CSC 551:Spring 2012

Homework 2: Recurrences, Divide and Conquer

Due at the start of class Thu, Feb 16.

Problem 1. Consider the following recurrence.

$$T(n) = \begin{cases} 1 & \text{if} \quad n = 1 \\ T(n) = 5T(n/2) + n^3 & \text{otherwise.} \end{cases} \tag{1}$$

(a) Solve this recurrence by the method of iteration. You may assume n is a power of 2.

(b) Solve it using the recursion tree method.

Problem 2. Use the Master Theorem to derive an asymptotic bound on the following recurrences.

(a) $T(n) = 2T(n/4) + n\sqrt{n}$.

(b) $T(n) = 2T(n/3) + 5^{\log_2 n}$.

Problem 3. Suppose you are given an array A containing $n-1$ unique integers in the range [0, n-1] in sorted order (smallest to largest). Give pseudocode for an $O(\log n)$ algorithm for finding the integer in the range [0, n-1] that is not in A.

Problem 4. Give an algorithm which given an array A[1..n] of numbers, a positive integer k ($1 \leq k \leq n$) and a number X outputs (in any order) the k numbers of A that are closest to X in value. (These numbers may be greater than, less than or equal to X.) For example if A=[3.5, 8.9, 7.6, 9.4, 1.5, 6.6, 2.0], $k = 3$ and $X = 7.0$, the algorithm should output 8.9, 7.6, 6.6 in any order. Your algorithm should run in $O(n)$ time. Explain your algorithm, give a small example and derive its running time.

Problem 5. Give an $O(n \lg n)$ divide-and-conquer algorithm for the 2D Maxima Problem we discussed in class. You are NOT allowed to invoke any sorting method. The input to the algorithm is an array P[1..n] of 2D points where P[i].x store the x-coordinate, P[i].y store the y-coordinate of the i-th point.