The Language of Proofs

Thus and therefore

- a statement that follows from the previous statement or previous statements, can be started with "Thus" or "Therefore"

    ex: n and m are integers. Therefore, $n+m$ is also an integer

    ex: n is a positive integer. Thus, $n \geq 1$

Let

- new variable names are often introduced with the word "let"

    ex: Let x be a positive integer

Suppose

- can also be used to introduce a new variable

    ex: Suppose that x is a positive integer

- can also be used to introduce a new assumption

    ex: Suppose that x is odd

    (assuming x has already been introduced as an integer earlier in the proof)

Since and Because we know that

- if a statement depends on a fact that appeared earlier in the proof or on assumptions of the theorem, it can be helpful to use "since" while reminding the reader of that fact

    ex: (assuming $x>0$ and $y>z$ were established earlier)

    Since $x>0$ and $y>z$, then $xy>xz$.

    ex: Because we know that $x>0$ and $y>z$, then $xy>xz$

By definition

if a fact is known because of a definition, it can be started with the phrase "By definition"

ex: The integer m is even. By definition, $m = 2k$ for some integer k

By assumption

a fact is known because of an assumption, can be started with the phrase "By assumption"

ex: By assumption, x is positive. Therefore, $x > 0$

In other words

when rephrasing a statement in a more specific way, the phrase "in other words" is useful

ex: we must show the average of x and y is positive. In other words, we must show that $(x+y)/2 > 0$.

Gives and yields

when a proof is clearer if even an algebraic step is justified, the words "gives" or "yields" are useful to say that one equation or inequality follows from another

ex: Multiplying both sides of the inequality $x > y$ by 2, gives $2x > 2y$

ex: Substituting $m = 2k$ into m^2 yields $(2k)^2$

ex: Since $z > 0$, we can multiply both sides of the inequality $x > y$ by z to get $xz > yz$