Counting subsets

- counting subsets is different from permutations in that
  - permutations – order matters (sequential)
    - ex: (Joshua, Karen, Ingrid) and (Karen, Ingrid, Joshua) are two different permutations
  - subsets – order doesn't matter (nonsequential)
    - ex: {Joshua, Karen, Ingrid} and {Karen, Ingrid, Joshua} are the same subset

r-subset or r-combination is a subset of size r

Using the K-to-1 rule to count subsets

1 subset can map to K permutations

ex: $\{\underbrace{a,b,c}_{n=3}\}$ can map to $\underbrace{(a,b,c),(a,c,b),(b,a,c),(b,c,a),(c,a,b),(c,b,a)}_{K=3!=6}$

one r-subset maps to r! permutations   r-subset = r! r-permutations

if we know the # of r-permutations, we can divide the total # of r-permutations by r! to get the total # of r-subsets

$$\#\text{ of r-permutations}\left[\frac{1\text{ r-subset}}{r!\text{ r-permutations}}\right] = \#\text{ of r-subsets}$$

$$\#\text{ of r-permutation of set } n = P(n,r) = \frac{n!}{(n-r)!}$$

# of r-subset "n choose r"

$$\#\text{ of r-subsets of set } n = \Big[\underbrace{\frac{n!}{(n-r)!}}_{\#\text{ of r-permutations}}\Big]\Big[\frac{1}{r!}\Big] = \frac{n!}{r!(n-r)!} \Rightarrow \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

To determine $\binom{n}{n-r}$ using $\binom{n}{r} = \frac{n!}{r!(n-r)!}$

$\Rightarrow$ n choose$(n-r)$ $= \binom{n}{n-r} = \frac{n!}{(n-r)!(n-(n-r))!} = \frac{n!}{(n-r)!(r!)} = \frac{n!}{r!(n-r)!}$

$\frac{n!}{r!(n-r)!} = \binom{n}{r}$

$$\binom{n}{n-r} = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

* there is a bijection b/w r-subsets of S and $(n-r)$-subsets of S.