Q1.

1. Show $m(a+bx) = a+b*m(x)$

$$m(a+bx) = \frac{1}{N}\sum_{i=1}^{N}(a+bx_i)$$

$$= \frac{1}{N}\left(\sum_{i=1}^{N} a + \sum_{i=1}^{N} bx_i\right) = \frac{1}{N}\left(Na + b*\sum_{i=1}^{N} x_i\right) =$$

$$= a + b\underbrace{\frac{1}{N}\sum_{i=1}^{N} x_i}_{m(x)} = a + bm(x)$$

2. Show $cov(x, a+by) = b * cov(x,y)$

$$cov(x,y) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))(y_i - m(y))$$

$$cov(x, a+by) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))((a+by_i) - m(a+by))$$

$$= \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))((a+by_i) - \overbrace{(a+bm(y))}^{\text{shown previously}})$$

$$= \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))(b(y_i - m(y)))$$

$$= b * \underbrace{\frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))(y_i - m(y))}$$

$$= b * cov(x,y)$$

3. Show that $cov(a+bx, a+bx) = b^2 cov(x,x)$, and in particular that $cov(x,x) = s^2$

$$cov(a+bx, a+bx) = \frac{1}{N}\sum_{i=1}^{N}\left((a+bx_i) - \overbrace{m(a+bx)}^{a+bm(x)}\right)^2$$

$$= \frac{1}{N}\sum_{i=1}^{N}\left((a+bx_i) - (a+bm(x))\right)^2 = \frac{1}{N}\sum_{i=1}^{N}\left(bx_i - bm(X)\right)^2 =$$

$$= \frac{1}{N}\sum_{i=1}^{N}\left(bx_i - bm(x)\right)\left(bx_i - bm(x)\right) = b^2 \underbrace{\frac{1}{N}\sum_{i=1}^{N}\left(x_i - m(x)\right)^2}_{cov(x,x) = s^2} =$$

$$= b^2 cov(x,x)$$

4. Is a non-decreasing transformation of the median the median of a transformed variable?

↳ A non-decreasing transformation preserves the order of the data ($x \geq x'$ and $g(x) \geq g(x')$) so therefore, the median is preserved or unchanged by the transformation. Thus, this applies to any quantile as order is unchanged. The IQR remains unchanged: $IQR(g(x)) = g(IQR(x))$. The range preserves the relative size (diff. between max and min).

5. Is it always true that $m(g(x)) = g(m(x))$?

↳ No, it is not always true that $m(g(x)) = g(m(x))$ Appying a non-decreasing transformation to data can affect the spread, which in turn affects the mean b/c the mean is sensitive to every point