# Solid Mechanics 2 Tutorial Sheets

Tutorial Sheets and Answers for DE2's Enjoyment

## Tutorial Sheet 5: 3D Kinematics

**Topics covered are:**

- 3D velocity and acceleration
- Reference frames

**Tips**

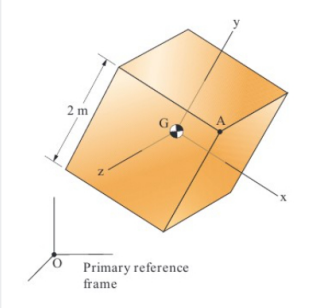
- Cross products will get much larger - the order of calculation matters, and you will need to use the full version of the acceleration equations.
- Mechanisms are now in 3D, so practice visualisiing the actual movements to avoid mistakes.
- Some will like to use column vectors, write it how you feel works best for you!

### Question 1

The aeroplane's angular velocity vector relative to an earth-fixed reference frame, expressed in terms of the body-fixed coordinate system shown, is $\omega$ = 0.62i+0.45j−0.23k rad/s. The coordinates of point A of the airplane are (3.6, 0.8, −1.2) m. What is the velocity of point A relative to the velocity of the aeroplane's center of mass?


### Question 2

The angular velocity of the cube relative to the primary reference frame, expressed in terms of the body-fixed coordinate system shown is $\omega$=−6.4i+8.2j+12k rad/s.The velocity of the center of mass G of the cube relative to the primary reference frame at the instant shown is $v_G$=26i+14j+32k m/s. What is the velocity of point A of the cube relative to the primary reference frame at the instant shown?



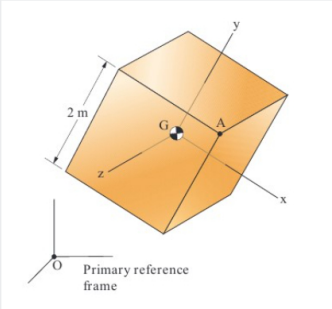

### Question 3

Using the cube in Q2, the coordinate system shown is fixed with respect to the cube. The angular velocity of the cube relative to the primary reference frame, $\omega$=−6.4i+8.2j+12k rad/s, is constant.The acceleration of the center of mass G of the cube relative to the primary reference frame at the instant shown is $a_G$=136i+76j−48k m/s $^2$. What is the acceleration of point A of the cube relative to the primary reference frame at the instant shown?

### Question 4

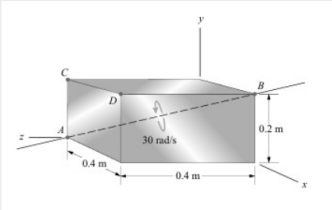
The origin of the secondary coordinate system shown is fixed to the center of mass G of the cube. The velocity of the center of mass G of the cube relative to the primary reference frame at the instant shown is $v_G$=26i+14j+32 m/s. The cube is rotating relative to the secondary coordinate system with angular velocity $\omega_{rel}$ =6.2i−5j+8.8k rad/s. The secondary coordinate system is rotating relative to the primary reference frame with angular velocity 2.2i+4j−3.6k rad/s.

**(a)** What is the velocity of point A of the cube relative to the primary reference frame at the instant shown?
**(b)** If the components of the vectors $\omega_{rel}$ and are constant, what is the cube's angular acceleration relative to the primary reference frame?




## Question 5

Relative to an earth-fixed reference frame, points A and B of the rigid parallelepiped are fixed and it rotates about the axis AB with an angular velocity of 30 rad/s. Determine the velocities of points C and D relative to the earth-fixed reference frame.



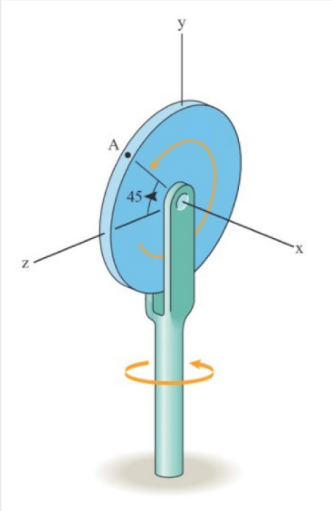

## Question 6

Using the parallelepiped in Q5, relative to the xyz coordinate system shown, points A and B of the rigid parallelepiped are fixed and the parallelepiped rotates about the axis AB with an angular velocity of 30 rad/s. Relative to an earth fixed reference frame, point A is fixed and the xyz coordinate system rotates with angular velocity −5i+8j+6k rad/s. Determine the velocities of points C and D relative to the earth-fixed reference frame.

## Question 7

Relative to an earth-fixed reference frame, the vertical shaft rotates about its axis with angular velocity $\omega_O$=4 rad/s. The secondary xyz coordinate system is fixed with respect to the shaft and its origin is stationary. Relative to the secondary coordinate system, the disk (radius 8 cm) rotates with constant angular velocity $\omega_d$=6 rad/s. At the instant shown, determine the velocity of point A

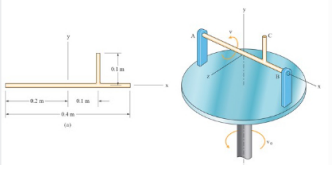
**(a)** Relative to the secondary reference frame.
**(b)** Relative to the earth-fixed reference frame.




## Question 8

The object in figure (a) is supported by bearings at A and B in figure (b). The horizontal circular disk is supported by a vertical shaft that rotates with angular velocity $\omega_O$=6 rad/s. The horizontal bar rotates with angular velocity $\omega$=10 rad/s. At the instant shown,

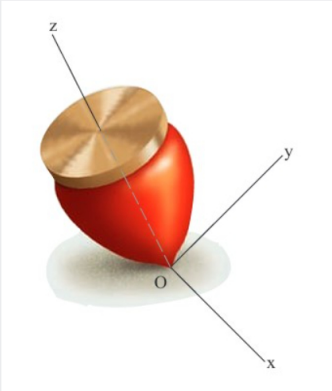
**(a)** What is the velocity relative to an earth-fixed reference frame of the end C of the vertical bar?
**(b)** What is the angular acceleration vector of the object relative to an earth-fixed reference frame?
**(c)** What is the acceleration relative to an earth-fixed reference frame of the end C of the vertical bar?



## Question 9

The point of the spinning top remains at a fixed point on the floor, which is the origin O of the secondary reference frame shown. The top's angular velocity vector relative to the secondary reference frame, $\omega_{rel}$=50k rad/s, is constant. The angular velocity vector of the secondary reference frame relative to an earth-fixed primary reference frame is $\omega$=2j+5.6k rad/s. The components of this vector are constant. (Notice that it is expressed in terms of the secondary reference frame.)

**(a)** Determine the velocity relative to the earth-fixed reference frame of the point of the top with coordinates (0, 20, 30) mm.
**(b)** What is the top's angular acceleration vector relative to the earth-fixed reference frame
**(c)** Determine the acceleration relative to the earth fixed reference frame of the point of the top with coordinates (0, 20, 30) mm




## Question 10

The radius of the circular disk is R=0.2 m, and b=0.3 m. The disk rotates with angular velocity $\omega_d$=6 rad/s relative to the horizontal bar. The horizontal bar rotates with angular velocity $\omega_b$=4 rad/s relative to the vertical shaft, and the vertical shaft rotates with angular velocity $\omega_O$=2 rad/s relative to an earth-fixed reference frame. Assume that the secondary reference frame shown is fixed with respect to the horizontal bar.

**(a)** What is the angular velocity vector $\omega_{rel}$ of the disk relative to the secondary reference frame?
**(b)** Determine the velocity relative to the earth-fixed reference frame of point P, which is the uppermost point of the disk.