LAWS OF PROPOSITIONAL LOGIC

Idempotent $p \vee p \equiv p$, $p \wedge p \equiv p$

* element in a set that is unchanged in value while multiplied by itself (ex: $1 \cdot 1 = 1$) *

Associative

$(p \vee q) \vee r \equiv p \vee (q \vee r)$

$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

* think grouping / associations *

Commutative $p \vee q \equiv q \vee p$, $p \wedge q = q \wedge p$

* think community = exchange of ideas *

Distributive

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

Identity $p \vee F \equiv p$, $p \wedge T \equiv p$

Domination $p \wedge F \equiv F$, $p \vee T \equiv T$

Double negation $\neg\neg p \equiv p$

Complement

$p \wedge \neg p \equiv F$, $\neg T \equiv F$

$p \vee \neg p \equiv T$, $\neg F \equiv T$

* think complementary colors are opposite / inverse *

Absorption $p \vee (p \wedge q) \equiv p$, $p \wedge (p \vee q) \equiv p$

* the meaning of q gets lost within p *

De Morgan's

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$
$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

* Not (p or q) means that both p and q must each be negated *
* Not (p and q) means that only one (p or q) must be negated *

Coditional

$$p \rightarrow q \equiv \neg p \vee q$$
$$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$$

De Morgan's for Quantified Statements

$$\neg \forall x\, F(x) \equiv \exists x\, \neg F(x)$$

"It's not true that for every x, F(x)" ≡ "There exists an x where F(x) is not true"

$$\neg \exists x\, P(x) \equiv \forall x\, \neg P(x)$$

"There is not an x where P(x)" ≡ "For all x, P(x) is not true"

Nested Quantifiers

$$\neg \forall x \forall y\, P(x,y) \equiv \exists x \exists y\, \neg P(x,y)$$
$$\neg \forall x \exists y\, P(x,y) \equiv \exists x \forall y\, \neg P(x,y)$$
$$\neg \exists x \forall y\, P(x,y) \equiv \forall x \exists y\, \neg P(x,y)$$
$$\neg \exists x \exists y\, P(x,y) \equiv \forall x \forall x\, \neg P(x,y)$$

*note: replace $\forall$ with $\exists$ and vice versa, then negate predicate