Introduction to Bayesian Analysis for Seabird Research

Pacific Seabird Group 2023 Annual Meeting
14 February 2023, 3:00-5:00 pm PT

Workshop Goals

1. Recognize **value of Bayesian framework** for seabird research.
2. Understand **fundamental differences** between Frequentist and Bayesian frameworks.
3. Learn **basic components** of a Bayesian analysis.
4. Learn how to **interpret results** from a Bayesian analysis.
5. Build **community and share resources** for future learning.

Dr Tim Jones
(He/Him)
Research Scientist
School of Aquatic & Fishery Sciences
University of Washington

Amelia DuVall
(She/Her)
Graduate Research Assistant
School of Aquatic & Fishery Sciences
University of Washington

Michelle Kissling
(She/Her)
PhD Student
University of Montana
Wildlife Biology Program

Molly McDevitt
(She/Her)
PhD Student
University of Montana
Wildlife Biology Program

Introductions

1. Name
2. Title/Affiliation
3. Pronouns
4. Study species/region/ecosystem, etc.
5. What brought you to this workshop?
6. On a scale of 1 (*not at all! I want to learn!*) to 5 (*pretty comfortable, I have run a few models on my own*), how familiar are you with Bayesian analyses?

Seabird data is messy.

Some of the challenges include:

- Difficult to get to colonies
- Hard to find individuals and/or hard to count them all
- Individuals are inaccessible during portions of life history

Which can lead to small sample sizes, gaps, and uncertainty in our datasets.

Good news, many of these issues can be addressed using Bayesian methods!

Overview

1. Welcome, Goals & Introductions
2. Bayes: An Updated Way of Thinking
3. Bayesian Data Analysis
4. Interpreting Bayesian Models
5. Building Your Own Models
6. Tips/Tricks and Further Information
7. Workshop Wrap-up & Optional Happy Hour

Bayes:
An Updated Way of Thinking

What is your belief?

I am going to show a video of some penguins attempting to ‘board’ an iceberg.

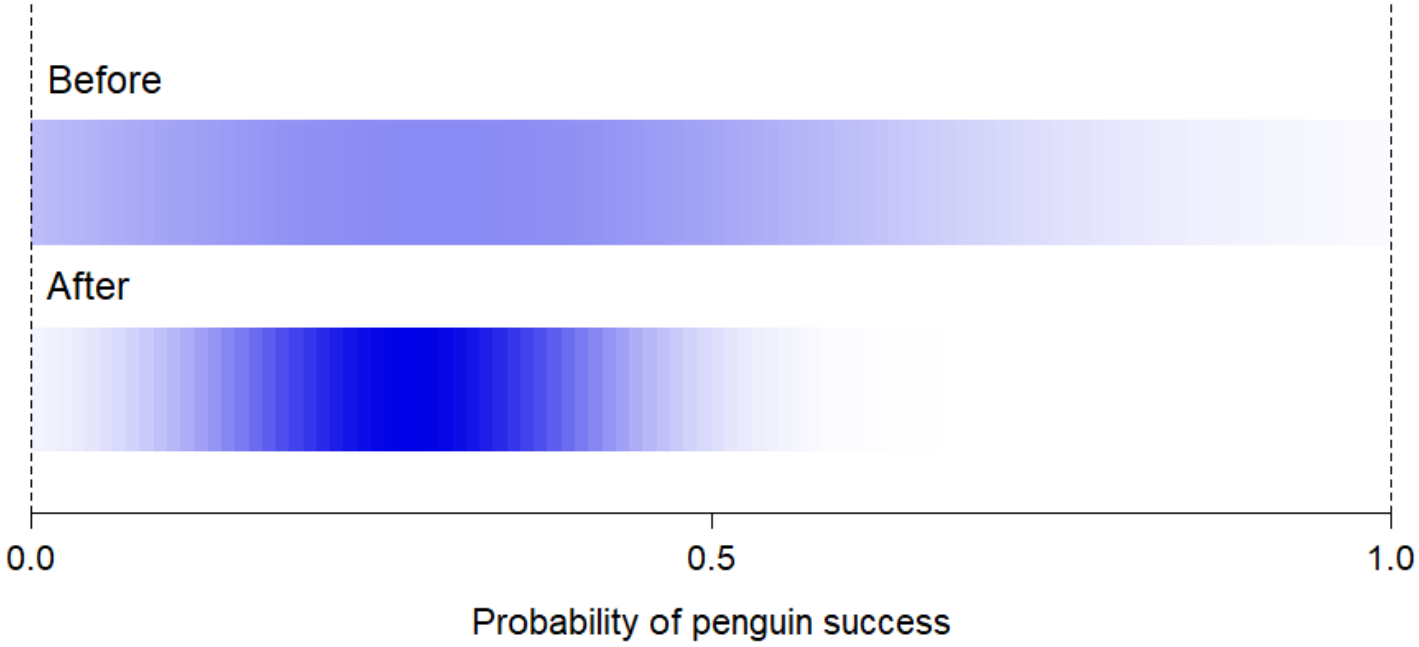
What is your initial expectation of the probability that an individual attempt is successful?

What is your belief?

I am going to show a video of some penguins attempting to 'board' an iceberg.

What is your initial expectation of the probability that an individual attempt is successful?

What do you think the probability is now that you have seen some attempts?


Before
After
0.0
0.5
1.0
Probability of penguin success

Thinking like a Bayesian

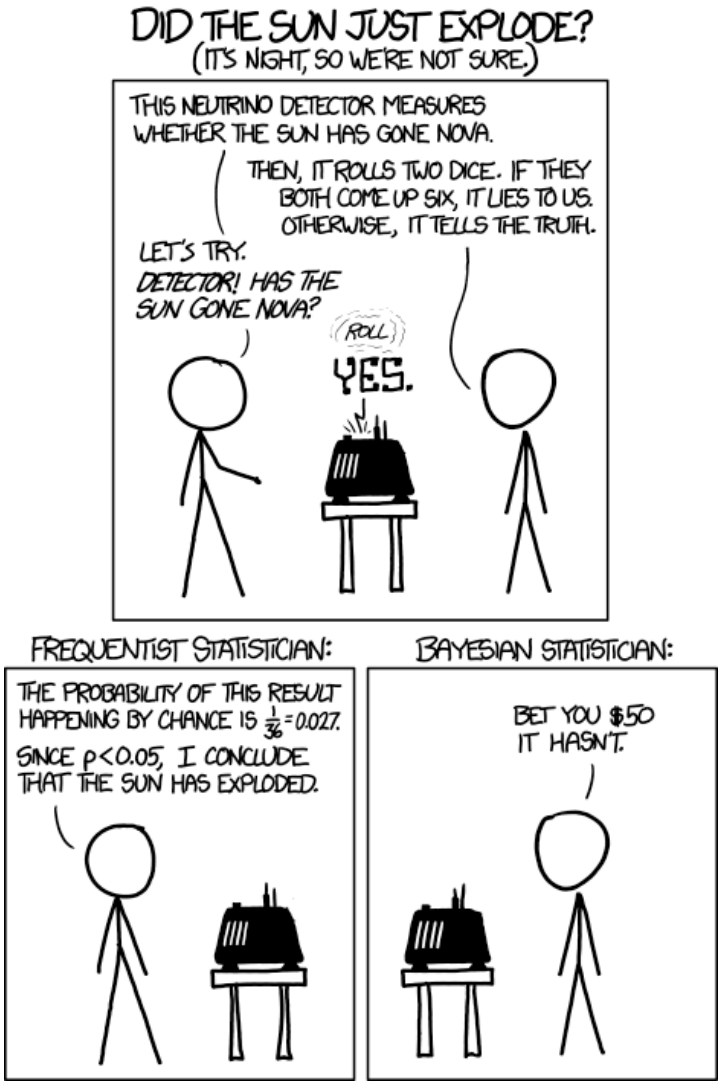
Bayesian thinking is a means of updating your certainty in something dependent on **data** and your **prior knowledge**.

It provides a natural way to incorporate existing knowledge into the estimation of processes and uncertainty.

It also provides a more natural way of thinking about uncertainty (we'll get to that…).

Schools of Thought:

Bayesian vs. Frequentist



	Bayesian	Frequentist
Data	Fixed	Repeatable random samples
Probability	Degrees of belief	Based on long-run frequency
Parameters	Random variables	Fixed but unknown
Calculates	P(hypothesis|data)	P(data|hypothesis)
Source of information	data(observations) + prior belief	data(observations)
Hypothesis testing	Bayes Factor (re-expression of Bayes' theorem)	p-values
Interval interpretation	Credible intervals	Confidence intervals
Point estimate	Mean, Median, Mode of posterior probability distribution	MLE, least squares estimate

Using a **frequentist** approach, a solution can be found for many (often linear) problems using either a closed form equation or a relatively simple algorithm.

For **Bayesian** analyses, even simple problems usually do not have an analytical solution and require modern computational techniques to find the solution(s).

In addition to debates about whether science should be subjective or objective (i.e. priors and belief??), this led to the dominance of **frequentist** thinking.

There’s no such thing as a Bayesian (only) model.

A Bayesian model is not a separate class of model.

- You can fit a simple linear regression using Bayesian techniques
- You can fit an age-structured population model using frequentist methods

The two approaches are more based on the philosophical differences between the two and what that means for interpretation of the model and it’s parameters.

Bayes Theorem

“Bayes’ theorem allows us to take our **beliefs** about the world, combine them with **data**, and then transform this combination into an estimate of the strength of our beliefs given the evidence we’ve observed.”

- *Bayesian Statistics The Fun Way* (Kurt 2019)

Notation Refresher

Before going any further, some terminology:

$p(\mathbf{A})$: probability of **A**

- **A** can be a value (i.e. a rate, or parameter), or occurrence

$p(\mathbf{A} \mid \mathbf{B})$: probability of **A** given that **B** is true

- This is a conditional probability

We will more often think of these in terms of probability distributions

- $p(\mathbf{A} = \boldsymbol{a})$: probability that **A** has a specific value $\boldsymbol{a}$

Bayes Theorem

$$p(A|B) =$$

The probability of **A** given **B** is equal to...

Bayes Theorem

$$p(A|B) = p(A)$$

The probability of **A** given **B** is equal to the probability of **A**...

Bayes Theorem

$$p(A|B) = p(A) \cdot p(B|A)$$

The probability of **A** given **B** is equal to the probability of **A** multiplied by the probability of **B** given **A**...

Bayes Theorem

$$p(A|B) = \frac{p(A) \cdot p(B|A)}{p(B)}$$

The probability of **A** given **B** is equal to the probability of **A** multiplied by the probability of **B** given **A** divided by the probability of **B**.

Bayes Theorem: Penguin example

We want to find the probability of different **success rates** (our parameter of interest) based on our observations of **data**.

$$p(A|B) = \frac{p(A) \cdot p(B|A)}{p(B)}$$

Bayes Theorem: Penguin example

We want to find the probability of different **success rates** (our parameter of interest) based on our observations of **data**.

probability of success rate, or initial knowledge about the success rate (e.g., not 0 or 1)

$$p(\textit{success rate}|\textit{data}) = \frac{p(\textit{success rate}) \cdot p(\textit{data}|\textit{success rate})}{p(\textit{data})}$$

probability distribution of success rate given our data

probability of our data

For a given success rate, we can calculate the probability of our data (e.g., if success rate were 0.5, what is the probability of observing 5 out of 18 successes?)

Questions?

Bayesian Data Analysis: The pieces of the model

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Key Takeaways

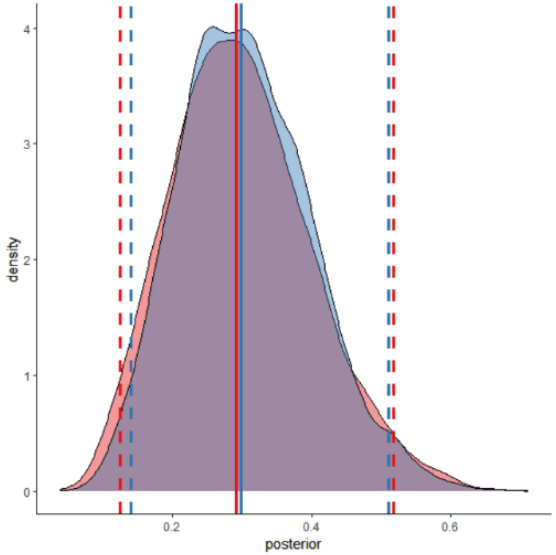
p(model | data) : posterior distribution

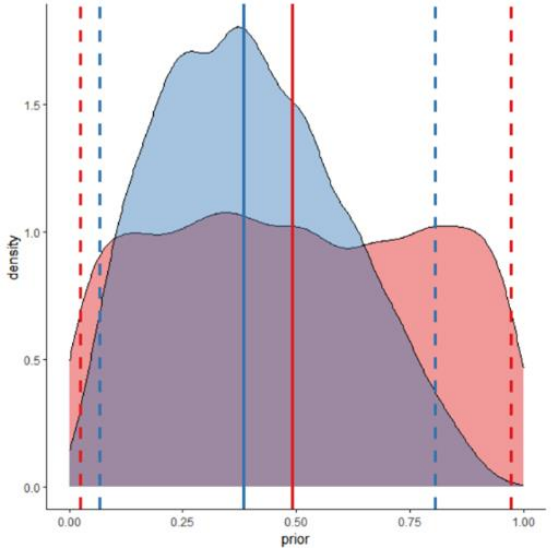
$\propto$

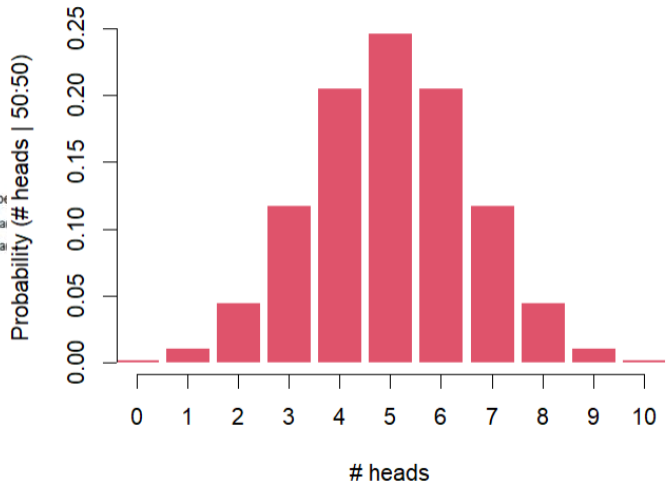
p(model) : prior distributions

$\cdot$

p(data | model) : model likelihood







Components of any Bayesian model: Inputs

Data ('**I cannot make bricks without clay**': Sherlock Holmes, The Adventure of the Copper Beeches, 1892)

Model structure ('**You know my methods, Watson**': Sherlock Holmes, The Crooked Man, 1893)

- This should encapsulate how you think the parameters talk to each other, and the data
  - Deterministic part (output is entirely determined by parameters - i.e. intercept/slope)
  - Stochastic part (given an input the outputs are random - aka the data generating component)

Priors ('**Education never ends, Watson. It is a series of lessons, with the greatest for the last**': Sherlock Holmes, His Last Bow, 1917)

- Probability distributions of model parameters based on a-priori knowledge

Components of any Bayesian model: Inputs & results

$$p(model|data) = \frac{p(model)p(data|model)}{p(data)}$$

p(data) : probability of the data

- is unknown, but importantly constant, so is usually omitted

$$p(model|data) \propto p(model)p(data|model)$$

is proportional to

p(model) : **prior** distributions (think of model as the collective structure and parameters)

p(data | model) : **model likelihood** (probability of data given model structure and parameters)

p(model | data) : **posterior** distribution (our updated belief in model parameters given the data)

Components of any Bayesian model: Inputs & results for penguins

$$p(success\ rate|data) = \frac{p(success\ rate) \cdot p(data|success\ rate)}{p(data)}$$

p(data) : probability of the data

- this is unknown, but importantly constant, so is usually omitted

$$p(success\ rate|data) \propto p(success\ rate)p(data|success\ rate)$$

p(success rate) : **prior** distributions : S ~ Beta(2,3)

p(data | model) : **model likelihood** : data ~ Binomial(n = 18, p = S)

p(success rate | data) : **posterior** distribution : p(S | data)

Key Takeaways

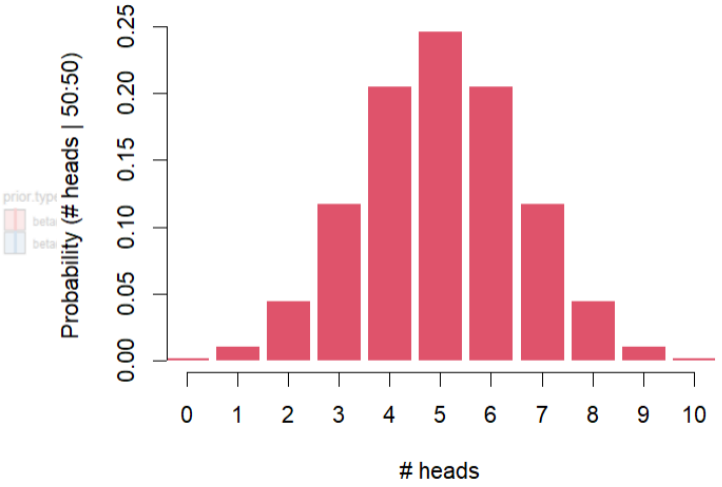
p(model | data) : posterior distribution

$\propto$

p(model) : prior distributions

$\cdot$

p(data | model) : model likelihood

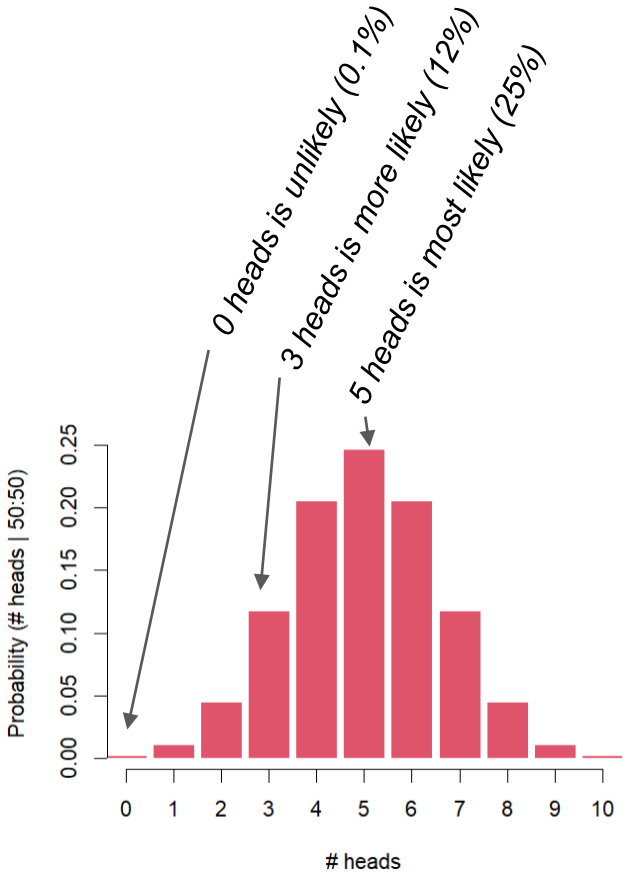


What is the likelihood?

The likelihood is the probability of the data given a particular model

Think of tossing a fair coin

- By assuming the coin is fair we are **fixing** the probability of heads at 0.5 - this is the **parameter** or **model**
- We toss the coin ten times and count the number of heads
- The number of heads we get is our **data**



Likelihood and likelihood function

Likelihood function

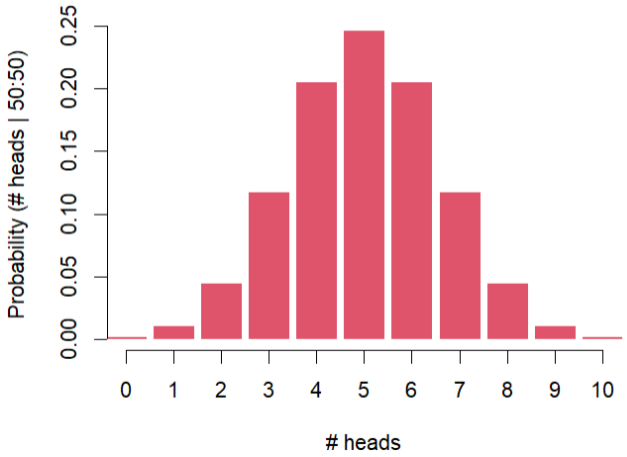
- How probability is shared across all possible observations

Quantifies how much a data point agrees with the fixed model

- E.g. 5 heads agrees with a p = 0.5 model more than an observation of 1 heads
- Used to judge among competing parameter values

Model can take any form, so long as it has parameters that relate to the data via the likelihood function

Likelihood function : binomial
(*One of a set of probability distributions*)



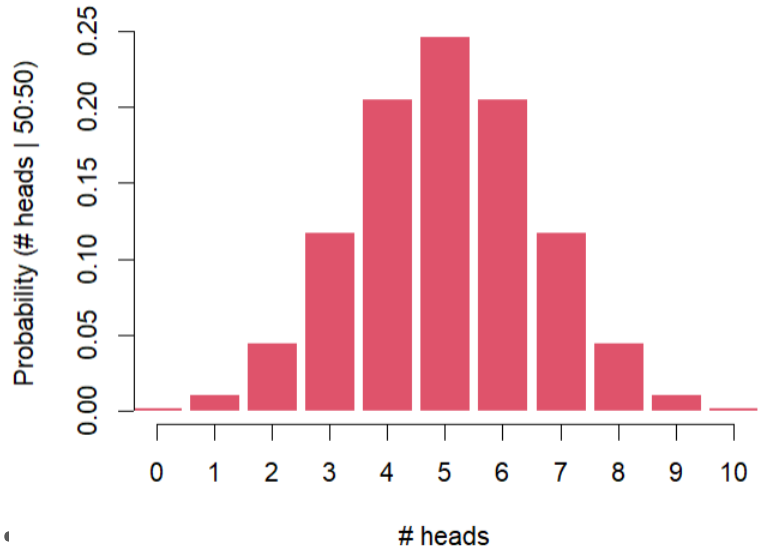
Closing likelihood statements

Likelihood = p(data | model)

Likelihood is simulated data as defined by the model

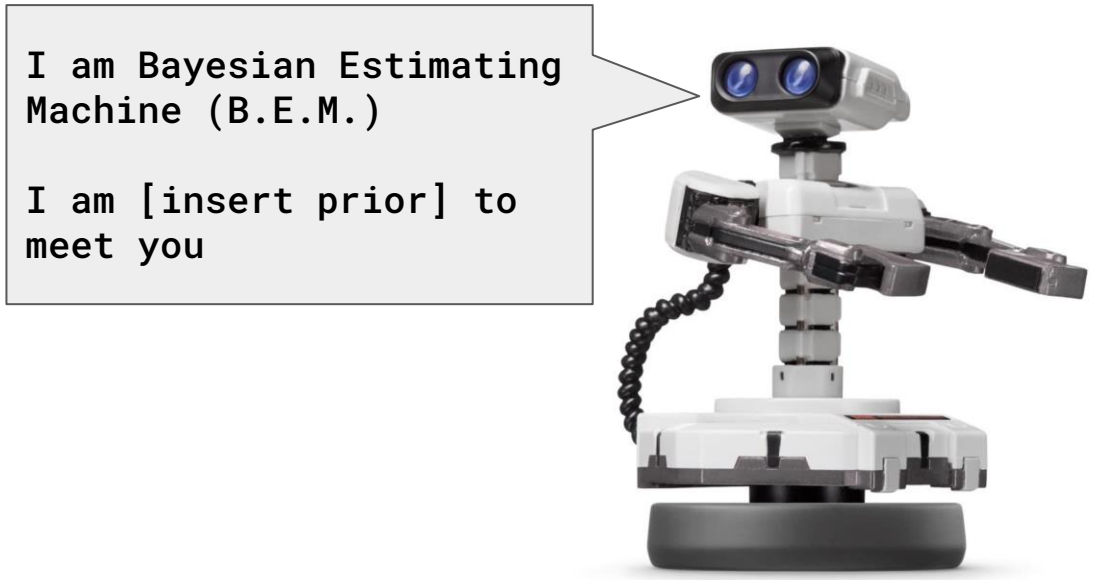
Likelihood can be calculated using:

- Trial and error (NOT efficient!)
- Compute the maximum of a function by hand (rarely doable in practice)
- **Iterative optimization algorithm**



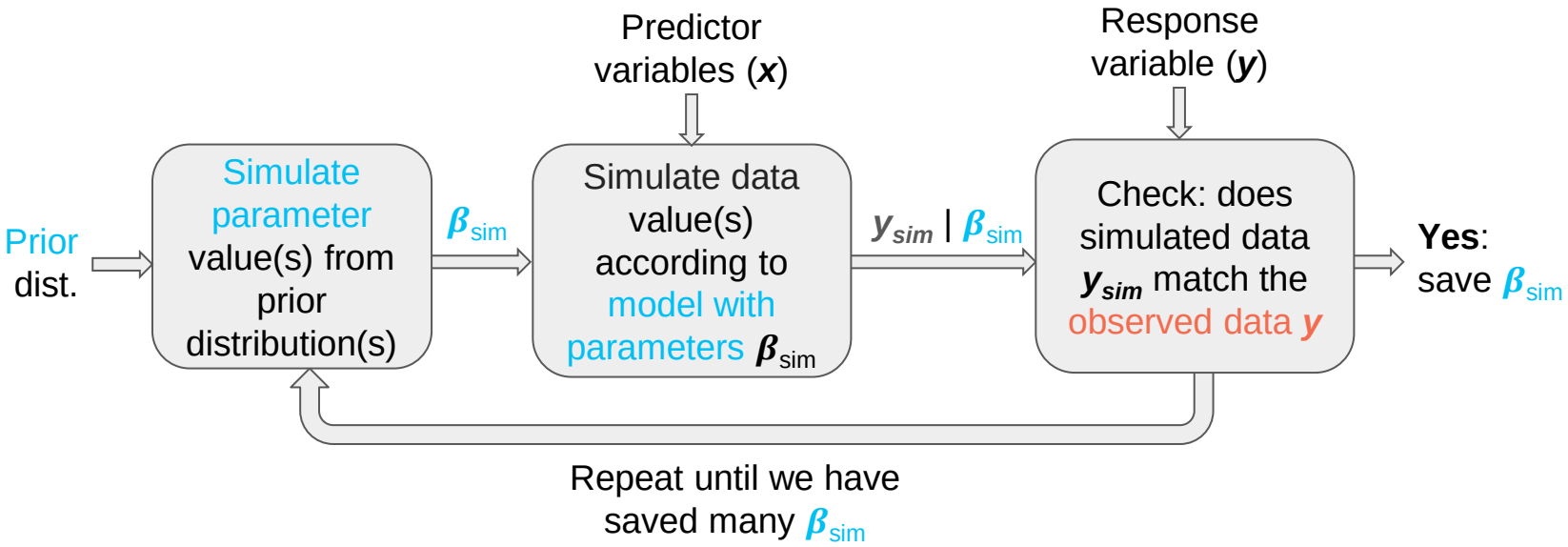
How do we estimate p(model | data)?

We have defined data, priors, model structure, how do we estimate posterior parameter estimates



B.E.M.

B.E.M is a silly, but illustrative, and very hard working machine for estimating posterior distributions



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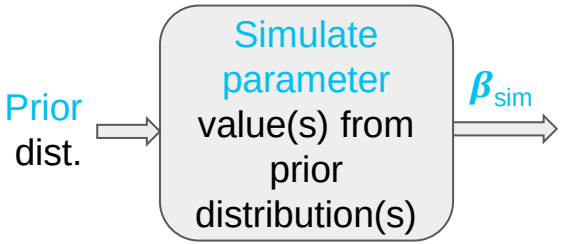

The distribution of $\boldsymbol{\beta}_{sim}$ values that are generated are informed only by prior knowledge - therefore a particular value β_1 will be generated at a rate p(β_1)

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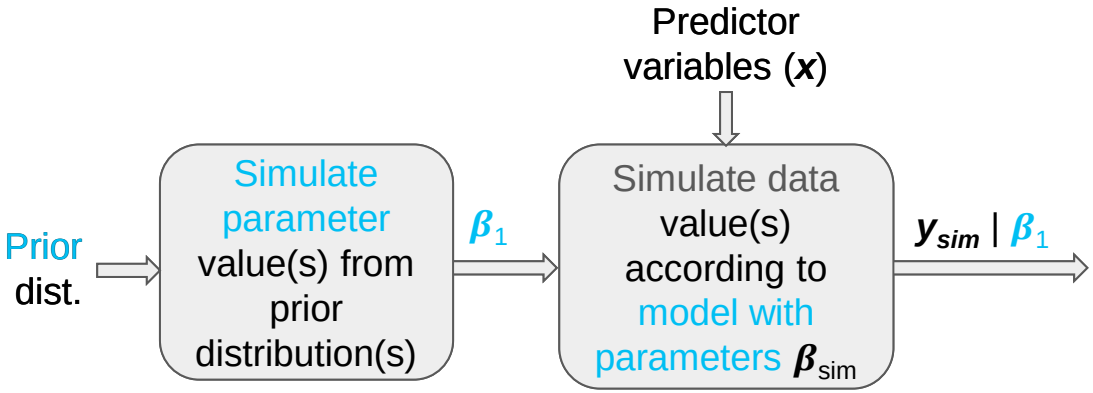

The distribution of data generated is informed by the model parameterised according to β_1. Therefore for any particular dataset, y, the probability that it is generated is $p(y \mid \beta_1)$

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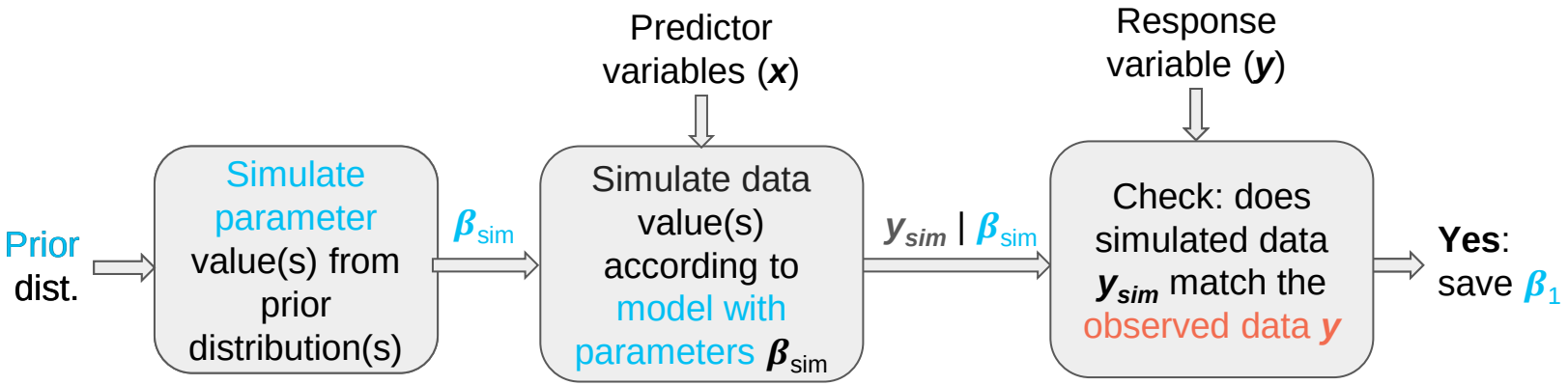

The number of times β_1 is saved is proportional to the rate it is generated $p(\beta_1)$ multiplied by the rate it generates the data $p(y \mid \beta_1)$

$$p(\beta_1 \mid y) \propto p(\beta_1)\, p(y \mid \beta_1)$$

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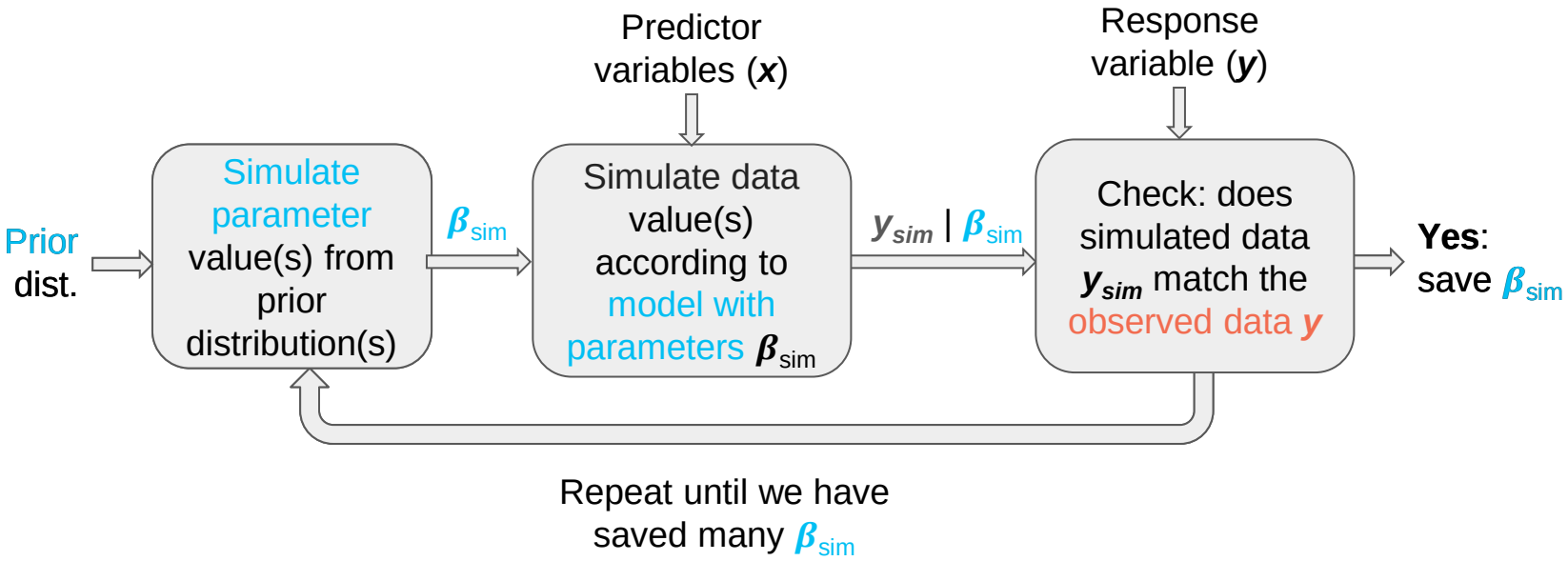


Applying B.E.M. to the penguin data

For the penguins we need to define a model for success rate

- We observed n attempts, of which y attempts were successful

$$y \sim Binomial(n, S)$$

- Where S is the success rate parameter we wish to estimate
- This defines our likelihood as p(data | model) = binomial(y | n, S)

Prior

- We were initially somewhat uninformed, but we know S must be between 0 and 1, and potentially skewed towards lower values

Bayesian Data Analysis: Defining priors

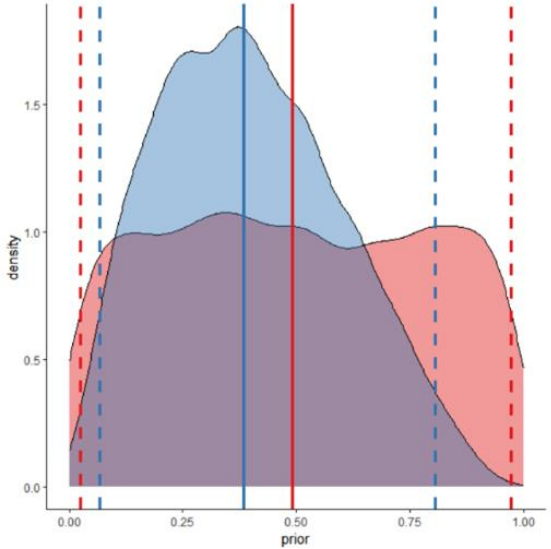
Key Takeaways

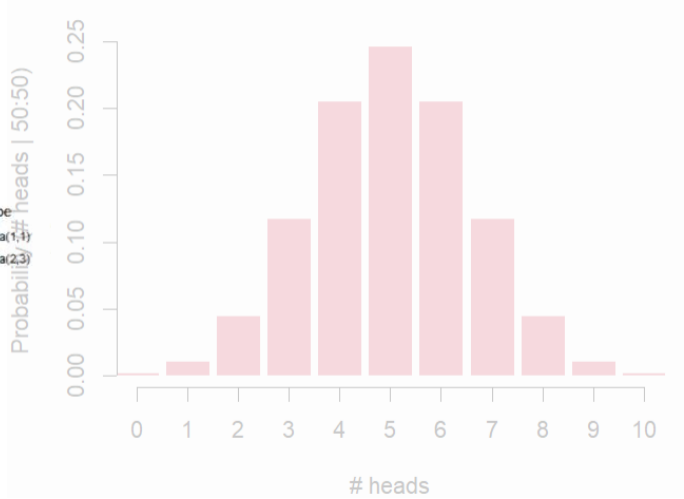
p(model | data) : posterior distribution

$\propto$ **p(model) : prior distributions**

$\cdot$

p(data | model) : model likelihood





Defining priors

Priors can, with sparse data, can enable inference on key parameters that might otherwise be impossible (cool!)

Priors are defined based on a probability distribution (i.e. normal…)

- Distribution parameters should reflect prior knowledge (i.e. distribution mean) and associated certainty (i.e. distribution width, shape)
    - Note that the way the model interprets your prior may not match your intention

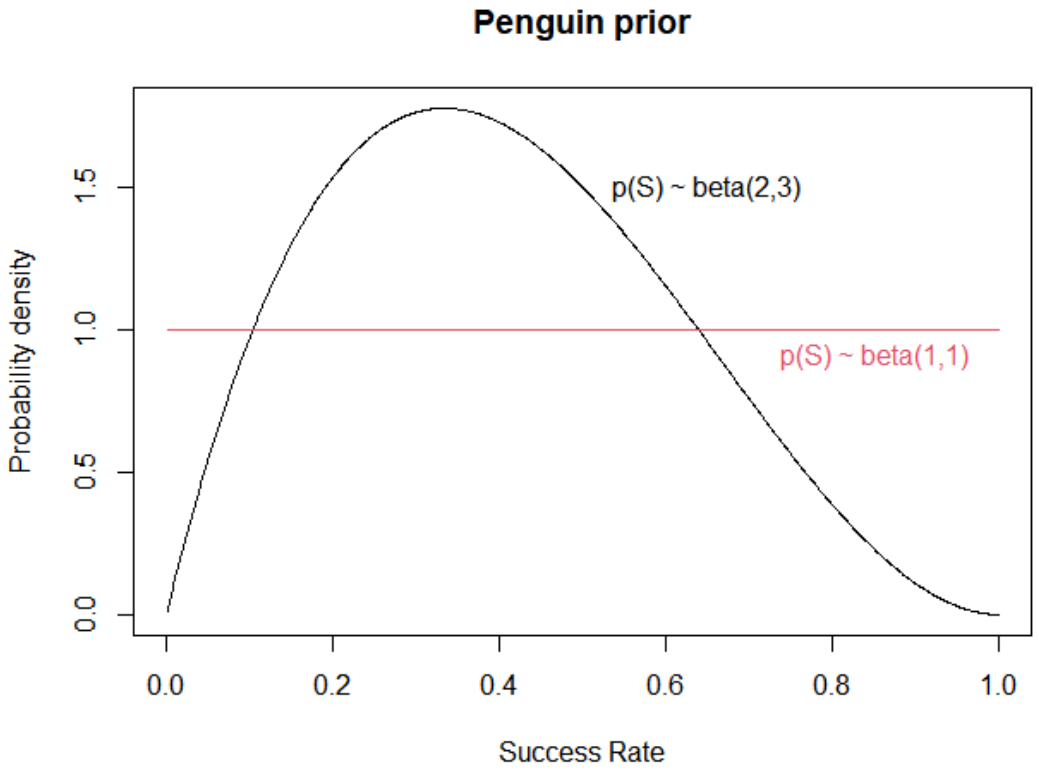
Priors could be informed by results from another study

- If not, we often know a realistic range (i.e. demographic rates)

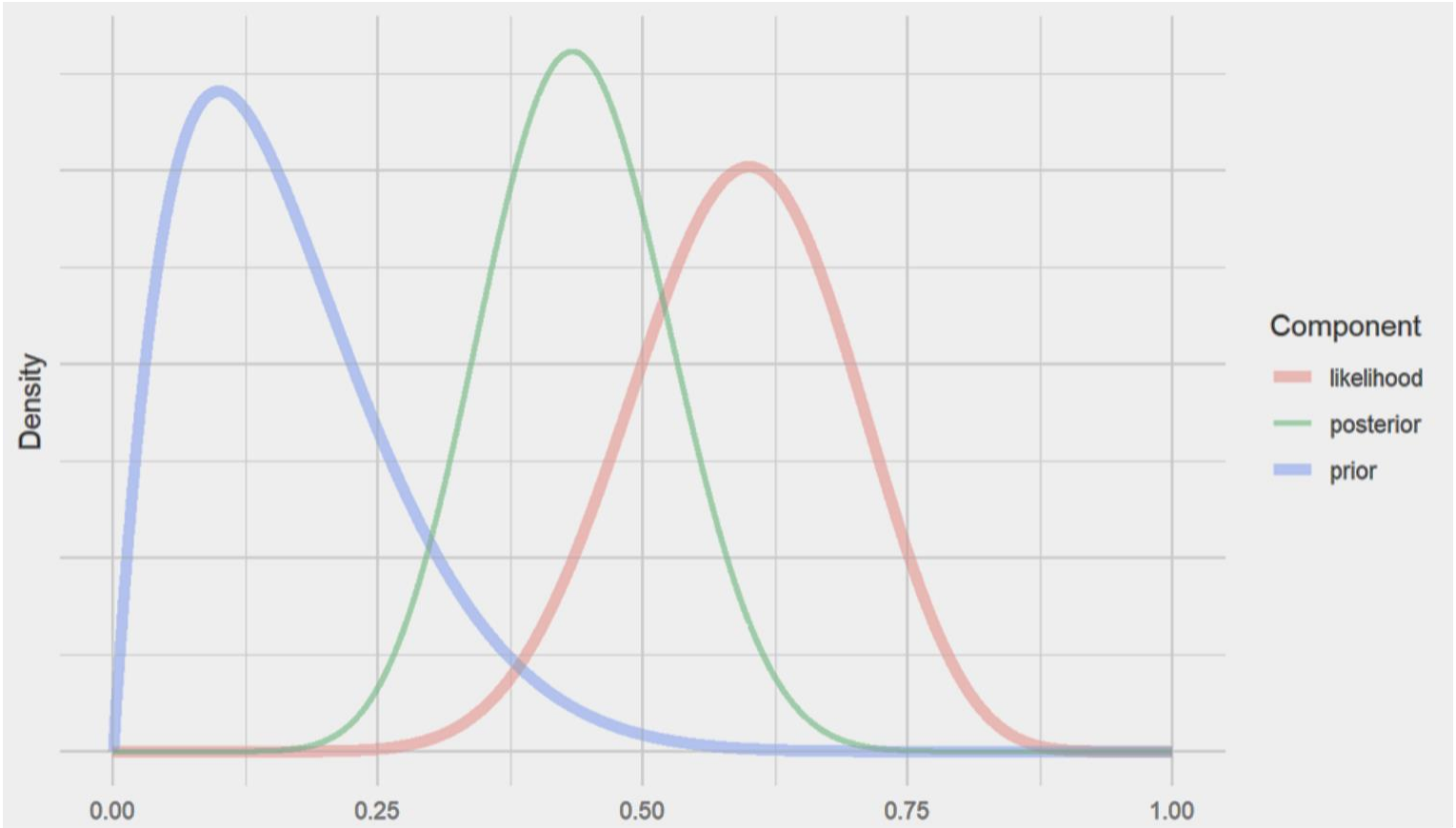
Priors are less influential and quickly overwhelmed by **informative data**

- If your main results depend on priors, then there either needs to be solid justification for them and/or suitable sensitivity analyses to show results are invariant to prior definition

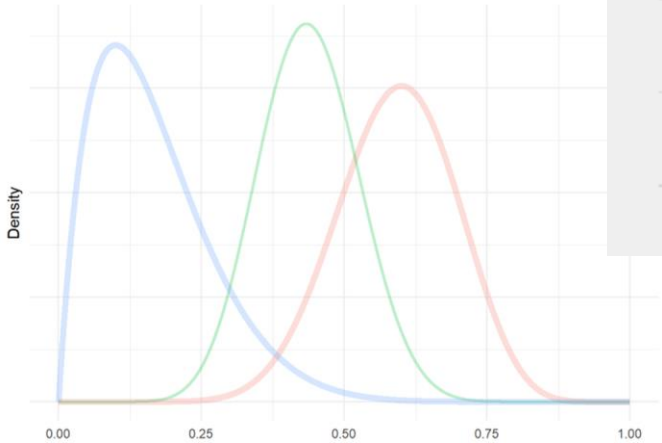
Priors for penguin success rate

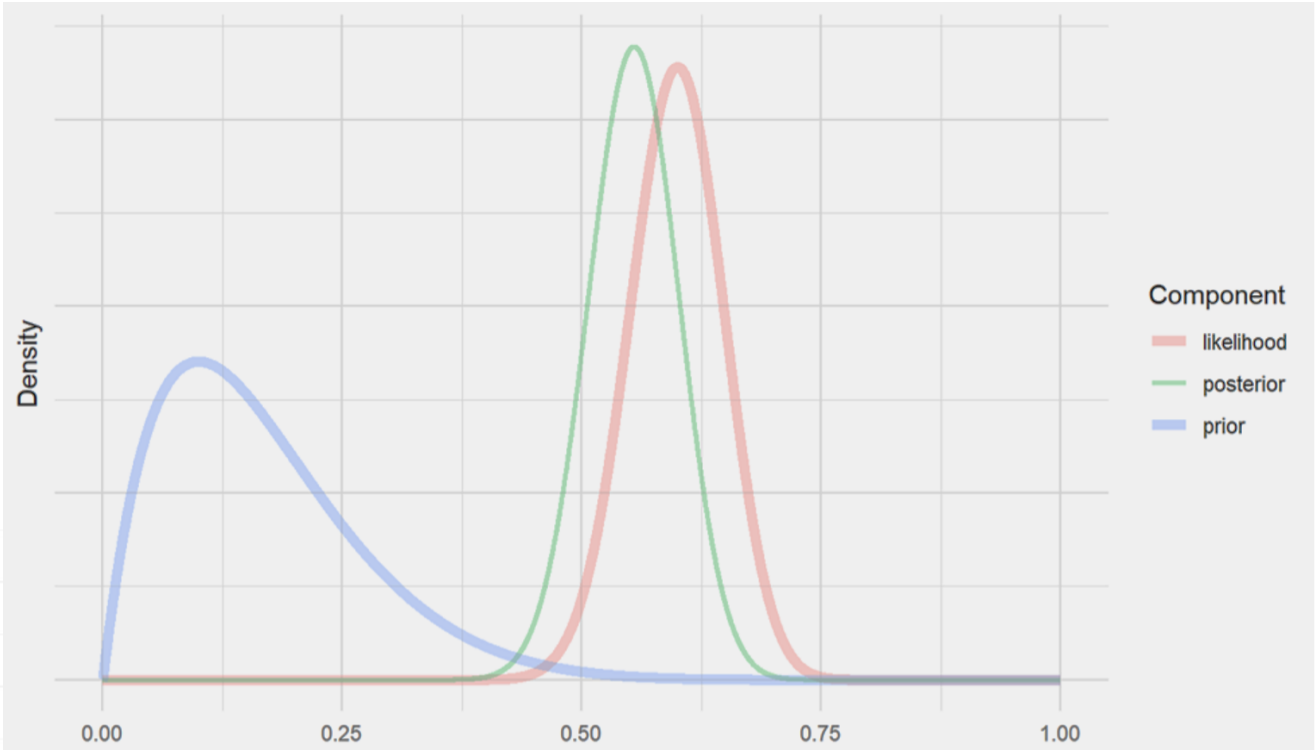


Priors, likelihood & posterior interactions

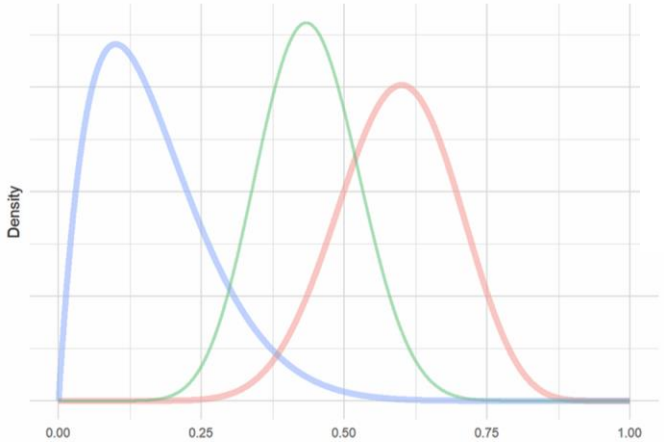


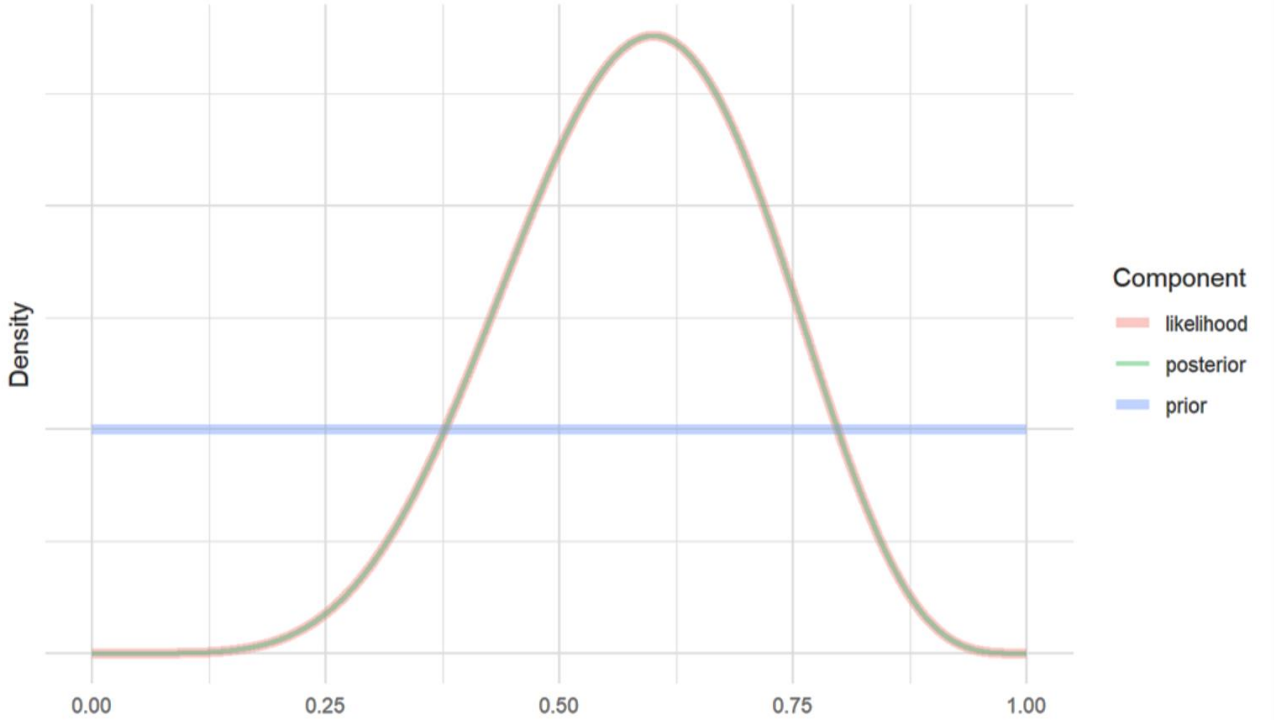
Priors are less influential and quickly overwhelmed by informative data





Likelihood has even more influence with uninformative prior





Bayesian Data Analysis: Model fitting/MCMC

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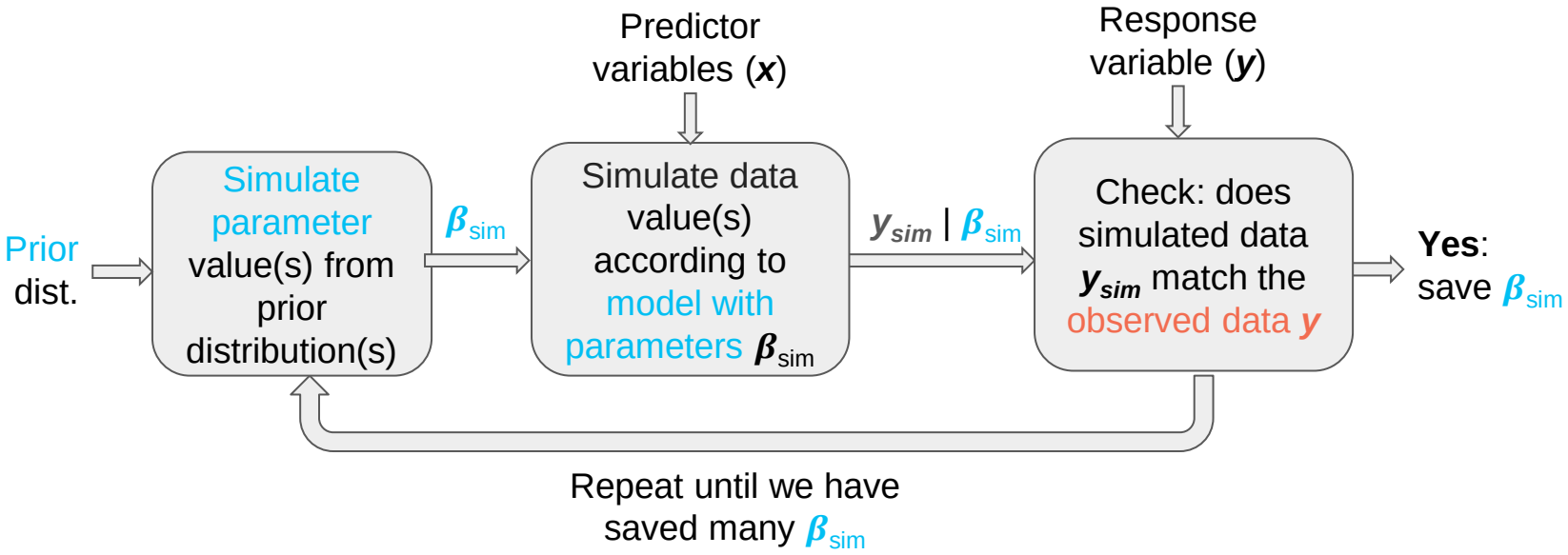
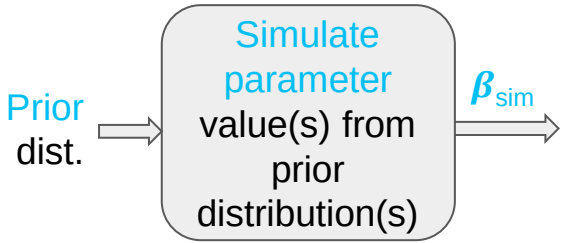


Applying B.E.M. to the penguin data

Simulated Success rates (according to prior p(S) ~ beta(2,3))

0.08 0.11 0.17 0.18 0.19 0.29 0.31 0.32 0.33 0.43 0.46 0.46 0.51 0.52 0.53 0.55 0.64 0.67 0.68 0.84



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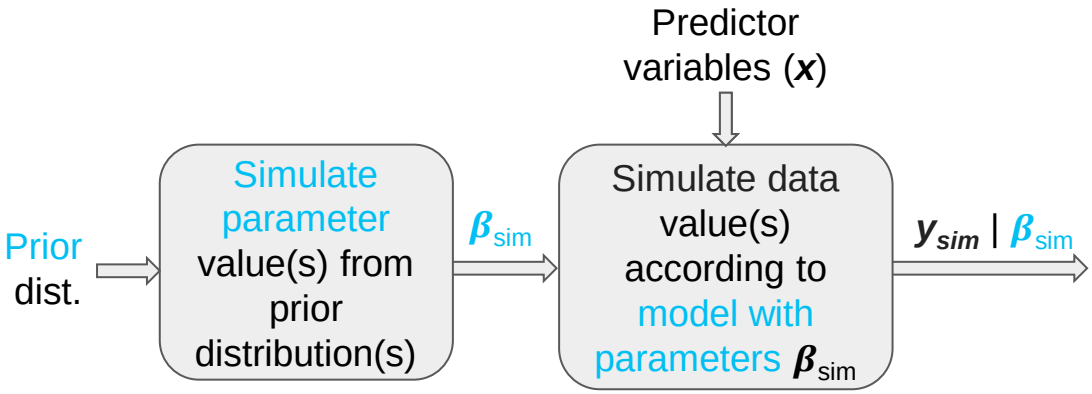
Simulated Success rates (according to prior p(S) ~ beta(2,3))

0.08 0.11 0.17 0.18 0.19 0.29 0.31 0.32 0.33 0.43 0.46 0.46 0.51 0.52 0.53 0.55 0.64 0.67 0.68 0.84

Simulated data (according to data ~ binomial(n = 18, p = S))

5 3 3 3 5 7 5 2 5 11 6 7 9 12 10 8 12 10 10 17



Applying B.E.M. to the penguin data

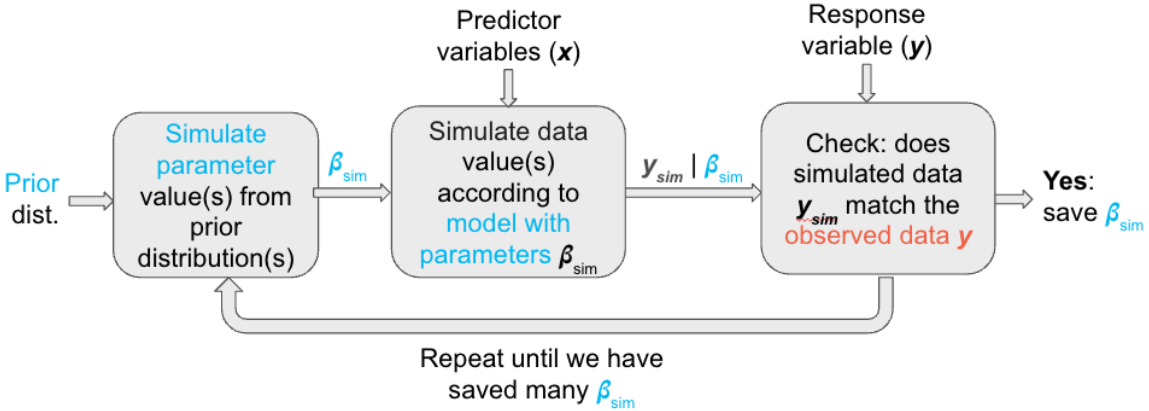
Simulated Success rates (according to prior p(S) ~ beta(2,3))

0.08 0.11 0.17 0.18 **0.19** 0.29 **0.31** 0.32 **0.33** 0.43 0.46 0.46 0.51 0.52 0.53 0.55 0.64 0.67 0.68 0.84

Simulated data (according to data ~ binomial(n = 18, p = S))

5 3 3 3 **5** 7 **5** 2 **5** 11 6 7 9 12 10 8 12 10 10 17

Only keep the values for S that are consistent with our data
The resulting distribution of values for S make up the posterior p(S | data)



How do we estimate p(model | data)?

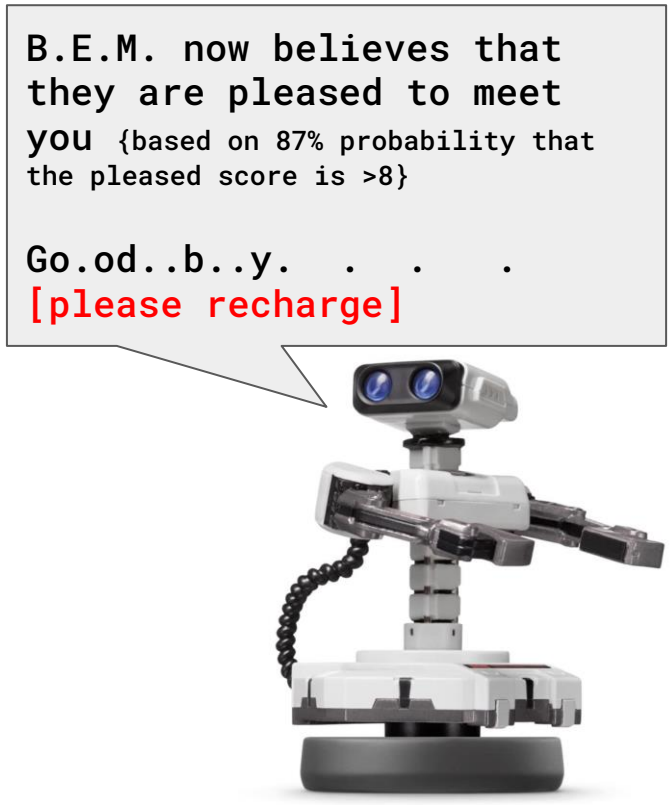
Some may have noticed that B.E.M isn't particularly efficient

- E.g. Even with one data point it required 50,000-100,000 simulations to get 5,000 estimates of success rate

So, we require a method that is smarter about choosing the parameter values to explore

Markov Chain Monte Carlo (MCMC)

- Monte Carlo - simulation of many random numbers
- Markov Chain - an elegant way of choosing those numbers



30 Second Intro to Markov Chain Monte Carlo

Propose a parameter value(s)

Evaluate likelihood at proposal

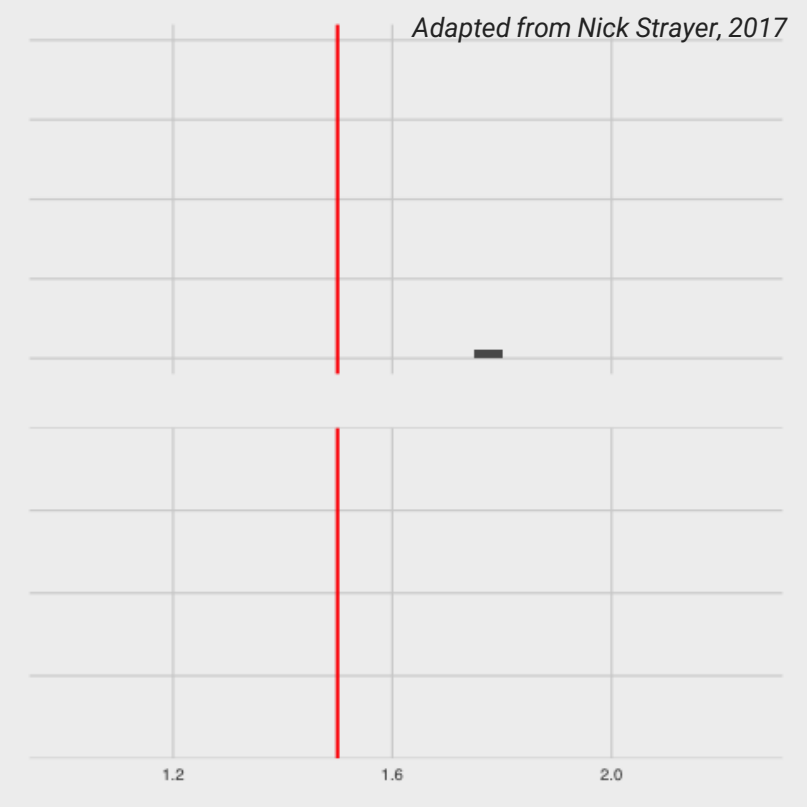
Propose an alternative parameter value(s)

Evaluate likelihood at alt proposal

Compare likelihoods

Save a parameter value

Repeat for a looooooooooooooong time



Bayesian model-fitting

Difference in model estimation

- Models fitted via maximum likelihood (frequentist) - find a **single value for each parameter**
- Bayesian models - find a **set of values for each parameter** consistent with the data and prior expectation
- Bayesian models are concerned with finding and exploring a **target set** (which makes up the posterior)

However, we can evaluate probabilities for a specific location and use that to guide exploration - enter MCMC

Key Takeaways

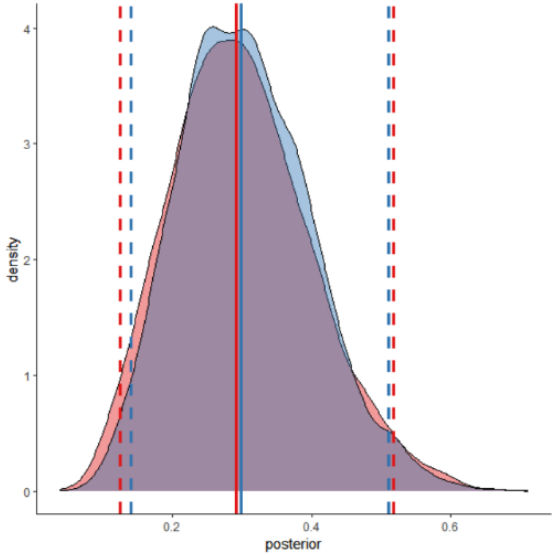
p(model | data) : posterior distribution

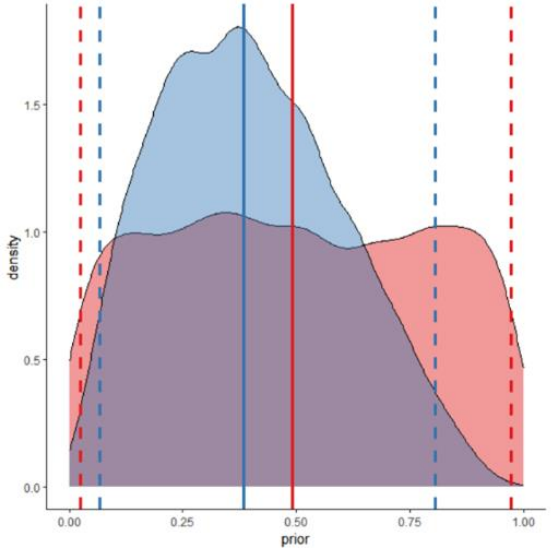
$\propto$

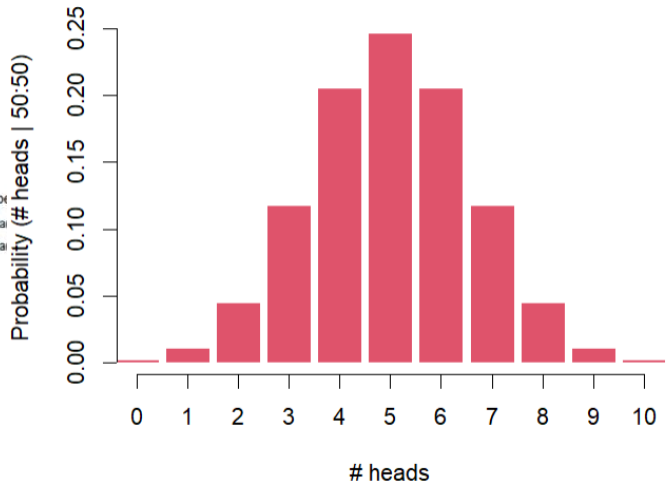
p(model) : prior distributions

$\cdot$

p(data | model) : model likelihood







Questions?

Interpreting Bayesian Models

Overview

1. Welcome, Goals & Introductions
2. Bayes: An Updated Way of Thinking
3. Bayesian Data Analysis
4. Interpreting Bayesian Models
5. Building Your Own Models
6. Tips/Tricks and Further Information
7. Workshop Wrap-up & Optional Happy Hour

Data analysis

Estimating things of interest

Quantifying uncertainty

Using models to make decisions/test hypotheses

Using models to make predictions

Model comparison

What you won't see

Test statistics and p-values

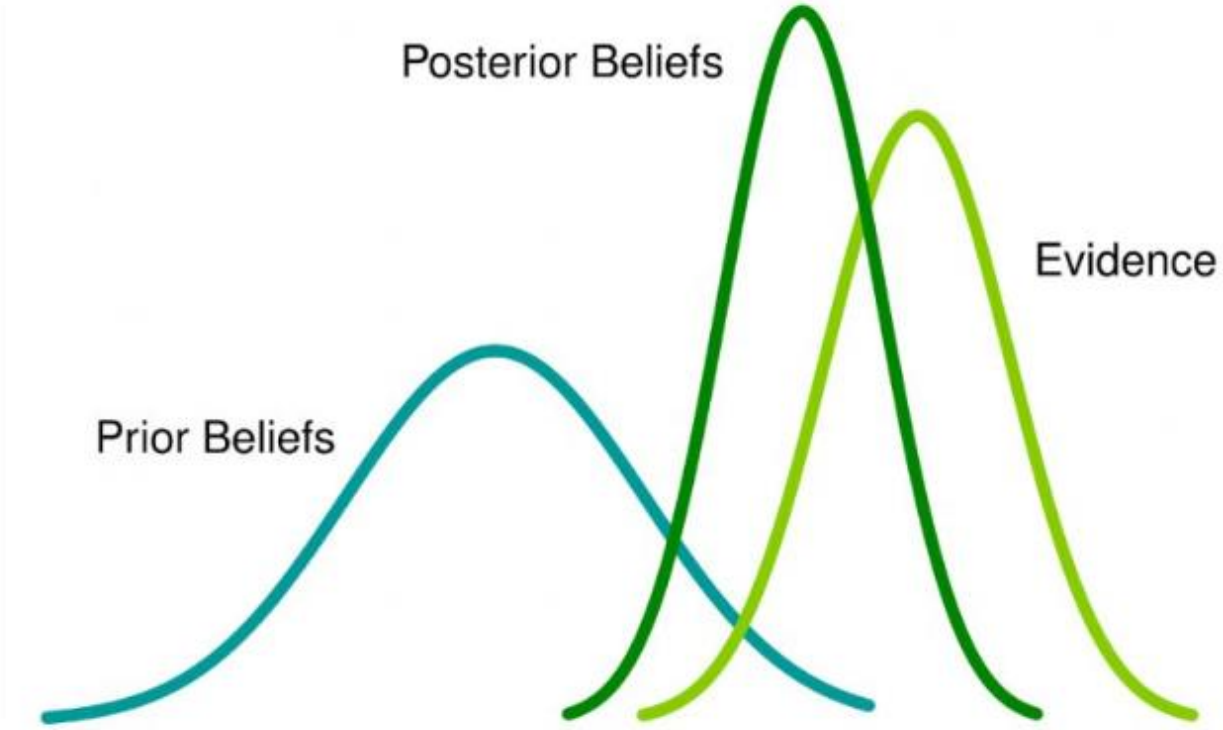
- These are grounded in null hypothesis (usually 0) significance testing

Model selection (AIC) or fit (R^2) criteria

- However, equivalent measures can be calculated

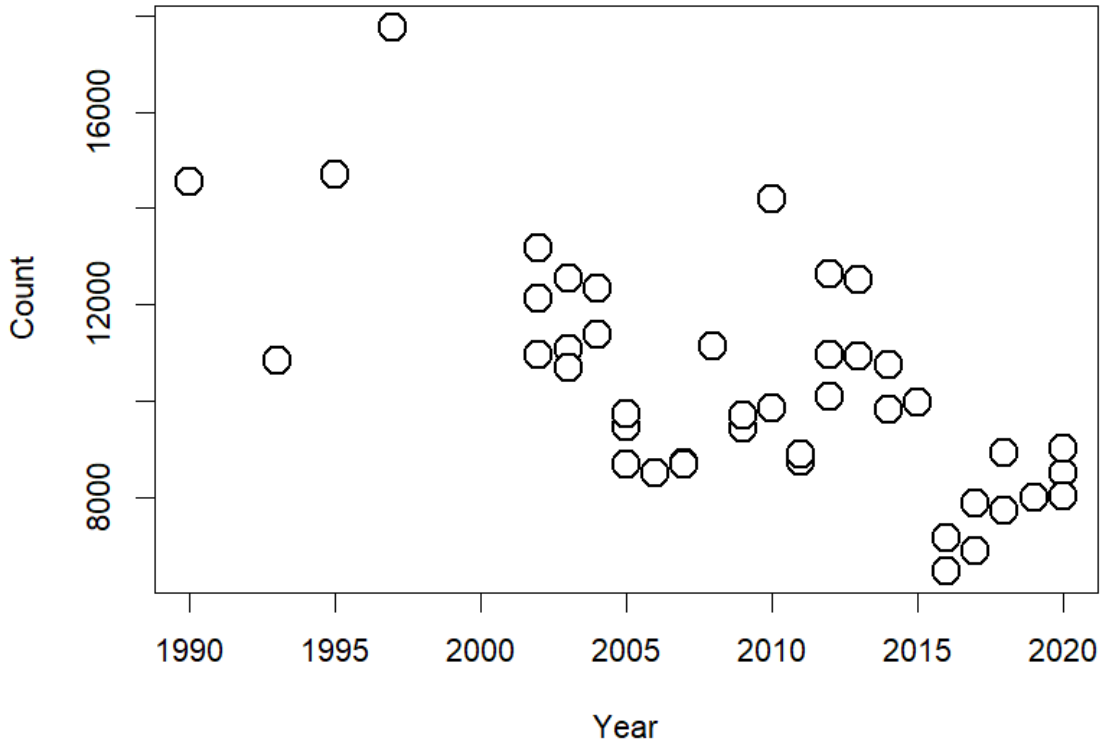
What do we use?

```
Coefficients:
             Estimate Std. Error z value Pr(>|z|)
(Intercept) 48.6190414  0.3953319  122.98   <2e-16 ***
year        -0.0196054  0.0001969  -99.58   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for poisson family taken to be 1)

    Null deviance: 21187  on 42  degrees of freedom
Residual deviance: 11484  on 41  degrees of freedom
AIC: 11963
```


Posterior Beliefs
Evidence
Prior Beliefs

Interpreting Bayesian Models ...with murres!

Example: Estimating trend in murre abundance



Example: Estimating trend in murre abundance

Need a model for Counts ~ f(time)

Bayesian version of a poisson GLM

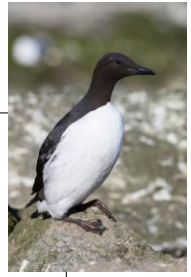
Functional part

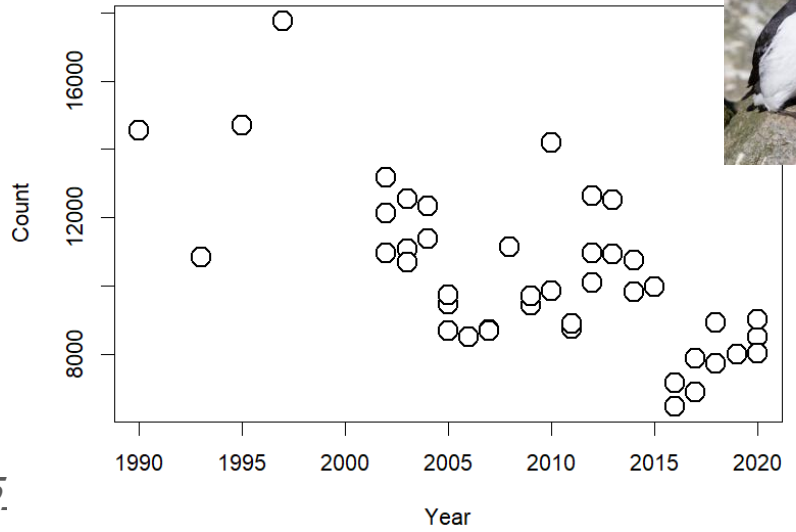
$$log(\lambda_t) = \beta_0 + \beta_1 t$$

Observation part

$$C_t \sim poisson(\lambda_t)$$

[For argument's sake, we'll set t = 0 @ 2005





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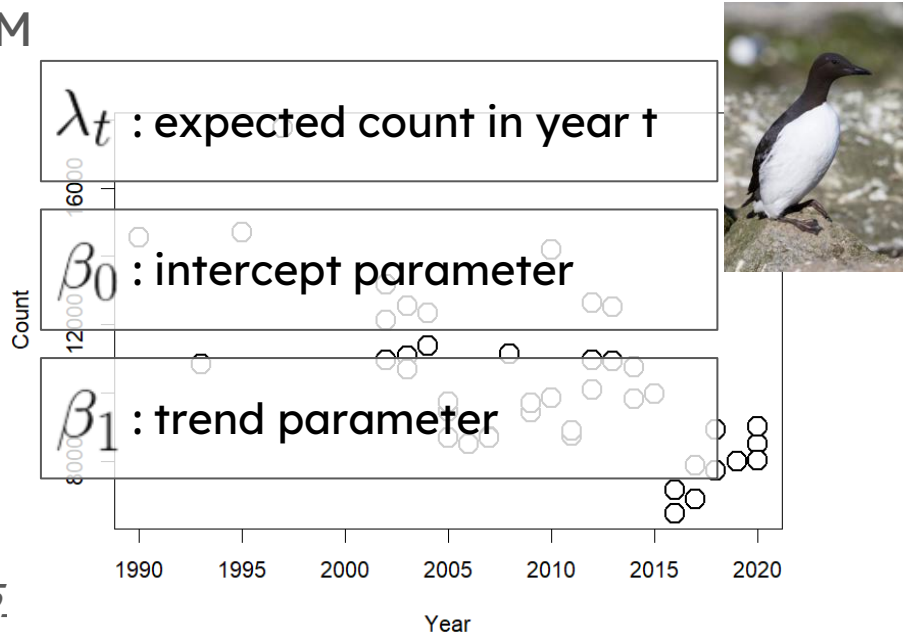


Example: Estimating trend in murre abundance

Need a model for Counts ~ f(time)

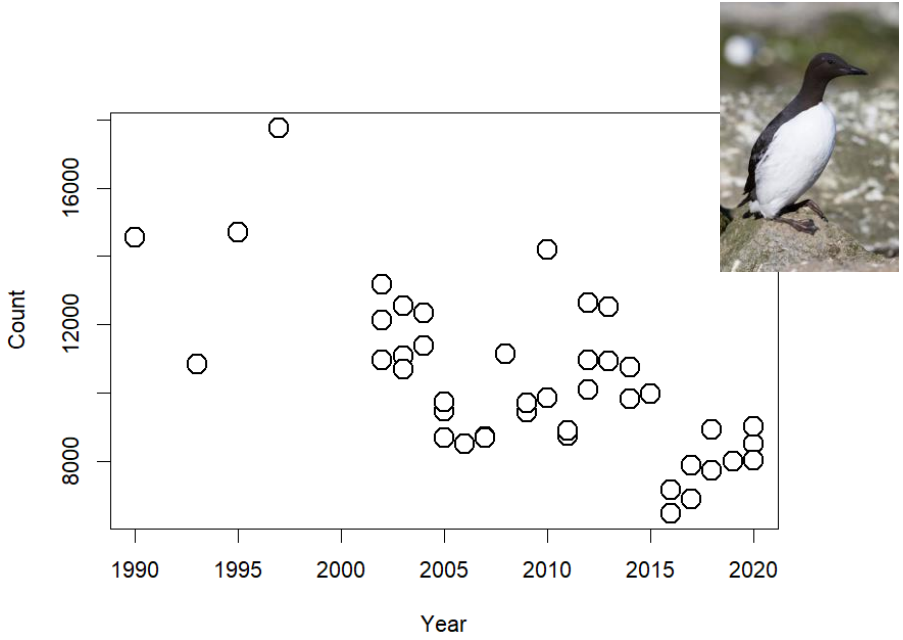
Model has two parameters

β_0 - Intercept (on log scale)

β_1 - Trend (on log scale)

Need to define priors for both

- What are our initial beliefs



Priors

Imagine you hadn't seen the data, but know the context

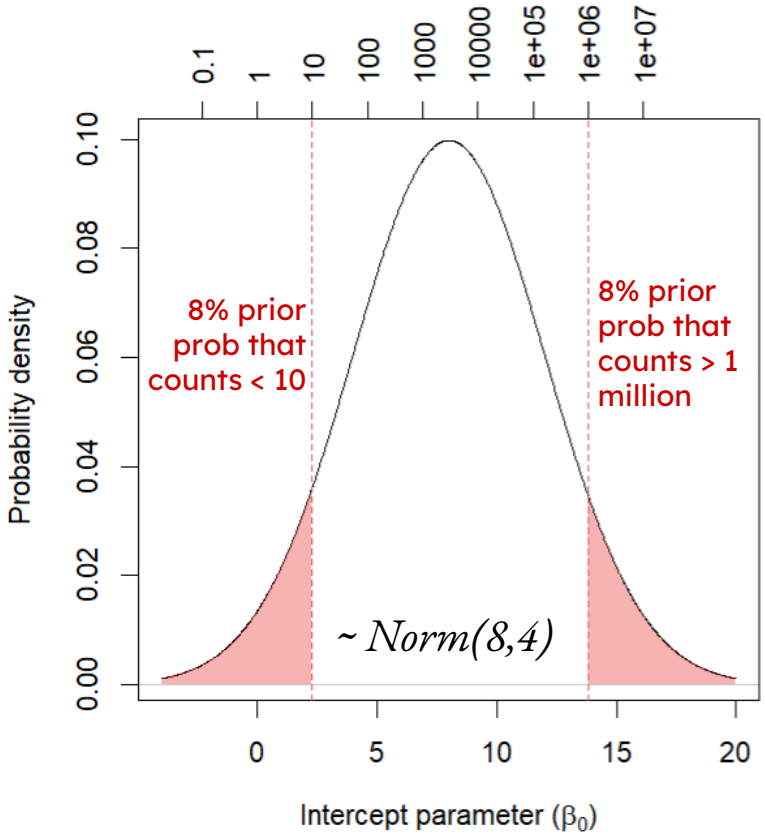
- Murre colony monitoring
- Estimation of trend through time

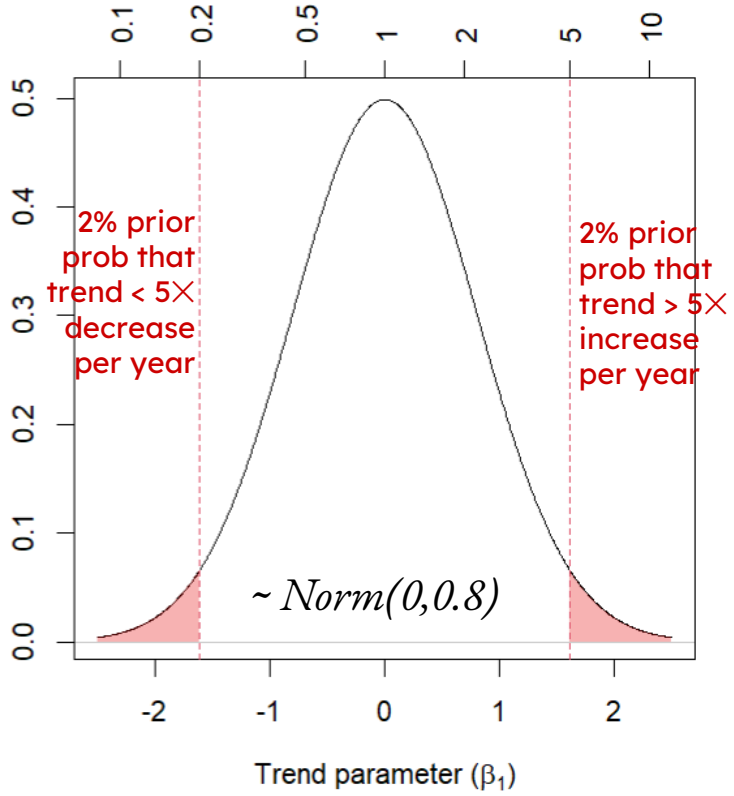
Our priors should encode some information we have initially

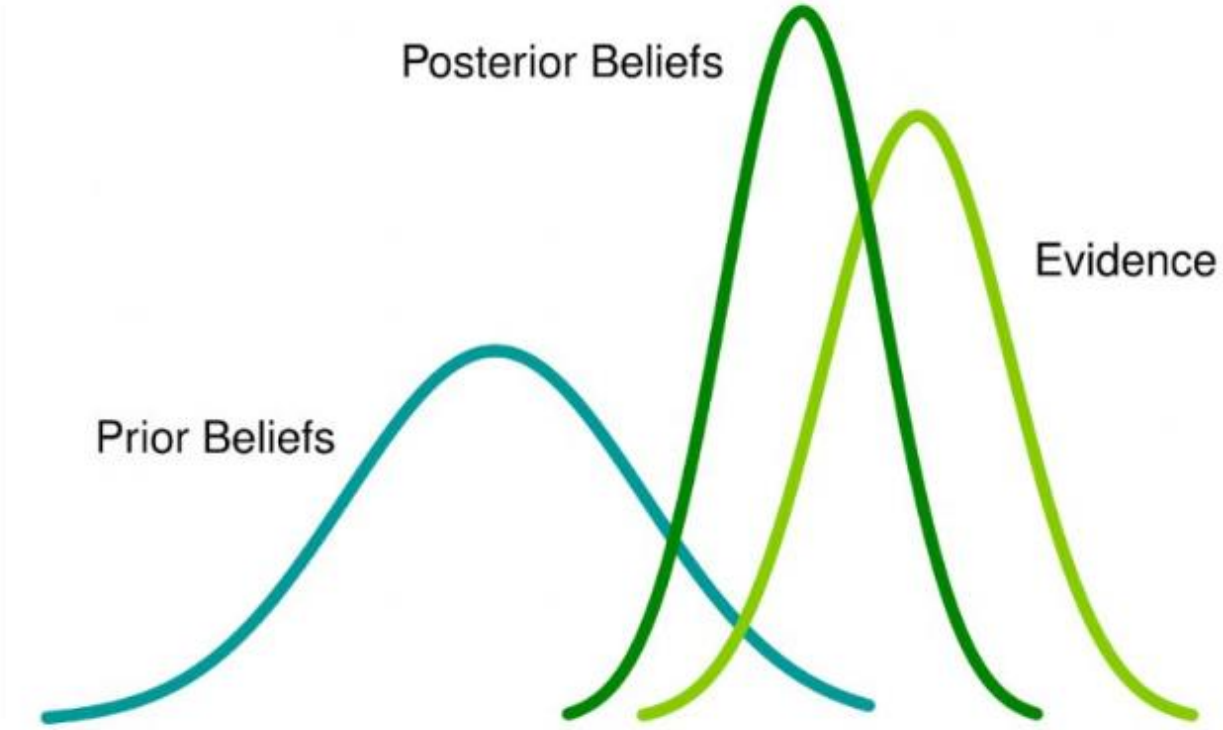
- Murre colonies: range in size from 10's to 100,000's of individuals
  - Our initial belief in the intercept should reflect this
- Trends: $exp(\beta_1)$ represents per annum (relative) growth rate
  - A consistent and sustained 5-fold per annum change in either direction seems unlikely

Within a Bayesian model we convert these to a probability distribution

Priors


0.1
1
10
100
1000
10000
1e+05
1e+06
1e+07
Probability density
0.00
0.02
0.04
0.06
0.08
0.10
8% prior prob that counts < 10
8% prior prob that counts > 1 million
~ Norm(8,4)
0
5
10
15
20
Intercept parameter (β0)

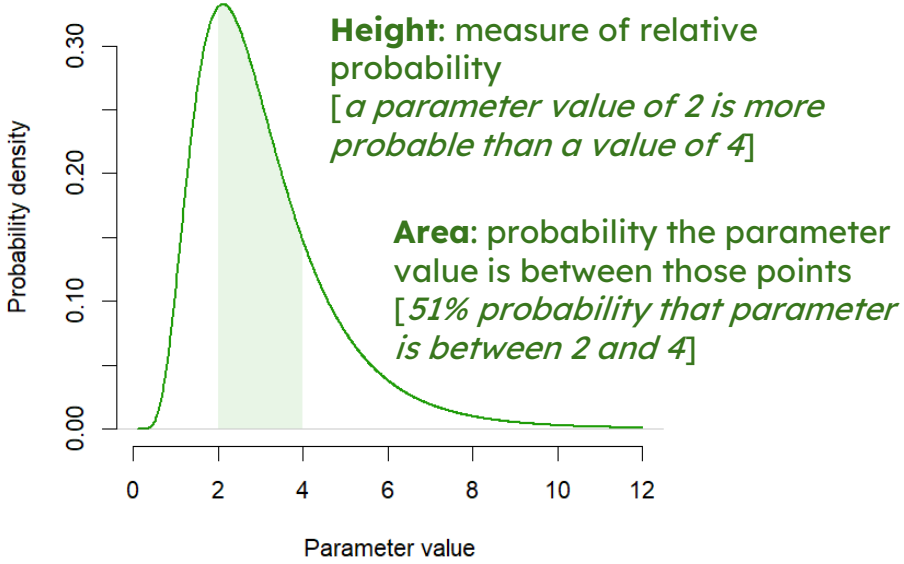

0.1
0.2
0.5
1
2
5
10
0.0
0.1
0.2
0.3
0.4
0.5
2% prior prob that trend < 5× decrease per year
2% prior prob that trend > 5× increase per year
~ Norm(0,0.8)
-2
-1
0
1
2
Trend parameter (β1)


Posterior Beliefs
Evidence
Prior Beliefs

What is a posterior distribution?

Posterior distribution - probability distribution representing our updated (prior + data) belief in a parameters value for a particular model

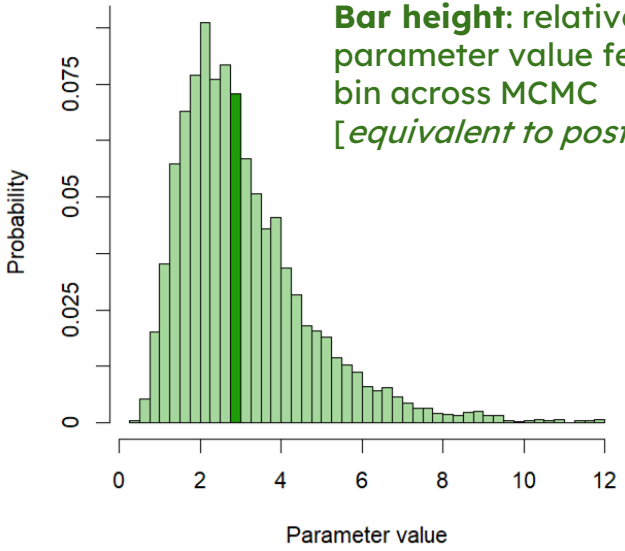
What we are trying to estimate



Height: measure of relative probability
[*a parameter value of 2 is more probable than a value of 4*]

Area: probability the parameter value is between those points
[*51% probability that parameter is between 2 and 4*]

What we get from MCMC



Bar height: relative frequency parameter value fell within bin across MCMC
[*equivalent to posterior prob*]

Data analysis

Estimating things of interest

Quantifying uncertainty

Using models to make decisions/test hypotheses

Using models to make predictions

Model comparison

Posterior distribution

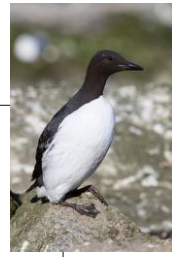
When interpreting a parameter estimated via a Bayesian model we use the posterior distribution

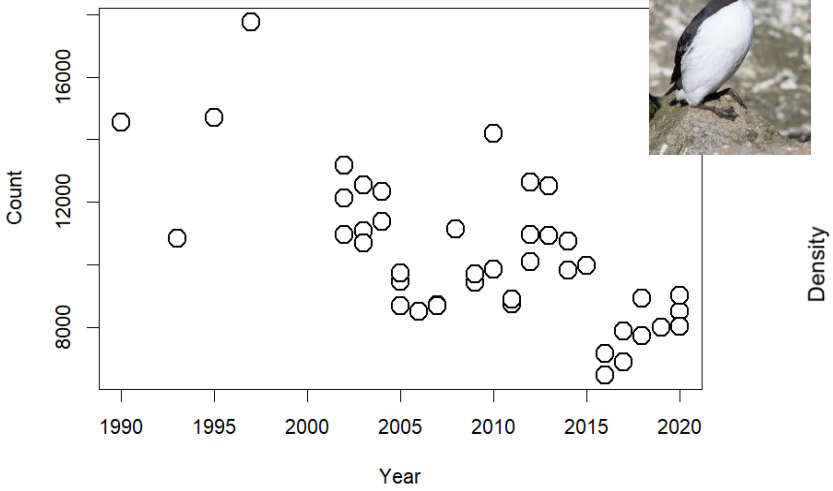
Important to note that the Bayesian interpretation of a parameter **IS the posterior distribution** - the probability distribution among different values

- Reduction to single point-value and/or interval will lose information
- But often it is a necessary step

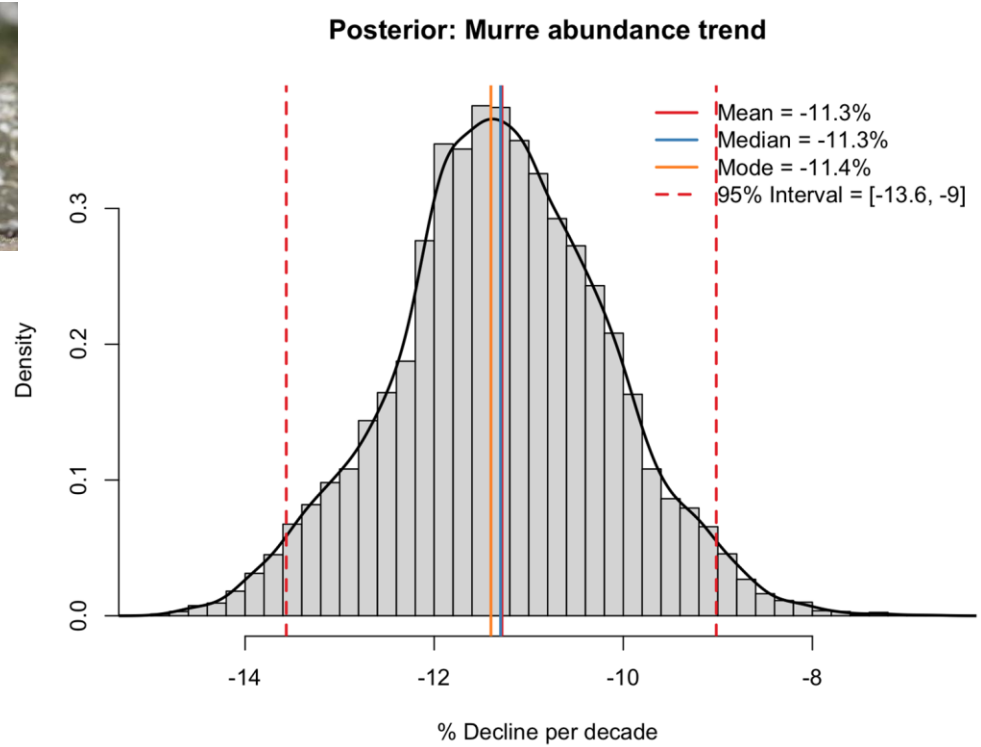
Should always evaluate the posterior distribution shape as part of analyses

Example: Estimating trend in murre abundance

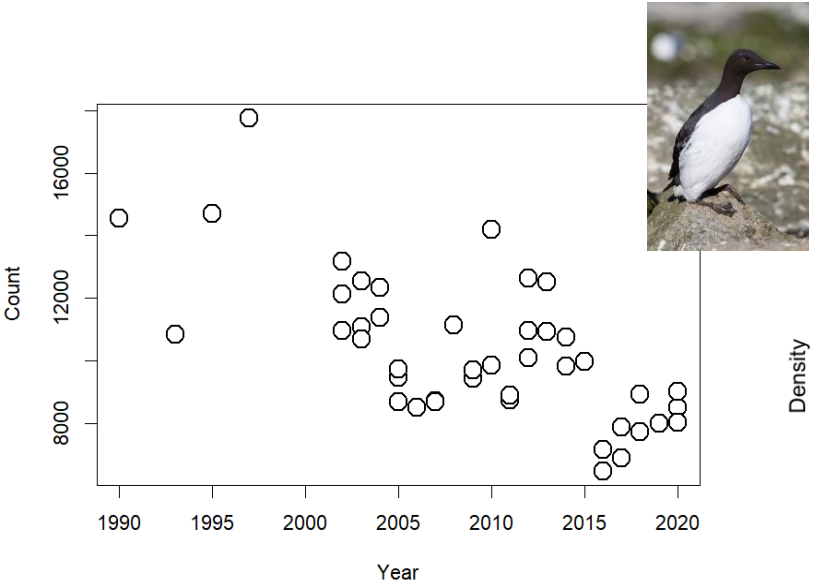




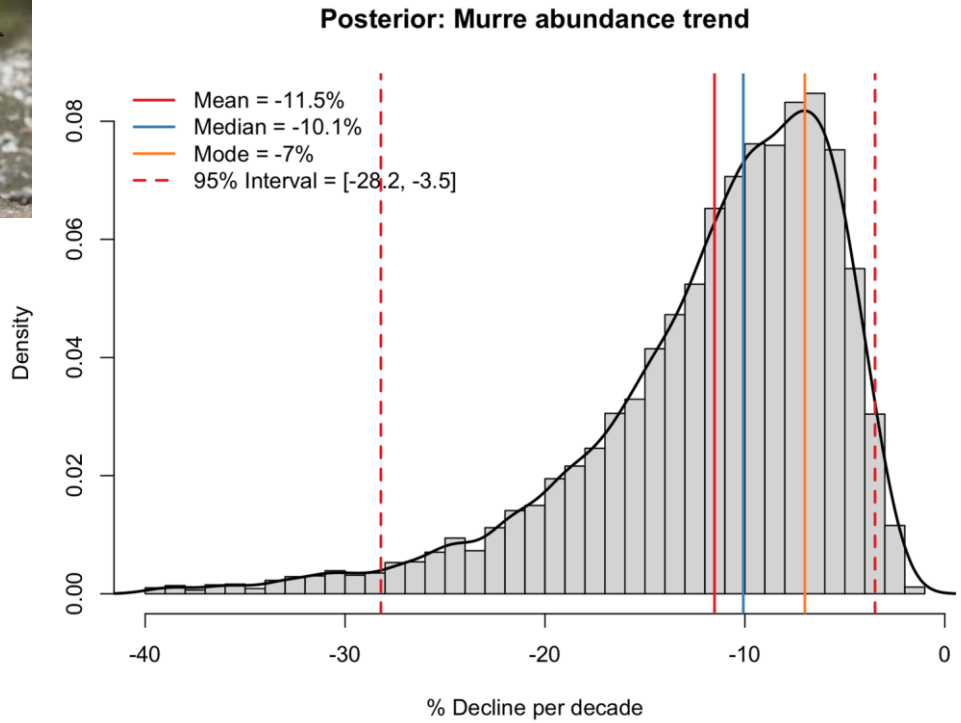
Point estimates: mean, median, mode



Example: Estimating trend in murre abundance



Point estimates: mean, median, mode



Data analysis

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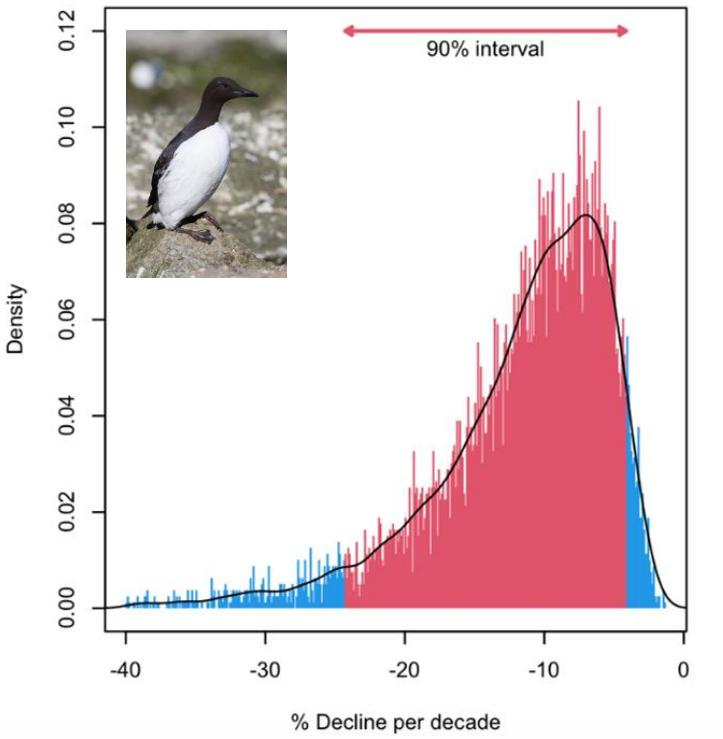
Model comparison

Bayesian credible intervals

Find the range of parameter values corresponding to certain cumulative probability

- 90% probability that trend is between 4% and 24% decline per decade
- 50% probability that trend is between 6% and 15% decline per decade etc



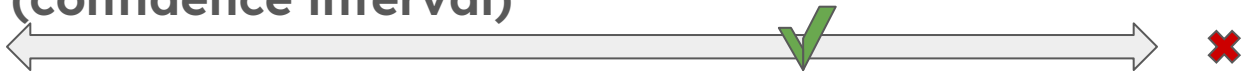
Confidence intervals and Bayesian intervals

Suppose a 95% interval

Bayesian (credible interval, compatibility interval)

Interpretation: there is 95% probability that the parameter lies in this range (given the model and data)

Frequentist (confidence interval)



Interpretation: in 95% of data collection repetitions the interval calculated from the data overlaps the ‘true’ value.

95% confidence does not mean there is a 95% probability that the parameter lies within the interval. Once an interval is calculated, it either covers the parameter value or it does not; it is no longer a matter of probability.

Data analysis

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Model comparison

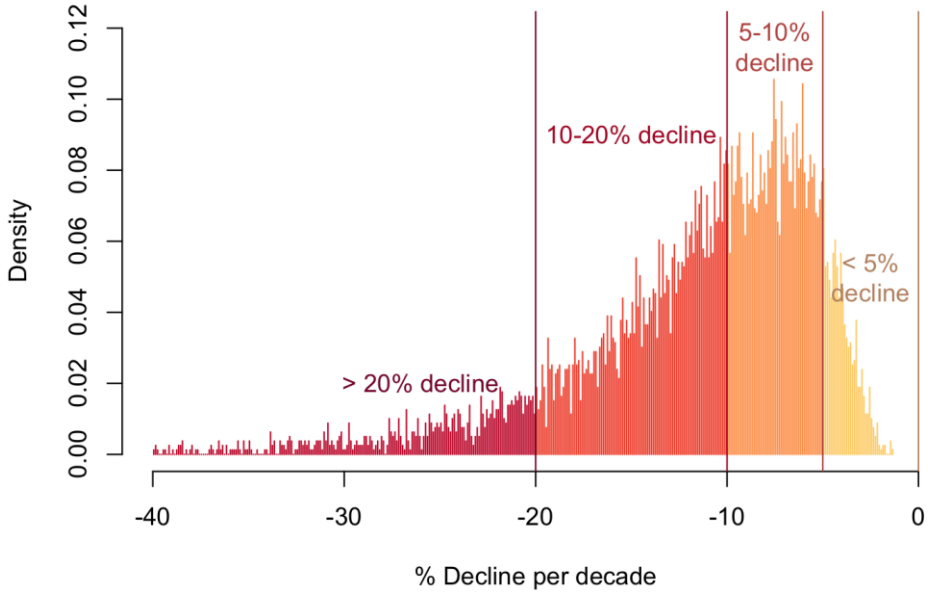
Measuring support for alternative rates

One advantage of Bayesian modelling comes in the ability to measure support for different ranges of parameter values

- i.e., what's the probability of decline > 10%

Allows for informed decision making based on probability

Can also be used for meaningful hypothesis testing



Data analysis

Estimating things of interest

Quantifying uncertainty

Using models to make decisions/test hypotheses

Using models to make predictions

Model comparison

Posterior prediction

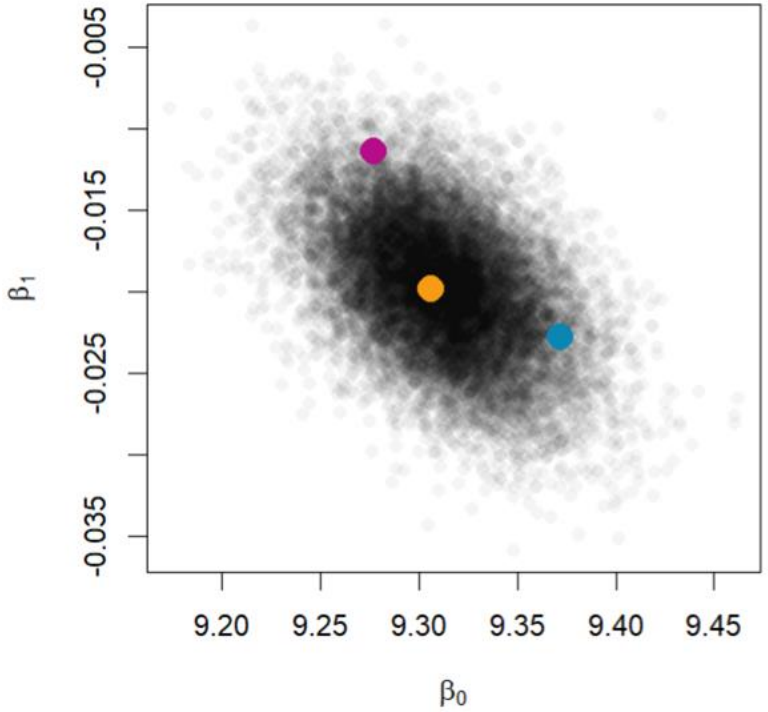
If you can fit a Bayesian model, you can generate values from it

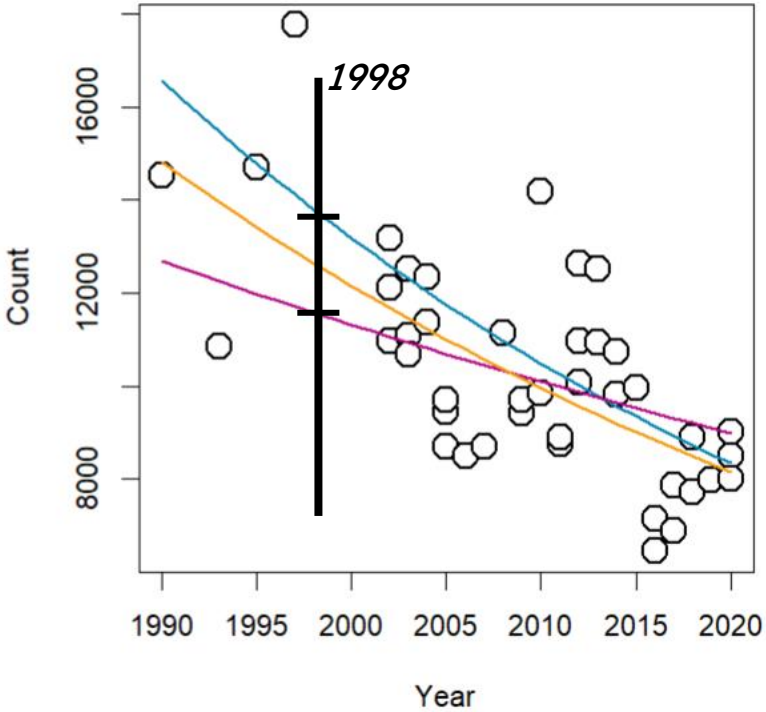
Posterior prediction

- Take parameter values from the posterior distribution and use those to generate values for a given scenario according to the model
- Propagates uncertainty of model parameters into predictions
    - i.e. fisheries model forecasts are built on this
- Posterior predictions can be treated as a posterior distribution
    - What is our posterior belief regarding a certain prediction
    - Allows identification of probability that predictions are above/below certain threshold

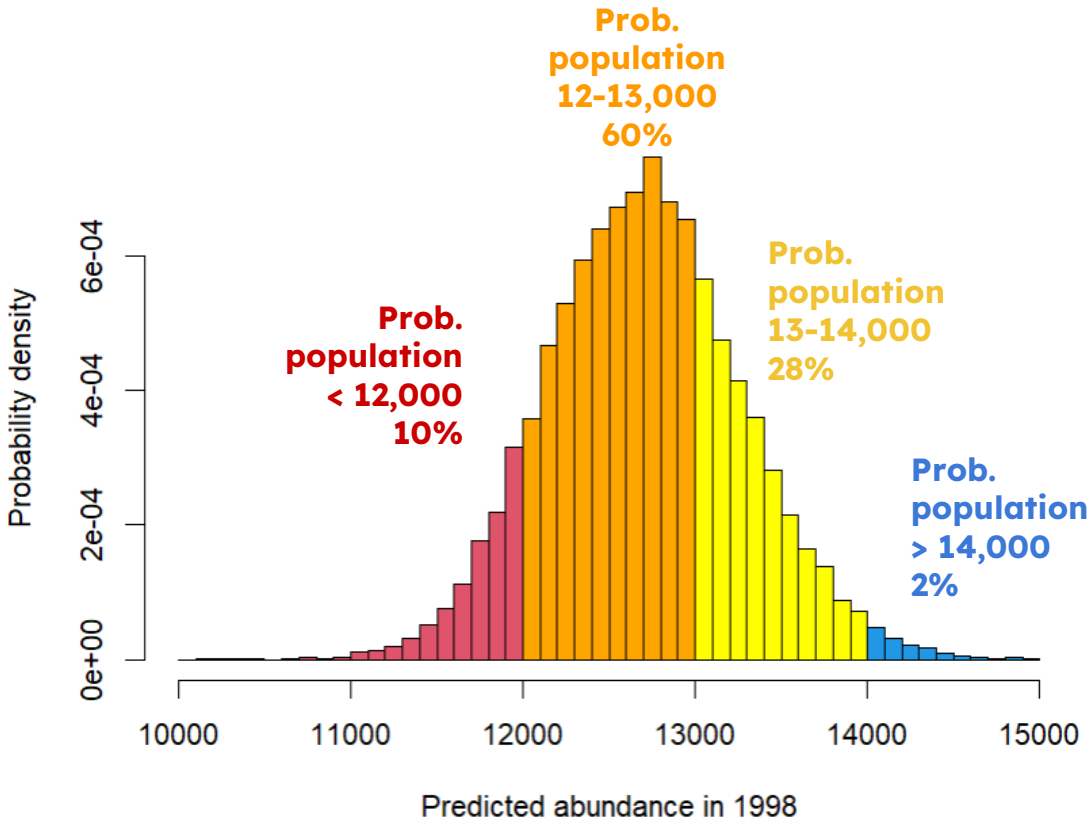
Posterior prediction

What was the expected population size in 1998?





Posterior prediction



Posterior prediction - assessing model fit

Posterior predictive distribution can be used to assess

- Adequacy of model fit (functional form)
- Assumptions of error distributions (how is variation accounted for)
- unusual observations not accounted for by the model

Data analysis

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Model comparison

Model comparison

Model comparison criteria choice should be driven by modelling purpose

- Predictive ability: cross-validation
- Model parsimony: information criterion

Both are possible using Bayesian models, and tools exist for each

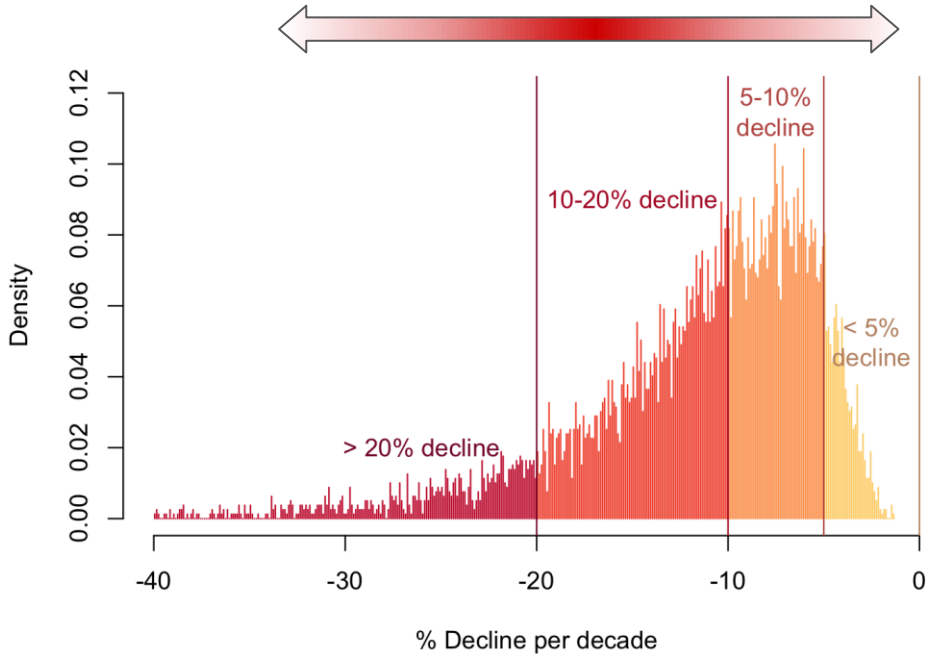
Criteria have different names and are calculated differently, but the principle is identical (model fit penalised by complexity)

- Deviance information criterion
- Widely applicable Bayesian information criterion (WBIC or WAIC)

Key takeaways

Posterior distribution: probabilities across different parameter values

- Point estimates: mode = most probable, median = 50:50% split
- Credible intervals: more natural way to interpret uncertainty
- Support for alternate values: allows more/better options for decision making
- Posterior predictive distribution: propagation of uncertainty
- Model comparison: comparable tools exist, and follow similar approach



Questions?

Building Your Own Models

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Building the murre model

We will show you the pieces of the murre model written in Stan

Why Stan?

- Quick and efficient
- Gives plenty of warnings when models are poorly specified
- Uses a fancy MCMC algorithm that can explore parameter space more completely
- Can handle classes of models that would take other software prohibitively long run-times

Building the murre model

Fitting a Bayesian GLM is almost as simple as using `glm()`

`rstanarm`

- Contains Bayesian equivalents of typical LMs, GLMs, GLMERs, GAMs
- Uses the same syntax, with the additions of defining prior distributions
- This might be all you need to get started

```
# The simple way
murre_stan <- stan_glm(count ~ year, data = murre.df,
                       family = poisson(link="log"),
                       prior_intercept = normal(8,4),
                       prior = normal(0,0.8),
                       iter = 2000,
                       warmup = 1000,
                       chains = 4)
```

Building the murre model

Some unfamiliar components

- `iter`: total number of iterations (including warm-up)
- `warmup`: number of warmup iterations
- `chains`: number of MCMC chains

```
# The simple way
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                       iter = 2000,
                       warmup = 1000,
                       chains = 4)
```

Model summary and diagnostics

```
> # examine coefficients
> print(murre_stan, digits=4)
stan_glm
 family:       poisson [log]
 formula:      count ~ year.off
 observations: 43
 predictors:   2
------
            Median  MAD_SD
(Intercept)  9.3102  0.0015
year.off    -0.0196  0.0002

------

> # create 95% credible interval
> posterior_interval(murre_stan, prob=0.95)
                   2.5%       97.5%
(Intercept)  9.30703001  9.31310158
year.off    -0.01998104 -0.01921781
```

Median (point estimate)
median posterior parameter estimate

MAD_SD (measure of uncertainty)
Median absolute deviation (MAD) from the median (think of this as a more robust measure of standard deviation for variation in MCMC parameter value)

Posterior interval (credible interval)
95% credible intervals of parameter values

Model summary and diagnostics

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                  2.5%       97.5%
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year.off    -0.01998104 -0.01921781
```

```
> # View model estimates and MCMC diagnostics
> launch_shinystan(murre_stan)
```

But, what if my model isn’t simple?

Even though `rstanarm` can do most of the workhorse basic models, it is worth looking inside the machine to how the model is actually setup

Pulling back the curtain

- Next time….

We don’t want to overwhelm you, there is enough here to digest

We have provided a code-file of the murre model written in base Stan, which you are welcome to explore

Key Takeaways

Now more than ever it is easy to start in Bayes

- Model specification: if you are comfortable with GLMs, then `rstanarm` is only a teeny-tiny step up
- Model types and evaluation: has all the benefits we discussed, with very few drawbacks - could easily replace your GLM/GLMER/GAM workflow
- Getting comfortable: allows for a gentle step into Bayes to build confidence
- So much more: once you feel comfortable, diving into a Bayesian model built from scratch allows for more complex models, and is incredibly rewarding

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                       prior_intercept = normal(8,4),
                       prior = normal(0,0.8),
                       iter = 2000,
                       warmup = 1000,
                       chains = 4)
```

Questions?

Tips/Tricks and Further Information

Troubleshooting and model evaluation

This is a vast topic - we will just cover the basics!

Model convergence

- When estimating a model we need to ensure the MCMC algorithm has converged on the posterior distribution
- Possible problems arise due to local maxima ‘trapping’ the exploration process
- Local maxima: https://chi-feng.github.io/mcmc-demo/app.html?algorithm=H2MC&target=multimodal

Troubleshooting and model evaluation

Model convergence

- Always start models with multiple parallel MCMC chains (usually 4 or more)
- Examine mixing of chains (e.g., with trace plots)
- If all of the chains end up in the same spot, then we are more confident that the model found the solutions
- Can also calculate measures of within/between chain parameter estimates: R-hat statistic (ideally as close to 1 as possible, or < 1.05)

Troubleshooting and model evaluation

Capturing the extent of possible solutions

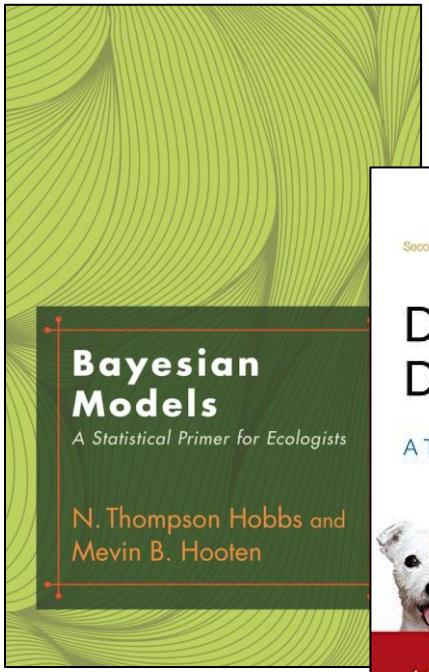
- Running a model for too few iterations can mean that there hasn't been enough time to explore the full range of solutions
    - We want to capture the most probable parameter values, but also the tails in order to estimate uncertainty
- Some software will complain about effective sample size (or tail effective sample size) - the number of estimates for a given parameter
- We ideally want this to be high
- Solution: run the model a little while longer

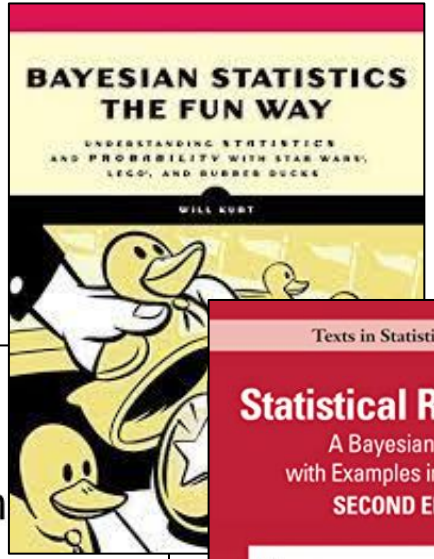
Troubleshooting and model evaluation

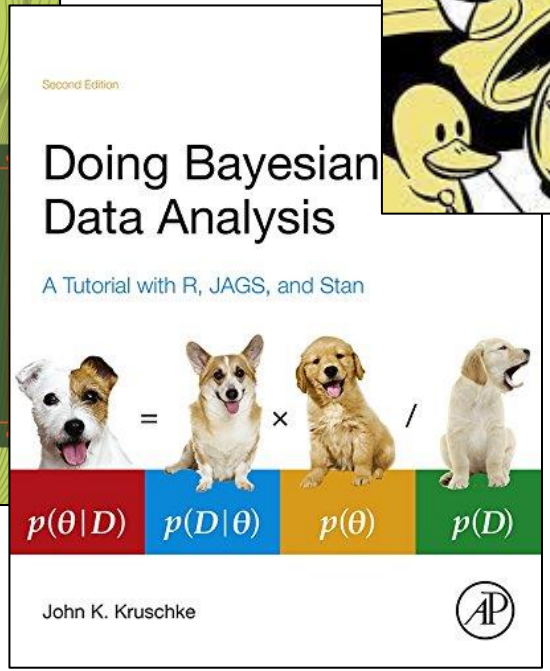
Model identifiability and re-parameterisation

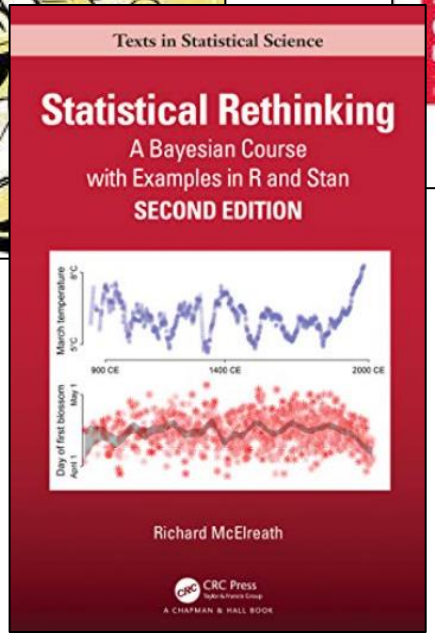
- Sometimes a model is so complex that parameters cannot be estimated effectively
- Often this can be remedied via a re-parameterisation (i.e. modelling log(B) rather than B), or re-scaling/standardisation of predictors/response

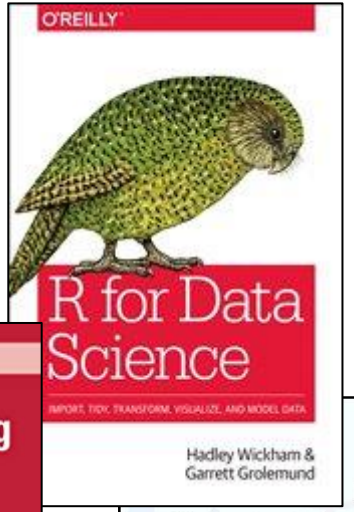
Resources

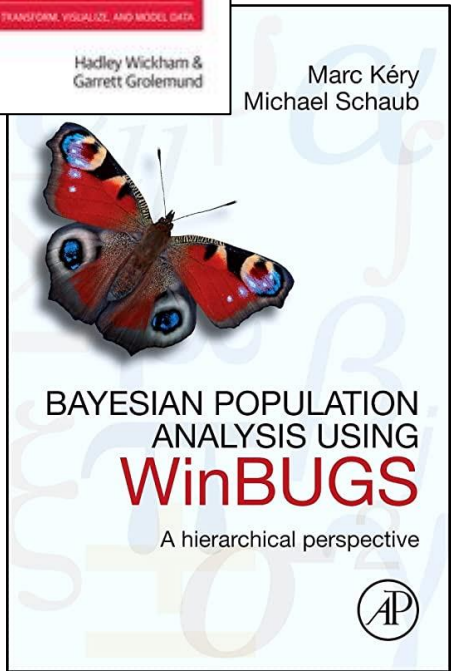

Bayesian Models
A Statistical Primer for Ecologists
N. Thompson Hobbs and Mevin B. Hooten


BAYESIAN STATISTICS THE FUN WAY
WILL KURT


Second Edition
Doing Bayesian Data Analysis
A Tutorial with R, JAGS, and Stan
$p(\theta|D)$ $p(D|\theta)$ $p(\theta)$ $p(D)$
John K. Kruschke


Texts in Statistical Science
Statistical Rethinking
A Bayesian Course with Examples in R and Stan
SECOND EDITION
Richard McElreath
CRC Press
A CHAPMAN & HALL BOOK


O'REILLY
R for Data Science
Hadley Wickham & Garrett Grolemund


Marc Kéry
Michael Schaub
BAYESIAN POPULATION ANALYSIS USING WinBUGS
A hierarchical perspective

Other online resources/examples

Bayesian statistics with R

Bayesian Inference 2019

MCMC and the case of the spilled seeds

A Zero-Math Introduction to Markov Chain Monte Carlo Methods

Further reading in the seabird field

Papers in our field that use Bayesian stats

Bertram, D. F., M. C. Drever, M. K. McAllister, B. K. Schroeder, D. J. Lindsay, and D. A. Faust. 2015. Estimation of coast-wide population trends of Marbled Murrelets in Canada using a Bayesian hierarchical model. *PLOS ONE* 10:e0134891.

Bertram, D. F., L. Wilson, K. Charleton, A. Hedd, G. J. Robertson, J. L. Smith, K. H. Morgan, and X. J. Song. 2021. Modelling entanglement rates to estimate mortality of marine birds in British Columbia commercial salmon gillnet fisheries. *Marine Environmental Research* 166:105268.

Good, T. P., J. E. Jannot, K. A. Somers, and E. J. Ward. 2022. Using Bayesian time series models to estimate bycatch of an endangered albatross. *Fisheries Research* 256:106492.

Horswill, C., M. J. Wood, and A. Manica. 2022. Temporal change in the contribution of immigration to population growth in a wild seabird experiencing rapid population decline. *Ecography* 2022:e05846.

Lahoz-Monfort, J. J., B. J. T. Morgan, M. P. Harris, F. Daunt, S. Wanless, and S. N. Freeman. 2013. Breeding together: modeling synchrony in productivity in a seabird community. *Ecology* 94:3–10.

Further reading in the seabird field

Richard, Y., L. Perriman, C. Lalas, and E. R. Abraham. 2015. Demographic rates of northern royal albatross at Taiaroa Head, New Zealand. *PeerJ* 3:e906.

Sarzo, B., C. Armero, D. Conesa, J. Hentati-Sundberg, and O. Olsson. 2019. Bayesian immature survival analysis of the largest colony of Common Murre (*Uria aalge*) in the Baltic Sea. *Waterbirds* 42:304.

Sherley, R. B., R. J. M. Crawford, A. D. de Blocq, B. M. Dyer, D. Geldenhuys, C. Hagen, J. Kemper, A. B. Makhado, L. Pichegru, D. Tom, L. Upfold, J. Visagie, L. J. Waller, and H. Winker. 2020. The conservation status and population decline of the African penguin deconstructed in space and time. *Ecology and Evolution* 10:8506–8516.

Stantial, M. L., J. B. Cohen, A. J. Darrah, S. Farrell, and B. Maslo. 2021. Habitat-specific behavior, growth rate, and survival of piping plover chicks in New Jersey, USA. *Ecosphere* 12:e03782.

Tinker, M. T., K. M. Zilliacus, D. Ruiz, B. R. Tershy, and D. A. Croll. 2022. Seabird meta-Population Viability Model (mPVA) methods. *MethodsX* 9:101599.

Further reading in the seabird field

Tucker, A. M., C. P. McGowan, J. E. Lyons, A. DeRose-Wilson, and N. A. Clark. 2021. Species-specific demographic and behavioral responses to food availability during migratory stopover. *Population Ecology*:1438–390X.12094.

van de Schoot, R., S. Depaoli, R. King, B. Kramer, K. Märtens, M. G. Tadesse, M. Vannucci, A. Gelman, D. Veen, J. Willemsen, and C. Yau. 2021. Bayesian statistics and modelling. *Nature Reviews Methods Primers* 1:1–26.

…and many more!

Workshop Wrap-Up

Seabird data is messy.

Some of the challenges include:

- Difficult to get to colonies
- Hard to find individuals and/or hard to count them all
- Individuals are inaccessible during portions of life history

Which can lead to small sample sizes, gaps, and uncertainty in our datasets.

Good news, many of these issues can be addressed using Bayesian methods!

Workshop Goals

1. Recognize **value of Bayesian framework** for seabird research.
2. Understand **fundamental differences** between Frequentist and Bayesian frameworks.
3. Learn **basic components** of a Bayesian analysis.
4. Learn how to **interpret results** from a Bayesian analysis.
5. Build **community and share resources** for future learning.

Where to from here?

- Additional materials to be shared soon (extended version of the workshop presentation, Stan installation information, R script to run the murre model)
- Opt-in email listserv to ask questions, share resources, etc. - let us know if you are interested!
- Gauging interest in establishment of Analytical Technical Committee
- Last but not least…

Optional Happy Hour
Shore Rider Bar
2168 Avenida De La Playa
La Jolla, CA 92037