Strict orders and directed acyclic graphs

a partial order acts similar to the $\leq$ operator, and a strict order acts similar to the $<$ operator

a relation R is a strict order if R is transitive and anti-reflexive.
- if an order is transitive and anti-reflexive <u>the result will also be anti-symmetric.</u>

<u>Notation</u>: $a \prec b = aRb$ if R is a strict order
$(A, \prec)$ where A is the domain
$(A, \prec)$ = the domain along w/ the strict order R defined on it

two elements are comparable if $x \prec y$ or $y \prec x$
two elements are incomparable if $\neg(x \prec y)$ AND $\neg(x \prec y)$
an element x is minimal if there is no y where $y \prec x$
an element x is maximal if there is no y where $x \prec y$

a total order is where every pair is comparable

<u>Directed Acylic Graphs</u>

a directed acyclic graph (or DAG) is a digraph that has no positive length cycles (i.e. a cycle's walk length is zero)
- useful for representing precedence relationships (ex: prerequisites)
- strict orders are closely related to DAGs

<u>Theorem: Directed acyclic graphs and strict orders</u>

Let G be a digraph. G has no positive length cykles if and only if G^+ is a strict order

if G is a digraph and G^+ is the transitive closure of G,
the minimal elements in G^+ have an in-degree = 0 in G
the maximal elements in G^+ have an out-degree = 0 in G