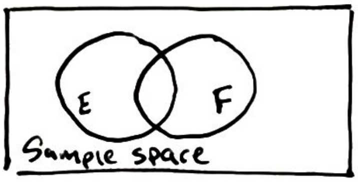
Conditional Probability and Independence

conditional probability of E, given event F has happened:

$$p(E|F) = \frac{p(E \cap F)}{p(F)}$$

- event F becomes the reduced sample space since event F has already happened
- $p(E \cap \bar{F}) = \emptyset$ because any elements in E that occur outside of F have a zero chance of occurring



Complement and conditional probability assuming E and F are both events in the same sample space S, then the probability of E and the probability of not E still sum to 1, even when conditioned on event F.

$$p(E|F) + p(\bar{E}|F) = 1$$

Independent events

if conditioning on one event does not change the probability of the other event, the two events are independent

Let E and F be two events in same sample space:

if:

1.) $p(E|F) = \frac{p(E \cap F)}{p(F)} = p(E)$

2.) $p(E \cap F) = p(E) \cdot p(F)$

3.) $p(F|E) = \frac{p(E \cap F)}{p(E)} = p(F)$

= equivalent statements and E and F are independent

Calculating probabilities of two independent events

If X and Y are two events in the same sample space, and X and Y are independent, then

$$p(X \cap Y) = p(X) \cdot p(Y)$$

Mutual Independence

For three events A, B, C in the same sample space: even if every pair ($A \cap B$, $A \cap C$, $B \cap C$) is independent, it is not always true that $p(A \cap B \cap C) = p(A) \cdot p(B) \cdot p(C)$

Events $A_1, \ldots, A_n$ are <u>mutually independent</u> if the probability of the intersection of <u>any</u> subset of the events is equal to the product of the probabilities of the events in that subset

If $A_1, \ldots, A_n$ are mutually independent events in the same sample space, then:

$$p(A_1 \cap A_2 \cap \ldots \cap A_n) = p(A_1) \cdot p(A_2) \cdot \ldots \cdot p(A_n)$$