Graph powers and the transitive closure

the power of G where G is a digraph of (V, E) is equal to the number of times E is composed with itself and is represented as K and G^K is a digraph of (V, E^K)

$$E^1 = E$$
$$E^K = E \circ E^{K-1}, \text{ for all } K \geq 2$$

The <u>Graph Power Theorem</u>:

Let G be a digraph Let u and v be any two vertices in G. There is an edge from u to v in G^K if and only if there is a walk of length K from u to v in G

<u>The Transitive Closure</u>

the transitive closure of R is the smallest relation of R^+ that is transitive and includes all pairs from R Notation: R^+

i.e $R^+ = R \cup R^2 \cup \ldots \cup R^n$ where $n \leq |D|$ w/ D: domain

the transitive closure of G is the digraph of (V, E^+) where E^+ is the transitive closure of E. Notation: G^+

The union of G^K for all $K \geq 1$

$$G^+ = G \cup G^2 \cup \ldots \cup G^n \quad \text{where } n \leq |V|$$

alternatively ...

Look for $x, y, z \in G$ such that (x,y) and $(y,z) \in E$ but $(x,z) \notin E$. If found, then add (x,z) to relation E repeat until no pair is added to E