

8. Use the given expressions:

$$A \uparrow A = \bar{A} \quad \text{--- (1)}$$

$$AB = (A \uparrow B) \uparrow (A \uparrow B) \quad \text{--- (2)}$$

$$A + B = (A \uparrow A) \uparrow (B \uparrow B) \quad \text{--- (3)}$$

Using: $A \uparrow A = \bar{A}$ --- (1)

Substitute (1) into equation:

$$\bar{A} \uparrow ((A \uparrow B) \uparrow (A \uparrow B)) \uparrow ((A \uparrow B) \uparrow (A \uparrow B))$$

Substitute (2) into eq:

$$A \uparrow ((AB) \uparrow (AB))$$

If we do $B = A$, we get

$$A + A = (A \uparrow A) \uparrow (A \uparrow A)$$

So,

$$AB \uparrow AB = \bar{A} \quad \therefore \bar{A} = A$$

Apply double negation

$$\uparrow (\bar{A} \uparrow \bar{A} = A)$$

$$\bar{A} \uparrow (AB \uparrow AB) = A$$

Therefore,

$$(A \uparrow A) \uparrow ((A \uparrow B) \uparrow (A \uparrow B)) \uparrow ((A \uparrow B) \uparrow (A \uparrow B)) = A$$