Inclusion-Exclusion Principle ##

With two sets

$$|A \cup B| = |A| + |B| - |A \cap B|$$

with three sets

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

The General inclusion-exclusion principle

1.) Add the size of each subset
2.) Subtract the size of the intersection of each set pair
3.) Add the three-way intersection of each set triplet
4.) Continue until the final term

      if (# of sets is even) → subtract last term

      if (# of sets is odd) → last term is added

Let $A_1, A_2, \ldots, A_n$ be a set of n finite sets

$$\left| A_1 \cup A_2 \cup \ldots \cup A_n \right| = \sum_{j=1}^{n} |A_j| \qquad \leftarrow \text{add each subset's cardinality}$$

then subtract the cardinality of each pair's intersection →

$$-\sum_{1 \leq j \leq n} |A_j \cap A_k|$$

the add the cardinality of each triplet's intersection →

$$+\sum_{1 \leq j < k < l \leq n} |A_j \cap A_k \cap A_l|$$

$$\cdots$$

Calculate cardinality of the intersection of all sets.
If even, subtract that value
if odd, add that value →

$$+(-1)^{n+1} |A_1 \cap A_2 \cap \ldots \cap A_n|$$