CSC 551:Spring 2012

Extra Credit Homework: Graph Algorithms

Due Final Exam day.. You are allowed to do this homework individually, or in groups of two or three. Each group will return a single homework and will get the same grade. But a group is NOT allowed to share anything written with another group. Remember that each group member has to fully participate in the solution and understand the solution to perform well in an exam.

Problem 1. An undirected graph is bipartite if it is possible to partition the vertices into two subsets, V1 and V2, such that all the edges go between V1 and V2, but not within V1 and within V2. Describe an O(V+E) algorithm to determine whether a given connected undirected graph is bipartite. Hint: Use BFS.

Problem 2. Let G=(V,E) be an undirected graph. Give an $O(V + E)$ time algorithm to identify the connected components of G. More precisely, your algorithm should assign every vertex v an integer label $cc[v]$ between 1 and k, where k is the number of connected components of G, such that $cc[u] = cc[v]$ if and only if u and v are in the same connected component. Hint: Use DFS.

Problem 3. (12 points) Given a free tree T, the *diameter* of T is the length of the longest path between two nodes of T. (The length of a path is the number of edges in that path.) Give an efficient algorithm for computing the diameter of T. (Hint: Use BFS or DFS.)

Problem 4. (13 points) Let $G = (V, E)$ be a graph, with weights assigned to its edges, and let T be an MST of G, which is given to you. Assume that (u, v) is an edge that is added to the graph after T is constructed. Give an algorithm that finds in time O(n) a MST of the graph $G(V, E \cup \{(u, v)\})$, where n denotes the number of vertices in V. (Hint: You should not run Kruskal's or Prim's again.) (Hint 2: Consider finding the path from u to v in T first.)