Solid Mechanics 2 Tutorial Sheets

Tutorial Sheets and Answers for DE2's Enjoyment

Tutorial Sheet 6: 3D Dynamics

Topics covered are:

- Euler equations
- Constructing inertia matrices in 3D

Tips

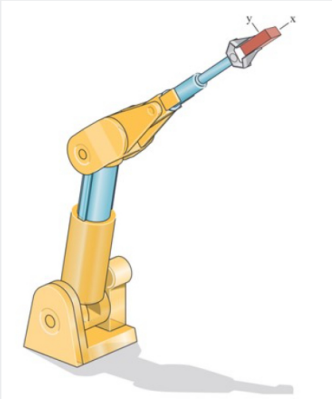
- Surprise, the equations are getting longer! I'd always have the Euler equation pulled up for reference so you can easily see what you need to calculate.
- The inertia matrices can get confusing if you need to construct them yourself. Be clear with directions and do more practice.
- Make sure you are clear with matrix manipulation.

Question 1

A robotic manipulator moves a casting. The inertia matrix of the casting in terms of a body-fixed coordinate system with its origin at the center of mass is

$$\begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{zy} & I_{zz} \end{bmatrix} = \begin{bmatrix} 0.05 & -0.03 & 0 \\ -0.03 & 0.08 & 0 \\ 0 & 0 & 0.04 \end{bmatrix} \text{kgm}^2$$

At the present instant, the angular velocity and angular acceleration of the casting are ω =1.2i+0.8j−0.4k rad/s and α=0.26i−0.07j+0.13k rad/s ^{2}. What moment is exerted about the center of mass of the casting by the manipulator?

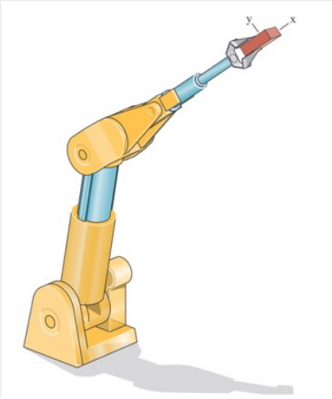


Question 2

A robotic manipulator holds a casting. The inertia matrix of the casting in terms of a body-fixed coordinate system with its origin at the center of mass is

$$\begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{zy} & I_{zz} \end{bmatrix} = \begin{bmatrix} 0.05 & -0.03 & 0 \\ -0.03 & 0.08 & 0 \\ 0 & 0 & 0.04 \end{bmatrix} \text{kgm}^2$$

At the present instant, the casting is stationary. If the manipulator exerts a moment $\sum M$= 0.042i+0.036j+0.066k Nm about the center of mass, what is the angular acceleration of the casting at that instant?

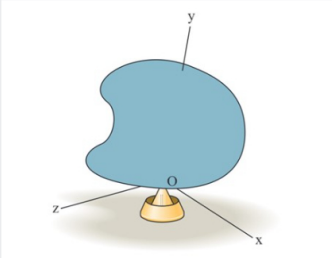


Question 3

The rigid body rotates about the fixed point O. Its inertia matrix in terms of the body-fixed coordinate system is

$$\begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{zy} & I_{zz} \end{bmatrix} = \begin{bmatrix} 4 & -2 & 0 \\ -2 & 3 & 1 \\ 0 & 1 & 5 \end{bmatrix} \text{kgm}^2$$

At the present instant, the rigid body's angular velocity is ω=6i+6j−4k rad/s and its angular acceleration is zero. What total moment about *O* is being exerted on the rigid body?

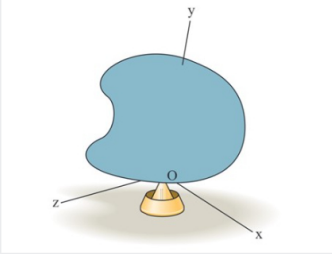


Question 4

The rigid body rotates about the fixed point O. Its inertia matrix in terms of the body-fixed coordinate system is

$$\begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{zy} & I_{zz} \end{bmatrix} = \begin{bmatrix} 4 & -2 & 0 \\ -2 & 3 & 1 \\ 0 & 1 & 5 \end{bmatrix} \text{kgm}^2$$

At the present instant, the rigid body's angular velocity is ω=6i+6j−4k rad/s. The total moment about O due to the forces and couples acting on the rigid body is zero. What is its angular acceleration?



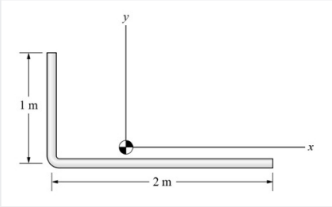
Question 5

In terms of the coordinate system shown, the inertia matrix of the 6-kg slender bar is

$$\begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{zy} & I_{zz} \end{bmatrix} = \begin{bmatrix} 0.5 & 0.667 & 0 \\ 0.667 & 2.667 & 0 \\ 0 & 0 & 3.167 \end{bmatrix} \text{kgm}^2$$

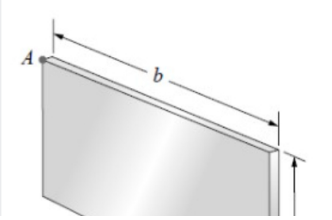
The bar is stationary relative to an inertial reference frame when the force F=12k N is applied at the right end of the bar. No other forces or couples act on the bar. Determine

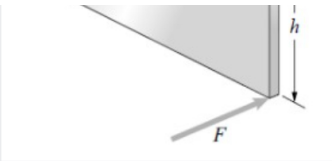
(a) The bar's angular acceleration relative to the inertial reference frame.
(b) The acceleration of the right end of the bar relative to the inertial reference frame at the instant the force is applied.



Question 6

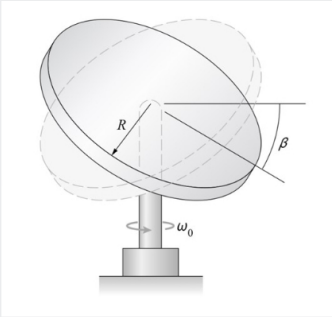
The dimensions of the 20 kg thin plate are h=0.4 m and b=0.6 m. The plate is stationary relative to an inertial reference frame when the force F=10 N is applied in the direction perpendicular to the plate. No other forces or couples act on the plate. At the instant F is applied, what is the magnitude of the acceleration of point A relative to the inertial reference frame?





Question 7

The thin circular disk of radius R=0.2 m and mass m=4 kg is rigidly attached to the vertical shaft. The plane of the disk is slanted at an angle β=30◦ relative to the horizontal. The shaft rotates with constant angular velocity ω_O=25 rad/s. Determine the magnitude of the couple exerted on the disk by the shaft.



Question 8

The vertical shaft rotates with constant angular velocity ω_O. The 35◦ angle between the edge of the 44.5 N thin rectangular plate pinned to the shaft and the shaft remains constant. Determine ω_O.