SENSITIVITY ANALYSIS IN ZIKA VIRUS DYNAMICS AND A MODEL DISCREPANCY APPROACH

SÃO PAULO SCHOOL OF ADVANCED SCIENCES ON NONLINEAR DYNAMICS
São Paulo, July 29th - August 9th, 2019

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INTRODUCTION

➔Zika virus: global widespread and connection with congenital diseases;

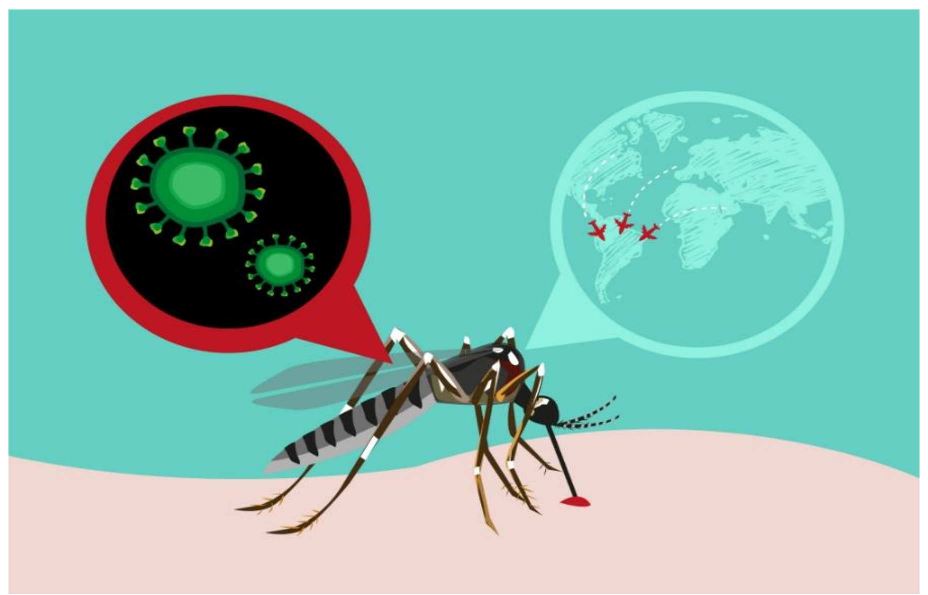
➔ 2016: Zika becomes a public health emergency of international concern;

➔ Main vector: Aedes mosquitoes;

➔A validated model can reveal new characteristics of the disease;

➔ Relations of model parameters are also of interest;

FIGURE 1 – Zika transmission representation.



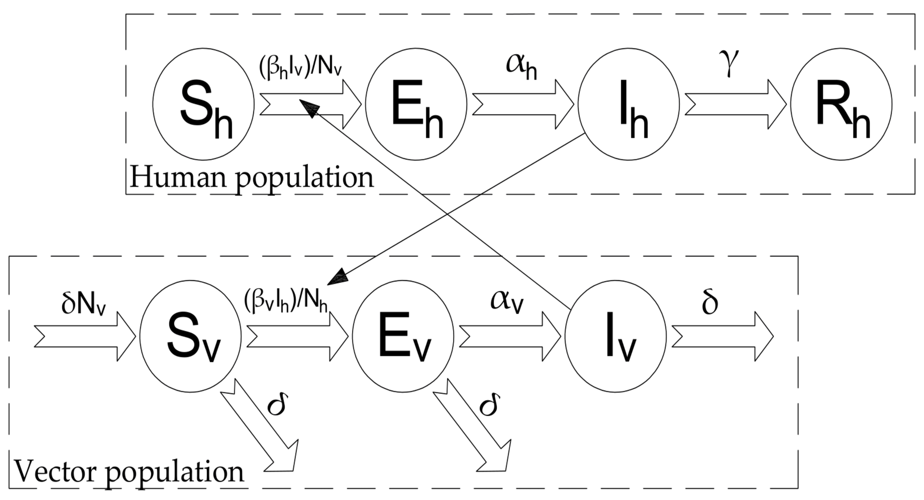
OBJECTIVE

◆Perform sensitivity analysis to compare the parameters' global effect under different scenarios;

◆ Develop a statistical framework using Bayesian Inference and Polynomial Chaos Expansion to quantify epidemic model discrepancies;

COMPUTATIONAL MODEL

FIGURE 2 – SEIR-SEI model schematic [1].



DYNAMICAL SYSTEM:

$$\frac{dS_h}{dt} = -\beta_h\, S_h\, \frac{I_v}{N_v}\,, \qquad \frac{dS_v}{dt} = \delta\, N_v - \beta_v\, S_v\, \frac{I_h}{N_h} - \delta\, S_v\,,$$

$$\frac{dE_h}{dt} = \beta_h\, S_h\, \frac{I_v}{N_v} - \alpha_h\, E_h\,, \qquad \frac{dE_v}{dt} = \beta_v\, S_v\, \frac{I_h}{N_h} - (\alpha_v + \delta)\, E_v\,,$$

$$\frac{dI_h}{dt} = \alpha_h\, E_h - \gamma\, I_h\,, \qquad \frac{dI_v}{dt} = \alpha_v\, E_v - \delta\, I_v\,,$$

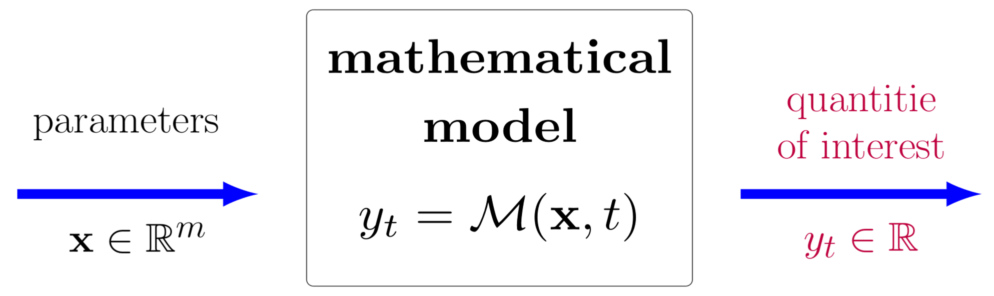
$$\frac{dR_h}{dt} = \gamma\, I_h\,, \qquad \frac{dC}{dt} = \alpha_h\, E_h\,.$$

+ Initial Conditions

QUANTITIES OF INTEREST:

- Cumulative cases of infectious:
  $C(t) = \int_{\tau=0}^{t} \alpha_h\, E_h(\tau)\, d\tau$
- New cases per week:
  $\mathcal{N}_w = C_w - C_{w-1}\,,\; w = 1 \ldots 52\,,\; \mathcal{N}_1 = C_1$

FIGURE 3 – Observation operator schematic [2].



SENSITIVITY ANALYSIS

The Hoeffding-Sobol' decomposition [3] for n iid $X_i \sim \mathcal{U}(0,1)$ gives

$$Y_t = \mathcal{M}_0 + \sum_{1\le i\le n} \mathcal{M}_i(X_i) + \sum_{1\le i<j\le n} \mathcal{M}_{ij}(X_i, X_j) + \cdots + \mathcal{M}_{1\cdots n}(X_1 \cdots X_n)\,,$$

$$\mathcal{M}_0 = \mathbb{E}[Y_t]\,,\; \mathcal{M}_i(X_i) = \mathbb{E}[Y_t|X_i] - \mathcal{M}_0\,,\; \mathcal{M}_{ij}(X_i, X_j) = \mathbb{E}[Y_t|X_i, X_j] - \mathcal{M}_0 - \mathcal{M}_i - \mathcal{M}_j.$$

Sobol' Indices: interaction effect of inputs in $\mathbf{u}$

$$S_{\mathbf{u}} = \mathrm{Var}\left[\mathcal{M}_{\mathbf{u}}(X_{\mathbf{u}})\right] / \mathrm{Var}\left[\mathcal{M}(\mathbf{X})\right]$$

The Polynomial Chaos Expansion [2] of model $Y = \mathcal{M}(\mathbf{X})$, for a multivariate orthonormal polynomial family $\Phi_{\boldsymbol{\alpha}}$ with coefficients $y_{\boldsymbol{\alpha}}$,

$$Y_t = \sum_{\boldsymbol{\alpha} \in \mathbb{N}^k} y_{\boldsymbol{\alpha}}(t)\, \Phi_{\boldsymbol{\alpha}}(\mathbf{X})\,,$$

enables analytic computation of Sobol Indices:

$$S_{\mathbf{u}} = \sum_{\boldsymbol{\alpha} \in \mathcal{A}_{\mathbf{u}}} y_{\boldsymbol{\alpha}}^2 \Big/ \sum_{\boldsymbol{\alpha} \in \mathcal{A}\setminus 0} y_{\boldsymbol{\alpha}}^2\,, \quad \mathcal{A}_{\mathbf{u}} = \{\boldsymbol{\alpha} \in \mathcal{A} : i \in \mathbf{u} \Longleftrightarrow \alpha_i \neq 0\}$$

FIGURE 4 – Sobol' indices of the model.

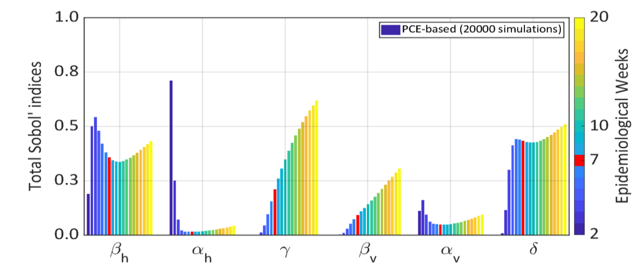


FIGURE 5 – Sobol' indices with α_v constant.

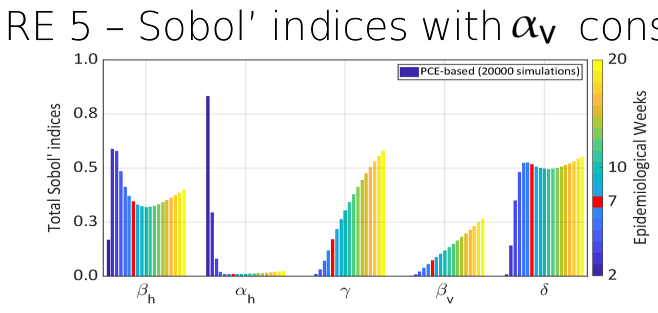


FIGURE 6 – Sobol' indices without α_v

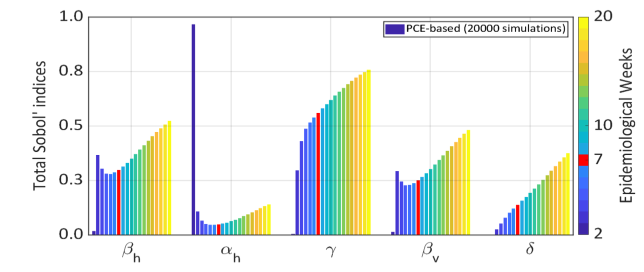
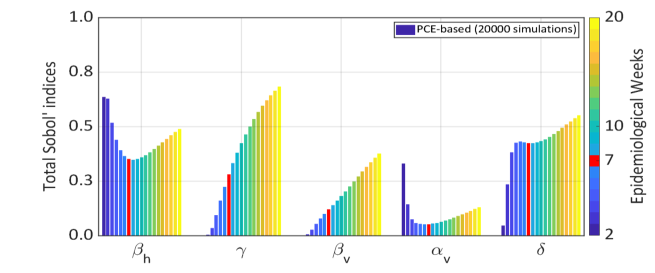


FIGURE 7 – Sobol' indices without α_h.



STATISTICAL INFERENCE (ONGOING RESEARCH)

DISCREPANCY CALCULATION:

Suppose a data set $\mathcal{D} = (t_1, y_1^{dat}), (t_2, y_2^{dat}), \ldots, (t_{N_d}, y_{N_d}^{dat})$ of measures y_t of the i. The i-th observation is given by

$$\underbrace{y_i^{dat}}_{reality} = \underbrace{\mathcal{M}(\mathbf{x}, t_i)}_{model} + \underbrace{\varepsilon_i}_{error}\,.$$

Sargsyan, Najm and Ghanem's [4] novel approach to deal with the model discrepancies is to adopt a metamodel structure which lumps the error into the parameters

$$Y^{dat} \approx \mathcal{M}(\mathbf{X}_\varepsilon, t)\,,\;\; \mathbf{X}_\epsilon = \sum_{\boldsymbol{\alpha}\in\mathcal{I}} \mathbf{X}_{\boldsymbol{\alpha}}(t)\, \boldsymbol{\Psi}_{\boldsymbol{\alpha}}(\boldsymbol{\xi}),$$

where $\mathbf{X}_{\boldsymbol{\alpha}}$ coefficients are defined as random to be able to be identified by using Bayesian Inference.

BAYESIAN INFERENCE:

- Inference problem become use data information to update the prior probability density func-tion (PDF) $\mathbf{X}_{\boldsymbol{\alpha}}$ defined for . The solution corresponds posterior PDF;
- From Bayes' rule,

$$\rho(\mathbf{X}_{\boldsymbol{\alpha}}|\mathcal{D}) = \frac{\rho(\mathcal{D}|\mathbf{X}_{\boldsymbol{\alpha}})\,\rho(\mathbf{X}_{\boldsymbol{\alpha}})}{\rho(\mathcal{D})}\,.$$

$\rightarrow \rho(\mathbf{X}_{\boldsymbol{\alpha}}|\mathcal{D})$: posterior distribution $\quad \rightarrow \rho(\mathbf{X}_{\boldsymbol{\alpha}})$: prior distribution

$\rightarrow \rho(\mathcal{D}|\mathbf{X}_{\boldsymbol{\alpha}})$: likelihood function $\quad \rightarrow \rho(\mathcal{D})$: evidence

To define a good point of start, the Maximum Entropy Principle is applied to construct the most informative prior distribution.

FINAL REMARKS

✓ Comparative results of global Sobol' Indices show how the lack of some parameters can change the sensibility effect of the others;

✓ A framework for statistical inference exploring Polynomial Chaos to measure the model discrepancies was presented;

✓ In future works, the authors intend explore this new framework to quantify model discrepancy and then improve its predictions;

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[4] K. Sargsyan, H. N. Najm and R. Ghanem. On the statistical calibration of physical models. *International Journal of Chemical Kinetics*, 47(4): 246-276, 2015. https://doi.org/10.1002/kin.20906

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