

Directed graphs, paths, and cycles

A directed graph/digraph consists of a pair (V, E) where

V : a set of vertices

E : a set of directed edges

and $E \subseteq V \times V$

a vertex is an individual element of V

- typically represented as a dot or circle labelled w/vertex name

an edge is an individual element of E

- is an ordered pair (u, v) where $u \in V$ and $v \in V$

- typically represented as an arrow from u to v

- the tail is the vertex u (where the arrow originates)

- the head is the vertex v (where arrow terminates)

a self-loop is when the head and tail are same vertex (i.e. $u=v$)

the in-degree of a vertex is the number of edges pointing into the vertex

the out-degree of a vertex is the number of edges pointing out of the vertex

mathematically:

$$\text{in-degree}(v) = |\{u \mid (u, v) \in E\}|$$

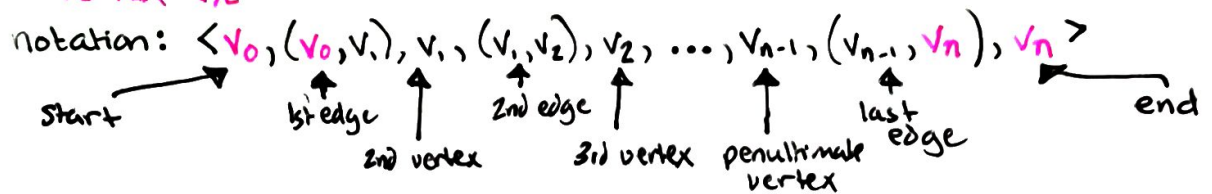
$$\text{out-degree}(v) = |\{u \mid (v, u) \in E\}|$$

A digraph is the same mathematically as a relation E on the set V !

- uEv if and only if there is a directed edge from u to v

- a picture of a digraph is equal to an arrow diagram for relation E

- a walk from v_0 to v_n in digraph G is a sequence of alternating vertices and edges that starts with vertex v_0 and ends with vertex v_n



- the length of a walk is the # of edges in the walk
- an open walk has different start and end vertices
- a closed walk has the same vertex for start and end

alternate notation for walk from v_0 to v_n :

$$\langle v_0, v_1, v_2, \dots, v_{n-1}, v_n \rangle$$

- a trail is an open walk in which no edge repeats
- a circuit is a closed walk in which no edge repeats
- a path is an open walk in which no vertex repeats
- a cycle is a closed walk of length ≥ 1 in which no vertex repeats except for the start and end vertex