Solid Mechanics 2 Tutorial Sheets

Tutorial Sheets and Answers for DE2's Enjoyment

Tutorial Sheet 5: 3D Kinematics Answers

Topics covered are:

- 3D velocity and acceleration
- Reference frames

Tips

- Cross products will get much larger - the order of calculation matters, and you will need to use the full version of the acceleration equations.
- Mechanisms are now in 3D, so practice visualisiing the actual movements to avoid mistakes.
- Some will like to use column vectors, write it how you feel works best for you!

Question 1

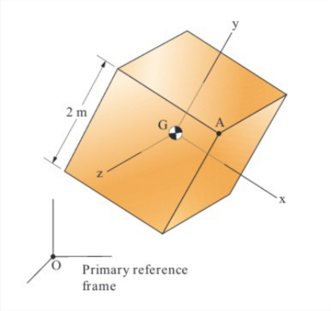
The aeroplane's angular velocity vector relative to an earth-fixed reference frame, expressed in terms of the body-fixed coordinate system shown, is ω = 0.62i+0.45j−0.23k rad/s. The coordinates of point A of the airplane are (3.6, 0.8, −1.2) m. What is the velocity of point A relative to the velocity of the aeroplane's center of mass?

Answer

$$v_A = v_O + \omega \times r_{A/O}$$
$$= 0 + \begin{vmatrix} i & j & k \\ 0.62 & 0.45 & -0.23 \\ 3.6 & 0.8 & -1.2 \end{vmatrix}$$
$$= -0.356i - 0.084j - 1.12k \text{ m/s}$$

Question 2

The angular velocity of the cube relative to the primary reference frame, expressed in terms of the body-fixed coordinate system shown is ω=−6.4i+8.2j+12k rad/s.The velocity of the center of mass G of the cube relative to the primary reference frame at the instant shown is v_G=26i+14j+32k m/s. What is the velocity of point A of the cube relative to the primary reference frame at the instant shown?



Answer

The vector A to G is

$$i + j + k$$

Then

$$v_A = v_G + \omega \times r_{A/G}$$
$$v_A = 26i + 14j + 32k + \begin{vmatrix} i & j & k \\ -6.4 & 8.2 & 12 \\ 1 & 1 & 1 \end{vmatrix}$$

$$v_A = 22.2i + 32.7j + 17.4k \text{ m/s}$$

Question 3

Using the cube in Q2, the coordinate system shown is fixed with respect to the cube. The angular velocity of the cube relative to the primary reference frame, ω=−6.4i+8.2j+12k rad/s, is constant.The acceleration of the center of mass G of the cube relative to the primary reference frame at the instant shown is a_G=136i+76j−48k m/s ^{2}. What is the acceleration of point A of the cube relative to the primary reference frame at the instant shown?

Answer

There is constant angular velocity so alpha is 0. The acceleration can be found

$$a_A = a_G + \alpha \times r_{A/G} + \omega \times (\omega \times r_{A/G})$$

$$a_A = (136i + 76j - 48k) + 0 + (-6.4i + 8.2j + 12k) \times \begin{vmatrix} i & j & k \\ -6.4 & 8.2 & 12 \\ 1 & 1 & 1 \end{vmatrix}$$

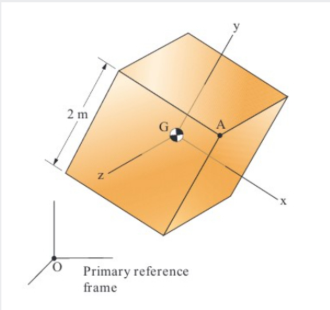
$$a_A = 136i + 76j - 48k + \begin{vmatrix} i & j & k \\ -6.4 & 8.2 & 12 \\ -3.8 & 18.4 & -14.6 \end{vmatrix}$$

$$v_A = -204.5i - 63.04j - 125k \text{ m/s}$$

Question 4

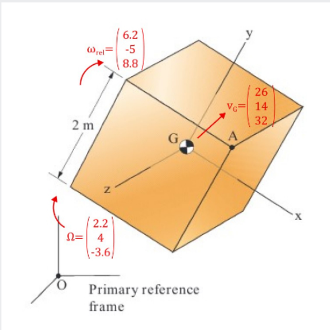
The origin of the secondary coordinate system shown is fixed to the center of mass G of the cube. The velocity of the center of mass G of the cube relative to the primary reference frame at the instant shown is v_G=26i+14j+32 m/s. The cube is rotating relative to the secondary coordinate system with angular velocity ω_{rel} =6.2i−5j+8.8k rad/s. The secondary coordinate system is rotating relative to the primary reference frame with angular velocity 2.2i+4j−3.6k rad/s.

(a) What is the velocity of point A of the cube relative to the primary reference frame at the instant shown?
(b) If the components of the vectors ω_{rel} and are constant, what is the cube's angular acceleration relative to the primary reference frame?



Answer

There is a lot going on here, so annotate



(a)

$$\omega = \Omega + \omega_{rel}$$

$$\omega = (22i + 4j - 3.6k) + (6.2i - 5j + 8.8k)$$

$$\omega = 8.4i - j + 5.2k$$

$$v_A = v_G + \omega \times r_{A/G}$$

$$v_A = 26i + 14j + 32k + \begin{vmatrix} i & j & k \\ 8.4 & -1 & 5.2 \\ 1 & 1 & 1 \end{vmatrix}$$

$$v_A = 19.8i + 10.8j + 41.4k \text{ m/s}$$

(b)

$$\alpha = \frac{d\omega}{dt} + \Omega \times \omega_{rel}$$

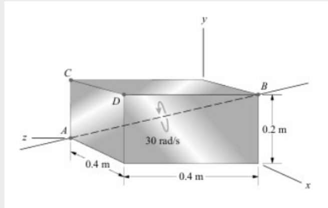
$$\alpha = 0 + \begin{vmatrix} i & j & k \\ 2.2 & 4 & -3.6 \\ 6.2 & -5 & 8.8 \end{vmatrix}$$

$$\alpha = 17.2i - 41.7j - 35.8k \text{ rad/s}^2$$

Question 5

Question 5

Relative to an earth-fixed reference frame, points A and B of the rigid parallelepiped are fixed and it rotates about the axis AB with an angular velocity of 30 rad/s. Determine the velocities of points C and D relative to the earth-fixed reference frame.



Answer

We need to know the vector ω as we only know the overall magnitude. This can be found using the unit vector of the axis of rotation.

$$(30)\frac{0.4i+0.2j-0.4k}{\sqrt{0.4^2+0.2^2+0.4^2}}$$
$$=20i+10j-20k$$

Then solving for velocity of C and D

$$v_C = v_A + \omega \times r_{C/A}$$
$$v_C = 0 + \begin{vmatrix} i & j & k \\ 20 & 10 & -20 \\ 0 & 0.2 & 0 \end{vmatrix}$$
$$v_C = 4i + 4k$$
$$v_D = v_A + \omega \times r_{C/A}$$
$$v_D = 0 + \begin{vmatrix} i & j & k \\ 20 & 10 & -20 \\ 0.4 & 0.2 & 0 \end{vmatrix}$$
$$v_D = 4i - 8j$$

Question 6

Using the parallelepiped in Q5, relative to the xyz coordinate system shown, points A and B of the rigid parallelepiped are fixed and the parallelepiped rotates about the axis AB with an angular velocity of 30 rad/s. Relative to an earth fixed reference frame, point A is fixed and the xyz coordinate system rotates with angular velocity −5i+8j+6k rad/s. Determine the velocities of points C and D relative to the earth-fixed reference frame.

Answer

Again, we need to find ω, but this time it becomes ω_{rel} as the coordinate system is rotating as well.

$$(30)\frac{0.4i+0.2j-0.4k}{\sqrt{0.4^2+0.2^2+0.4^2}}$$
$$\omega_{rel}=20i+10j-20k$$

ω is then

$$\omega = \Omega + \omega_{rel}$$
$$\omega = (-5i+8j+6k)+(20i+10j-20k)$$
$$\omega = 15i+18j-14k$$

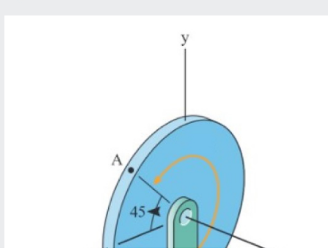
Then solving for velocity of C and D

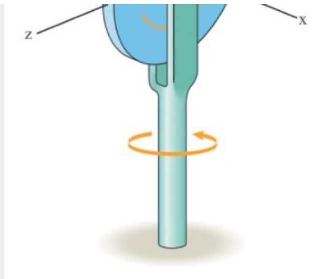
$$v_C = v_A + \omega \times r_{C/A}$$
$$v_C = 0 + \begin{vmatrix} i & j & k \\ 15 & 18 & -14 \\ 0 & 0.2 & 0 \end{vmatrix}$$
$$v_C = 2.8i + 3k$$
$$v_D = v_A + \omega \times r_{C/A}$$
$$v_D = 0 + \begin{vmatrix} i & j & k \\ 15 & 18 & -14 \\ 0.4 & 0.2 & 0 \end{vmatrix}$$
$$v_D = 2.8i - 5.6j - 4.2k$$

Question 7

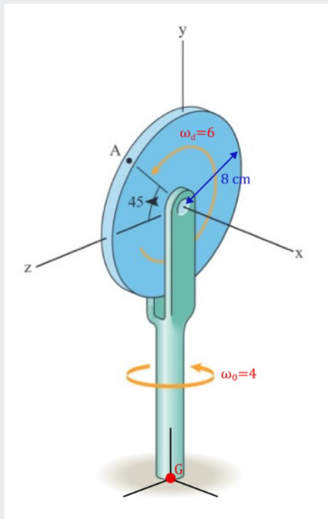
Relative to an earth-fixed reference frame, the vertical shaft rotates about its axis with angular velocity ω_O=4 rad/s. The secondary xyz coordinate system is fixed with respect to the shaft and its origin is stationary. Relative to the secondary coordinate system, the disk (radius 8 cm) rotates with constant angular velocity ω_d=6 rad/s. At the instant shown, determine the velocity of point A

(a) Relative to the secondary reference frame.
(b) Relative to the earth-fixed reference frame.





Answer



(a)

$$v_A = v_O + \omega_{rel} \times r_{A/O}$$

$$v_A = 0 + \begin{vmatrix} i & j & k \\ 6 & 0 & 0 \\ 0 & 8\sin(45) & 8\cos(45) \end{vmatrix}$$

$$v_A = \mathbf{-33.9}j + \mathbf{33.9}k \text{ cm/s}$$

(b)

$$\omega = \Omega + \omega_{rel}$$

$$\omega = 4j + 6i$$

$$v_A = v_O + \omega_{rel} \times r_{A/G}$$

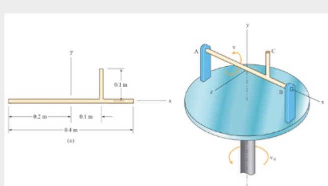
$$v_A = 0 + \begin{vmatrix} i & j & k \\ 6 & 4 & 0 \\ 0 & 8\sin(45) & 8\cos(45) \end{vmatrix}$$

$$v_A = \mathbf{-33.9}j + \mathbf{33.9}k \text{ cm/s}$$

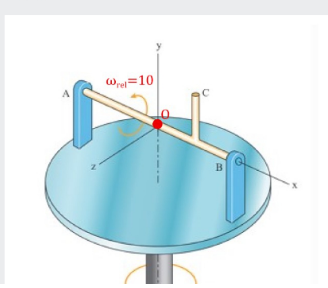
Question 8

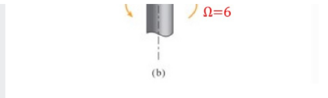
The object in figure (a) is supported by bearings at A and B in figure (b). The horizontal circular disk is supported by a vertical shaft that rotates with angular velocity ω_O=6 rad/s. The horizontal bar rotates with angular velocity ω=10 rad/s. At the instant shown,

(a) What is the velocity relative to an earth-fixed reference frame of the end C of the vertical bar?
(b) What is the angular acceleration vector of the object relative to an earth-fixed reference frame?
(c) What is the acceleration relative to an earth-fixed reference frame of the end C of the vertical bar?



Answer





(a)

$$\omega = \Omega + \omega_{rel}$$
$$\omega = 6j + 10i$$

$$v_C = v_O + \omega \times r_{C/O}$$
$$v_C = 0 + \begin{vmatrix} i & j & k \\ 10 & 6 & 0 \\ 0.1 & 0.1 & 0 \end{vmatrix}$$
$$v_C = 0.4k \text{ m/s}$$

(b)

$$\alpha = \frac{dw}{dt} + \Omega \times \omega$$
$$\alpha = 0 + \begin{vmatrix} i & j & k \\ 0 & 6 & 0 \\ 10 & 0 & 0 \end{vmatrix}$$
$$\alpha = -60k \text{ rad/s}^2$$

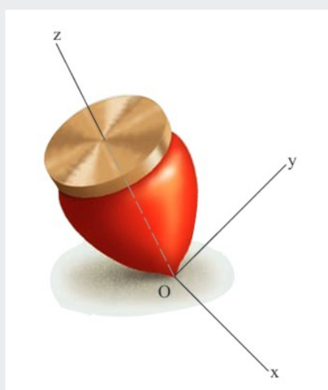
(c) We've worked out the last sum in part (a) so you can shortcut it

$$a_C = a_O + \alpha \times r_{C/O} + \omega \times (\omega \times r_{C/O})$$
$$a_C = a_O + \begin{vmatrix} i & j & k \\ 0 & 0 & -60 \\ 0.1 & 0.1 & 0 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 10 & 6 & 0 \\ 0 & 0 & 0.4 \end{vmatrix}$$
$$a_C = 8.4i - 10j \text{ m/s}^2$$

Question 9

The point of the spinning top remains at a fixed point on the floor, which is the origin O of the secondary reference frame shown. The top's angular velocity vector relative to the secondary reference frame, ω_{rel}=50k rad/s, is constant. The angular velocity vector of the secondary reference frame relative to an earth-fixed primary reference frame is ω=2j+5.6k rad/s. The components of this vector are constant. (Notice that it is expressed in terms of the secondary reference frame.)

(a) Determine the velocity relative to the earth-fixed reference frame of the point of the top with coordinates (0, 20, 30) mm.
(b) What is the top's angular acceleration vector relative to the earth-fixed reference frame
(c) Determine the acceleration relative to the earth fixed reference frame of the point of the top with coordinates (0, 20, 30) mm



Answer

(a)

$$\omega = \Omega + \omega_{rel}$$
$$\omega = 50k + 2j + 5.6k$$
$$\omega = 2j + 55.6k$$

$$v_A = v_O + \omega \times r_{A/O}$$
$$v_A = 0 + \begin{vmatrix} i & j & k \\ 0 & 2 & 55.6 \\ 0 & 0.02 & 0.03 \end{vmatrix}$$
$$v_A = -1.05i \text{ m/s}$$

(b)

$$\alpha = \frac{dw}{dt} + \Omega \times \omega$$
$$\alpha = 0 + \begin{vmatrix} i & j & k \\ 0 & 2 & 5.6 \\ 0 & 0 & 50 \end{vmatrix}$$
$$\alpha = 100i \text{ rad/s}^2$$

(c) Shortcut like in the previous question

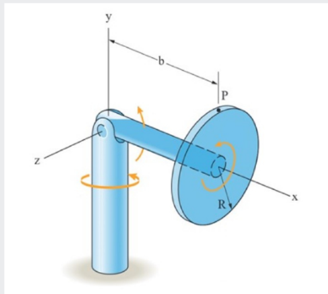
$$a_A = a_C + \alpha \times r_{A/C} + \omega \times (\omega \times r_{A/C})$$
$$a_A = 0 + \begin{vmatrix} i & j & k \\ 100 & 0 & 0 \\ 0 & 0.02 & 0.03 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 0 & 2 & 55.6 \\ 1.05 & 0 & 0 \end{vmatrix}$$

$$a_C = -61.4j + 4.1k \text{ m/s}^2$$

Question 10

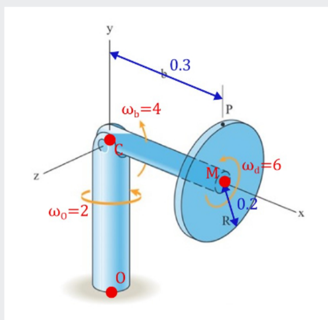
The radius of the circular disk is R=0.2 m, and b=0.3 m. The disk rotates with angular velocity ω_d=6 rad/s relative to the horizontal bar. The horizontal bar rotates with angular velocity ω_b=4 rad/s relative to the vertical shaft, and the vertical shaft rotates with angular velocity ω_O=2 rad/s relative to an earth-fixed reference frame. Assume that the secondary reference frame shown is fixed with respect to the horizontal bar.

(a) What is the angular velocity vector ω_{rel} of the disk relative to the secondary reference frame?
(b) Determine the velocity relative to the earth-fixed reference frame of point P, which is the uppermost point of the disk.



Answer

There's a lot of moving parts, so I'm going to label it so algebra symbols don't get mixed up.



(a) This is simply

$$\omega_{rel} = 6i \text{ rad/s}$$

(b) The angular velocity of the disk to the primary reference frame is

$$\omega = \omega_{rel} + \omega_{rel} + \Omega$$
$$\omega = 6i + 2j + 4k$$

The velocity of M (middle of the disk) is

$$v_M = v_O + \Omega \times r_{C/O}$$
$$v_M = 0 + \begin{vmatrix} i & j & k \\ 0 & 2 & 4 \\ 0.3 & 0 & 0 \end{vmatrix}$$
$$v_M = 1.2j - 0.6k$$

The velocity of P is

$$v_P = v_M + \omega \times r_{P/M}$$
$$v_P = 1.2j - 0.6k + \begin{vmatrix} i & j & k \\ 6 & 2 & 4 \\ 0 & 0.2 & 0 \end{vmatrix}$$
$$v_P = -0.8i + 1.2j + 0.6k \text{ m/s}$$