Matrix multiplication and graph powers

A matrix is a rectangular array of elements with n rows and m columns.

each element in a matrix is called an entry

the address of an element, denoted e_{ij}, is the element's location in row i and column j

in a square matrix, the rows and columns are equal

an adjacency matrix is a square matrix that corresponds to a digraph, G. There is a row and column for each vertex in the digraph.

if there is an edge from vertex i to vertex j, $e_{ij} = 1$

if there is NOT edge from vertex i to vertex j, $e_{ij} = \varnothing$

a boolean matrix only has entries from $\{0,1\}$

Matrix multiplication

to multiply matrix A and B...

matrix A must be a $m \times K$ matrix

AND B must be a $K \times n$ matrix

the product $A \times B$ will be a $m \times n$ matrix

to calculate $A \times B$,

- the entry in the address ij of $A \times B$ is determined by multiplying row i of A with column j of B

  i.e the first element of i with first element of j

- then get the sum of the ij products

  i.e $A_{i1}B_{1j} + A_{i2}B_{2j} + \ldots + A_{ik}B_{kj}$

  → this is the dot product

the power of a matrix is the product of k copies of that matrix

ex: $A^k = \underbrace{A \cdot A \cdot \quad \cdot A}_{k \text{ times}}$

if adjacency matrix A represents graph G, then A^k is the adjacency matrix for graph G^k.

- there is a walk of length k from u to v in G if and only if the entry in row u, colum v = 1 in matrix A^k

Theorem: Relationship b/w the powers of a graph and the powers of its adjacency matrix:

Let G be a digraph with n vertices, and let A be the $n \times n$ adjacency matrix for G.

→ for any $k \geq 1$, A^k is the adjacency matrix for G^k, where Boolean addition and multiplication are used to compute A^k

Matrix addition

matrices of the same dimensions can be added by finding the sum of the elements of the same address

addition and graph union

Let G and H be two digraphs with the same vertex set. Let A and B the adjacency matrices for G and H. The adjacency matrix for $G \cup H$ is $A+B$, where boolean addition is used on the entries of A and B.