Composition of functions

Composition - process of applying a function to the result of another function

Composition of functions

f and g are two functions where $f: X \to Y$ and $g: Y \to Z$.
The composition of g with f, denoted $g \circ f$ is the function
$(g \circ f): X \to Z$
such that for all $x \in X$ $(g \circ f) = g(f(x))$

Order is important !!!! $f \circ g$ is not equivalent to $g \circ f$!!!

Composition is associative
$f \circ g \circ h = (f \circ g) \circ h = f \circ (g \circ h) = f(g(h(x)))$

Identity function

always maps a set unto itself and maps every element onto itself
Notation $I_A: A \to A$, where $I_A(a) = a$ for all $a \in A$

the identity function on A

If a function $f: A \to B$ has an inverse, then $f^{-1} \circ f$ is the identity function.

ex: $f(a) = b$, then $f^{-1}(b) = a$
$(f^{-1} \circ f)(a) = f^{-1}(f(a)) = f^{-1}(b) = a$
$(f \circ f^{-1})(b) = f(f^{-1}(b)) = f(a) = b$

Let $f: A \to B$ be a bijection. Then $f^{-1} \circ f = I_A$ and $f \circ f^{-1} = I_B$