

## ## Linearity of Expectations ##

The expectation of the sum of two random variables is equal to the sum of the expectations of each random variable

If  $X$  and  $Y$  are two random variables defined on the same sample space  $S$ , and  $c$  is a real number:

$$\begin{aligned}E[X + Y] &= E[X] + E[Y] \\E[cX] &= cE[X]\end{aligned}$$

example:

a blue die and a red die are rolled

$B$ : the outcome of the blue die

$R$ : the outcome of the red die

$D$ : the sum of the outcomes of the blue and red die =  $B + R$

$$E[D] = E[B + R] = E[B] + E[R]$$

Linearity of expectations can be shown by induction to apply to more than two variables:

If  $X_1, \dots, X_n$  are  $n$  variables defined on the same sample space, then

$$E\left[\sum_{j=1}^n X_j\right] = \sum_{j=1}^n E[X_j]$$

If  $X = X_1 + \dots + X_n$  and all the  $X_j$  have the same expectation, then  $E[X] = nE[X_1]$

Linearity of expectations does NOT require random variables be independent !!!