## Directed graphs, paths, and cycles ##

A directed graph / digraph consists of a pair $(V, E)$ where
- $V$: a set of vertices
- $E$: a set of directed edges
- and $E \subseteq V \times V$

a vertex is an individual element of $V$
- typically represented as a dot or circle labelled w/ vertex name

an edge is an individual element of $E$
- is an ordered pair $(u, v)$ where $u \in V$ and $v \in V$
- typically represented as an arrow from $u$ to $v$
- the tail is the vertex $u$ (where the arrow originates)
- the head is the vertex $v$ (where arrow terminates)

a self-loop is when the head and tail are same vertex (i.e. $u = v$)

the in-degree of a vertex is the number of edges pointing into the vertex
the out-degree of a vertex is the number of edges pointing out of the vertex

mathematically:

$$\text{in-degree}(v) = |\{u \mid (u, v) \in E\}|$$
$$\text{out-degree}(v) = |\{u \mid (v, u) \in E\}|$$

A digraph is the same mathematically as a relation $E$ on the set $V$
- $u E v$ if and only if there is a directed edge from $u$ to $v$
- a picture of a digraph is equal to an arrow diagram for relation $E$

- a walk from $v_0$ to $v_n$ in digraph $G$ is a sequence of alternating vertices and edges that starts with vertex $v_0$ and ends with vertex $v_n$

notation: $\langle v_0, (v_0, v_1), v_1, (v_1, v_2), v_2, \ldots, v_{n-1}, (v_{n-1}, v_n), v_n \rangle$

start — 1st edge — 2nd vertex — 2nd edge — 3rd vertex — penultimate vertex — last edge — end

- the length of a walk is the # of edges in the walk
- an open walk has different start and end vertices
- a closed walk has the same vertex for start and end

alternate notation for walk from $v_0$ to $v_n$:

$\langle v_0, v_1, v_2, \ldots, v_{n-1}, v_n \rangle$

- a trail is an open walk in which no edge repeats
- a circuit is a closed walk in which no edge repeats
- a path is an open walk in which no vertex repeats
- a cycle is a closed walk of length $\geq 1$ in which no vertex repeats except for the start and end vertex