

# Math and the Mona Lisa

## A Study of the Golden Ratio

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# Statement

Mathematical “truths” have always been “discovered” therefore we may conclude by definition of the word ‘discover’ that all of these “truths” are already “divinely” in place just waiting for the mathematician to “find” it.

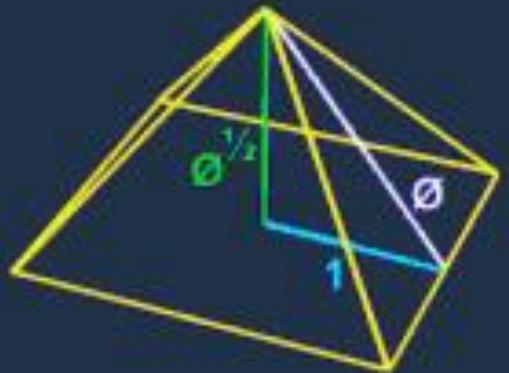
M. Mendoza

# Introduction: History of the Golden Ratio

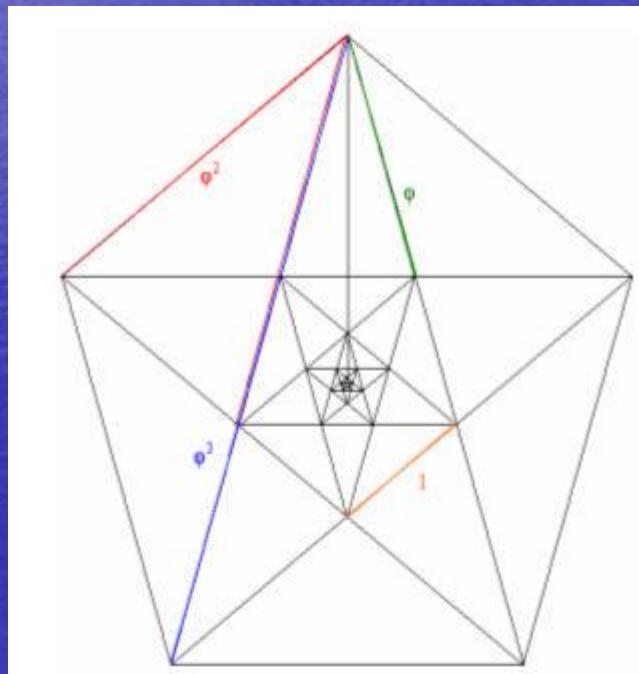
While the proportion known as the *Golden Mean* has always existed in mathematics and in the physical universe, it is unknown exactly when it was first discovered and applied by mankind. It is reasonable to assume that it has perhaps been discovered and rediscovered throughout history, which explains why it goes under several names.

# Uses in architecture date to the ancient Egyptians and Greeks

- It appears that the Egyptians may have used both pi and phi in the design of the Great Pyramids. The Greeks based the design of the Parthenon on this proportion.
- Phidias (500 BC - 432 BC), a Greek sculptor and mathematician, studied phi and applied it to the design of sculptures for the Parthenon.



- Euclid (365 BC - 300 BC), in "Elements," referred to dividing a line at the 0.6180399... point as "dividing a line in the extreme and mean ratio." This later gave rise to the use of the term mean in the golden mean. He also linked this number to the construction of a pentagram.



# The Fibonacci Series was discovered around 1200 AD

- Leonardo Fibonacci, an Italian born in 1175 AD discovered the unusual properties of the numerical series that now bears his name, but it's not certain that he even realized its connection to phi and the Golden Mean. His most notable contribution to mathematics was a work known as Liber Abaci, which became a pivotal influence in adoption by the Europeans of the Arabic decimal system of counting over Roman numerals.

# Fibonacci numbers

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233,...

The Fibonacci numbers form what we call a recurrent sequence. Beginning with 1, each term of the Fibonacci sequence is the sum of the *two preceding numbers*.

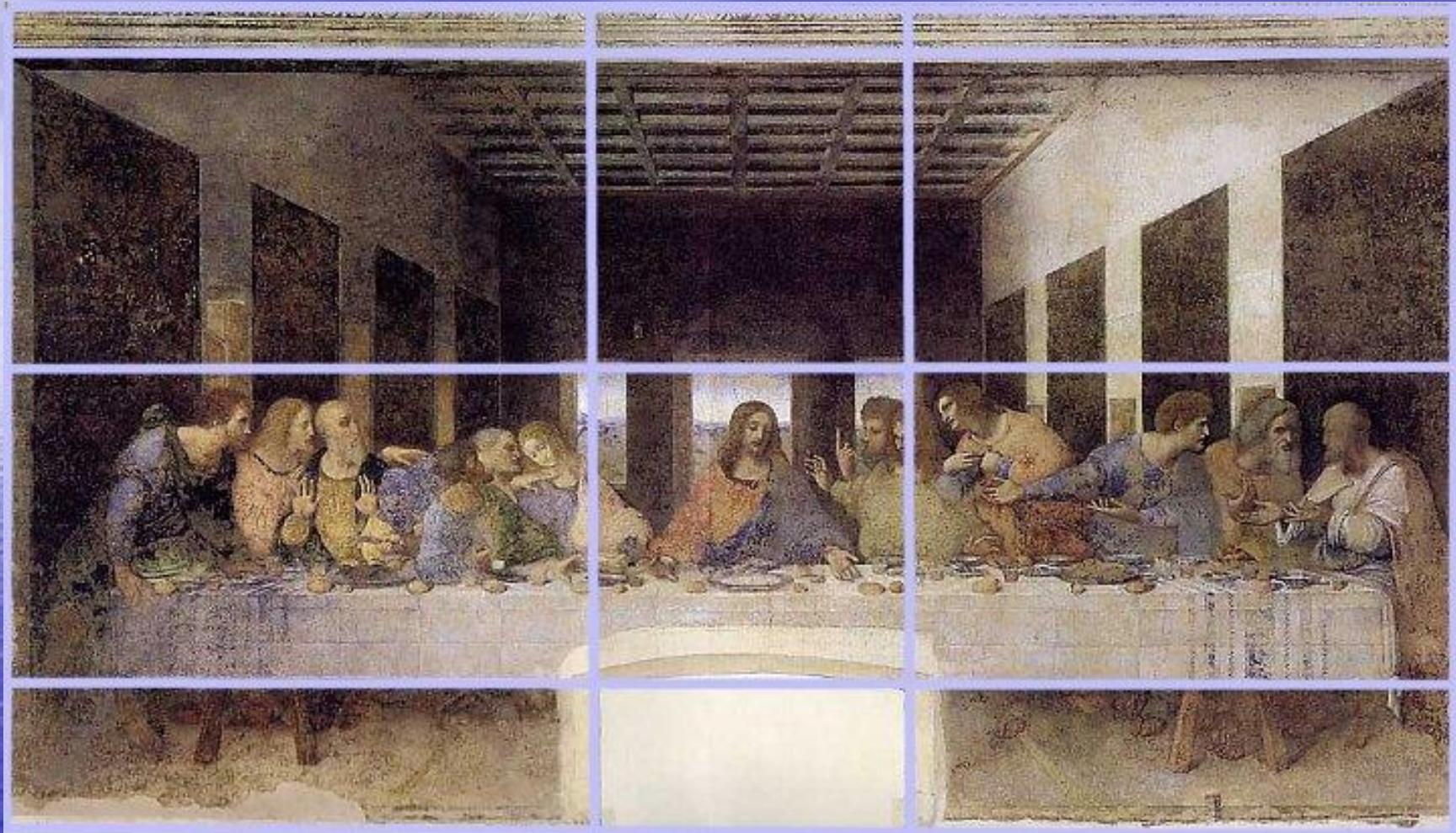
$$0+1=1 \quad 1+1=2 \quad 1+2=3 \quad 2+3=5$$

$$3+5=8 \quad 5+8=13\dots$$

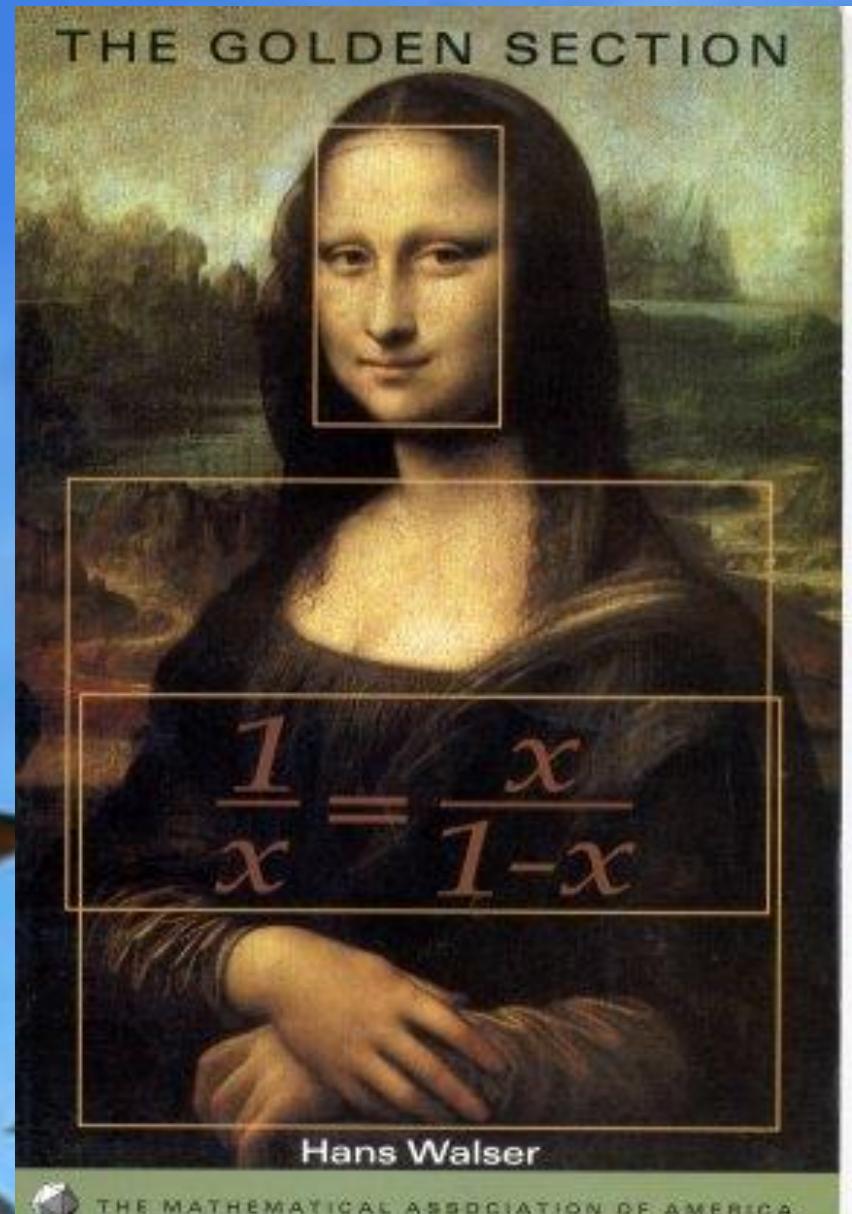
# **It was first called the "Divine Proportion" in the 1500's**

- Da Vinci provided illustrations for a dissertation published by Luca Pacioli in 1509 entitled "De Divina Proportione", perhaps the earliest reference in literature to another of its names, the "Divine Proportion." This book contains drawings made by Leonardo da Vinci of the five Platonic solids. It was probably da Vinci who first called it the "sectio aurea," which is Latin for golden section.

# ● The Last Supper

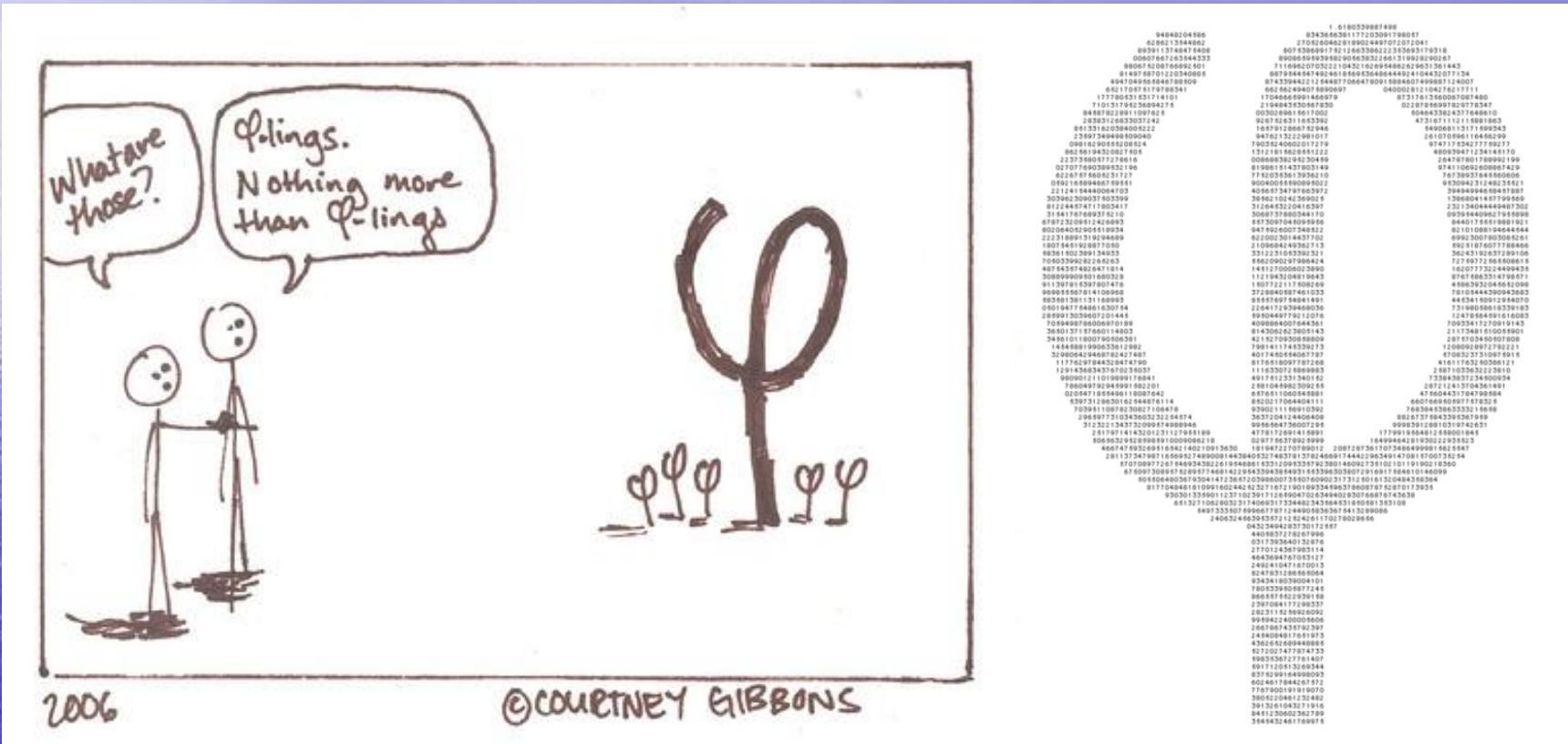


# Mona Lisa



- If I could write the beauty of your eyes  
And in fresh numbers, number all your graces  
The age to come would say, "This poet lies;"  
His heavenly touches ne'er touch'd earthly faces.  
-William Shakespeare

# Phi



# What is Phi?

Phi ( $\phi = 1.618033988749895\dots$ ), most often pronounced fi like "fly," is simply an irrational number like pi ( $\pi = 3.14159265358979\dots$ ), but one with many unusual mathematical properties. Unlike pi, which is a transcendental number, phi is the solution to a quadratic equation.

# The quadratic equation :

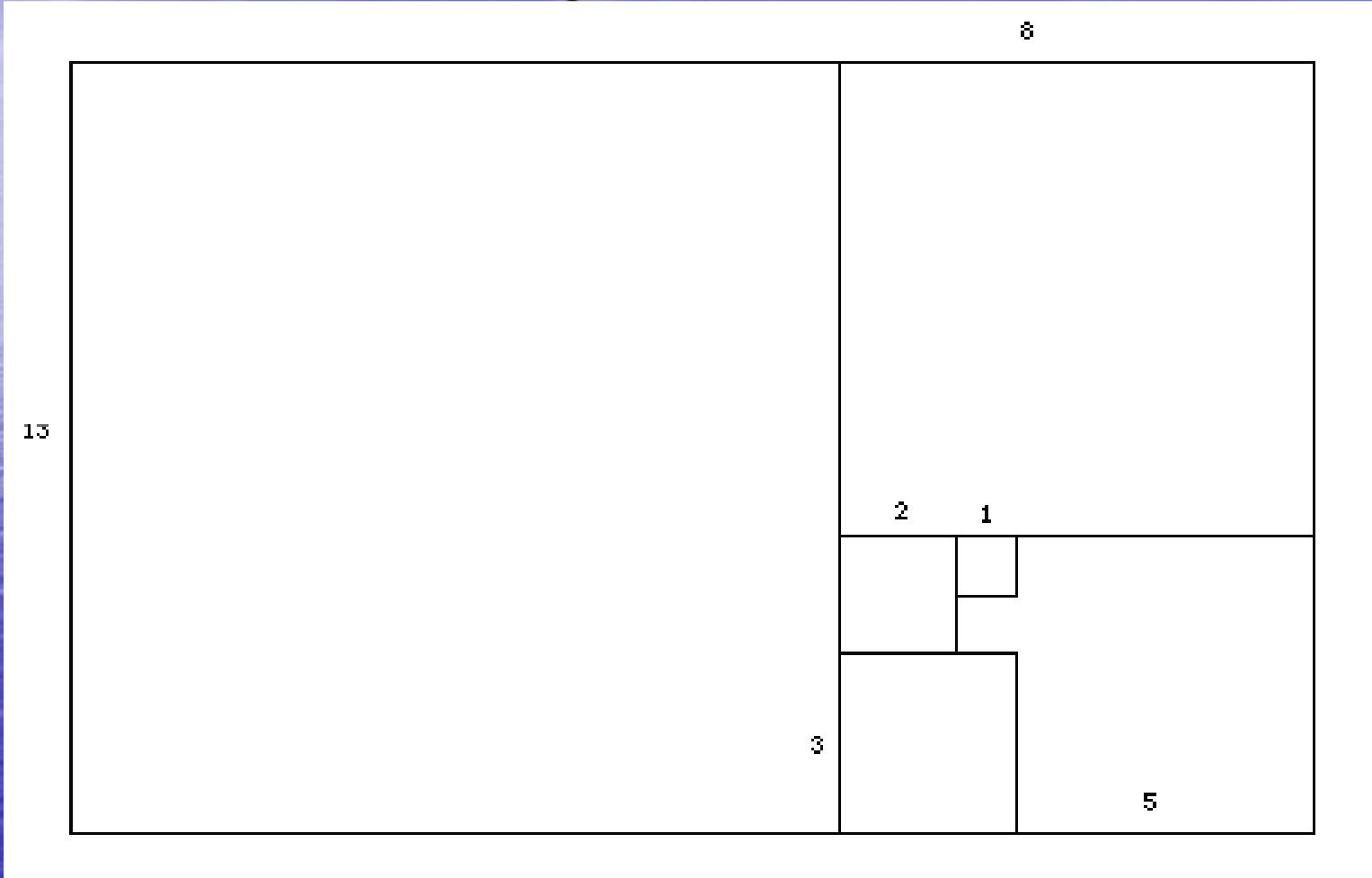
$$l^2 - lw - w^2 = 0$$

- One property of golden rectangles is that their length  $(length + width)$
- $\frac{length}{width} = \frac{length + width}{length}$
- When we cross multiply the above proportion, we get
- $l^2 = lw + w^2$  or
- $l^2 - lw - w^2 = 0.$
- Solving the equation gives us
- $l = w(1 + \sqrt{5})$
- $\frac{l}{w} = \frac{w(1 + \sqrt{5})}{w} = 1 + \sqrt{5}$
- Divide both sides by  $w^2$ :
- $\frac{l^2}{w^2} = \frac{(1 + \sqrt{5})^2}{w^2}$
- Hence;  $\Phi = 1.61803398874989484820.....$

# Phi and the Fibonacci Series

- **Leonardo Fibonacci discovered the series which converges on phi**
- In the 12th century, Leonardo Fibonacci discovered a simple numerical series that is the foundation for an incredible mathematical relationship behind phi.
- Starting with 0 and 1, each new number in the series is simply the sum of the two before it.
- 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, . . .
- The ratio of each successive pair of numbers in the series approximates phi (1.618. . .) , as 5 divided by 3 is 1.666..., and 8 divided by 5 is 1.60.
- The table below shows how the ratios of the successive numbers in the Fibonacci series quickly converge on Phi. After the 40th number in the series, the ratio is accurate to 15 decimal places.
- **1.618033988749895 . . .**

# The Golden Rectangle thru Fibonacci's sequence:



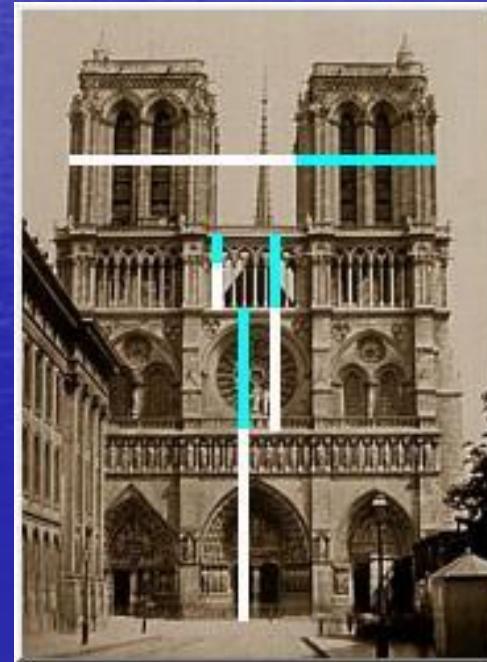
# Determining the nth number of the Fibonacci series

- You can use phi to compute the nth number in the Fibonacci series ( $f_n$ ):
- $f_n = \Phi^n / 5^{1/2}$
- As an example, the 40th number in the Fibonacci series is 102,334,155, which can be computed as:
- $f_{40} = \Phi^{40} / 5^{1/2} = 102,334,155$
- This method actually provides an estimate which always rounds to the correct Fibonacci number.
- You can compute any number of the Fibonacci series ( $f_n$ ) exactly with a little more work:
- $f_n = [\Phi^n - (-\Phi)^{-n}] / (2\Phi - 1)$
- Note:  $2\Phi - 1 = 5^{1/2} = \text{The square root of } 5$

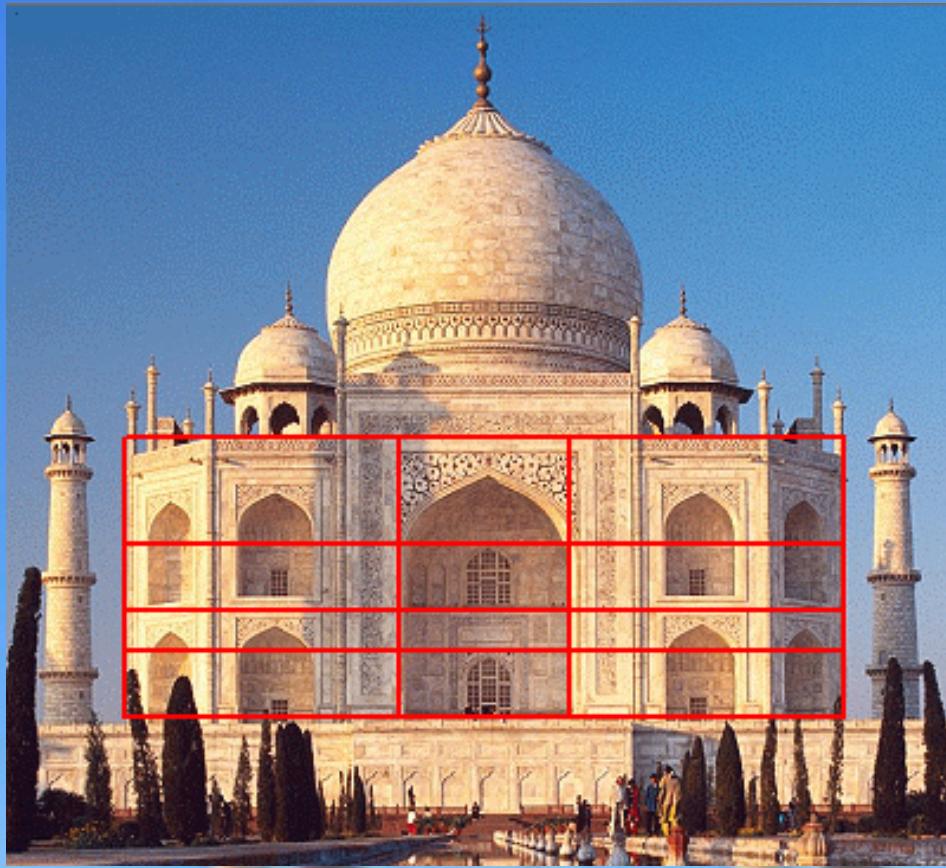
# Here a phi, there a phi, everywhere a phi, phi...

- In Architecture:

It was used it in the design of Notre Dame in Paris, which was built in the 1163 and 1250.



In India, it was used in the construction of the Taj Mahal, which was completed in 1648.

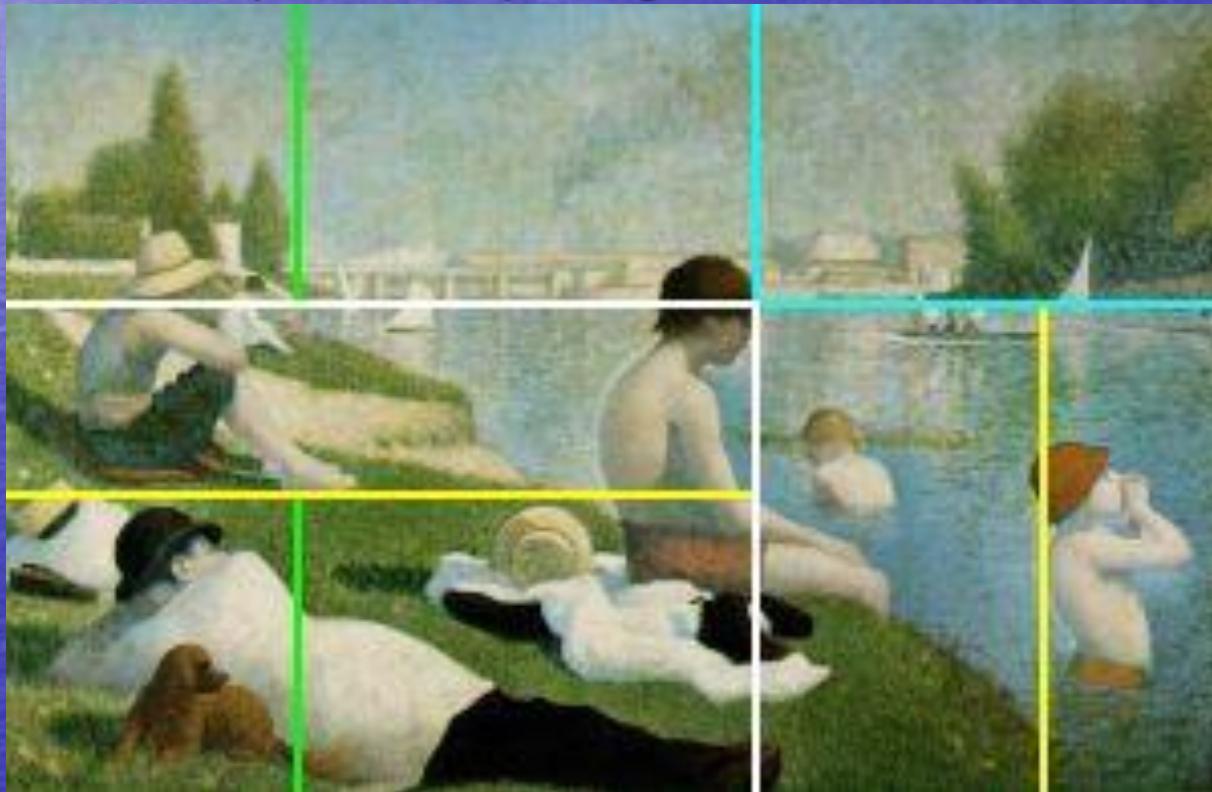


# In Art

"Without mathematics there is no art."

--Luca Pacioli

- The French impressionist painter Georges Pierre Seurat is said to have "attacked every canvas by the golden section," as illustrated below.



- In "The Sacrament of the Last Supper," Salvador Dali framed his painting in a golden rectangle. Following Da Vinci's lead, Dali positioned the table exactly at the golden section of the height of his painting. He positioned the two disciples at Christ's side at the golden sections of the width of the composition. In addition, the windows in the background are formed by a large dodecahedron. Dodecahedrons consist of 12 pentagons, which exhibit phi relationships in their proportions



# In Nature:

"Nature hides her secrets because of her essential loftiness, but not by means of ruse."

--Einstein, Albert (1879-1955)

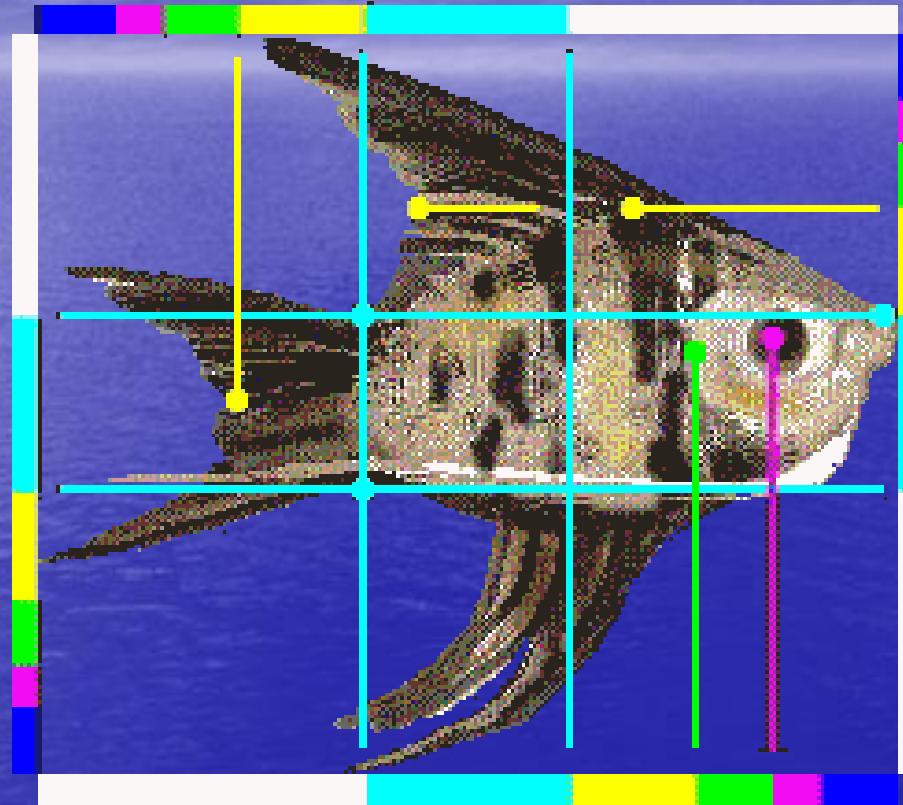
- sunflowers have a Golden Spiral seed arrangement. This provides a biological advantage because it maximizes the number of seeds that can be packed into a seed head



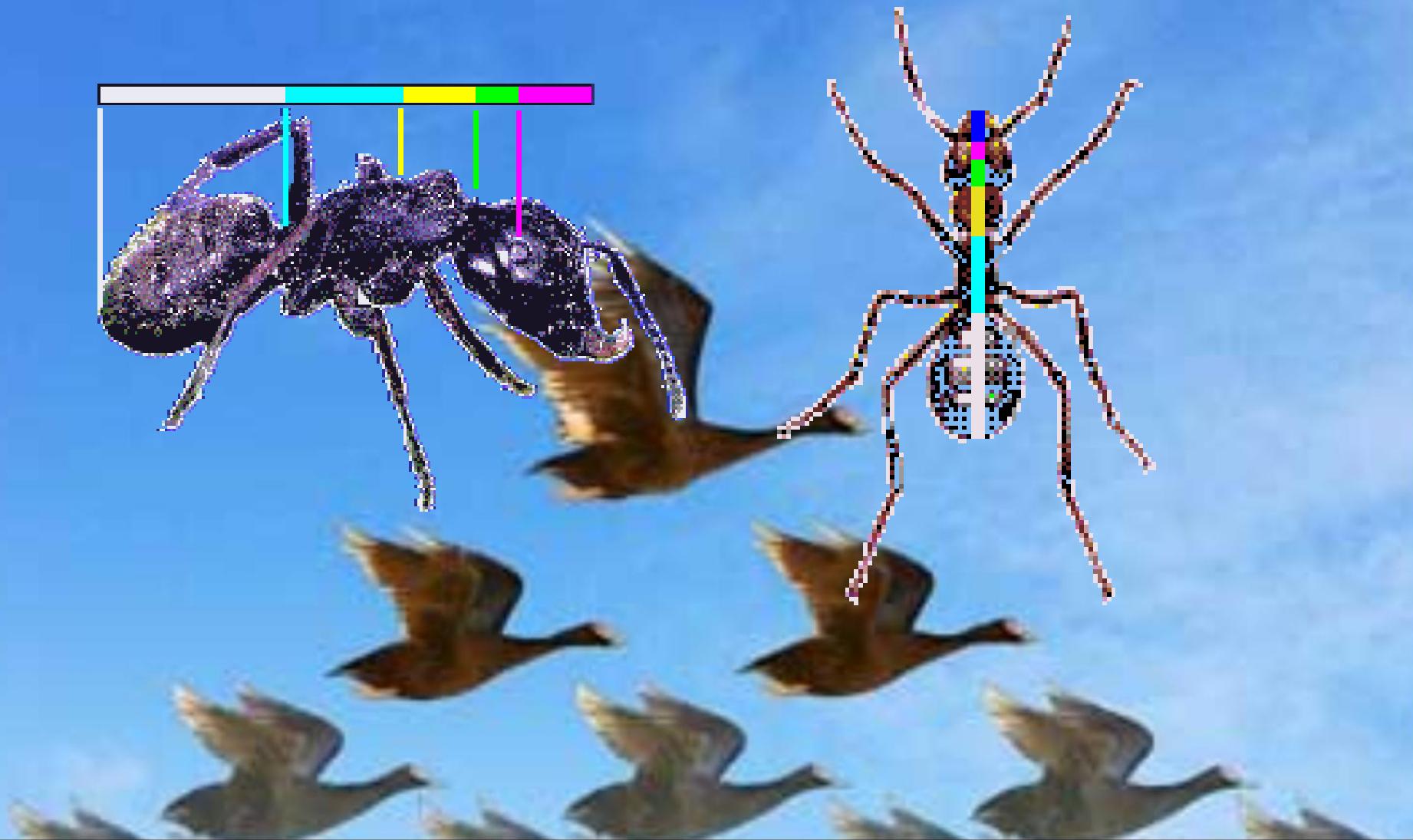
- many flowers have a Fibonacci number of petals. Some, like this rose, also have Fibonacci, or Golden Spiral, petal arrangement



- Every key body feature of the angel fish falls at golden sections of its width and length



- The body sections of an ant are defined by the golden sections of its length. Its leg sections are also golden sections of its length.

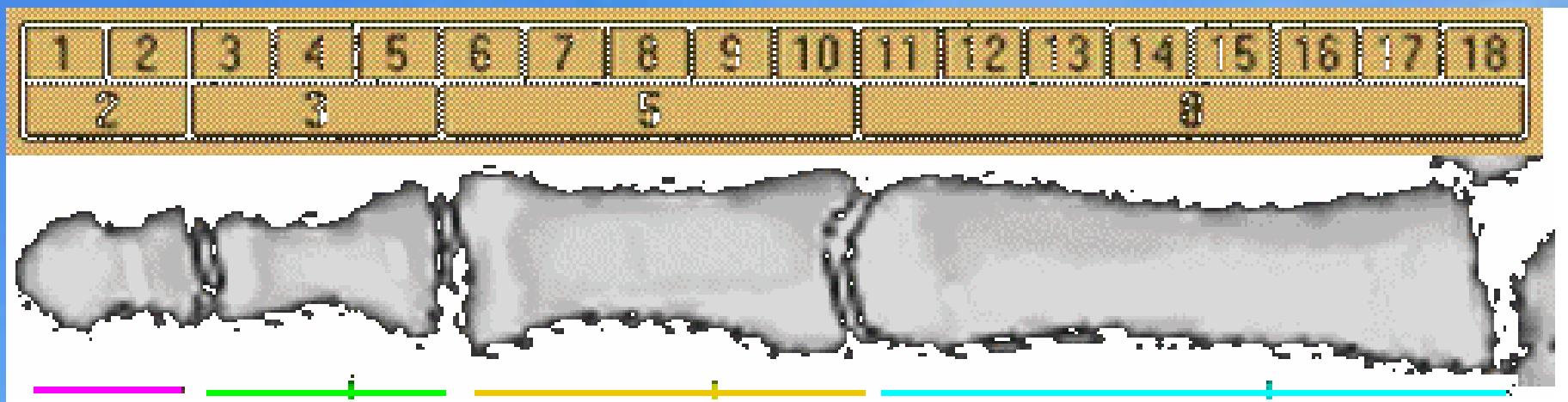


- The shell of the chambered Nautilus has Golden proportions. It is a logarithmic spiral.



- "[The universe] cannot be read until we have learnt the language and become familiar with the characters in which it is written. It is written in mathematical language, and the letters are triangles, circles and other geometrical figures, without which means it is humanly impossible to comprehend a single word."  
--Galilei, Galileo (1564 - 1642), Opere Il Saggiatore p. 171.

- Humans exhibit Fibonacci characteristics, too. The Golden Ratio is seen in the proportions in the sections of a finger.

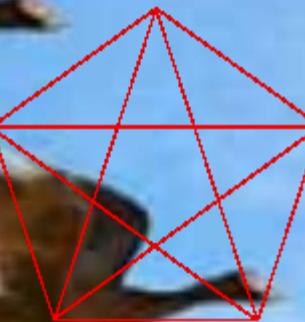
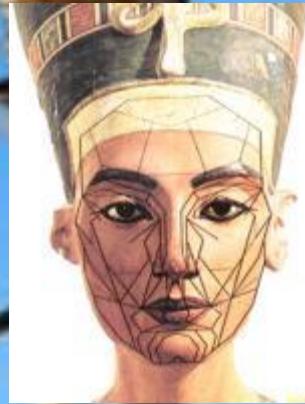
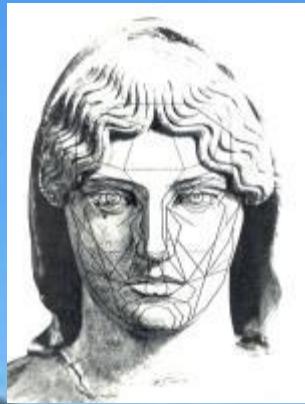
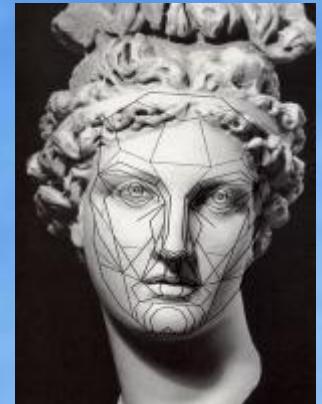
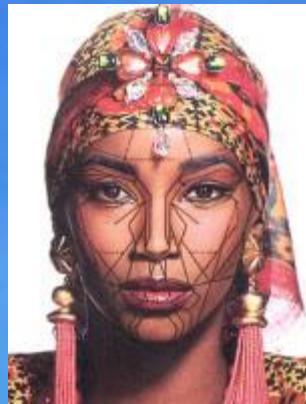
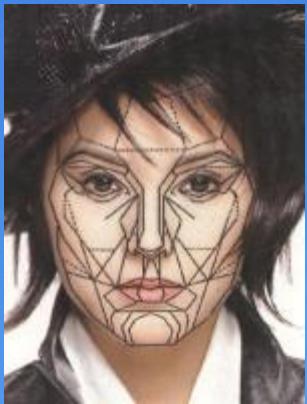


**In Beauty:** "The good, of course, is always beautiful, and the beautiful never lacks proportion."  
--Plato

## **"Beauty is in the phi of the beholder."**

- It has long been said that beauty is in the eye of the beholder and thought that beauty varies by race, culture or era. The evidence, however, shows that our perception of physical beauty is hard wired into our being and based on how closely one's features reflect phi in their proportions.
- A template for human beauty is found in phi and the pentagon

# Take another look at beauty through the eyes of medical science



# Conclusion

- We now have a deeper appreciation of the things around us as a result of our study. It is our hope that your senses may also be opened through the mathematics of science and art.

# Sources used in this study:

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