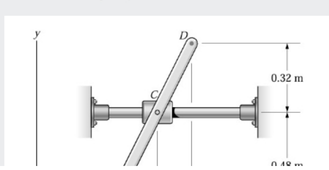
$$v_G = v_B + \omega \times r_{G/B}$$

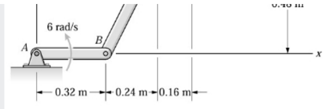
$$v_G = 3i + \begin{vmatrix} i & j & k \\ 0 & 0 & 1.6 \\ -\cos(70) & \sin(70) & 0 \end{vmatrix}$$

$$= 1.5i - 0.547j \text{ m/s}$$

Question 10

Bar AB rotates in the counterclockwise direction at 6 rad/s. Determine the angular velocity of bar BD and the velocity of point D.





Answer

Looking at the mechanism we can determine some key constraints that will help us solve this. A is a fixed centre of rotation so has zero velocity. C has only an i component.

First calculate v_B from point A

$$\begin{aligned} v_B &= v_A + \omega_{AB} \times r_{B/A} \\ &= 0 + 6k \times 0.32i \\ &= 1.92j \end{aligned}$$

We can then need to calculate ω_{BD} (which is the same as ω_{BC}). We can do this using the calculated v_B and the constraints we know about C

$$\begin{aligned} v_C &= v_B + \omega_{BC} \times r_{C/B} \\ &= 0.48\omega_{BC} i + 0.24\omega_{BC} j + 1.92j \end{aligned}$$

Now analyse components

$$\begin{aligned} (i) v_C &= 0.48\omega_{BC} \\ (j) 0 &= 1.92 + 0.24\omega_{BC} \end{aligned}$$

Solving

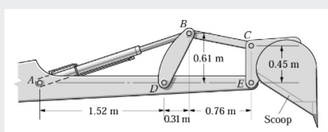
$$\begin{aligned} \omega_{BC} &= -8k \\ v_C &= 3.84i \end{aligned}$$

Now to calculate velocity of D

$$\begin{aligned} v_D &= v_B + \omega_{BD} \times r_{D/B} \\ &= 1.92j + \begin{vmatrix} i & j & k \\ 0 & 0 & -8 \\ 0.4 & 0.8 & 0 \end{vmatrix} \\ v_D &= 6.4i - 1.28j \end{aligned}$$

Question 11

The horizontal member ADE supporting the scoop is stationary. If the link BD is rotating in the clockwise direction at 1 rad/s,what is the angular velocity of the scoop?



Answer

This question relies on calculating a lot of velocities relative to each other, so be careful with carrying over errors!

$$\begin{aligned} v_B &= v_D + \omega_{BD} \times r_{B/D} \\ &= 0 + \begin{vmatrix} i & j & k \\ 0 & 0 & -1 \\ 0.31 & 0.61 & 0 \end{vmatrix} = 0.61i - 0.31j \end{aligned}$$

Then if we write multiple expressions for v_C we can make simultaneous equations to solve the question

$$\begin{aligned} v_C &= v_B + \omega_{BC} \times r_{C/B} \\ 0.61i - 0.31j &+ \begin{vmatrix} i & j & k \\ 0 & 0 & \omega_{BC} \\ 0.76 & -0.15 & 0 \end{vmatrix} \\ &= 0.61i - 0.31j + 0.15\omega_{BC} i + 0.76\omega_{BC} j \\ v_C &= v_E + \omega_{CE} \times r_{C/E} \\ 0 &+ \begin{vmatrix} i & j & k \\ 0 & 0 & \omega_{CE} \\ 0 & 0.46 & 0 \end{vmatrix} \\ &= -0.46\omega_{CE} i \end{aligned}$$

Equating v_C expressions in terms of i and j components

$$\begin{aligned} (i) - 0.46\omega_{CE} &= 0.61 + 0.15\omega_{BC} \\ (j) 0 &= 0.31 + 0.76\omega_{BC} \end{aligned}$$

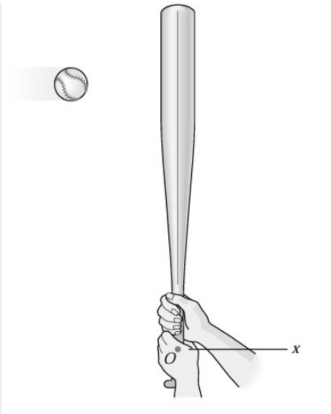
Hence

$$\begin{aligned} \omega_{BC} &= 0.4 \text{ rad/s} \\ \omega_{CE} &= -1.47 \text{ rad/s} \end{aligned}$$

Where CE is the scoop!

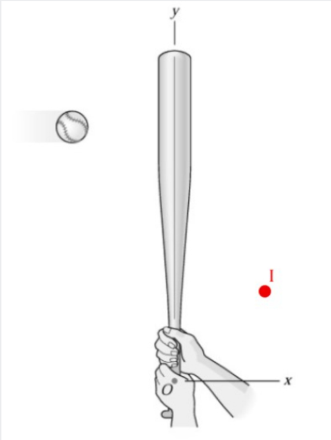
Question 12

The velocity of point O of the bat is v_O =−1.83i− 4.27j m/s, and the bat rotates about the z axis with a counterclockwise angular velocity of 4 rad/s. What are the x and y coordinates of the bat's instantaneous center?



Answer

Arbitrarily place the instentaneous center. It doesn't really matter where - as long as you have signs right the maths will correct itself.



Say the coordinates of the instantaneous center is (x_I, y_I), you then work it all out as O from I (as I has zero velocity)

$$v_O = -1.83i - 4.27j = \omega \times r_{O/I} = \begin{vmatrix} i & j & k \\ 0 & 0 & 4 \\ -x_I & -y_I & 0 \end{vmatrix} = 4y_I i - 4x_I j$$

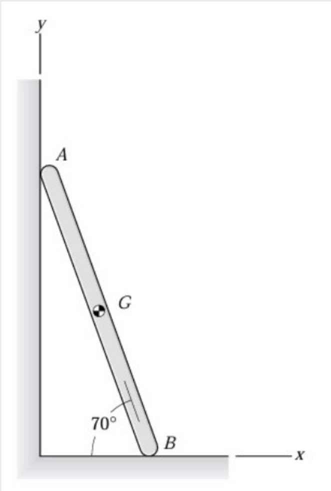
Equate terms to find the coordinates

$$(i) - 1.83 = 4y_C \rightarrow y_C = -0.46$$
$$(j) - 4.27 = -4x_C \rightarrow x_C = 1.07$$
$$(1.07, -0.46)\ \mathrm{m}$$

Question 13

Points A and B of the 1m bar slide on the plane surfaces. The velocity of B is v_B = 2i m/s.

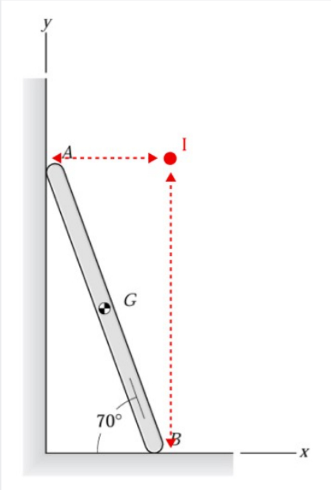
(a) What are the coordinates of the instantaneous center of the bar?
(b) Use the instantaneous center to determine the velocity at A.



Answer

Just as in Q9, the bar is constrained A in j, B in i.

(a) Work this out using geometry. Draw perpendiculars to the velocity vectors at A and B.



$$\begin{aligned}&(\sin(20), \cos(20))\\&= (0.34, 0.94)\end{aligned}$$

(b) Find the angular velocity of the bar

$$\begin{aligned}v_B = 2i &= v_I + \omega \times r_{O/I}\\2 = 0.94\omega &\to \omega = 2.13\ \mathrm{rad/s}\end{aligned}$$

Then the velocity of A from the instantaneous center

$$\begin{aligned}v_A &= v_I + \omega \times r_{A/I}\\&= 0 + 2.13k \times -0.34i\\&= -0.23j\ \mathrm{m/s}\end{aligned}$$