Rules of Inference for Quantified Statements

c is an element (arbitrary or particular)
$\forall_x P(x)$
$\therefore P(c)$

Universal Instantiation

* if c is an element in the domain, and $P(x)$ is true for all elements in the domain, then $P(c)$ is true

$\exists_x P(x)$
$\therefore$ (c is a particular element) $\wedge P(c)$

Existential instantiation

* each use of existential instantiation must define a new element with its own name (ex: 'c', or 'd')
* There is a case in the domain where $P(x)$ is true, then c is a particular element where $P(x)$ is true, so $P(c)$ is true

c is an arbitrary element
$P(c)$
$\therefore \forall_x P(x)$

Universal generalization

* c is an element representative of all domain elements, and $P(c)$ is true, there fore $P(x)$ is true for all domain elements

c is an element (arbitrary or particular)
$P(c)$
$\therefore \exists_x P(x)$

Existential generalization

* c is an element in the domain, and $P(c)$ is true, therefore there exists a case in the domain where $P(x)$ is true