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```
clear all; close all;
%parameters
syms R0;
syms r theta phi x y z;

assumeAlso(0<=phi<=2*pi);
assumeAlso(0<=theta<=2*pi);
assumeAlso(r>0);
assumeAlso(R0>r);
assumeAlso(R0-r>0);
assumeAlso(R0>0);
```

Coordinate transformation

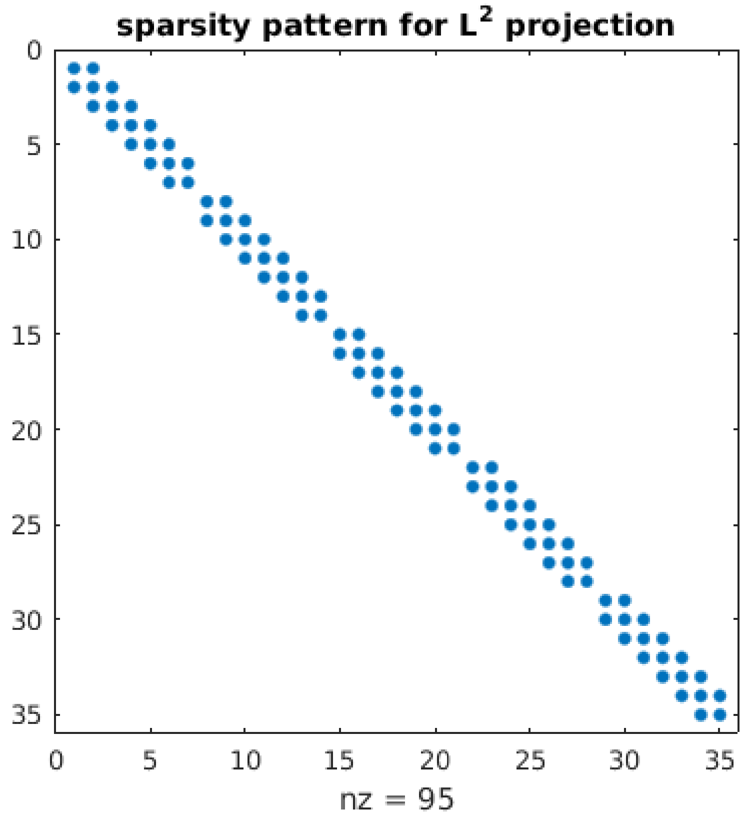
The coordinate transformation for a torus

```
T=[(R0+ r.*cos(theta)).*cos(phi)    ; ...
   (R0+ r.*cos(theta)).*sin(phi);...
   r.*sin(theta)];

 %T=[r.*cos(theta); ...
 %    phi;...
 %   r.*sin(theta)];



% Cartesian coordinates as function Handles
X=matlabFunction( T(1,:),'Vars',[r,theta,phi,R0]);
Y=matlabFunction( T(2,:),'Vars',[r,theta,phi,R0]);
Z=matlabFunction( T(3,:),'Vars',[r,theta,phi,R0]);
```

Jacobi matrix of the transformation

```
J_T= simplify(jacobian(T', [r theta phi]));
% Inverse of Jacobi matrix
J_T_inv=simplify(inv(J_T),'Steps', 10);

L=-simplify(J_T_inv*J_T_inv'*det(J_T));

% Tnteger modes
syms n m integer;
```

```
% Basis function constant in r
psi=symfun(exp(1j*(n*phi+m*theta)),[m,n]);
% Gradient of basis function
psi_grad=gradient(psi,[r theta phi]);

syms m1 n1 m2 n2;
%
% K=symfun(simplify(psi_grad(m1,n1)'*L*psi_grad(m2,n2)),...
%       [m1,n1,m2,n2]);


M=simplify(psi(m1,n1)'*psi(m2,n2)*-det(J_T));
% M=simplify(subs(subs(M,'r',1),'R0',5));

M=symfun(M,[m1,m2,n1,n2]);

num_theta=3; num_phi=2;
[m,n]=ndgrid( -num_theta:1:num_theta, -num_phi:1:num_phi);
m=reshape(m,numel(m),1);
n=reshape(n,numel(n),1);
nm=[n,m];



MM=sym(zeros((num_theta*2+1)*(num_phi*2+1)));
for idx=1:length(nm)
   for jdx=idx:length(nm)
       MM(idx,jdx)=int(int(...
           M(nm(idx,2),nm(jdx,2),nm(idx,1),nm(jdx,1)),...
           phi,0,2*sym('pi')),theta,0,2*sym('pi'));
   end
end
MM=MM+MM'-diag(diag(MM));

% Plot sparsity pattern
spy(MM);
title('sparsity pattern for L^2 projection');
```



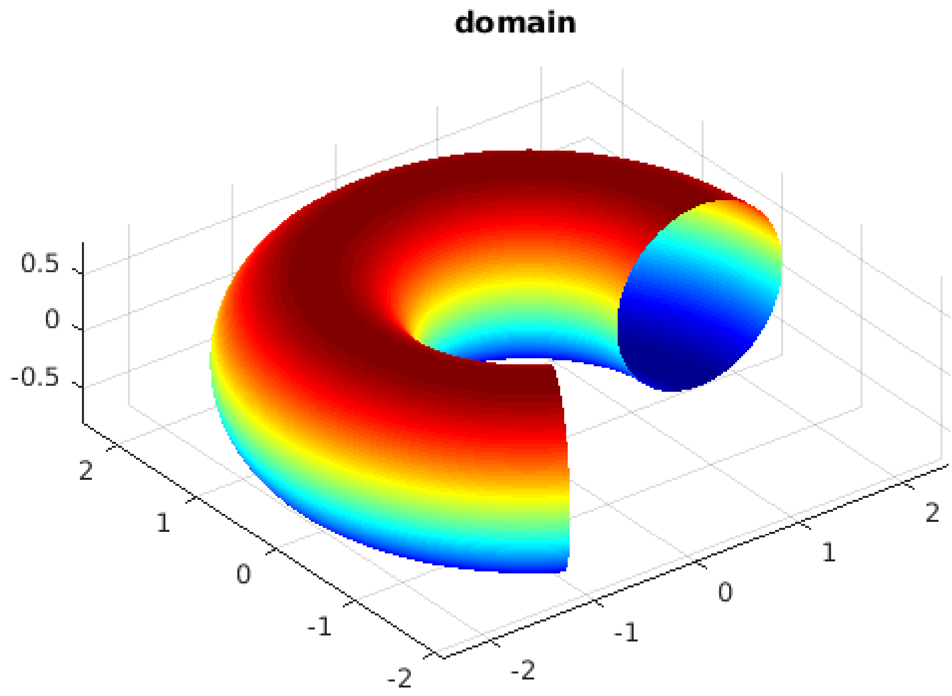
Visualization

Set up grid in theta-phi plane for visulization

```
[THETA,PHI]=ndgrid(linspace(0,2*pi,200),...
     linspace(0,2*pi*2/3,200));
```

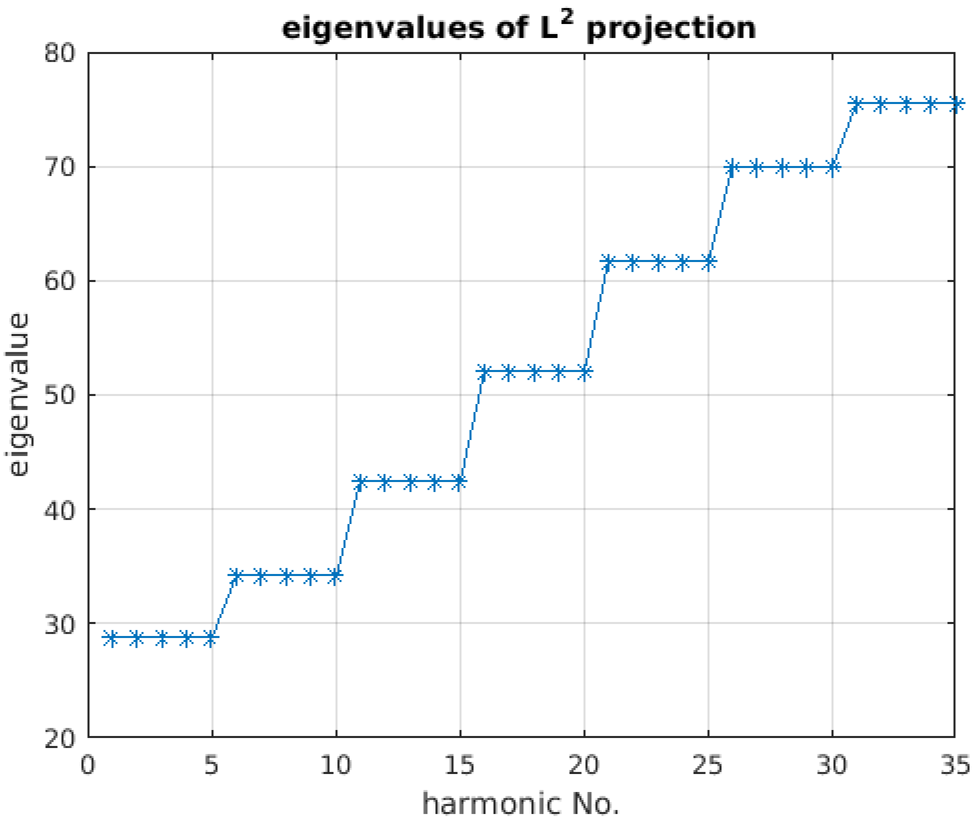
ASDEX Upgrade geometry

```
minorR=0.8; %minor radius
majorR=1.65; %major radius

% Get cartesian coordinates
XX=X(minorR, THETA,PHI,majorR);
YY=Y(minorR, THETA,PHI,majorR);
ZZ=Z(minorR, THETA,PHI,majorR);

figure;
surf(XX,YY, ZZ);
axis equal; grid on; colormap jet; shading interp;
title('domain');
```



Spectrum of the L^2 projection

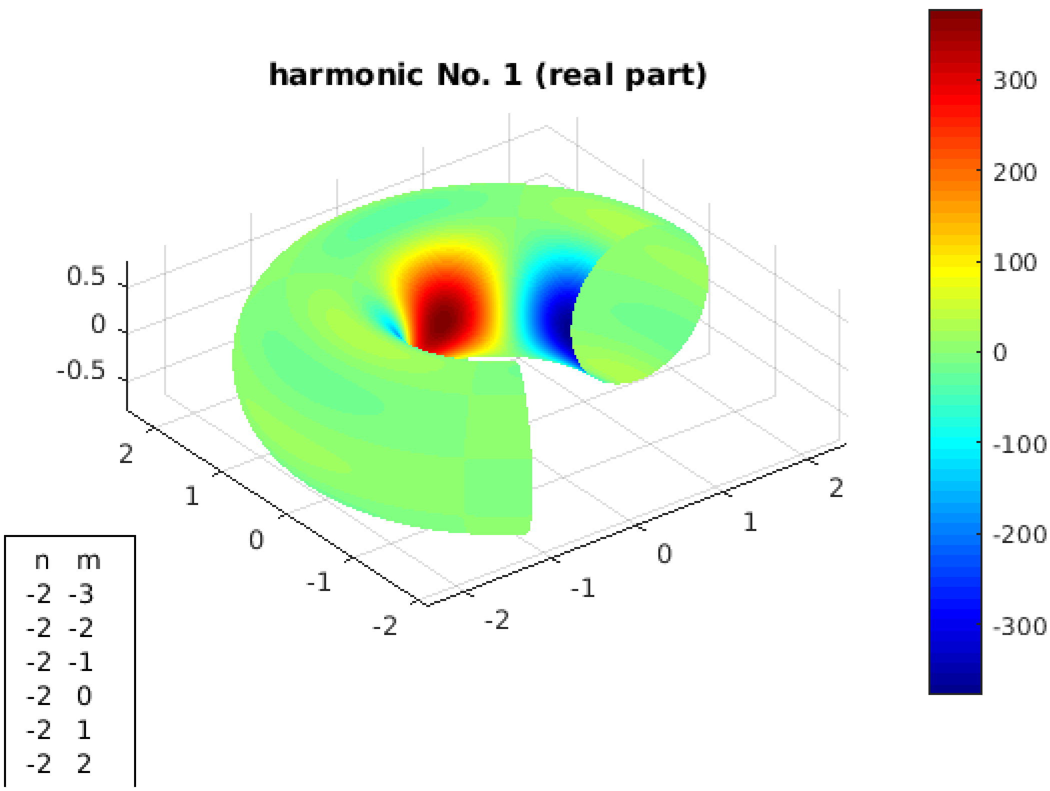
```
%Get symbolic eigenvectors and eigenvalues
[MM_V,MM_D]=eig(MM);

% Sort eigenvalues, eigenvectors for the given parameters
[D,idx]=sort(double(subs(subs(diag(MM_D),r,minorR),R0,majorR)));
V=MM_V(:,idx);
V=double(subs(subs(V,r,minorR),R0,majorR));

figure;
plot(D,'*-');
xlabel('harmonic No.');
ylabel('eigenvalue');
%axis([1,length(D),-inf,inf]);
title('eigenvalues of L^2 projection'); grid on;
```

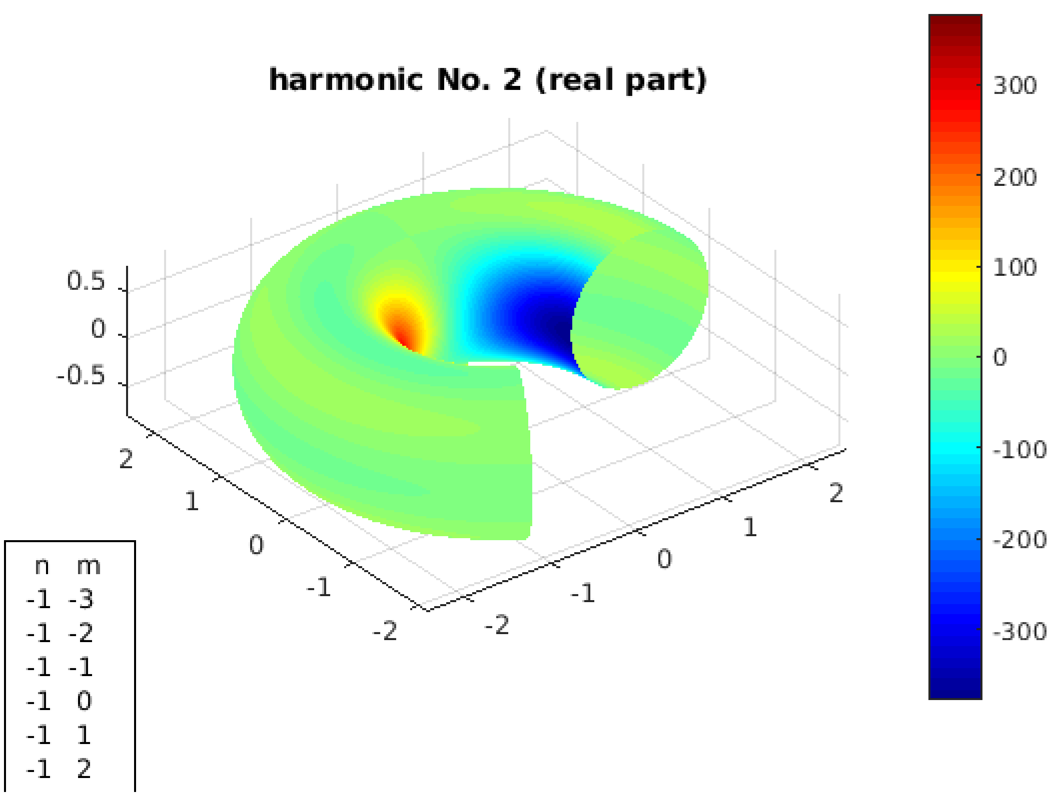


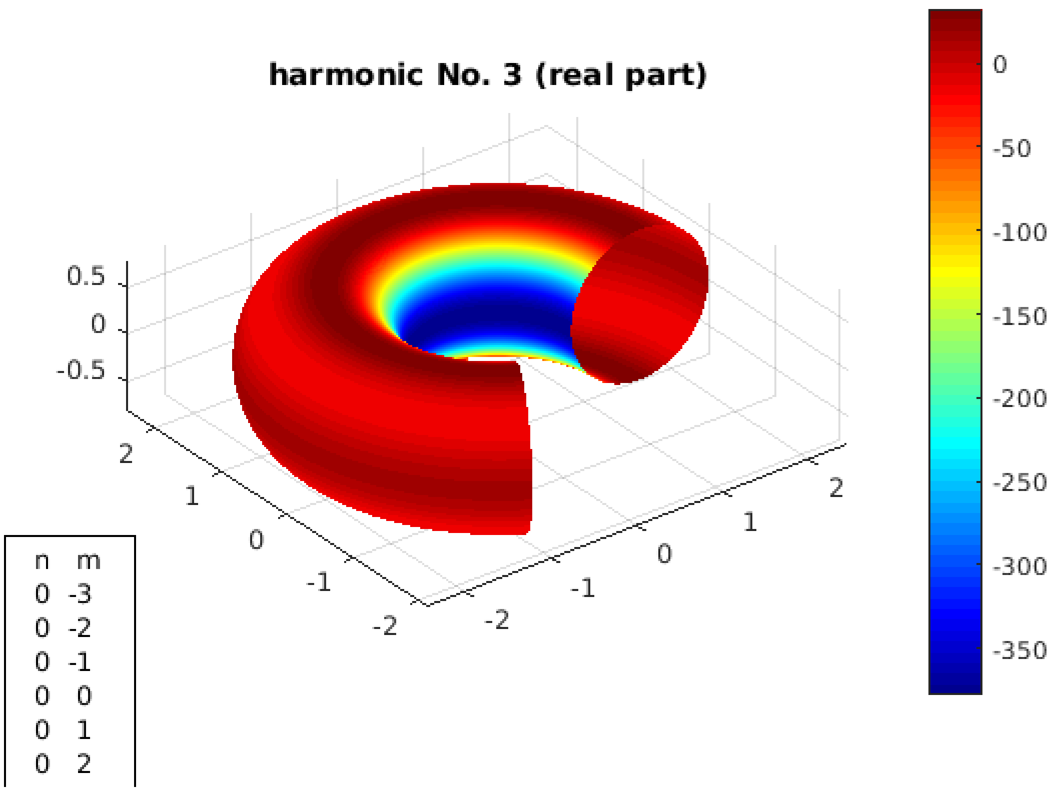
Harmonics of L^2 projection

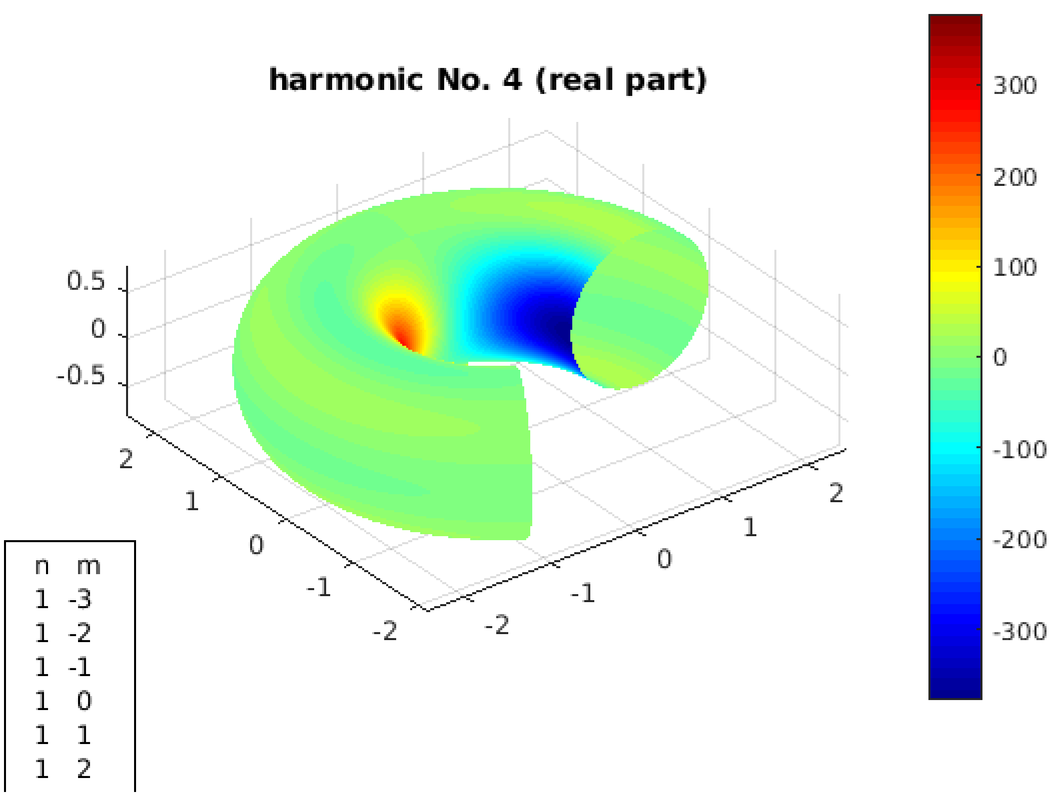
```
figure;
for idx=1:size(V,2)

    figure;
    coeffs=V(:,idx);
    modes=nm(coeffs~=0,:);
    coeffs=coeffs(coeffs~=0);
    fun=0;
    for jdx=1:size(modes,1)
        fun=fun+coeffs(jdx)*exp(1j*...
            (modes(jdx,1)*PHI+(modes(jdx,2)*THETA)));
    end
    surf(XX,YY, ZZ,real(fun)*D(idx));
    axis equal; grid on; colormap jet; shading interp;
    colorbar;
    title(sprintf('harmonic No. %d (real part)', idx));


str=cell(size(modes,1)+1,1);
str{1}='  n    m';
for kdx=1:size(modes,1)
str{kdx+1}=sprintf('%3d %3d', modes(kdx,1),modes(kdx,2));
end
```

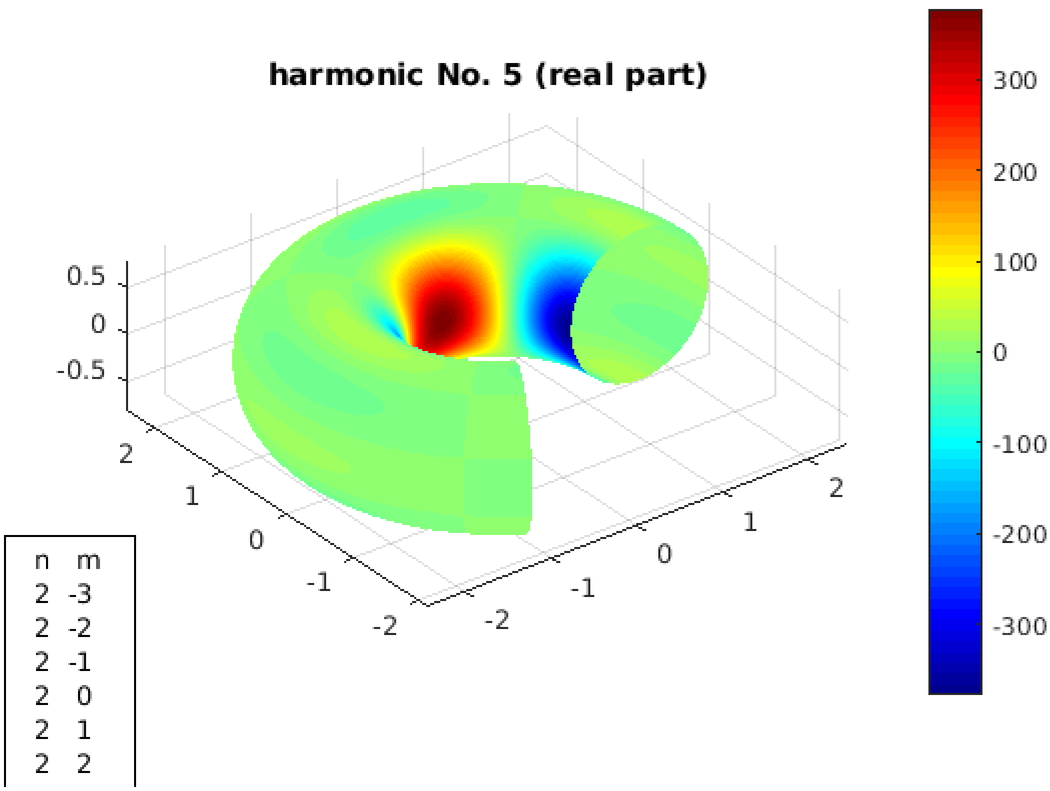
```
dim = [0 0 0.3 0.3];

annotation('textbox',dim,'String',str,'FitBoxToText','on');

end
```

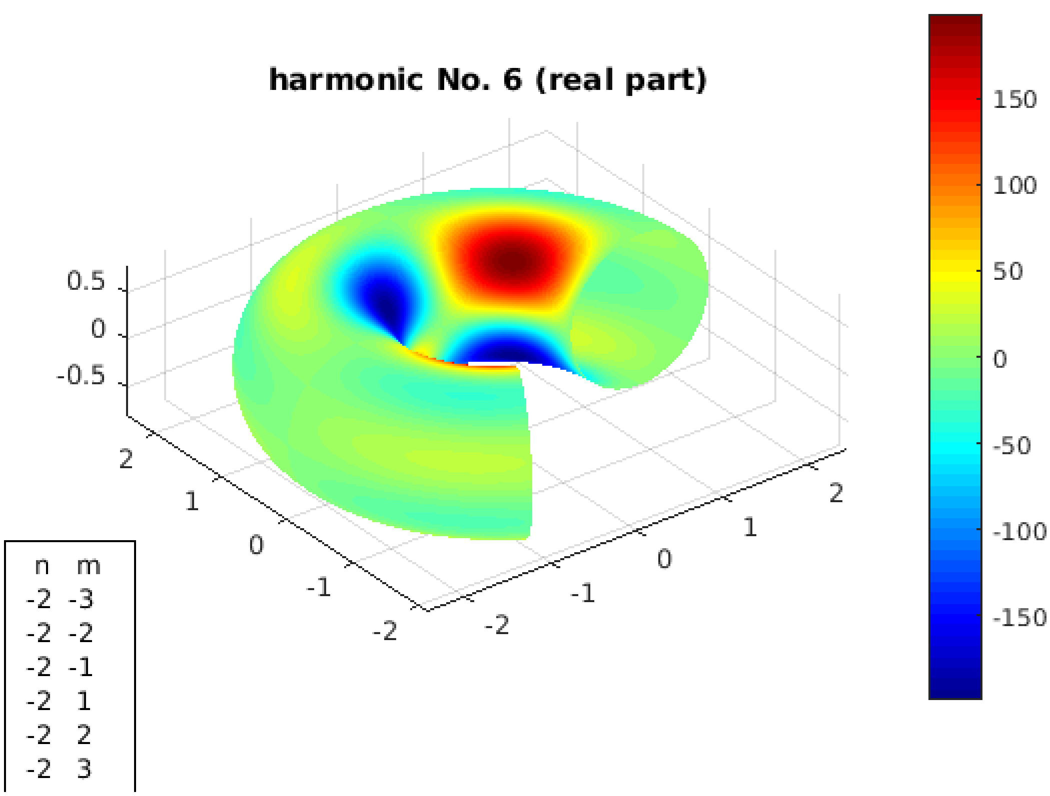

harmonic No. 1 (real part)
0.5
0
-0.5
2
1
0
-1
-2
-2
-1
0
1
2
300
200
100
0
-100
-200
-300
n m
-2 -3
-2 -2
-2 -1
-2 0
-2 1
-2 2

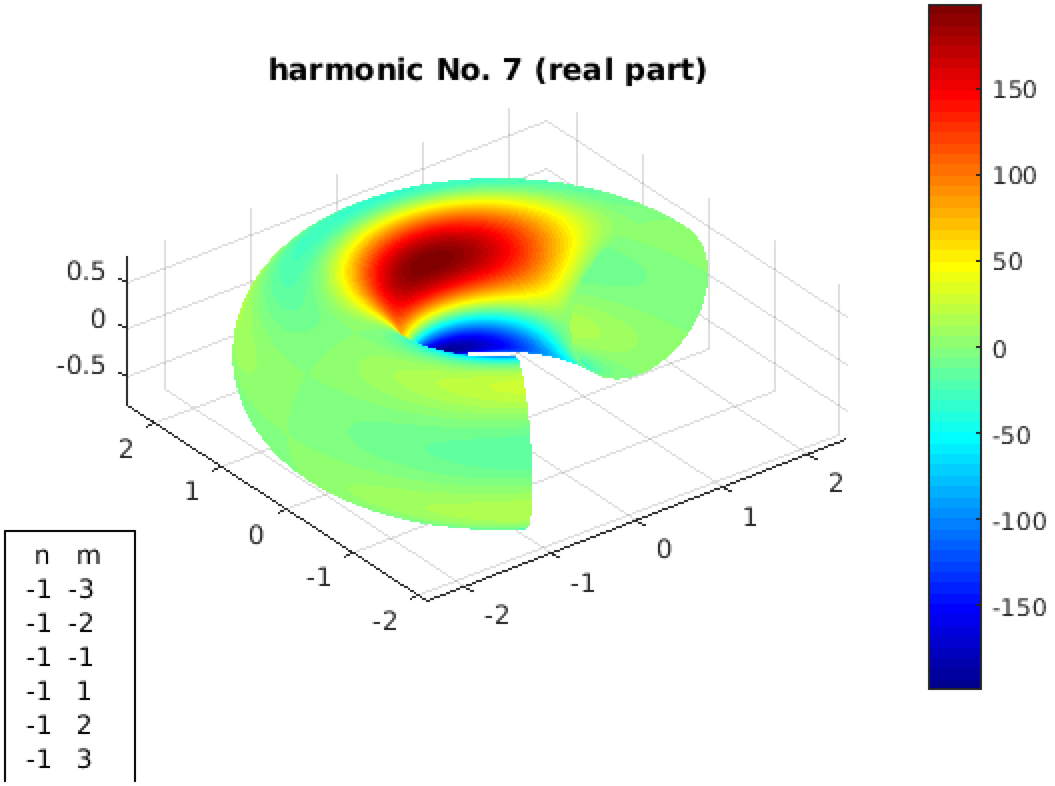

harmonic No. 2 (real part)
0.5
0
-0.5
2
1
0
-1
-2
-2
-1
0
1
2
300
200
100
0
-100
-200
-300
n m
-1 -3
-1 -2
-1 -1
-1 0
-1 1
-1 2

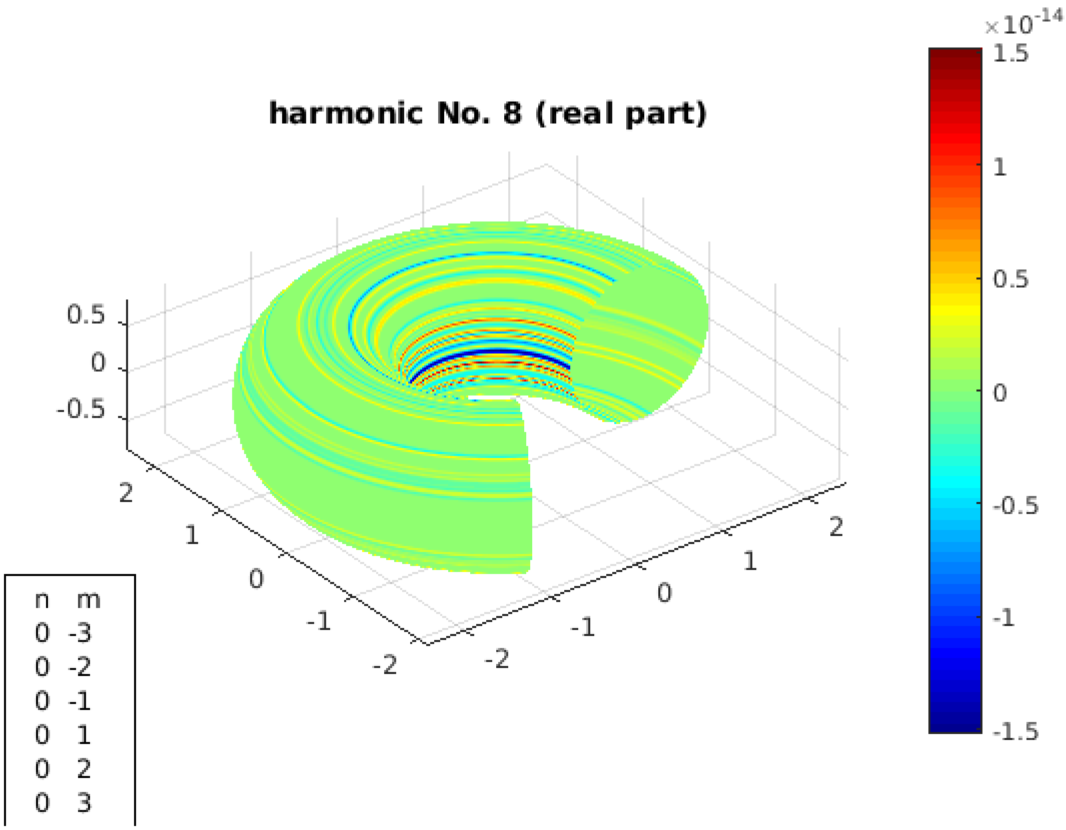

harmonic No. 3 (real part)
0
-50
-100
-150
-200
-250
-300
-350
0.5
0
-0.5
2
1
0
-1
-2
2
1
0
-1
-2
n m
0 -3
0 -2
0 -1
0 0
0 1
0 2

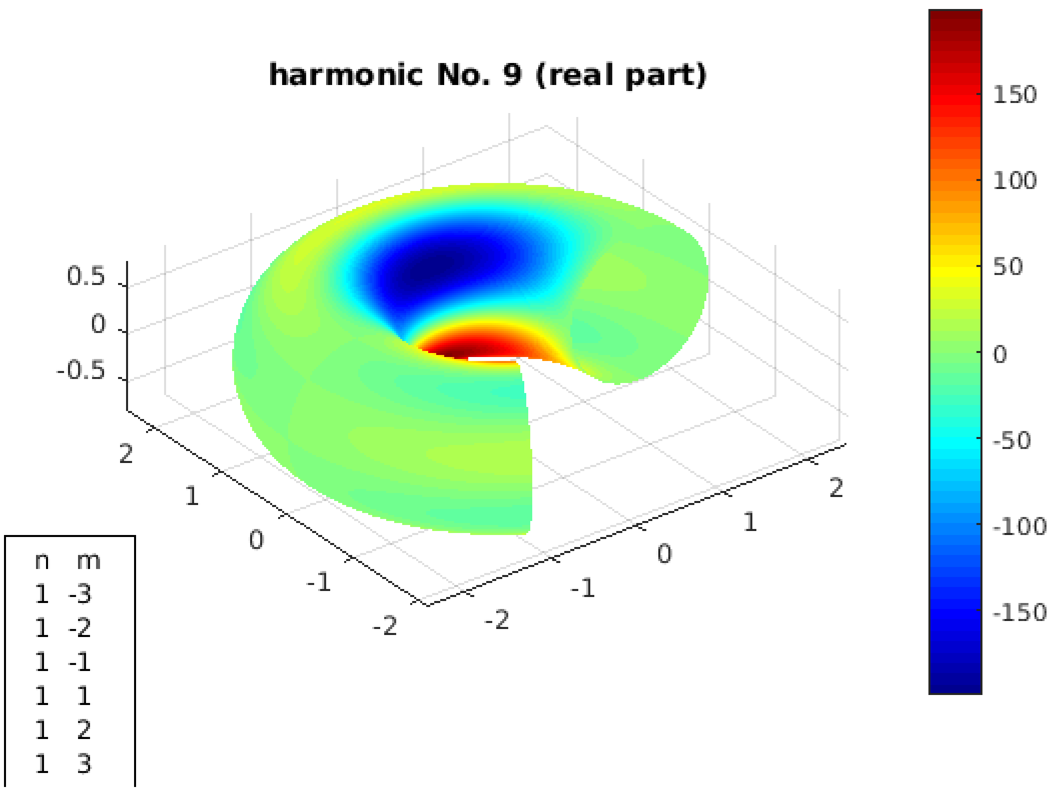

harmonic No. 4 (real part)
300
200
100
0
-100
-200
-300
0.5
0
-0.5
2
1
0
-1
-2
2
1
0
-1
-2
n m
1 -3
1 -2
1 -1
1 0
1 1
1 2

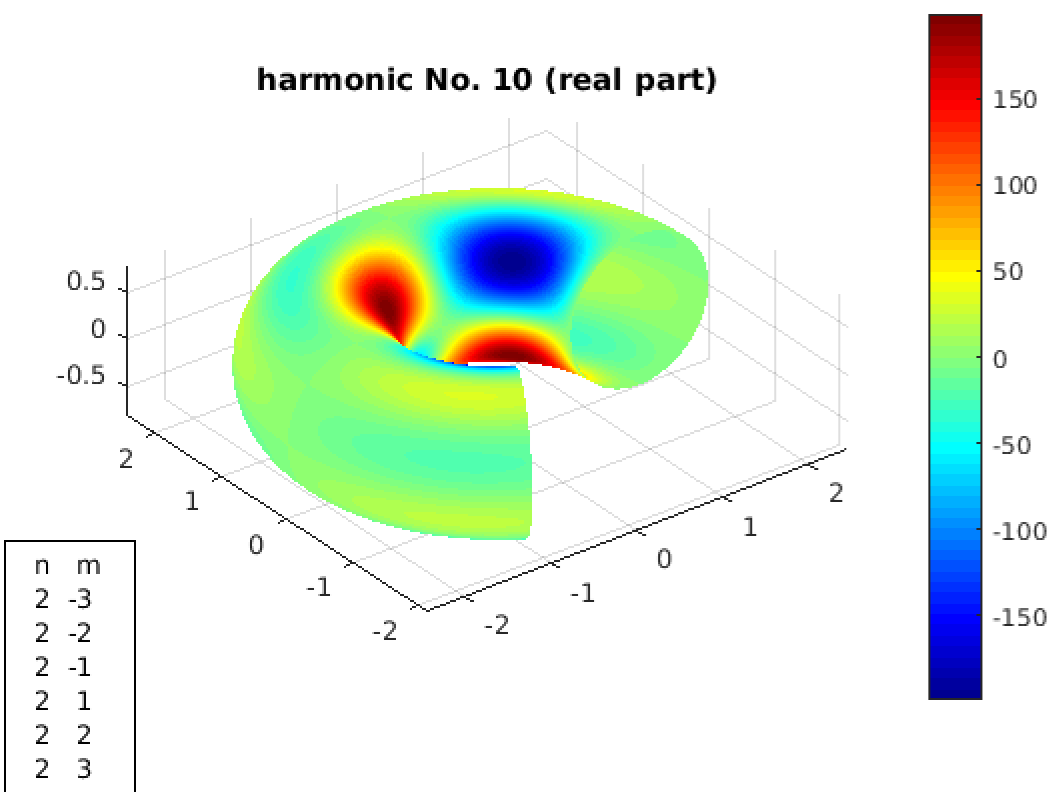

harmonic No. 5 (real part)
0.5
0
-0.5
2
1
0
-1
-2
-2
-1
0
1
2
n m
2 -3
2 -2
2 -1
2 0
2 1
2 2
300
200
100
0
-100
-200
-300

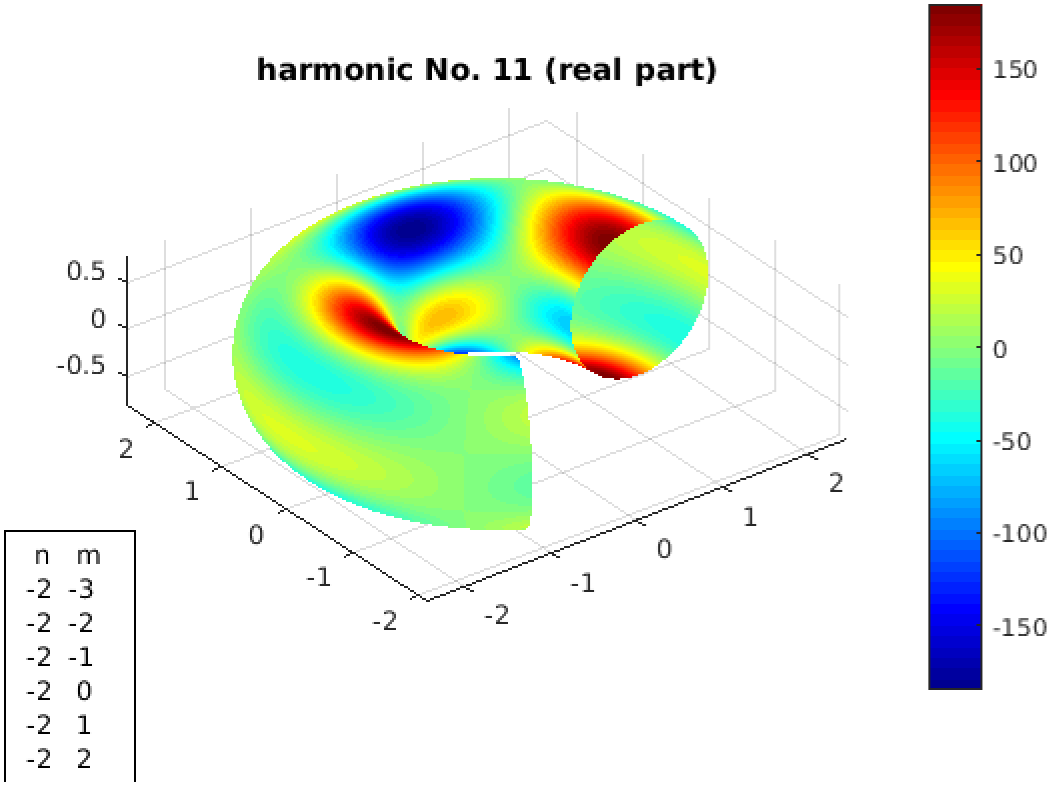

harmonic No. 6 (real part)
0.5
0
-0.5
2
1
0
-1
-2
-2
-1
0
1
2
n m
-2 -3
-2 -2
-2 -1
-2 1
-2 2
-2 3
150
100
50
0
-50
-100
-150

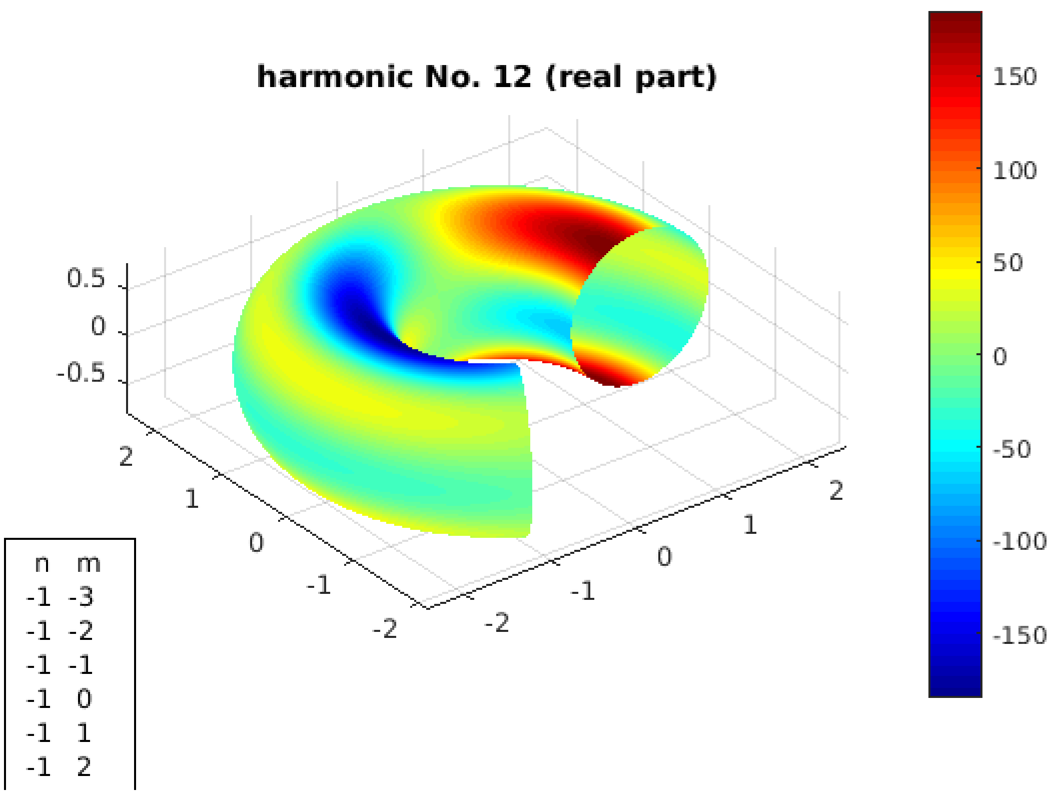

harmonic No. 7 (real part)
0.5
0
-0.5
2
1
0
-1
-2
-2
-1
0
1
2
150
100
50
0
-50
-100
-150
n m
-1 -3
-1 -2
-1 -1
-1 1
-1 2
-1 3

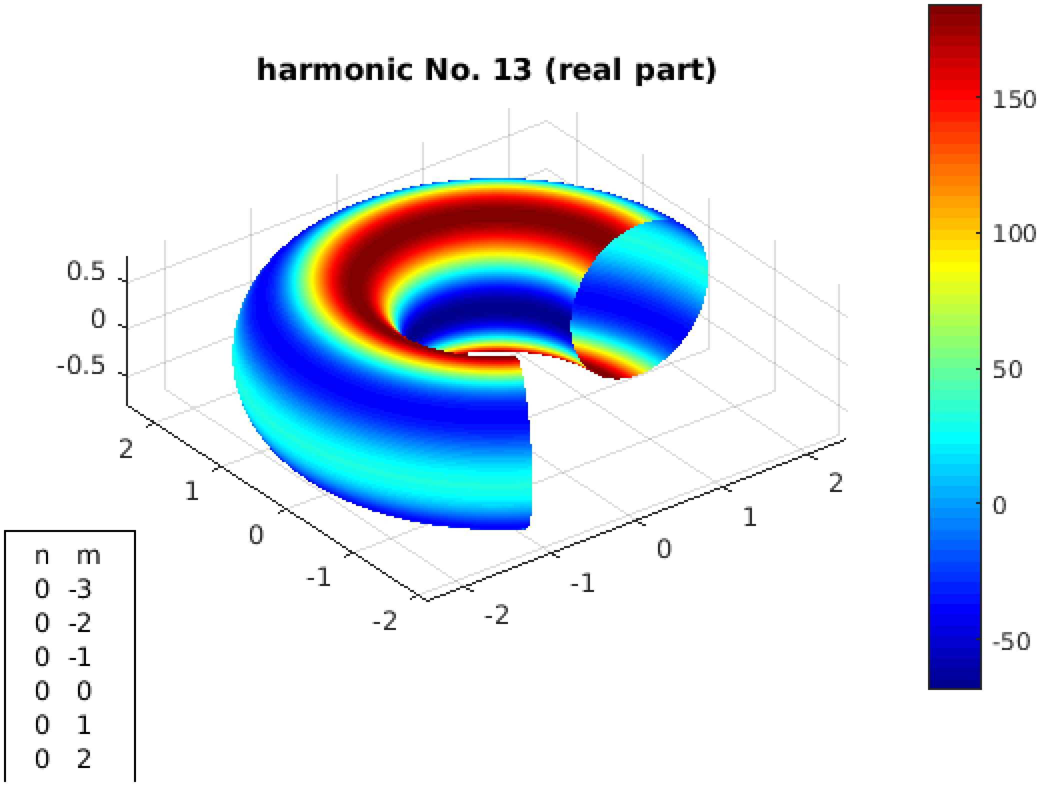

harmonic No. 8 (real part)
×10^-14
1.5
1
0.5
0
-0.5
-1
-1.5
0.5
0
-0.5
2
1
0
-1
-2
-2
-1
0
1
2
n m
0 -3
0 -2
0 -1
0 1
0 2
0 3

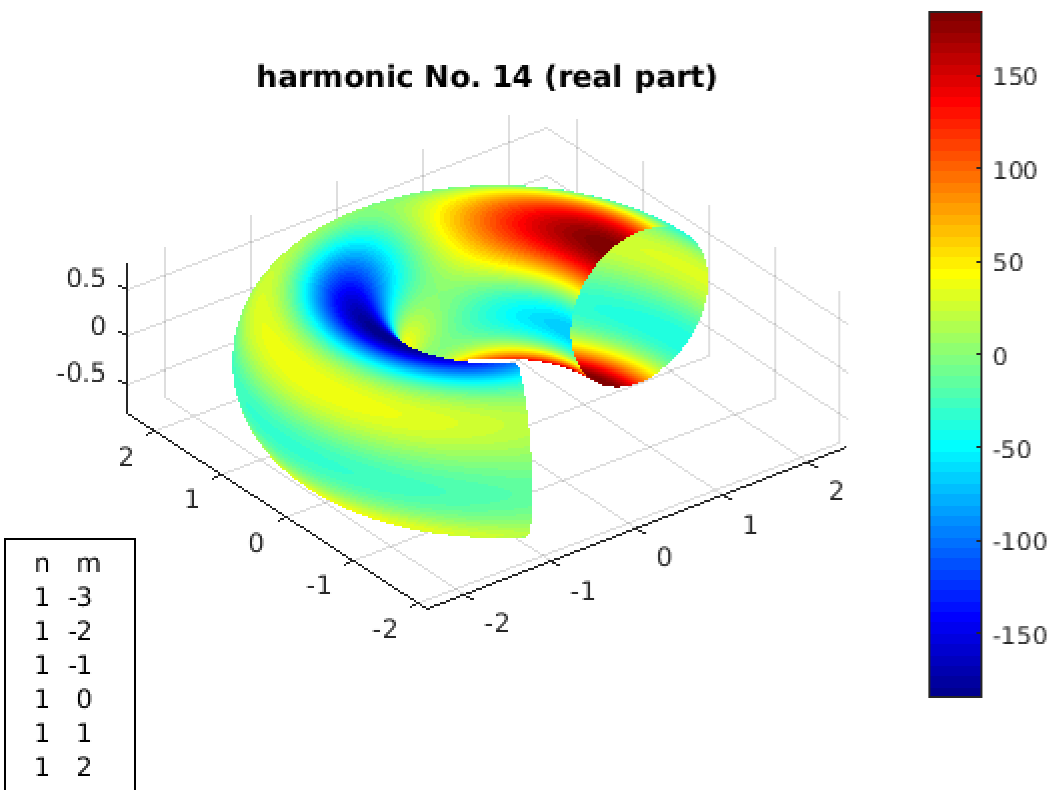

harmonic No. 9 (real part)
0.5
0
-0.5
2
1
0
-1
-2
-2
-1
0
1
2
150
100
50
0
-50
-100
-150
n m
1 -3
1 -2
1 -1
1 1
1 2
1 3

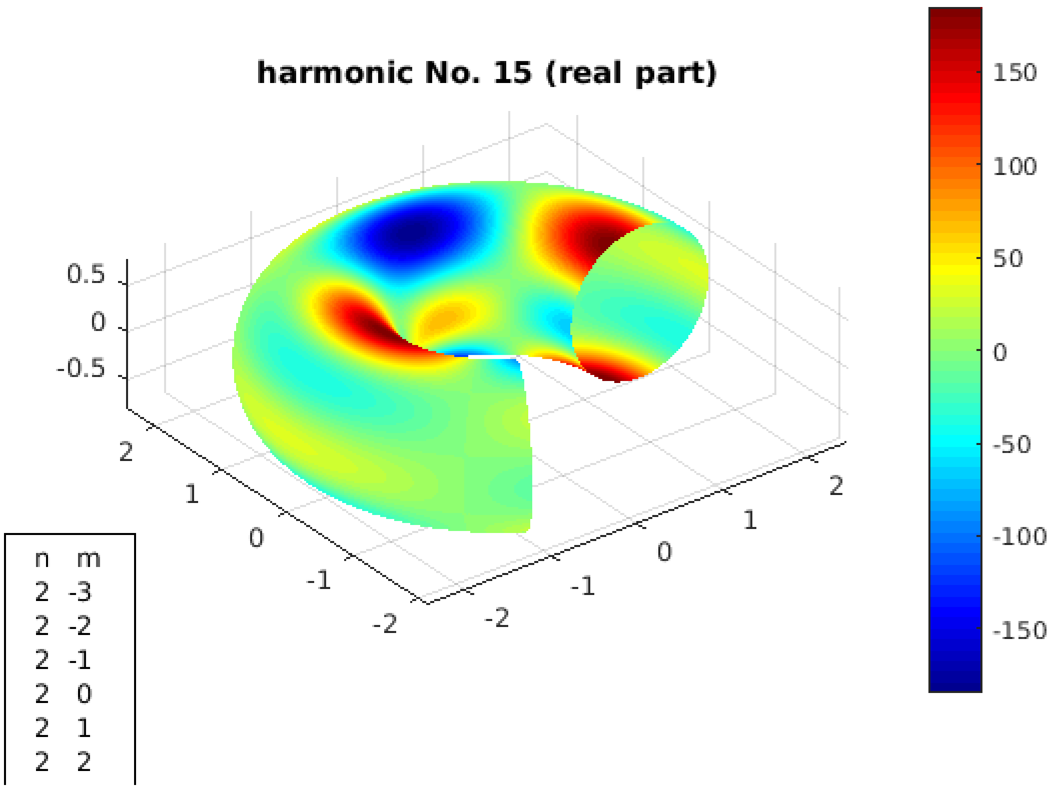

harmonic No. 10 (real part)
0.5
0
-0.5
2
1
0
-1
-2
-2
-1
0
1
2
150
100
50
0
-50
-100
-150
n m
2 -3
2 -2
2 -1
2 1
2 2
2 3

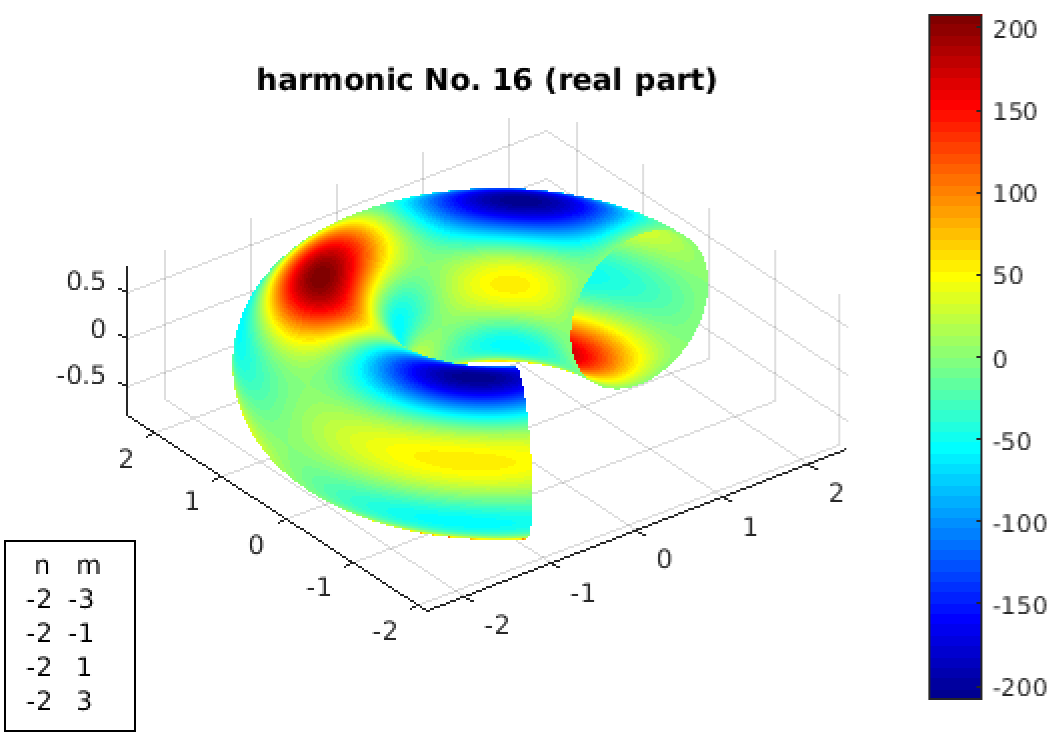

harmonic No. 11 (real part)
0.5
0
-0.5
2
1
0
-1
-2
-2
-1
0
1
2
150
100
50
0
-50
-100
-150
n m
-2 -3
-2 -2
-2 -1
-2 0
-2 1
-2 2

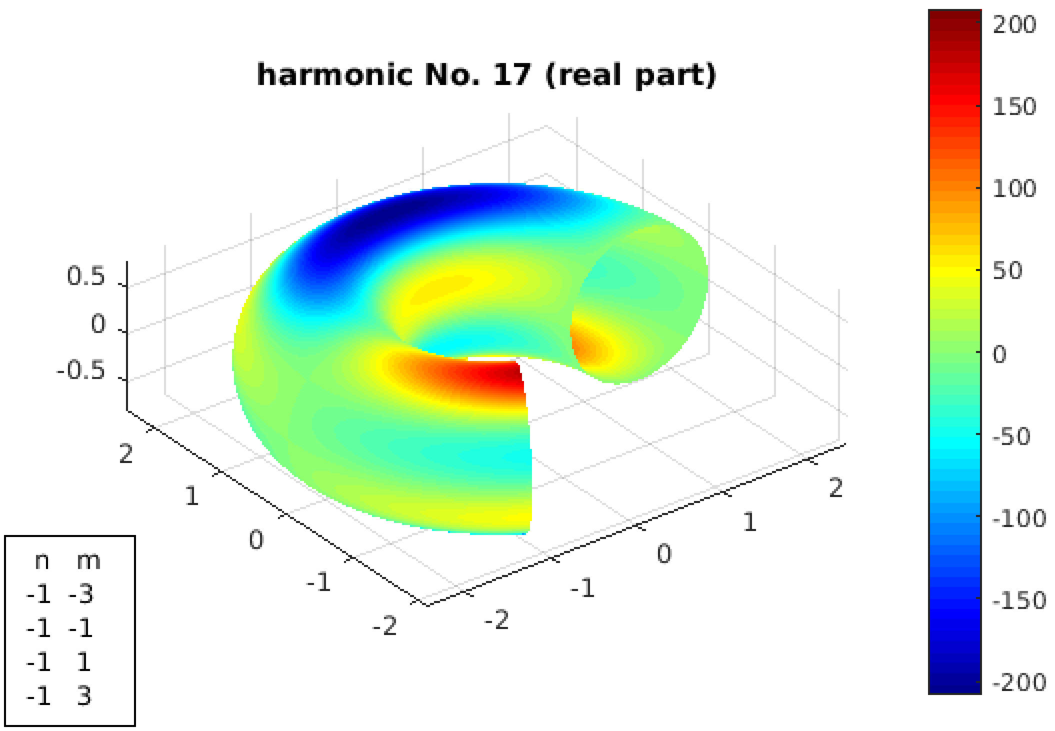

harmonic No. 12 (real part)
0.5
0
-0.5
2
1
0
-1
-2
-2
-1
0
1
2
150
100
50
0
-50
-100
-150
n m
-1 -3
-1 -2
-1 -1
-1 0
-1 1
-1 2

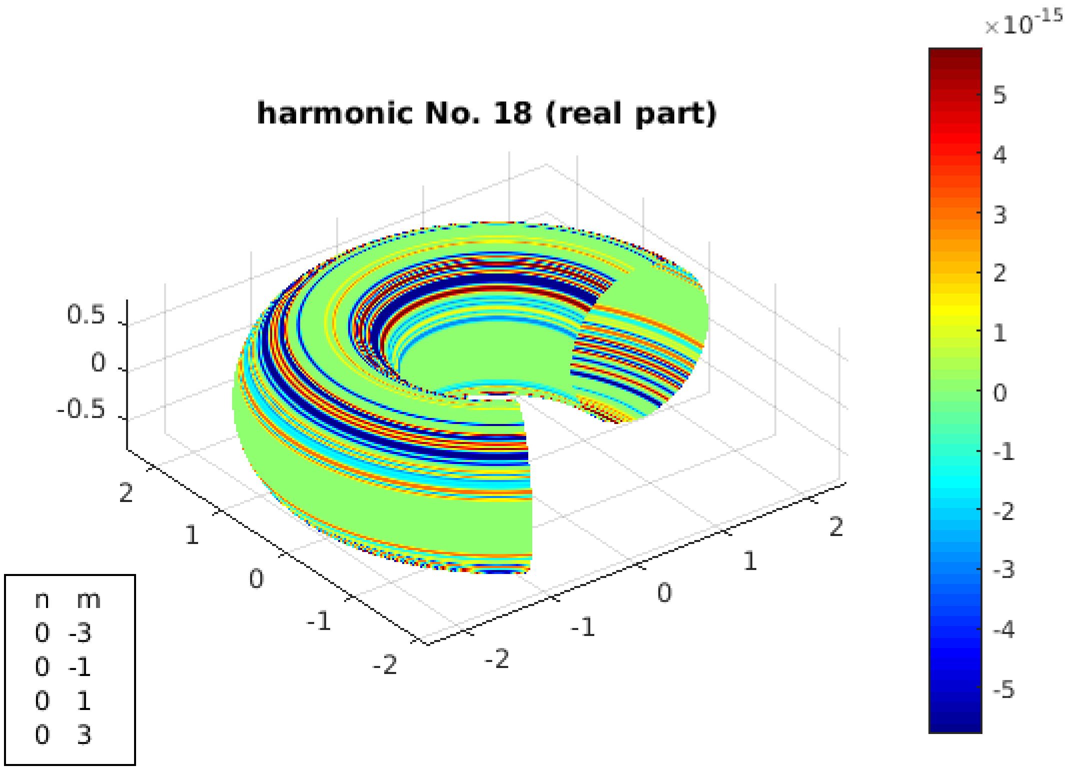

harmonic No. 13 (real part)
0.5
0
-0.5
2
1
0
-1
-2
-2
-1
0
1
2
150
100
50
0
-50
n m
0 -3
0 -2
0 -1
0 0
0 1
0 2

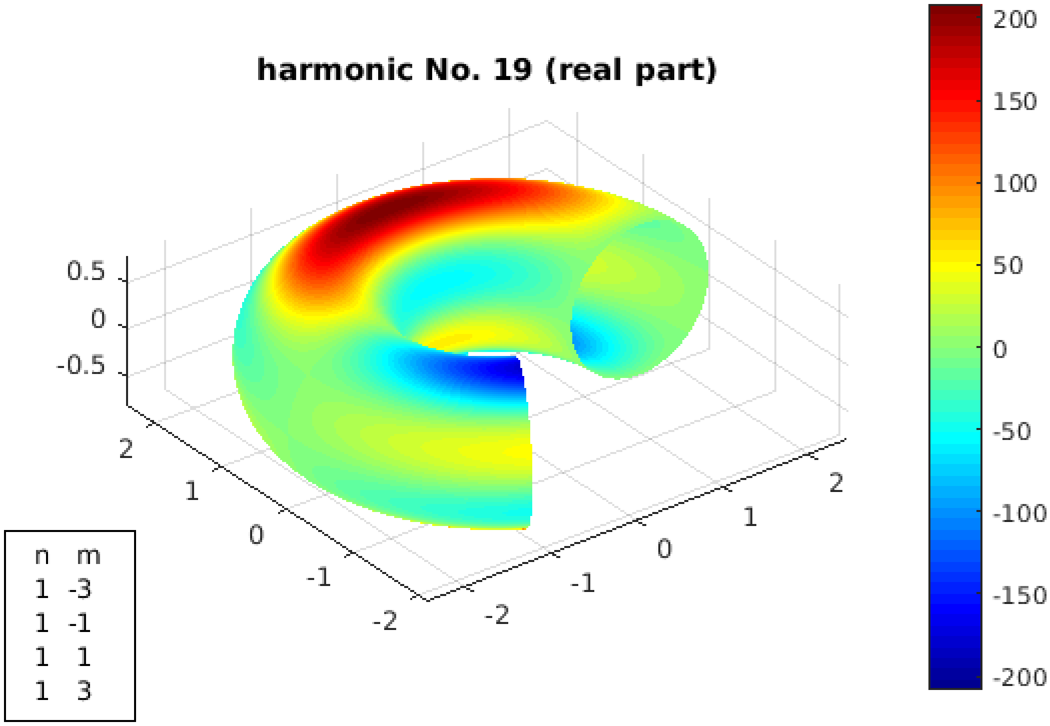

harmonic No. 14 (real part)
0.5
0
-0.5
2
1
0
-1
-2
-2
-1
0
1
2
150
100
50
0
-50
-100
-150
n m
1 -3
1 -2
1 -1
1 0
1 1
1 2

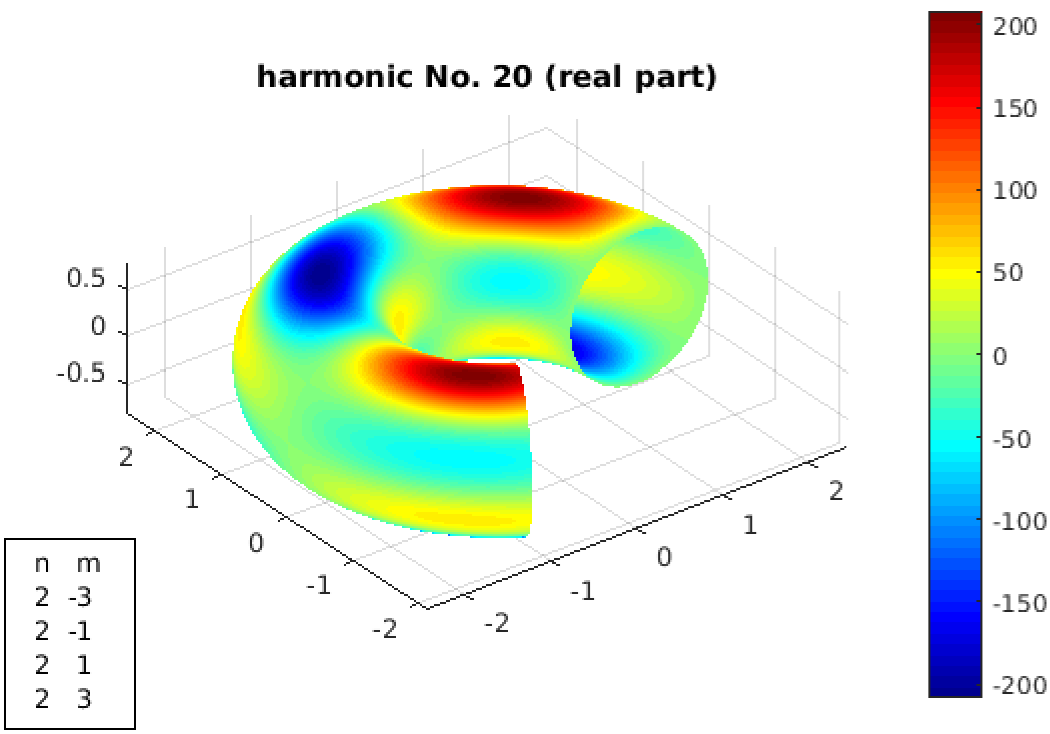

harmonic No. 15 (real part)
n m
2 -3
2 -2
2 -1
2 0
2 1
2 2
150
100
50
0
-50
-100
-150
0.5
0
-0.5
2
1
0
-1
-2

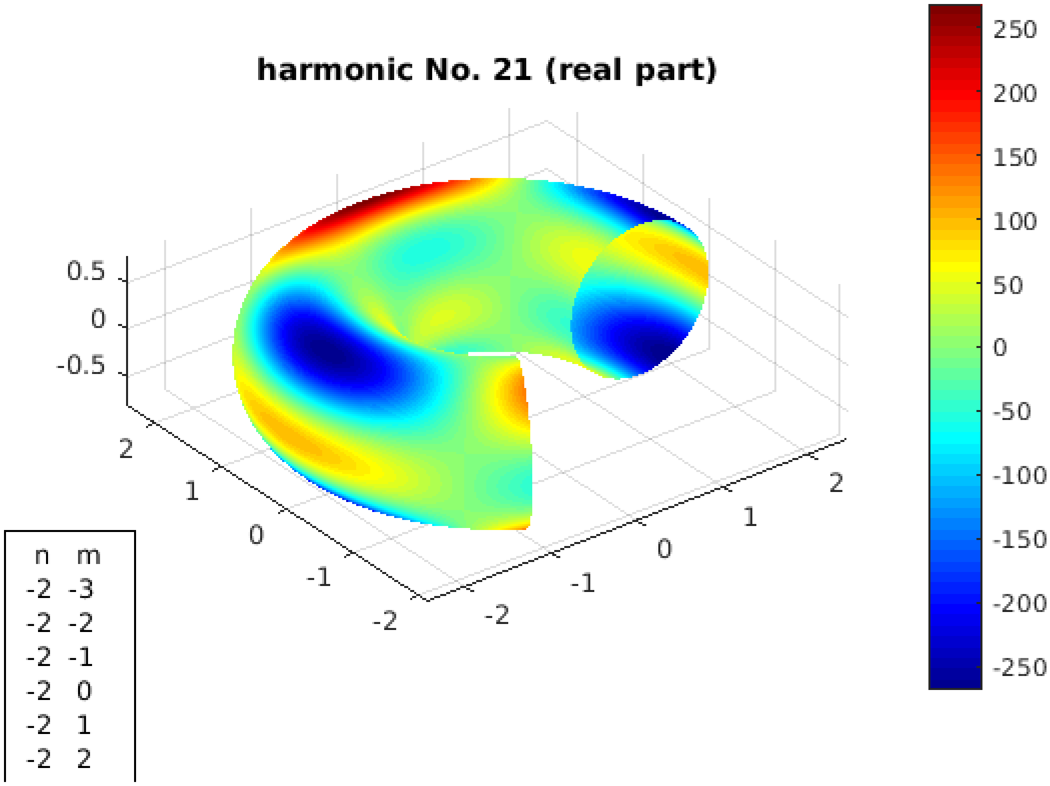

harmonic No. 16 (real part)
n m
-2 -3
-2 -1
-2 1
-2 3
200
150
100
50
0
-50
-100
-150
-200
0.5
0
-0.5
2
1
0
-1
-2

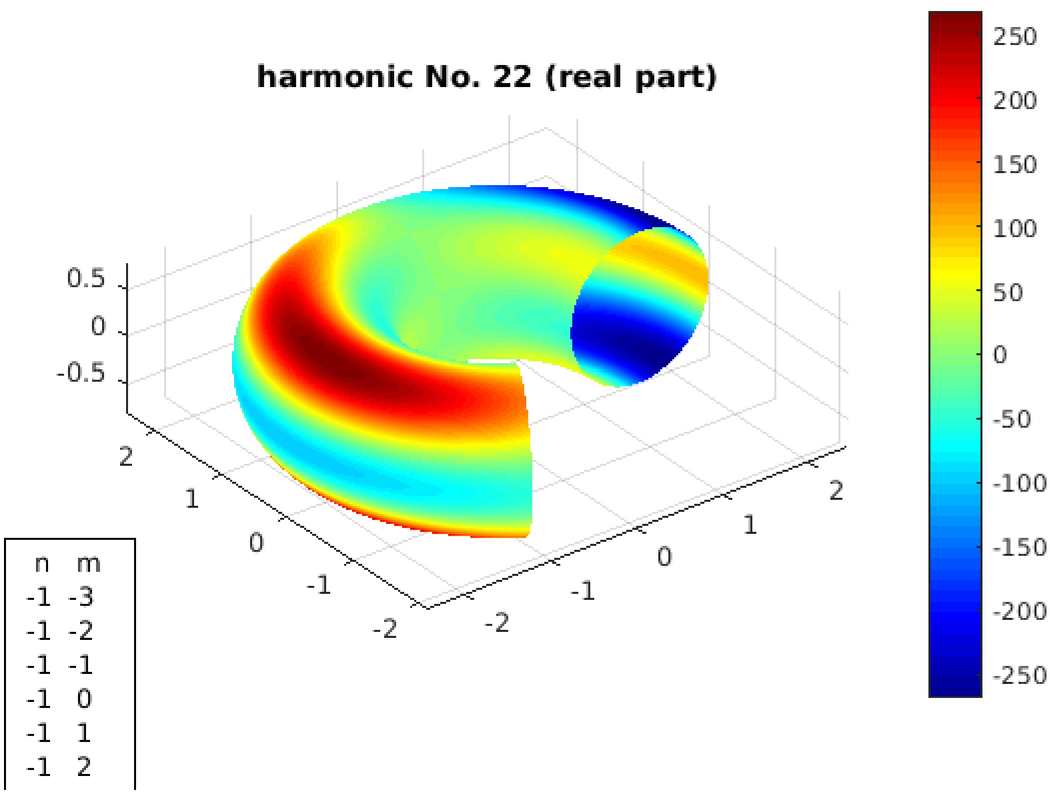

harmonic No. 17 (real part)
0.5
0
-0.5
2
1
0
-1
-2
-2
-1
0
1
2
n m
-1 -3
-1 -1
-1 1
-1 3
200
150
100
50
0
-50
-100
-150
-200

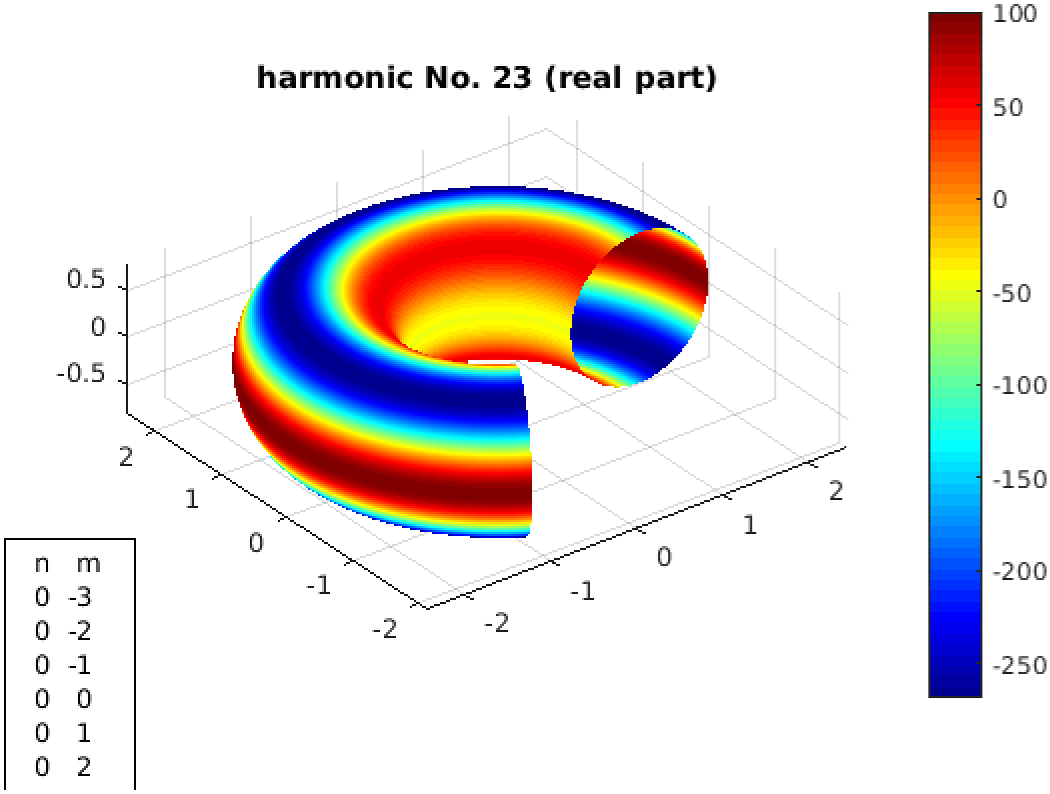

harmonic No. 18 (real part)
0.5
0
-0.5
2
1
0
-1
-2
-2
-1
0
1
2
n m
0 -3
0 -1
0 1
0 3
×10-15
5
4
3
2
1
0
-1
-2
-3
-4
-5

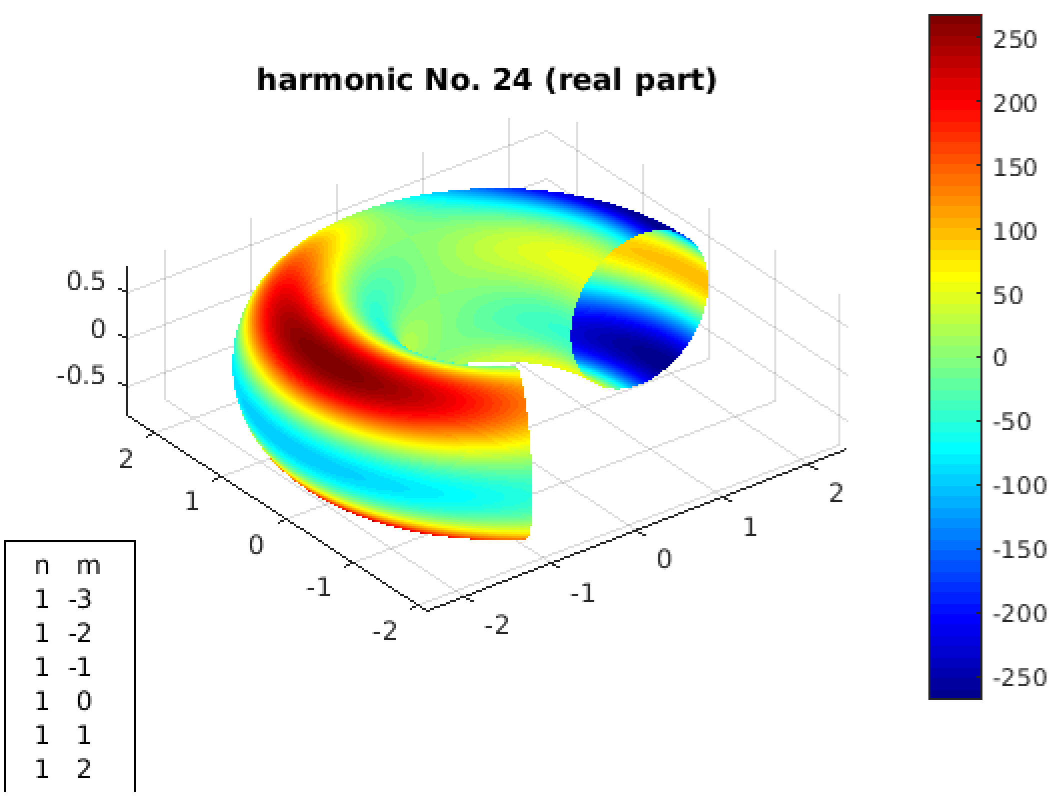

harmonic No. 19 (real part)
n m
1 -3
1 -1
1 1
1 3
200
150
100
50
0
-50
-100
-150
-200
0.5
0
-0.5
2
1
0
-1
-2

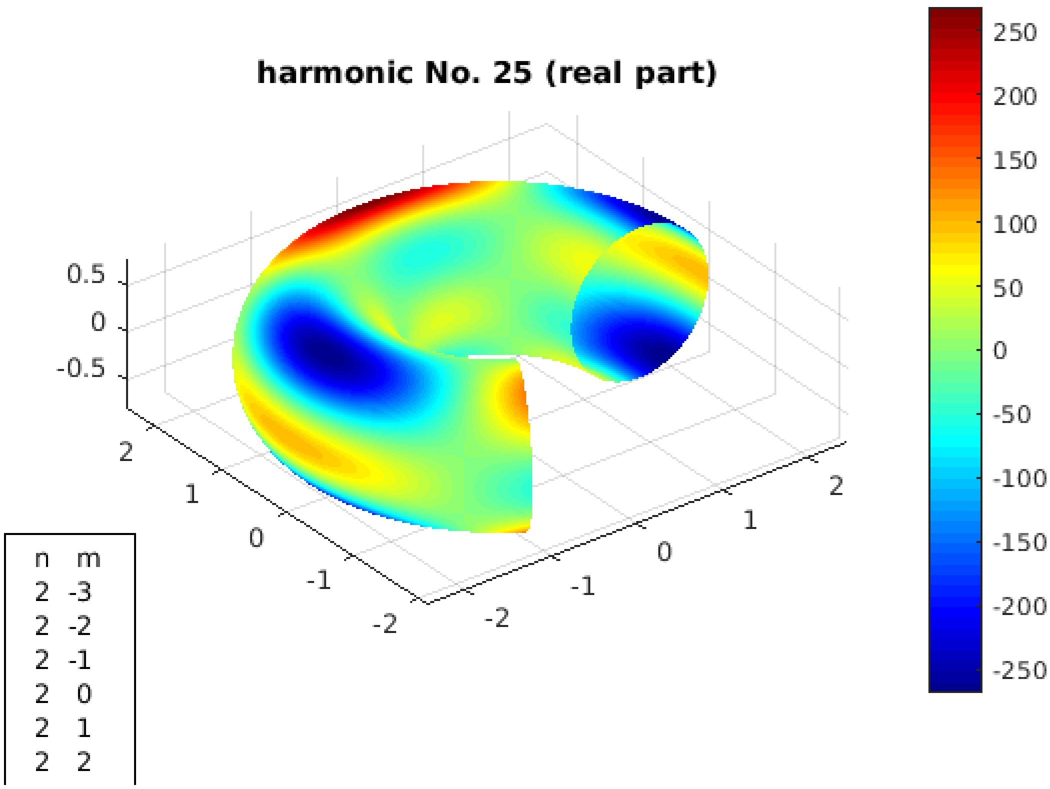

harmonic No. 20 (real part)
n m
2 -3
2 -1
2 1
2 3
200
150
100
50
0
-50
-100
-150
-200
0.5
0
-0.5
2
1
0
-1
-2

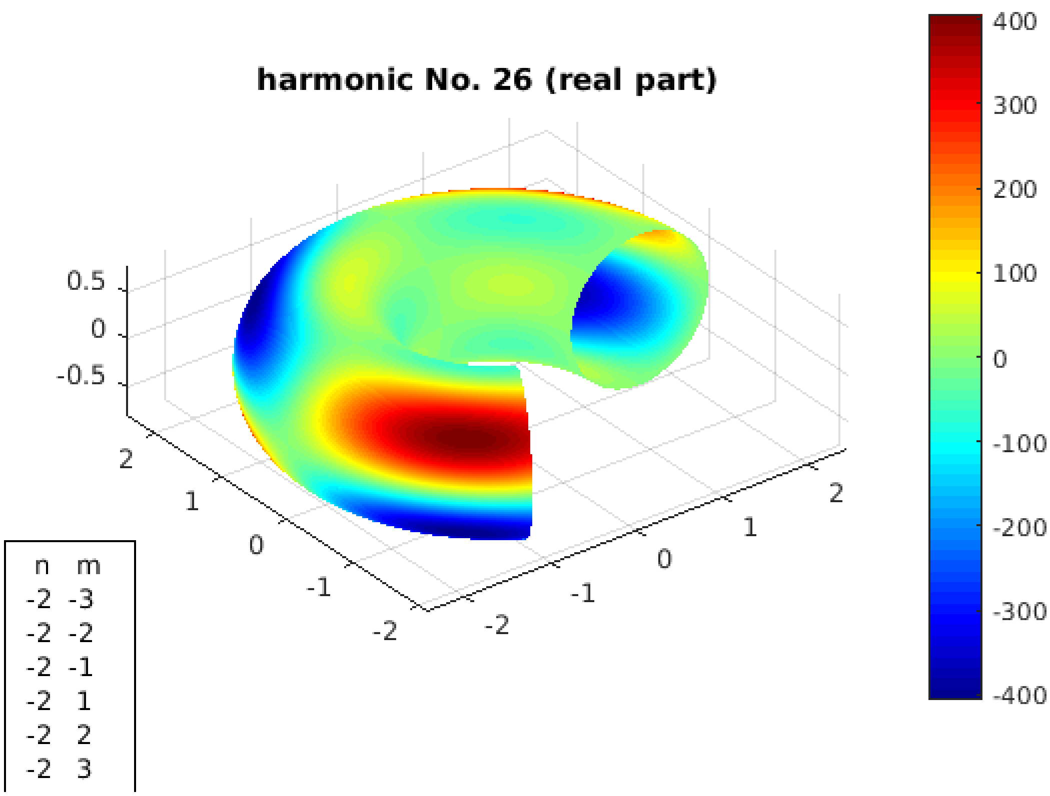

harmonic No. 21 (real part)
250
200
150
100
50
0
-50
-100
-150
-200
-250
0.5
0
-0.5
2
1
0
-1
-2
-2
-1
0
1
2
n m
-2 -3
-2 -2
-2 -1
-2 0
-2 1
-2 2

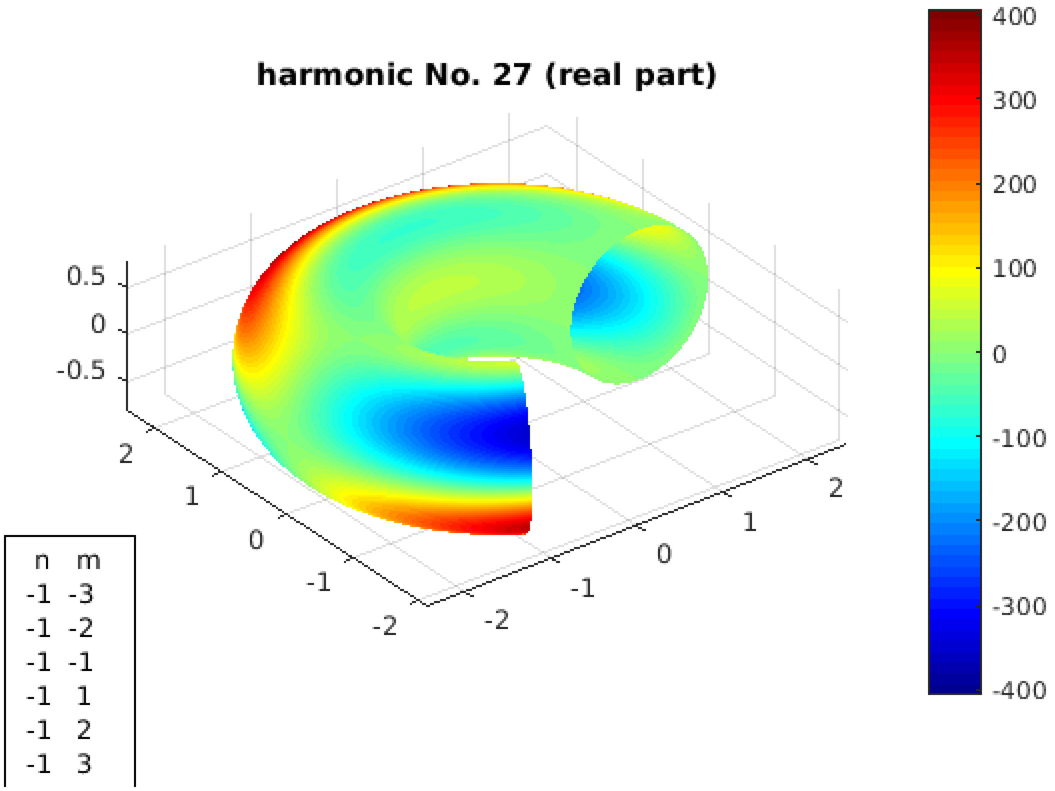

harmonic No. 22 (real part)
250
200
150
100
50
0
-50
-100
-150
-200
-250
0.5
0
-0.5
2
1
0
-1
-2
-2
-1
0
1
2
n m
-1 -3
-1 -2
-1 -1
-1 0
-1 1
-1 2

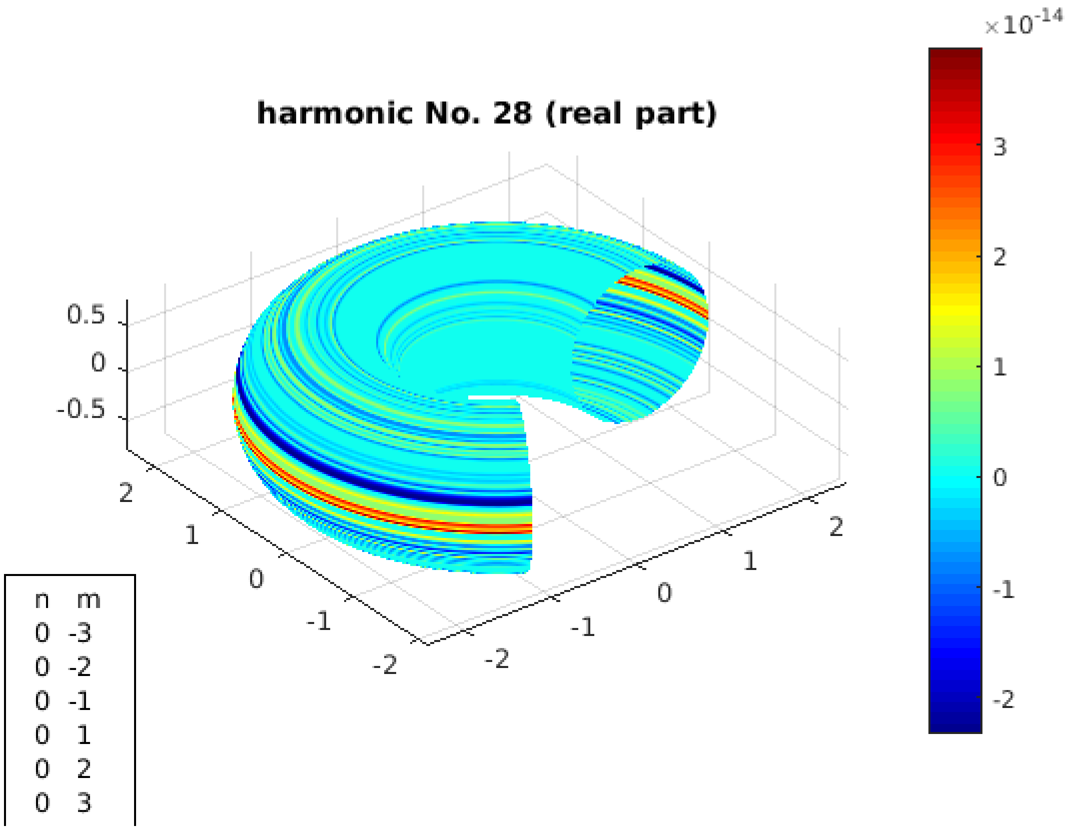

harmonic No. 23 (real part)
100
50
0
-50
-100
-150
-200
-250
0.5
0
-0.5
2
1
0
-1
-2
-2
-1
0
1
2
n m
0 -3
0 -2
0 -1
0 0
0 1
0 2

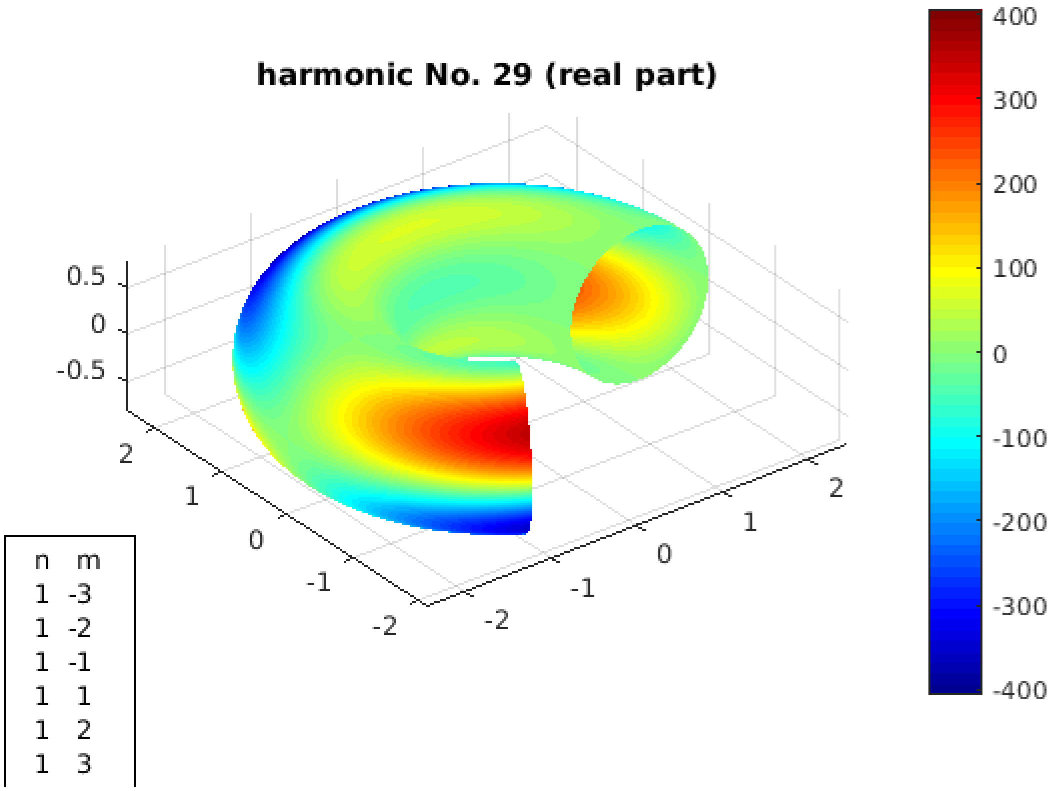

harmonic No. 24 (real part)
250
200
150
100
50
0
-50
-100
-150
-200
-250
0.5
0
-0.5
2
1
0
-1
-2
-2
-1
0
1
2
n m
1 -3
1 -2
1 -1
1 0
1 1
1 2

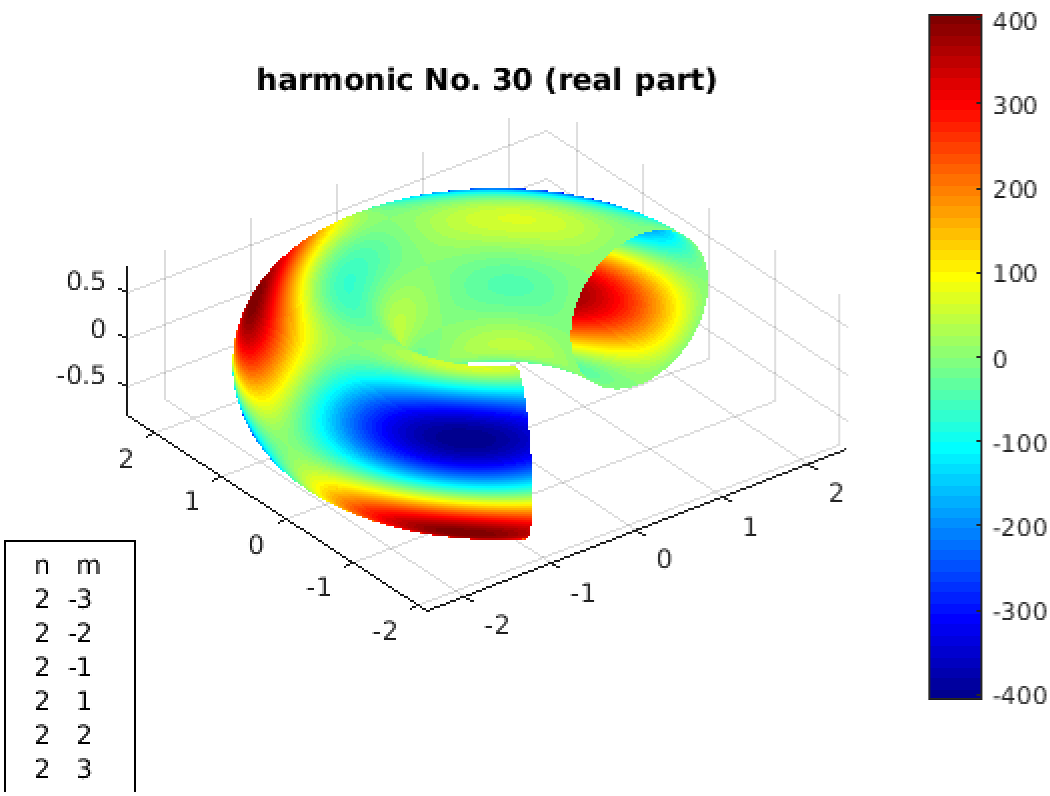

harmonic No. 25 (real part)
250
200
150
100
50
0
-50
-100
-150
-200
-250
0.5
0
-0.5
2
1
0
-1
-2
-2
-1
0
1
2
n m
2 -3
2 -2
2 -1
2 0
2 1
2 2

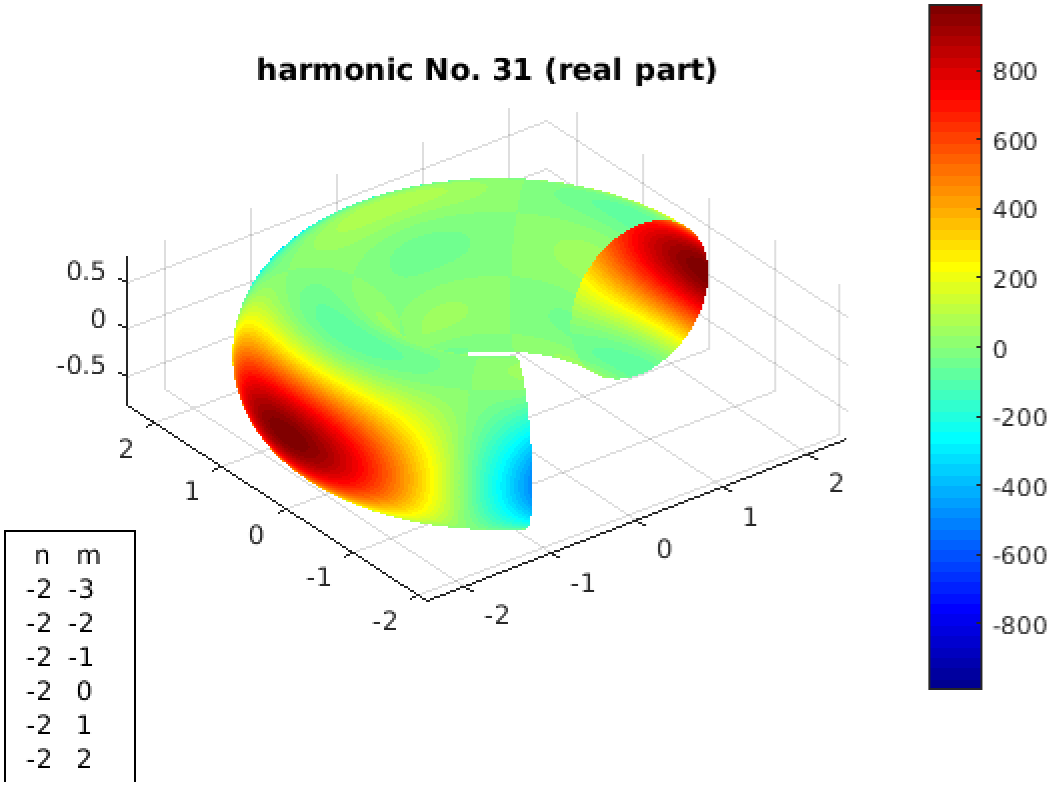

harmonic No. 26 (real part)
400
300
200
100
0
-100
-200
-300
-400
0.5
0
-0.5
2
1
0
-1
-2
-2
-1
0
1
2
n m
-2 -3
-2 -2
-2 -1
-2 1
-2 2
-2 3

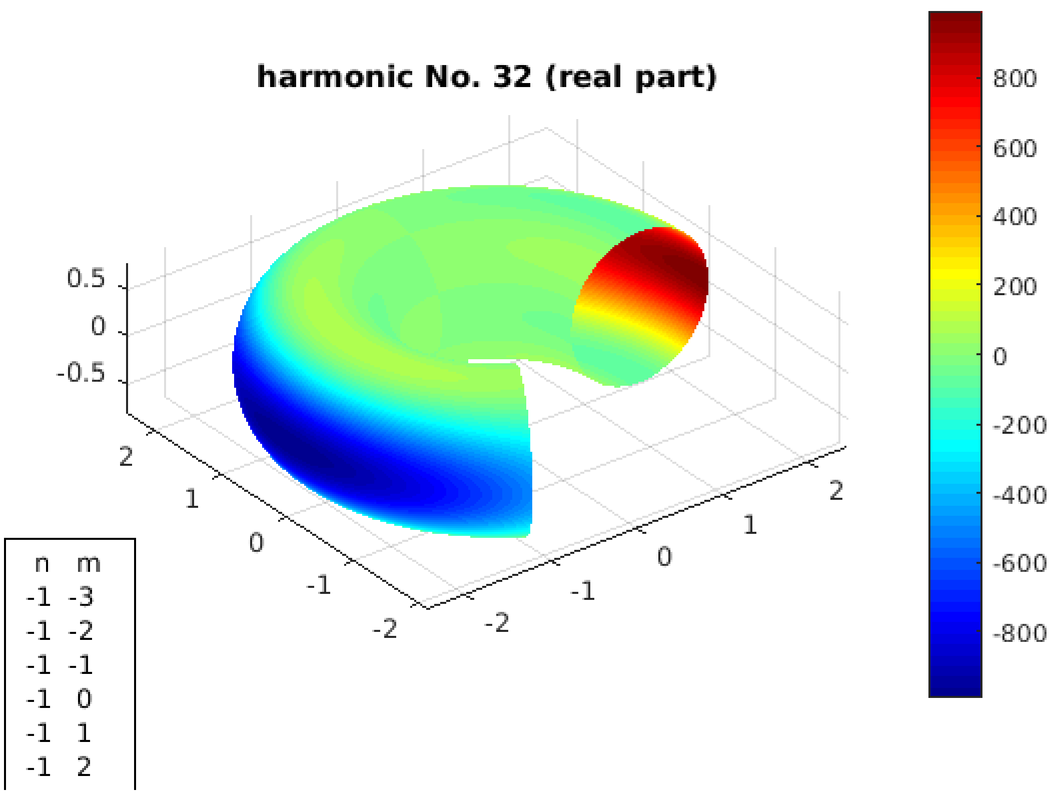

harmonic No. 27 (real part)
400
300
200
100
0
-100
-200
-300
-400
0.5
0
-0.5
2
1
0
-1
-2
-2
-1
0
1
2
n m
-1 -3
-1 -2
-1 -1
-1 1
-1 2
-1 3

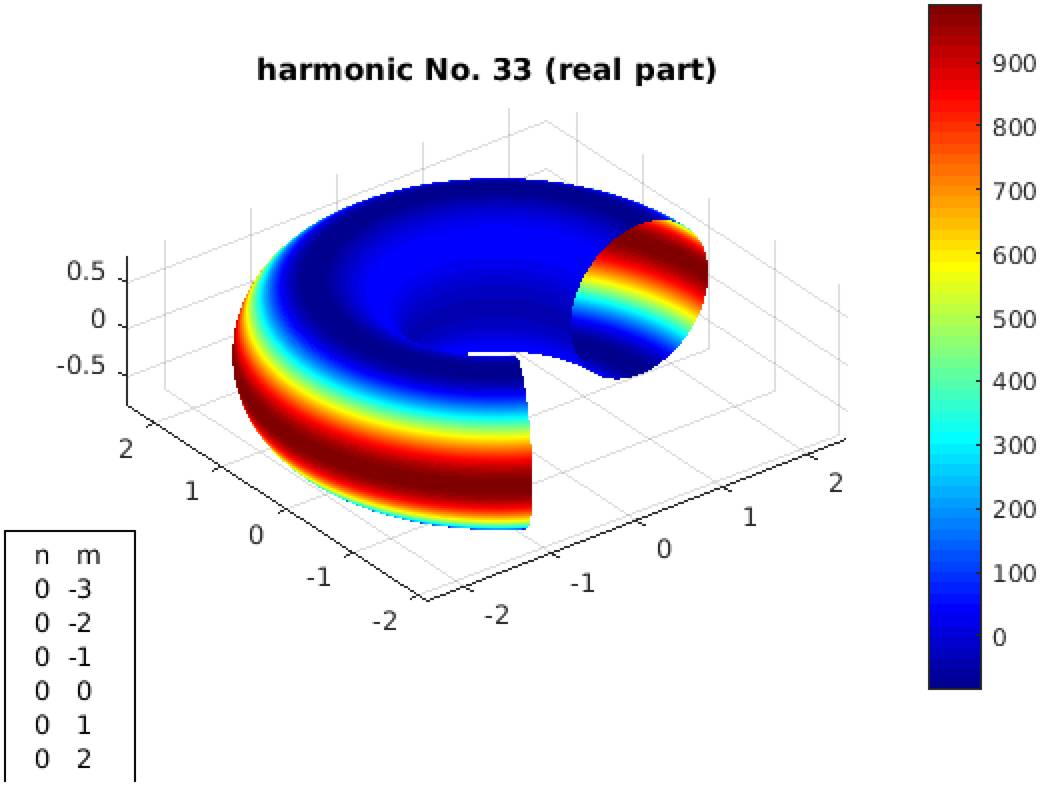

harmonic No. 28 (real part)
×10^-14
3
2
1
0
-1
-2
0.5
0
-0.5
2
1
0
-1
-2
-2
-1
0
1
2
n m
0 -3
0 -2
0 -1
0 1
0 2
0 3

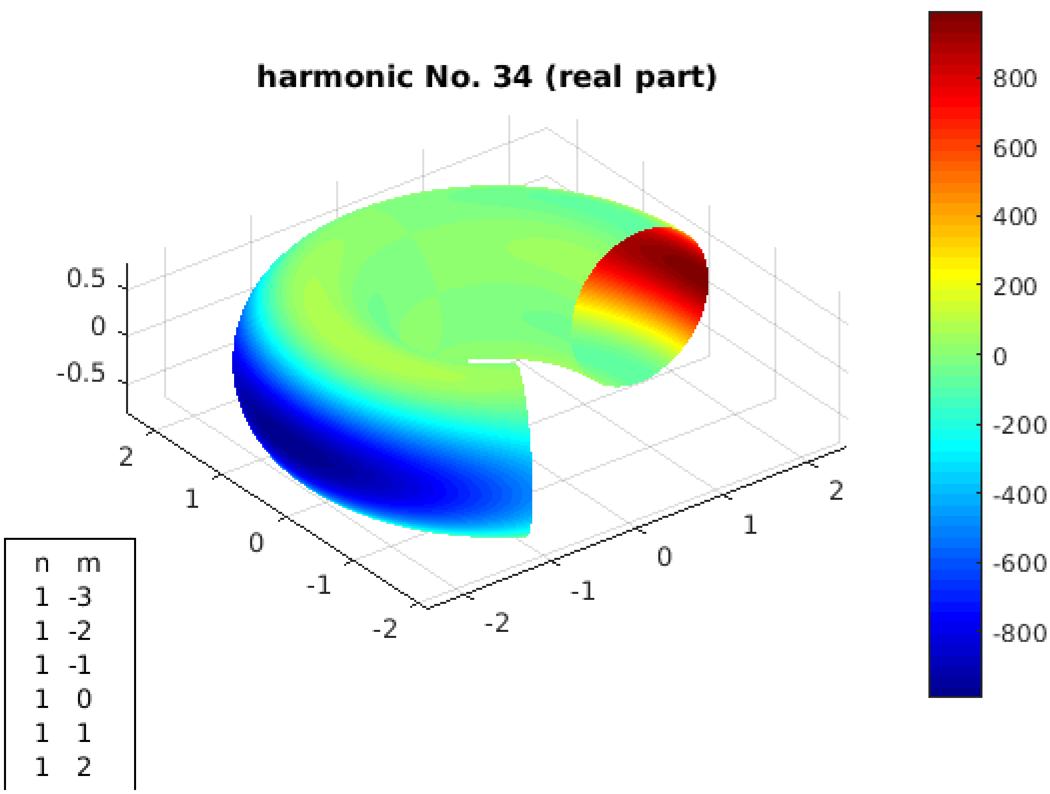

harmonic No. 29 (real part)
n m
1 -3
1 -2
1 -1
1 1
1 2
1 3

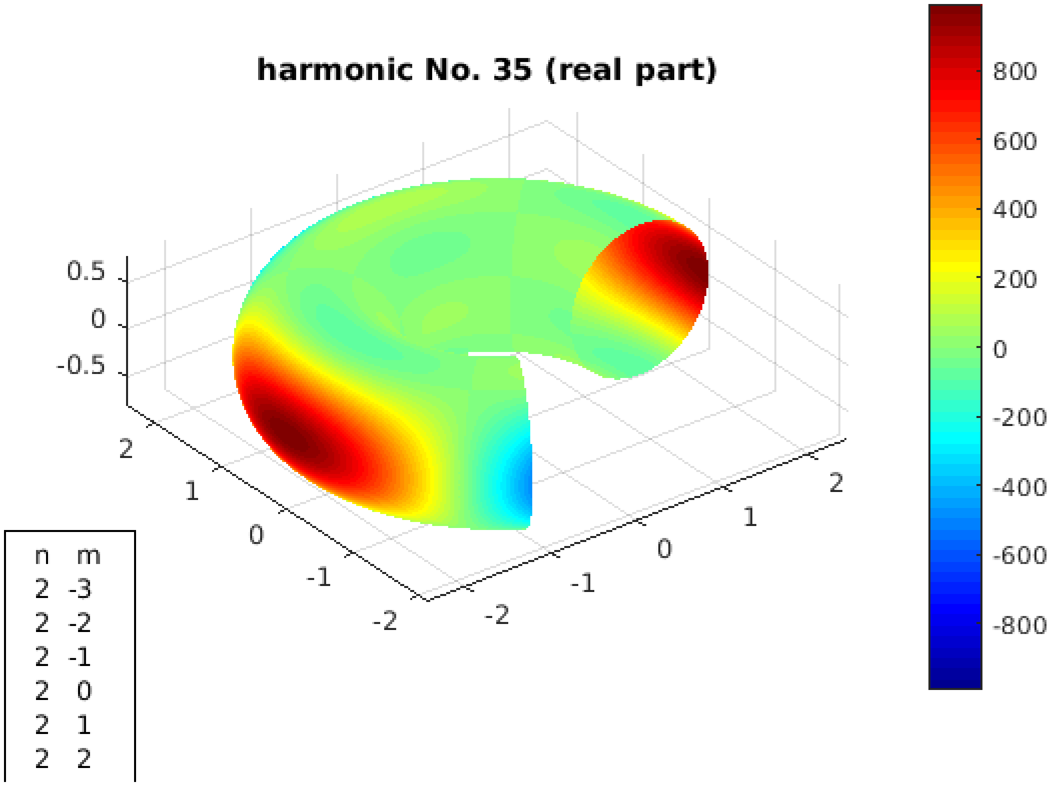

harmonic No. 30 (real part)
n m
2 -3
2 -2
2 -1
2 1
2 2
2 3


harmonic No. 31 (real part)
800
600
400
200
0
-200
-400
-600
-800
0.5
0
-0.5
2
1
0
-1
-2
-2
-1
0
1
2
n m
-2 -3
-2 -2
-2 -1
-2 0
-2 1
-2 2


harmonic No. 32 (real part)
800
600
400
200
0
-200
-400
-600
-800
0.5
0
-0.5
2
1
0
-1
-2
-2
-1
0
1
2
n m
-1 -3
-1 -2
-1 -1
-1 0
-1 1
-1 2


harmonic No. 33 (real part)
0.5
0
-0.5
2
1
0
-1
-2
-2
-1
0
1
2
n m
0 -3
0 -2
0 -1
0 0
0 1
0 2
900
800
700
600
500
400
300
200
100
0


harmonic No. 34 (real part)
0.5
0
-0.5
2
1
0
-1
-2
-2
-1
0
1
2
n m
1 -3
1 -2
1 -1
1 0
1 1
1 2
800
600
400
200
0
-200
-400
-600
-800



Filtering

The standard approach is to keep just a certain set of Fourier modes this corresponds to ommiting one basis function and its complex conjugate

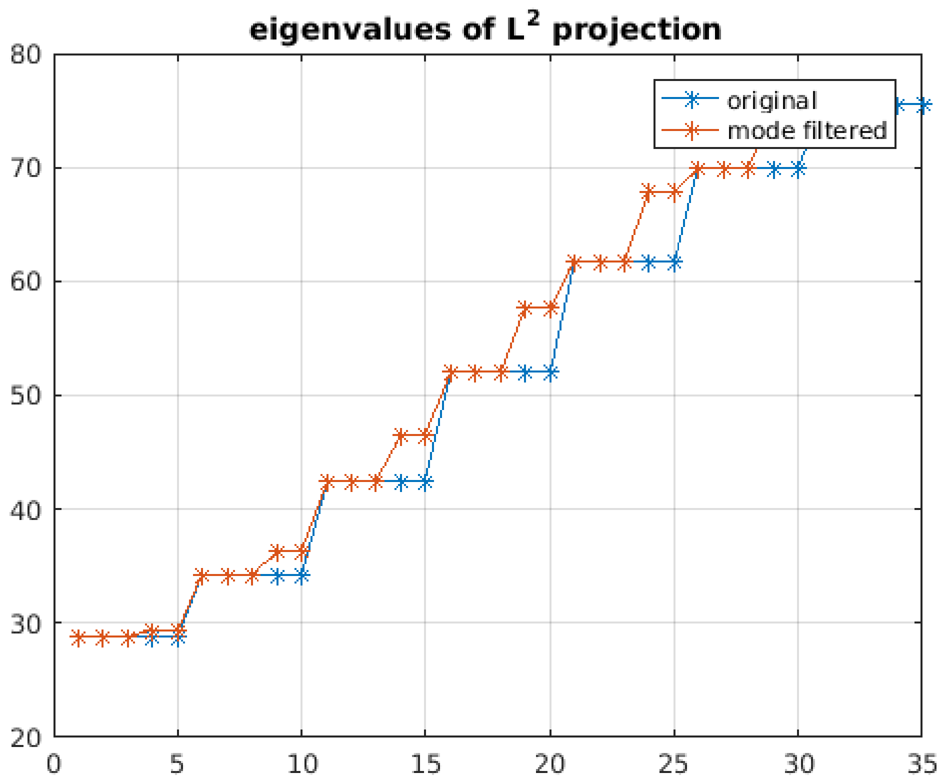
```
filter.nm=[2 3];

% assemble filter matrix
filter.mat=eye(size(MM));

%delete mode ...
idx=find((nm(:,1)==filter.nm(1) & nm(:,2)==filter.nm(2)));
filter.mat(idx,idx)=0;
%and complex conjugate
idx=find((nm(:,1)==-filter.nm(1) & nm(:,2)==-filter.nm(2)));
filter.mat(idx,idx)=0;

%Apply filter
filter.MM=filter.mat'*MM*filter.mat;


%Calculate eigenvalues of filtered matrix
filter.D=eig(double(subs(subs(filter.MM,r,minorR),R0,majorR)));
filter.D(filter.D==0)=[];

% Eigenvalues of original matrix
```

```
d1=eig(double(subs(subs(MM,r,minorR),R0,majorR)));
% Eigen
d2=eig(double(subs(subs(filter.MM,r,minorR),R0,majorR)));
d2(d2==0)=[];
```

Filtering changes global spectrum of operator

Whereas filtering directly the eigenvalues leaves the spectrum untouched and acts only locally. This also works in radial direction

```
figure
plot(D,'*-'); hold on;
plot(filter.D,'*-');
legend('original','mode filtered');
title('eigenvalues of L^2 projection'); grid on;
```



Harmonics of Laplace operator

Todo

```
%
%
%
%
% sort(diag(U))
```

```
%
%
% Mint=matlabFunction(M,'Vars',[theta,phi,m1,m2,n1,n2]);
%
% num_theta=5; num_phi=5;
% [m,n]=ndgrid( -num_theta:1:num_theta, -num_phi:1:num_phi);
% m=reshape(m,numel(m),1);
% n=reshape(n,numel(n),1);
% nm=[n,m];
%
%
%
% M=zeros((num_theta*2+1)*(num_phi*2+1));
% for idx=1:length(nm)
%    for jdx=idx:length(nm)
%        M(idx,jdx)=sum(sum(Mint(THETA,PHI,...
%            nm(idx,2),nm(jdx,2),nm(idx,1),nm(jdx,1))))...
%            *(2*pi/length(int_grid))^2;
%    end
% end
% M(abs(M)<1e-13)=0;
% M=M+M'-diag(diag(M));
%
%
% abs(nm)==0
% size(M)
% rank(M)
%
%
%
% K=psi_grad(m1,n1)'*L*psi_grad(m2,n2);
% K=subs(subs(K,'r',1),'R0',5);
% K=simplify(K);
%
% disp(K)
% %
 int(int(psi_grad(m1,n1)'*L*psi_grad(m2,n2),phi,0,2*pi),theta,0,2*pi)
%
% %K_mn=int(int(K_mn, theta,0,2*pi),phi,0,2*pi);
%
% Kint=matlabFunction(K,'Vars',[theta,phi,m1,n1,m2,n2]);
%
% real(K)
% integral2(@(x,y)real(Kint(x,y,1,1,3,3)),0,2*pi,0,2*pi)
%
%
% integral( @(x)exp(1j*(1)*x),0,2*pi)
%
%
%
% K_mn=simplify(int(K, phi,0,2*pi));
%
% simplify(K_mn)
%
```

```
% simplify(K_mn(2,2,3,3))
%
% int(K
%
%
%
%
%
% int(exp(1j*(n1-n2)*theta),0,2*pi)
%
% K_mn(nm)
%
% K=zeros(num_phi*num_theta);
%
%
%
%
% nm=1;
% n=1;
% for idx=1:size(K,1)
%
% for jdx=idx:size(K,1)
%
% end
%
% end
%
% subs(L)
%
%
%
%
%
%
% %% Flux surface
% % We fix _r_ to describe a single flux surface.
%
%
%
%
%
% r=1;
% R0=4;
%
% syms R0 r;
%
% [V,D]=eig([ 2*pi*R0*r, pi*r^2; pi*r^2, 2*pi*R0*r]);
%
%
%
```

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