

## ## The Division Algorithm

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$$\begin{array}{lcl} \text{Quotient} = q = n \text{ div } d & \left\{ \begin{array}{l} \text{div} = \text{integer division} \\ \text{mod} = \text{modulus operation} \end{array} \right. & \\ \text{remainder} = r = n \text{ mod } d & & \\ n = qd + r & \left\{ \begin{array}{l} 0 \leq r \leq (d-1) \end{array} \right. & \end{array}$$

### The Division Algorithm

Theorem: Let  $n$  be an integer and let  $d$  be a positive integer. Then, there are unique integers  $q$  and  $r$ , with  $0 \leq r \leq (d-1)$ , such that  $n = qd + r$ .

### Procedural version of the Division Algorithm

Input: Integers  $n$  and  $d$ , with  $d > 0$

Output:  $q = n \text{ div } d$  and  $r = n \text{ mod } d$

LET  $q$  equal zero

LET  $r$  equal  $n$

IF  $n \geq 0$ :

WHILE  $(r \geq d)$ :

$q := q + 1$

$r := r - d$

END-WHILE

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ELSE (i.e.  $n < 0$ ):

WHILE  $(r < 0)$ :

$q := q - 1$

$r := r + d$

END-WHILE

END IF