Accelerating Eulerian Fluid Simulation with CNN

2020.05.18 손서영

Before getting into it..

Navier Stokes equation

#zero viscosity

#imcompressible

#neglible air pressure

$$\frac{\partial u}{\partial t} = -u \cdot \nabla u - \frac{1}{\rho}\nabla p + f$$ → external force (Newton's law)

$$\nabla \cdot u = 0$$ → incompressible

- u = velocity
- t = time
- f = summation of external forces
- p = fluid density

$$r : \rho\left(\frac{\partial u_r}{\partial t} + u_r\frac{\partial u_r}{\partial r} + \frac{u_\phi}{r\sin(\theta)}\frac{\partial u_r}{\partial \phi} + \frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta} - \frac{u_\phi^2 + u_\theta^2}{r}\right) = -\frac{\partial p}{\partial r} + \rho g_r +$$
$$\mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u_r}{\partial r}\right) + \frac{1}{r^2\sin(\theta)^2}\frac{\partial^2 u_r}{\partial \phi^2} + \frac{1}{r^2\sin(\theta)}\frac{\partial}{\partial \theta}\left(\sin(\theta)\frac{\partial u_r}{\partial \theta}\right) - 2\frac{u_r + \frac{\partial u_\theta}{\partial \theta} + u_\theta\cot(\theta)}{r^2} - \frac{2}{r^2\sin(\theta)}\frac{\partial u_\phi}{\partial \phi}\right]$$
$$\phi : \rho\left(\frac{\partial u_\phi}{\partial t} + u_r\frac{\partial u_\phi}{\partial r} + \frac{u_\phi}{r\sin(\theta)}\frac{\partial u_\phi}{\partial \phi} + \frac{u_\theta}{r}\frac{\partial u_\phi}{\partial \theta} + \frac{u_r u_\phi + u_\phi u_\theta\cot(\theta)}{r}\right) = -\frac{1}{r\sin(\theta)}\frac{\partial p}{\partial \phi} + \rho g_\phi +$$
$$\mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u_\phi}{\partial r}\right) + \frac{1}{r^2\sin(\theta)^2}\frac{\partial^2 u_\phi}{\partial \phi^2} + \frac{1}{r^2\sin(\theta)}\frac{\partial}{\partial \theta}\left(\sin(\theta)\frac{\partial u_\phi}{\partial \theta}\right) + \frac{2\sin(\theta)\frac{\partial u_r}{\partial \phi} + 2\cos(\theta)\frac{\partial u_\theta}{\partial \phi} - u_\phi}{r^2\sin(\theta)^2}\right]$$
$$\theta : \rho\left(\frac{\partial u_\theta}{\partial t} + u_r\frac{\partial u_\theta}{\partial r} + \frac{u_\phi}{r\sin(\theta)}\frac{\partial u_\theta}{\partial \phi} + \frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r u_\theta - u_\phi^2\cot(\theta)}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \rho g_\theta +$$
$$\mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u_\theta}{\partial r}\right) + \frac{1}{r^2\sin(\theta)^2}\frac{\partial^2 u_\theta}{\partial \phi^2} + \frac{1}{r^2\sin(\theta)}\frac{\partial}{\partial \theta}\left(\sin(\theta)\frac{\partial u_\theta}{\partial \theta}\right) + \frac{2}{r^2}\frac{\partial u_r}{\partial \theta} - \frac{u_\theta + 2\cos(\theta)\frac{\partial u_\phi}{\partial \phi}}{r^2\sin(\theta)^2}\right].$$

???

Some Approaches

Lagrangian method

Eulerian method

- Lagrangian method

→ Treats fluid as continuum particle system

: Tracking each particle

- $\boxed{\text{Eulerian method}}$

→ Observe fixed points in space change over time

ex. velocity, density, temperature

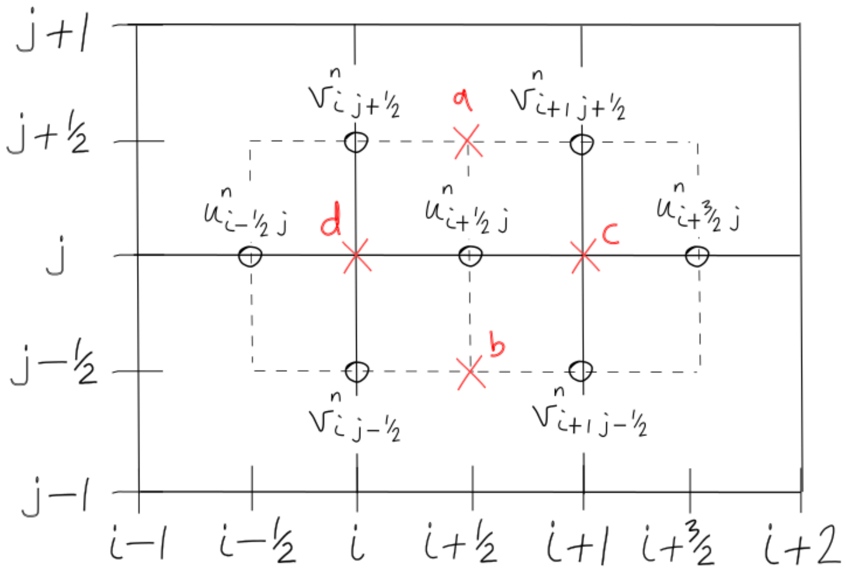
→ temperature of individual particle hasn't changed, however, in fixed point it did!

Navier Stokes equation

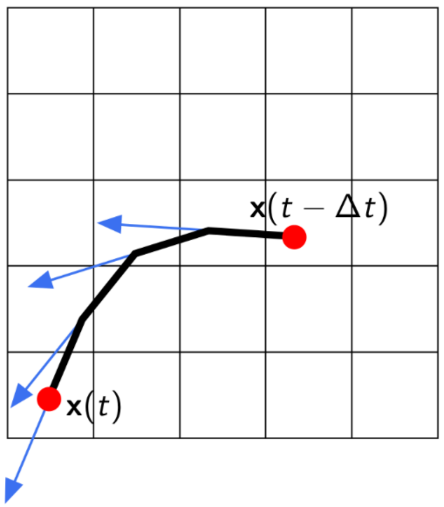
#zero viscosity

#imcompressible

#neglible air pressure

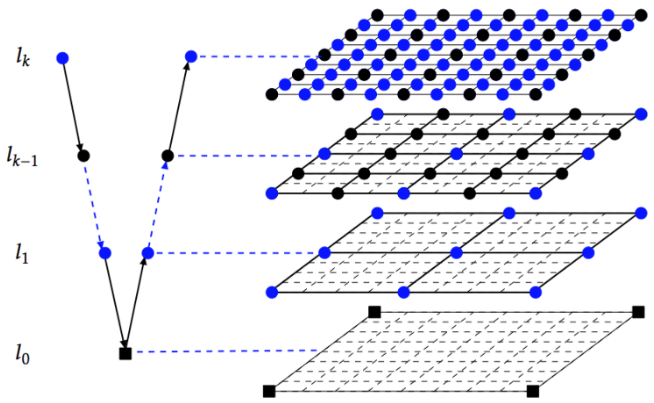


- Staggered grid: Marker and Cell



Other methods

What' s new



- Multi-grid method

  → difficult to implement, hard to parallelize on GPU

- Inexact approximate solution

  → low computational cost, but data … ?

- Supervised regression

  → predict output by NN but accumulated errors…

Algorithm

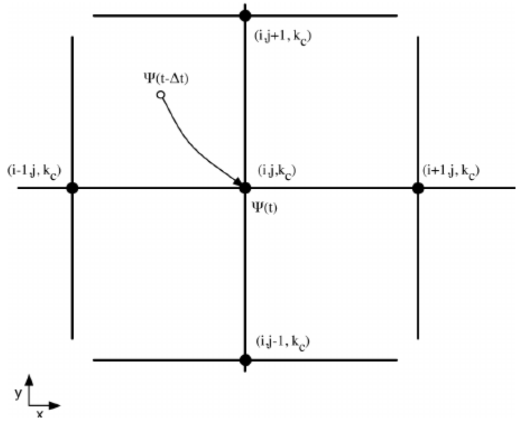
Euler Equation Velocity Update

Algorithm 1 Euler Equation Velocity Update

Advection and Force Update to calculate $u_t^\star$:
  (optional) Advect scalar components through u_{t-1}
  Self-advect velocity field u_{t-1}
  Add external forces f_{body}
  Add vorticity confinement force f_{vc}
  Set normal component of solid-cell velocities.
Pressure Projection to calculate u_t:
  Solve Poisson eqn, $\boxed{\nabla^2 p_t = \frac{1}{\Delta t}\nabla \cdot u_t^\star}$ to find p_t
  Apply velocity update $u_t = u_{t-1} - \frac{1}{\rho}\nabla p_t$

$$w = \nabla \times u, \quad f_{vc} = \lambda h \,(N \times w)$$

$N = {}^{\nabla|w|}/_{\|\nabla|w|\|}$, λ controls the amplitude of vorticity confinement, and h is the grid size (typically $h = 1$).



$$\nabla^2 p_t = \frac{1}{\Delta t}\nabla \cdot u_t^\star$$ → Poisson equation

$$Ap_t = b,$$

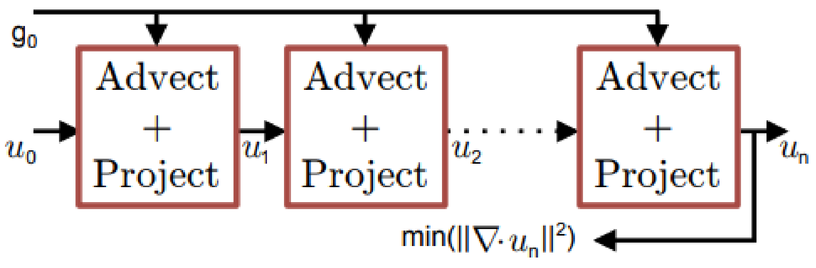
1. Ignore ▽ p: includes unwanted divergence
2. p will be calculated for incompressible condition = divergence free velocity field

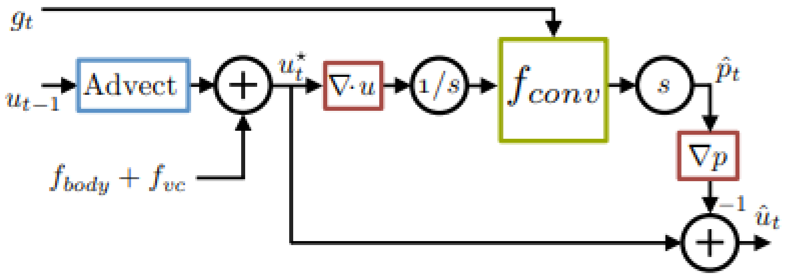
Algorithm

Pressure Model

$$f_{obj} = \sum_i w_i \left\{\nabla \cdot \hat{u}_t\right\}_i^2 \qquad w_i = \max\left(1, k - d_i\right)$$

$$= \sum_i w_i \left\{\nabla \cdot \left(u_t^\star - \frac{1}{\rho}\nabla \hat{p}_t\right)\right\}_i^2 \qquad \hat{p}_t = f_{conv}\left(c, \nabla \cdot u_t^\star, g_{t-1}\right)$$





Algorithm

Convolutional

Neural

Network

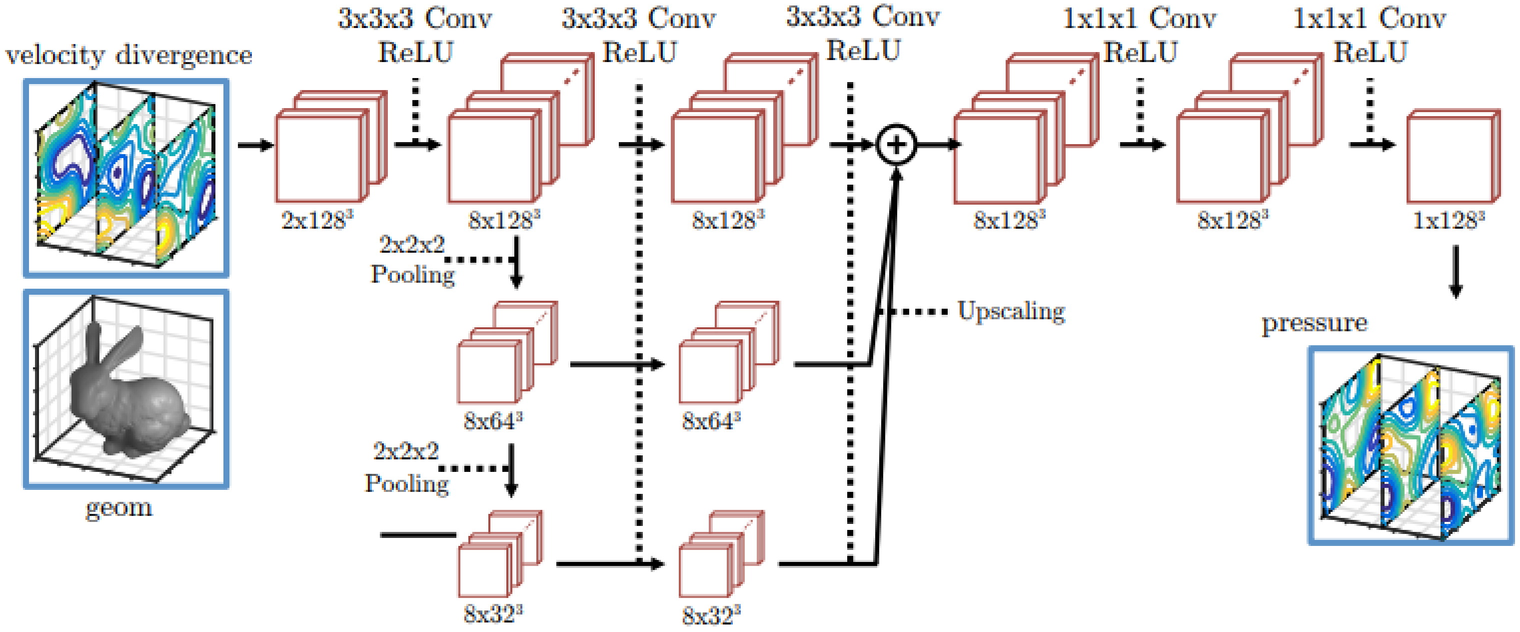


Figure 3. Convolutional Network for Pressure Solve


Result

Thank you

Github: @sju-coml/SAI-Team-S

나비에 스토크스 방정식 유도: https://casterian.net/archives/544

http://leo1984.impa.br/fluidsurf/