Partial orders

a partial order is a relation on a set that is reflexive, transitive, and anti-symmetric

Notation: $a \preceq b = aRb$ if R is a partial order
Let a and b be in set A and let R be a relation on A that is partially ordered.

a partially ordered set/poset consists of the partial order and its domain

Notation $(A, \preceq)$ where A is the domain for the partial order

Examples:

- the $\leq$ operator acting on a set of real numbers
- the domain is $\mathbb{Z}$, if $x|y$ then $x \preceq y$ (i.e. xRy)

two elements are comparable if $x \preceq y$ or $y \preceq x$
two elements are incomparable if $\neg(x \preceq y)$ AND $\neg(y \preceq x)$
a total order is when every two elements in the domain is comparable

an element x is minimal if $(y \preceq x) \rightarrow (y = x)$
i.e. there is no $y \neq x$ such that $y \preceq x$
an element x is maximal if $(x \preceq y) \rightarrow (y = x)$
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Hasse diagram rules

- do not draw self-loops
- omit arrows
- if $x \preceq y$, then make x appear lower than y
- if $x \preceq z$ such that $x \preceq y$ AND $y \preceq z$, then only draw the (x, y) and (y, z) edges