Definition of Functions

A function f that maps elements of a set X to elements of a set Y, is a subset of X×Y such that for every $x \in X$ there is exactly one $y \in Y$ for which $(x,y) \in f$.

Notation: $f: X \rightarrow Y$ says f maps from set X to set Y
- X is the domain + Y is the target

Notation: $(x,y) \in f$ or $f(x) = y$ says that f maps element x to element y

f is NOT well-defined if:
- f maps an element from the domain to zero elements, OR
- f maps an element from the domain to >1 element in target

arrow diagram - elements of domain on left and elements of target are on the right. An arrow goes from $x \in X$ to $y \in Y$ if $f(x) = y$
- B/c f is a function: there is exactly one arrow pointing out of every element in domain

range - the elements in Y that are mapped to from X for $f: X \rightarrow Y$
Notation: Range of $f = \{ y : (x,y) \in f, \text{ for some } x \in X \}$

A mathematical function's definition is NOT complete until the domain of f is specified
ex: not complete: $f(x) = x^2 - 2$ $g(x) = |x|$
     complete: $f: \mathbb{Z} \rightarrow \mathbb{Z}$, where $f(x) = x^2 - 2$ $g: \mathbb{R} \rightarrow \mathbb{R}$, where $g(x) = |x|$

f and g are EQUAL functions if:
f and g have the same domain and target AND
$f(x) = g(x)$ for every element x in the domain
Notation: $f = g$