

## ## Properties of Functions ##

one-to-one or injective function: is a function  $f: X \rightarrow Y$

where  $x_1 \neq x_2$  implies  $f(x_1) \neq f(x_2)$

i.e. different elements in  $X$  will map to different elements in  $Y$

i.e. no element in  $Y$  is mapped upon by two elements in  $X$

onto or surjective function is a function  $f: X \rightarrow Y$  where

the range of  $f$  is equal to the target  $Y$

i.e. for every  $y \in Y$ , there is an  $x \in X$  such that  $f(x) = y$

i.e. there are no elements in  $Y$  that are unmapped

one-to-one correspondence or bijective function or bijection

is a function that is both one-to-one and onto

i.e. every element in  $Y$  is mapped upon by exactly one element in  $X$ .

(i.e.) no element in  $Y$  has zero or  $>1$  arrows to it

If the domain and target are finite sets:

relative sizes can be inferred if the equations are one-to-one and/or onto

• if  $f: D \rightarrow T$  is onto,  $|D| \geq |T|$

- for every element in target, there is at least 1 element in domain (all elements in  $T$  must be mapped at least once)

• if  $f: D \rightarrow T$  is one-to-one,  $|T| \geq |D|$

- for every element in domain, there is a unique element in the target (but target can exceed range)

• if  $f: D \rightarrow T$  is a bijection,  $|D| = |T|$