Equivalence relations

a relation R is an equivalence relation if R is reflexive, symmetric, and transitive

Notation: $a \sim b$ = aRb and R is an equivalence relation

example(s):

- B is a relation on all people, where xBy if x and y have the same birthday
  - reflexive: xBx
  - symmetric: $(xBy) \rightarrow (yBx)$
  - transitive: $(xBy \wedge yBz) \rightarrow (xBz)$
- $R: \mathbb{Z} \rightarrow \mathbb{Z}$, where xRy if $x^2 = y^2$

an equivalence class is the set of all x in the domain such that $a \sim x$ (i.e aRx) is true, where a is an element in the domain and R is an equivalence relation on the domain

Notation: $[a]$ is the equivalence class for a

Theorem: Structure of equivalence relations

Consider an equivalence relation on set A. Let $x, y \in A$:

- If $x \sim y$ then $[x] = [y]$
- If $\neg(x \sim y)$ then $[x] \cap [y] = \emptyset$

a partition of the domain set A is a set of non-empty subsets of A that are pairwise disjoint and whose union is A.

Theorem: Equivalence relations define a partition

Consider an equivalence relation over a set A. The set of all distinct equivalence classes defines a partition of A. "Distinct" means that if there are two equal equivalence classes $[a]=[b]$, the set $[a]$ is included only once.