

Properties of binary relations

Suppose R is a binary relation on set A :

- R is **reflexive** if and only if for every $x \in A$, xRx
- R is **anti-reflexive** if and only if for every $x \in A$, $\neg(xRx)$
- R is **symmetric** if and only if for every pair, x and $y \in A$,
 xRy if and only if yRx
- R is **anti-symmetric** if and only if for every pair, x and $y \in A$,
 $(x \neq y) \rightarrow \neg(xRy \wedge yRx)$
i.e. $(xRy \wedge yRx) \rightarrow (x=y)$
- R is **transitive** if and only if for every three elements, $x, y, z \in A$,
 $[(xRy) \wedge (yRz)] \rightarrow xRz$

Proving and disproving properties of binary relations:

- Because each property is a universal condition, only one counter example is needed to show the relation does not have the property
- In order to establish a relation has a property, the condition must be shown to be true for all elements in the domain