Cardinality of a union by complement

Example:

Let P_i be a set of four digit pins with an 8 in the ith digit

P_1 = all pins w/ $8***$ format
P_2 = all pins w/ $*8**$ format
P_3 = all pins w/ $**8*$ format
P_4 = all pins w/ $***8$ format

The goal is to find $|P_1 \cup P_2 \cup P_3 \cup P_4|$

· using inclusion-exclusion will result in many terms and is not the most efficient way to determine the solution

· instead we can <u>count by complement</u> by

1.) Determining the total # of pins possible w/ 4 digits
   ↰ this is the Universe set = U

2.) Determining the total # of pins possible that have zero 8's
   ↰ this is the Complement = $\overline{P_1 \cup P_2 \cup P_3 \cup P_4}$

3.) Subtract the complement from the Universe set
   ↳ $|U| - |\overline{P_1 \cup P_2 \cup P_3 \cup P_4}| = |P_1 \cup P_2 \cup P_3 \cup P_4|$

order matters → count permutations

$|U| = 10^4$ b/c $****$ and each $*$ has 10 options → $(10)(10)(10)(10)$

$|\overline{P_1 \cup P_2 \cup P_3 \cup P_4}| = 9^4$ b/c $****$ and each $*$ has 9 options (no 8 possible)

⇒ $|P_1 \cup P_2 \cup P_3 \cup P_4| = |U| - |\overline{P_1 \cup P_2 \cup P_3 \cup P_4}| = 10^4 - 9^4$