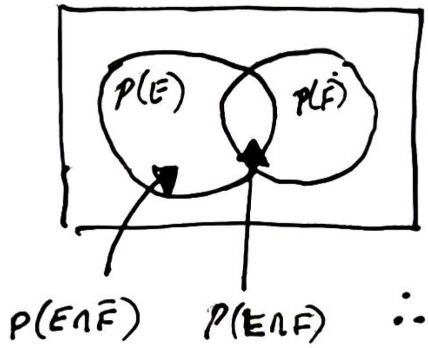
Bayes' Theorem

we know that: $p(E|F) = \dfrac{p(E\cap F)}{p(F)}$ and $p(F|E) = \dfrac{p(E\cap F)}{p(E)}$

$\therefore \; p(E\cap F) = p(E|F)\cdot p(F) = p(F|E)\cdot p(E)$

$\therefore \; p(E|F) = \dfrac{p(F|E)\cdot p(E)}{p(F)}$ and $p(F|E) = \dfrac{p(E|F)\cdot p(F)}{p(E)}$



Because $p\big((E\cap F)\cup(E\cap \bar{F})\big) = p(E)$

$\Rightarrow p(E) = p(E\cap F) + p(E\cap \bar{F})$

$\Rightarrow p(E) = p(E|F)p(F) + p(E|\bar{F})p(\bar{F})$

$\therefore \; p(F|E) = \dfrac{p(E|F)\cdot p(F)}{p(E|F)p(F) + p(E|\bar{F})p(\bar{F})}$