Definitions #

proposition - a statement that is either true or false
(note: all values must be known for a statement to be determined as true or false)

truth value whether proposition is actually true or false

logical operation - combines propositions using a particular composition rule
- conjunction operation - AND - $\wedge$
- disjunction operation - OR - $\vee$
- exclusive OR - XOR - $\oplus$
- negation operation - NOT - $\neg$
  - * negation only acts on single proposition *

compound proposition - individual propositions are connected with logical operations

conditional operation - IF this THEN that - $\rightarrow$
- hypothesis - follows IF
- conclusion - follows THEN

example: $p \rightarrow q$ (ONLY False if p=True and q=false)

- converse $q \rightarrow p$
- inverse $\neg p \rightarrow \neg q$
- contra positive $\neg q \rightarrow \neg p$

biconditional operation - IF and ONLY IF - $\leftrightarrow$

$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

Tautology - a compound proposition whose truth value is always true

Contradiction - a compound proposition whose truth value is always false

Logically equivalent propositions are two compound propositions whose truth table values are equivalent

Predicate - a logical statement whose truth value is a function of one or more variables

ex: $P(x)$ = P of x = $x + 3 = 6$

$Q(x, y)$ = Q of x and y = $x^2 = y$

* a predicate must have a defined value to be a proposition

ex: $P(5) = 5 + 3 = 6$ is false *

Domain - set of all possible values for the variable

(ex: all positive integers, all students in a class, or all cities in the United States)

Universal Quantifier - "for all values in the domain" - $\forall$

asserts that a predicate is true for all values of a variable

ex: $\forall x\, P(x)$ = "for all x, P(x)"

* the universally quantified statement is a proposition *

* a counterexample is a case for which the predicate is false *

Existential Quantifier - "there exists a value in the domain" - $\exists$

asserts a predicate is true for at least one value of a variable

ex: $\exists x\, P(x)$ = "there exists an x, such that P(x)"

* the existentially quantified statement is a proposition *