

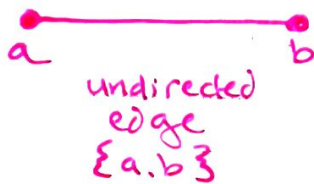
Introduction to Graphs

an **undirected graph** consists of a set of vertices, V , and a set of edges, E where each edge is an **unordered** pair of vertices

• useful for modelling symmetric relationships

ex: edges $ab = ba$

edge notation: $\{a, b\}$ (as opposed to (a, b) in a digraph)



vs.



parallel edges are multiple edges between the same pair of vertices

a **self-loop** is an edge between a vertex and itself

a **simple graph** has no parallel edges and no self-loops

adjacent vertices are vertices that share an edge

an **endpoint vertex** of an edge is a vertex connected to the edge

an **incident edge** to a vertex is a edge connected to that vertex

a vertex v is a **neighbor** to vertex u if an edge connects them

the **degree** of a vertex is the number of neighbors the vertex has

the **total degree** of a graph is the sum of the degrees of all vertices

in a **regular graph**, all vertices have the same degree

in a **d -regular graph**, all vertices have degree of d

a **subgraph** H of graph G uses edges that are a subset of G 's edges and vertices that are a subset of G 's vertices

* note: G is always a subgraph of G

Theorem: Number of edges and total degree

Let $G = (V, E)$ be an undirected graph. **Total degree = $2 \cdot (\# \text{ of edges})$**

i.e. $\sum_{v \in V} \deg(v) = 2 \cdot |E|$