Mathematical Definitions

Even integer - an integer x where there is an integer k such that:

$$x = 2k$$

Odd integer - an integer x where there is an integer k such that

$$x = 2k + 1$$

Parity - whether a number is odd or even

Rational number - a number r where there exists integers x and y such that

$$(y \neq 0) \wedge r = \frac{x}{y}$$

Divides - an integer x divides an integer y if and only if

$$(x \neq 0) \wedge (y = kx) \quad \text{for some integer } k$$

Notation:

$x \mid y$: x divides y

$x \nmid y$: x does not divide y

- if $x \mid y$, then y is a multiple of x
- if $x \mid y$, then x is a factor (or divisor) of y

Prime integer - an integer n is prime if and only if $n > 1$, and for every positive integer m,

$$(m \mid n) \rightarrow ((m = 1) \vee (m = n))$$

$$\Rightarrow (n > 1) \wedge ((m \mid n) \rightarrow ((m = 1) \vee (m = n)))$$

Composite integer - an integer n is composite if and only if

$$(n>1) \wedge ((1<m<n) \wedge (m|n))$$

Inequalities

$$(x \geq c) \leftrightarrow ((x=c) \vee (x>c))$$
$$(x \leq c) \leftrightarrow ((x=c) \vee (x<c))$$
$$\neg(x>c) \equiv (x=c) \vee (x<c) \equiv (x \leq c)$$
$$\neg(x<c) \equiv (x=c) \vee (x>c) \equiv (x \geq c)$$

Positive number $x>0$
Non-negative number $x \geq 0$
Negative number $x<0$
Non-positive number $x \leq 0$

Theorem - a statement that can be proven to be true

Proof - a series of steps, each following logically from assumptions or from previously proven statements, whose final step should result in the statement of the theorem being proven

Axioms - statements assumed to be true.

Perfect Square - a number n is a perfect square if there is a integer k such that

$$n=k^2$$

Proof by exhaustion · a proof that checks each element in the domain individually This option is good for

Consecutive numbers - two integers are consecutive if for integers m and n:

$$m = n+1$$

Existence Proof - a proof that shows an existential statement is true

Constructive proof of existence - an existence proof that gives a specific example of an element in the domain or a set of directions to construct an element in the domain that has the required properties

Nonconstructive proof of existence - proves that an element with the required properties exists without giving a specific example

* most common method is to show that the existence of an element with the required properties leads to a contradiction