Bayes Theorem - YouTube notes

Probability of event A given event B has occurred

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Probability of event B given event A has occurred

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$\Rightarrow$ $P(A \cap B) = P(A|B) \cdot P(B)$ and $P(A \cap B) = P(B|A) \cdot P(A)$ (i)

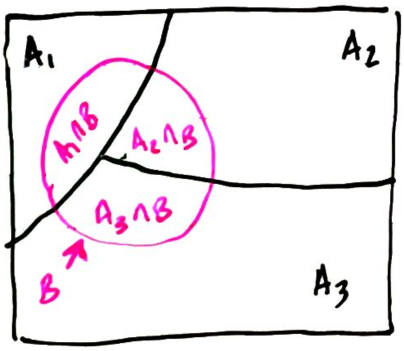
$\Rightarrow$ $P(A|B) \cdot P(B) = P(B|A) \cdot P(A)$

$\Rightarrow$ $P(A|B) = \frac{P(B|A \cdot P(A)}{P(B)}$ and $P(B|A) = \frac{P(A|B) \cdot P(B)}{P(A)}$ (ii)

If events are collectively exhaustive, then combining the elements from all events results in the sample space (i.e. there is no element in the sample space that is outside of the events)

in other words, adding up the probabilities of collectively exhaustive events results in 1 (or 100% chance).

ex:



- $\underbrace{A_1, A_2, \text{and } A_3}$ are collectively exhaustive
  - ↳ called prior probabilities
- since event B overlaps w/ all 3 events,
  - $P(B) = P(A_1 \cap B) + P(A_2 \cap B) + P(A_3 \cap B)$
- using equation (ii):

$$P(B) = [P(B|A_1) \cdot P(A_1)] \cdot [P(B|A_2) \cdot P(A_2)] \cdot [P(B|A_3) \cdot P(A_3)]$$

$\rightarrow$ $P(B) = \sum_{i=1}^{n} P(B|A_i) \cdot P(A_i)$ if $A_1 - A_i$ are mutually disjoint + collectively exha...